Computer Aided Engineering Design
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Lecture - 4

Hello and welcome to NPTEL video lecture number 4 - can solids be generalized, that is can be come up with a single statement that covers valid solids of all possible shapes and sizes.

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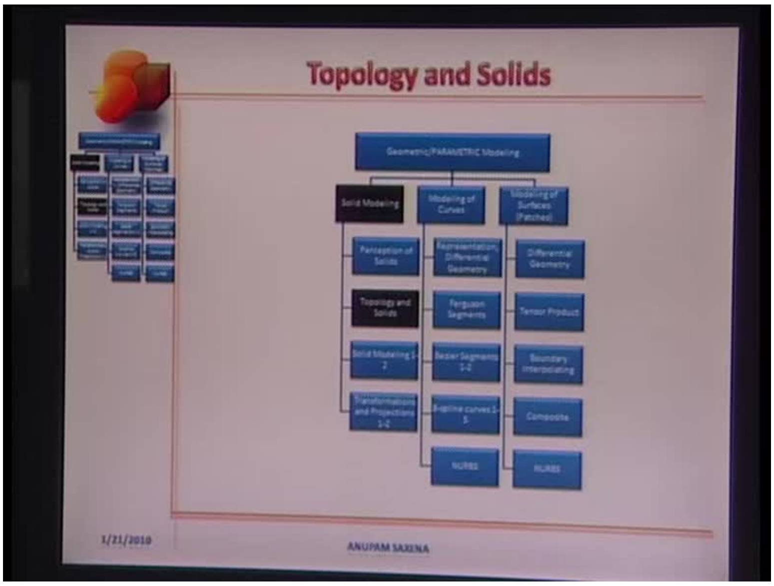

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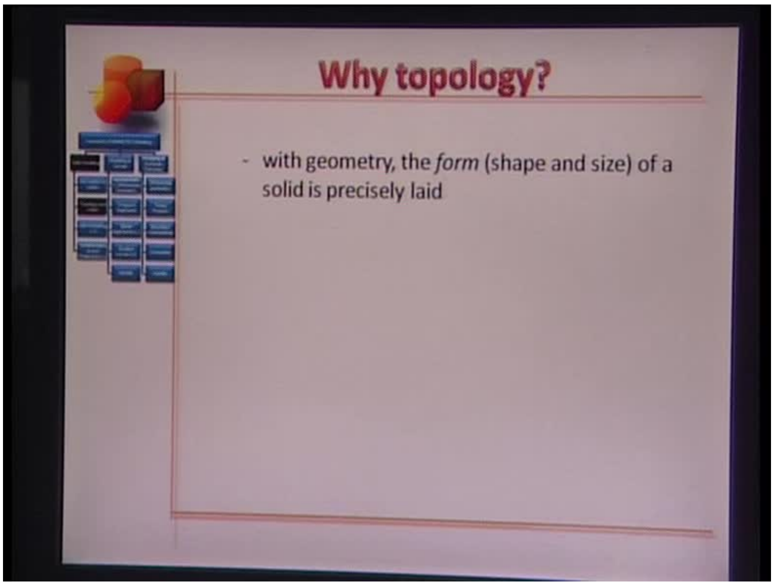

We will answer that question by studying topology of solids.Why do we need to study topology, because description of solids is quite broad; they are plentiful, and they come in variety of shapes. With geometry, they form which is the shape and size of a solid is precisely laid, this is a constraint for us. Geometry does not give us the flexibility for generalizing solid.

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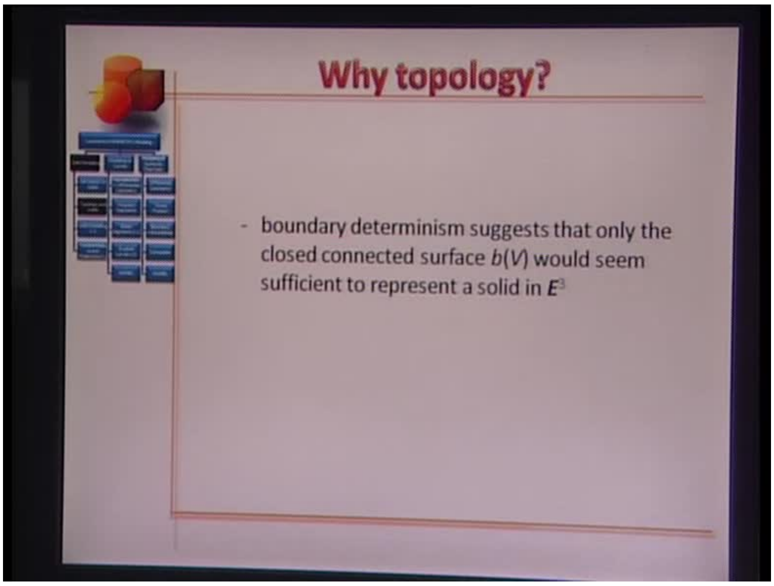

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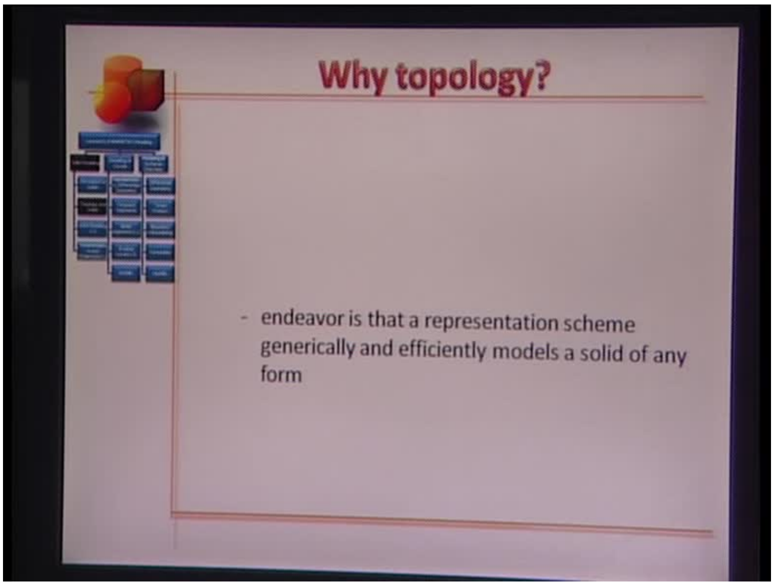

The boundary determinism theory suggests that only the closed connected surface b of v, would seem sufficient to represent a solid in the Euclidian space. Looks like only bounding surfaces need to be considered. The effort is that the representation scheme is required that generically, and efficiently models a solids of any form.

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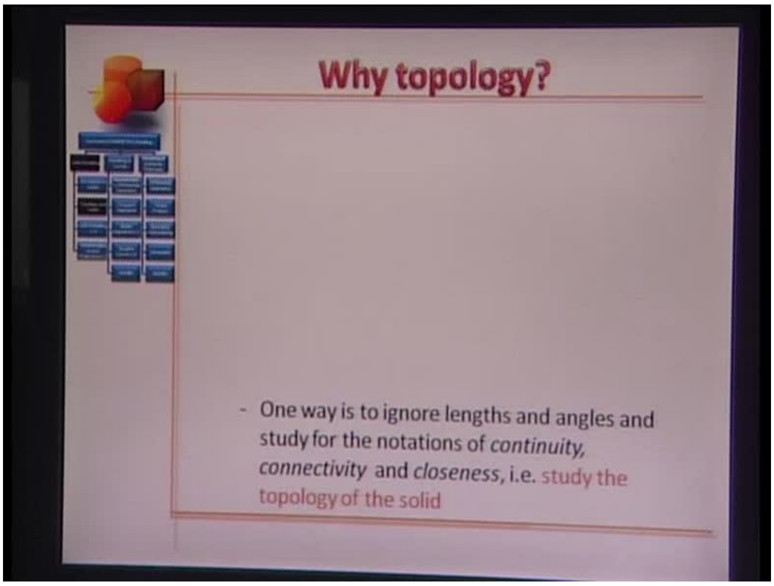

One way is to ignore lengths and angles and study for the notations of continuity, connectivity and closeness; that is study the topology of a solid with topology, one less go in specificity of size and shape and discusses only connectivity. We will try to answer in this lecture, if it is possible for us to describe solids generically. The next question what is topology?

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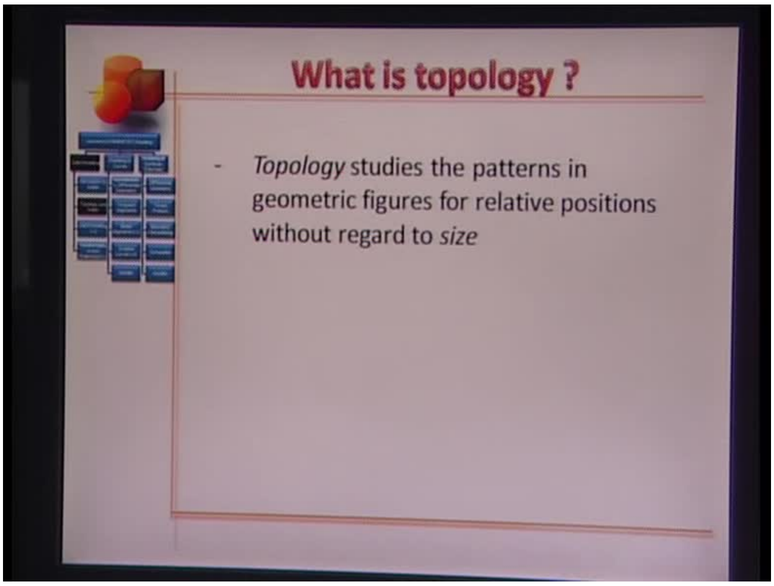

Topology studies the patterns in geometric figures for relative positions, without regard to size. In a sense, topology studies connectivity and the associated properties. Consider a paper band for example, one can get this paper band in any shape one wants, but primarily it is a loop had this paper band be a rubber band, one could have even stretched it out.

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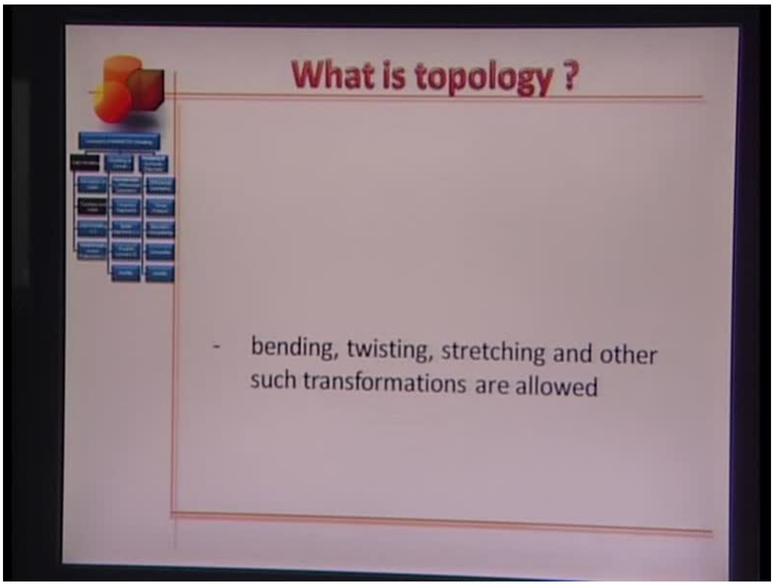

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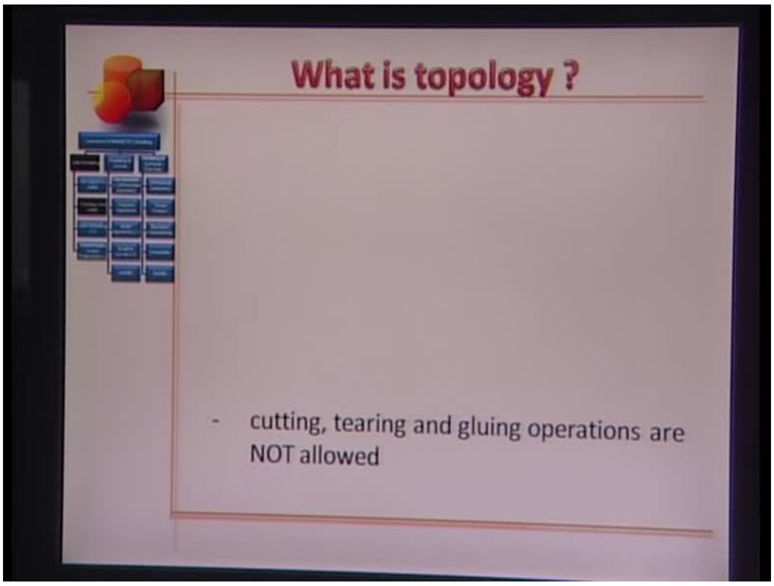

Topology is sometimes referred to as the rubber sheet geometry. When studying topological properties, bending, twisting, stretching and other such transformations are allowed. Cutting, tearing and gluing operations are not allowed. Consider this tennis ball, I can squeeze this ball in any way I like to distort its shape, this is allowed the paper band example again I can bend it, I can twist it any way I like, but tearing is not allowed.

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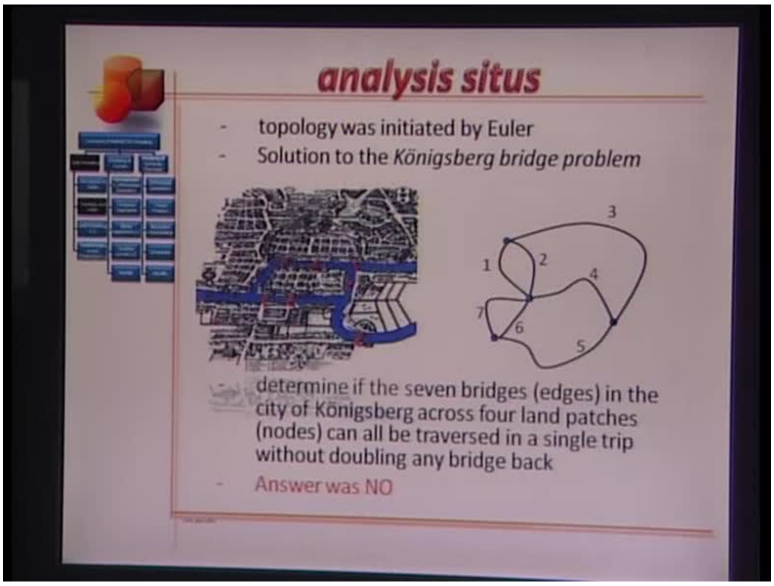

Topology or analysis situs, situs refers to site perhaps was initiated by Euler, Euler was challenged if he could solve Konigsberg bridge problem, this is the Konigsbergs bridge problem. This is the Konig's statement determine if the seven bridges or edges in the city of Konigsberg across four land patches, which are nodes can all be traversed in a single trip without doubling any bridge back. In this slide here these are four land patches and these strips in red they are the seven bridges. In this figure here, the land patches are represented by nodes 1, 2, 3, and 4 and the other seven bridges are represented by edges. Euler talked about this problem for a while and then he answered, no it is not possible.

Homeomorphism, this is one of the topological properties that solids share loose meaning of the term homeomorphic is of the same shape. Two solids belong to the same topological class if they have the same basic overall structure, even though differing much in details. Consider this cube for example, this has a side of size this is a block of side rather sides a, b and c. So, what I have done here is, I have retained the size of one of the sides and I have stretched the other two sides to lengths b and c. As you know stretching in topological study is allowed and that is the reason, why this cube and this block are homeomorphic (( )).

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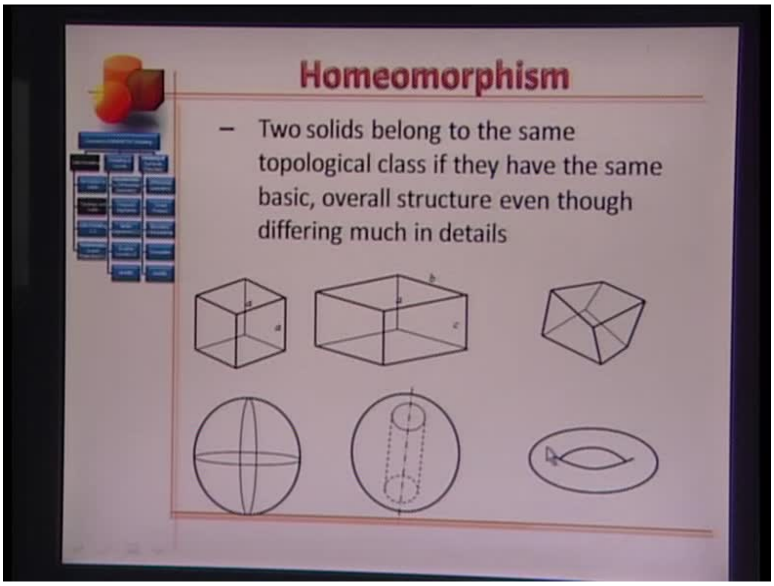

Likewise, what I have done now is that I have distorted the internal angles as well, if you look at the cube and this distorted cube, this is also allowed in topology study. Therefore, even this distorted cube is homeomorphic to a regular cube, question is a sphere homeomorphic to a cube the answer is of course. Yes, how's that so well, what one can do since, distortion is allowed one can distort all the edges and all the faces such that they form equations of the spherical surfaces in part.

The other way to look at this is considered this cube to be like a balloon, and place it in such a way that these edges they happen to be placed exactly on the surface of the sphere and so, the faces they happen to be placed exactly over the surface of the sphere. Therefore, once again a sphere would be homeomorphic to a cube and therefore, all the other distorted versions of the cube.

Now, consider a hole drilled to a sphere is this solid homeomorphic to all the other four solids, the answer to this question is no, it is not. No matter what distortions you perform on any of these four solids, you will never be able to get this solid it is ators homeomorphic to a solid a sphere with a hole weld in it the answer is yes, you can distort a solid to get ators and vice versa.

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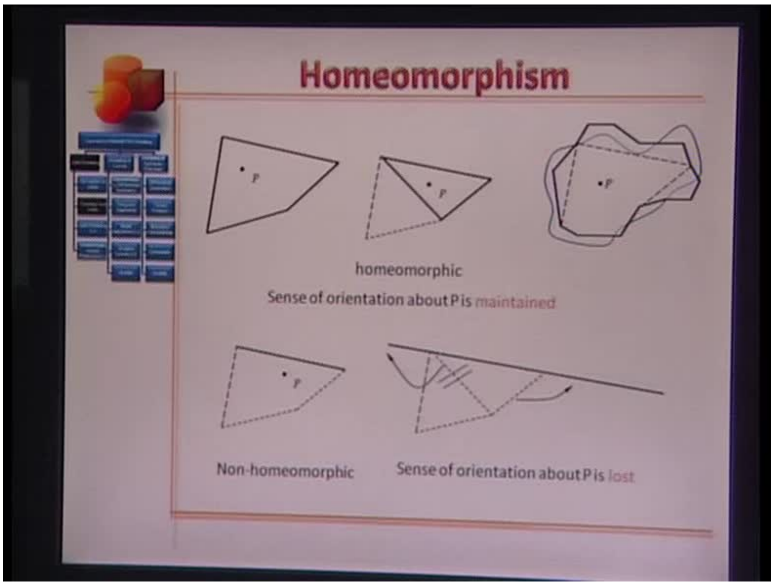

A little more on homeomorphism, let us look at how poly lines or polygons differ; consider this example for instance, this is a quadrilateral with some internal point p. Since, distortions, bending, twisting, extra like operations are allowed what I can do is I can straighten this angle, in the sense I can make the internal angle between these two lines as 180 degrees, and I can also shorten the length of the resultant edge to get this edge, are these two polygons homeomorphic to each other.

The answer is yes they are likewise; all simple closed polygons will be homeomorphic to each other. And in general they will all be homeomorphic to a circle in other words, you can take any closed simple poly line and stretch it out to form a circle vice a versa, you can take the circle and distort the pheri or the perimeter of the circle in any manner to get any closed loop.

There is another observation that one can consider that of course, all these figures are homeomorphic, if you consider an internal point p in all these polygons of closed loops. If you start from a point and say it travels counter clock wise on the framing you would observe that this point p will always lye to your left. Here as well if you start from this point say and travel counter clock wise, point p will be towards your left you have to come back to the same point from where you started.

The point I am trying to make here is the sense of orientation about a point p is maintained, with all these closed polygons. Now, a question is a straight line homeomorphic to any of these closed polygons, I can think of getting the straight line as follows, I can shrink this edge length to almost 0, I can shrink this edge length to almost 0 and so this. In the limit I do that will I be getting a straight line.

We get back to the paper loop example now with four edges duly visible this is similar to the quadrilateral you see in the slide, I can shrink this length, I can shrink this length and I can shrink this length, but whatever I do so, long as I am not performing any cutting or gluing operations. If I stretch it fully this will always be a loop no matter what, this will not become a straight line.

To be able to make it a straight line, I have to perform a tearing operation; this is now a straight line; this is a straight line, I had to cut the loop to be able to make a straight line all over. Now, if you start from a point here and traverse the line and come back, let us say you do the same in this figure here, you start from this point traverse through the line stop and come back.

What you would see or what you would observe is when you had gone from bottom towards top point p was towards your left, and when you traverse backwards point p is towards your right side. So, the sense of orientation about p which was there in these closed poly lines is now lost. So, we would say that a straight line is not homeomorphic to these closed polygons, we had mentioned before that the bounding surfaces are adequate enough to represent the respective solids.

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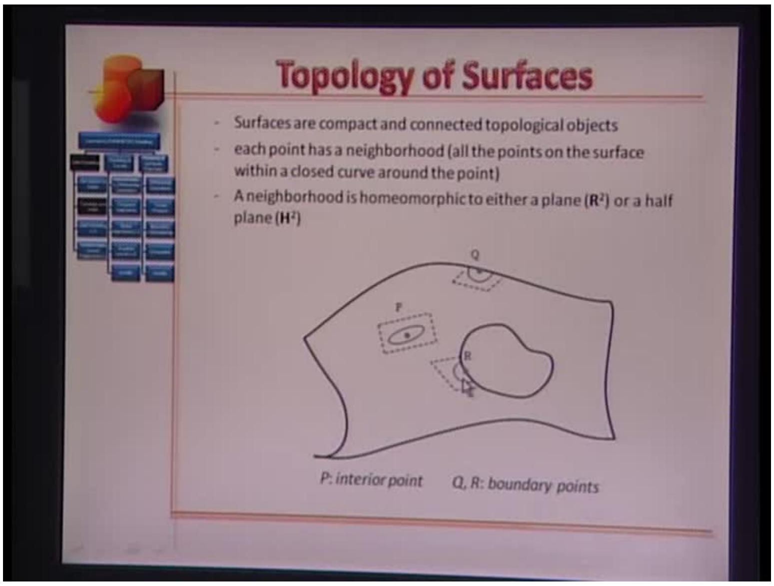

Surfaces are compact and connected topological objects. Each point has a neighborhood. A neighborhood is homeomorphic to either a plane which is R 2 or a half plane, which is H 2. Let us see this figure here it is a typical surface, you would see two kinds of points here one would be the eternal point and the other would be a boundary point. For point p draw any disc of radius R. What one can do is one can stretch it infinitely in all possible directions. For points Q and R it is not possible for us to draw a full disc around it.

It is only possible for us to draw a sketch of a partial disc. Now, this disc can be stretched only in a few directions and not all directions. For example, I cannot consider the neighborhood of R in this region, it seems like an internal void for which reason the neighborhood of the boundary points Q and R are half planes. Let us discuss surfaces and their boundaries.

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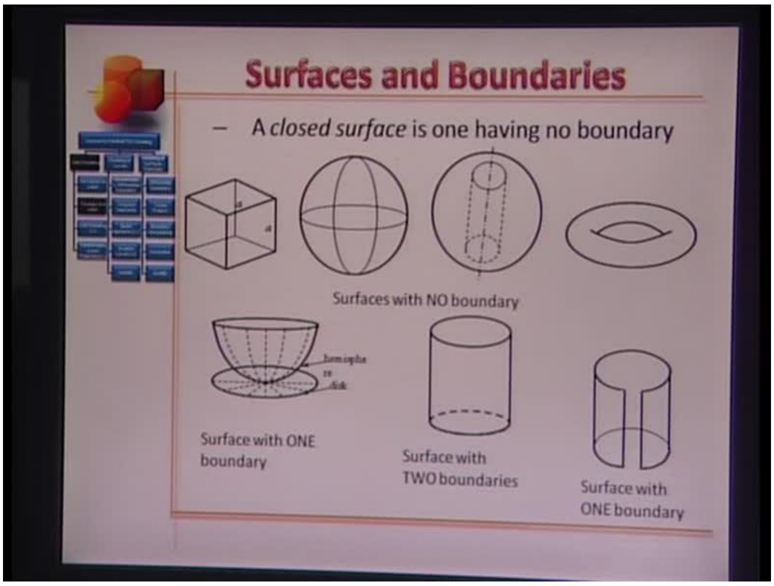

A closed surface is a one that has no boundaries a cube for example, it does not have any boundary. Another example a sphere likewise it does not have any boundary, a sphere with a hole drilled no boundary. A torus no boundary as well, all these are surfaces with no boundary. Now, consider a disc you can deform the disc to get a hemispherical shape in that matter both disc and the hemisphere are homeomorphic to each other, as you would realize both of these would have one boundary. This is an open cylinder, this could have two boundaries; this is a rectangular plane folded into a cylindrical form, this shape will have one boundary.

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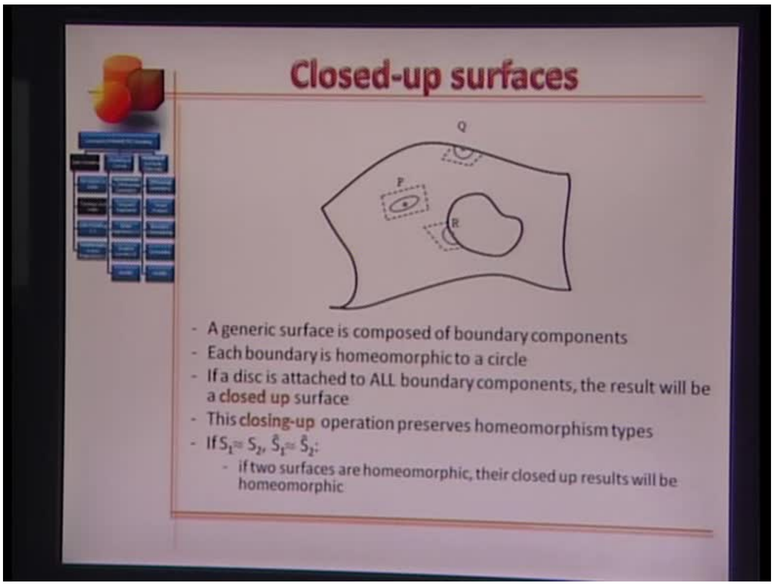

How many boundaries would you think are there in this surface. You are right, there are 2, 1 would be the internal boundary and the other would be the external boundary. Boundaries are like closed (( )) both interior ones and the exterior ones. Now the question is can a surface with boundaries in closed volume to represent a bounding surface of the solid?

The answer is no as of now, but you can perform certain closing operations. Well we all know that a generic surface may be composed of boundary components, each boundary is homeomorphic to a circle this is clear because we can modify, distort then twist each boundary and get the shape of a circle. Now, if a disc is also a circular boundary is attached to all boundary components 1 by 1, the result will be a closed up surface. In a sense, what we will get will be a surface with no boundaries and therefore, a solid it is like eliminating the boundaries 1 by 1.

This closing up operation preserves homeomorphism types. If surface S 1 is homeomorphic to surface S 2, S 1 cap the closed up surface of S 1 will be homeomorphic to S 2, cap the closed up surface of S 2, S cap refers to the closed up surface corresponding to S. Once again, when two surfaces are homeomorphic their closed up results will also be homeomorphic.

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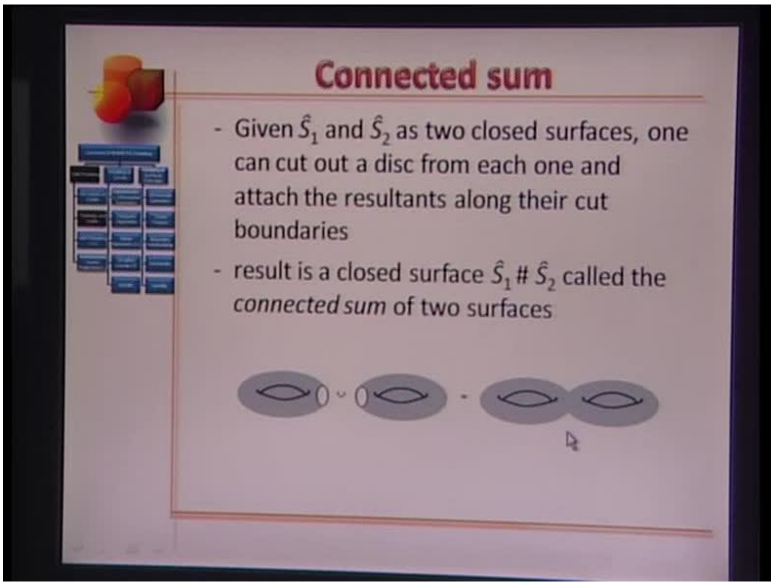

We can extend this concept to that of a connected sum. Given S 1 cap and S 2 cap as 2 closed surfaces or two closed up surfaces, one can cut out a disc from each one and attach the resultants along their cut boundaries. The result is a closed surface S 1 cap ash S 2 cap called the connected sum of two surfaces. If you consider a torus for example, and another torus and cut out a disc from each of them and then attach these two solids at these two boundaries circle the boundaries take the union, what you would get is the connected sum of two torii. Two questions here would this result the connected sum of two torii have any boundaries and question two, can this result behave as a bounding surface of a solid, you might going to think about that.

Let us now try to study some basic surfaces and try to classify them. Basic surfaces can be of four or five types. A sphere a torus a mobious strip and a cross cap. Sphere and torus some examples of basic closed surfaces, this is how a torus can be constructed. Take a rectangular piece of paper, join the two opposite ends to form an open cylinder, and then glue the two boundaries of the open cylinder together to get a torus. A mobious strip can be constructed by a rectangular piece of paper in this manner, take this rectangular piece of paper and flip this edge.

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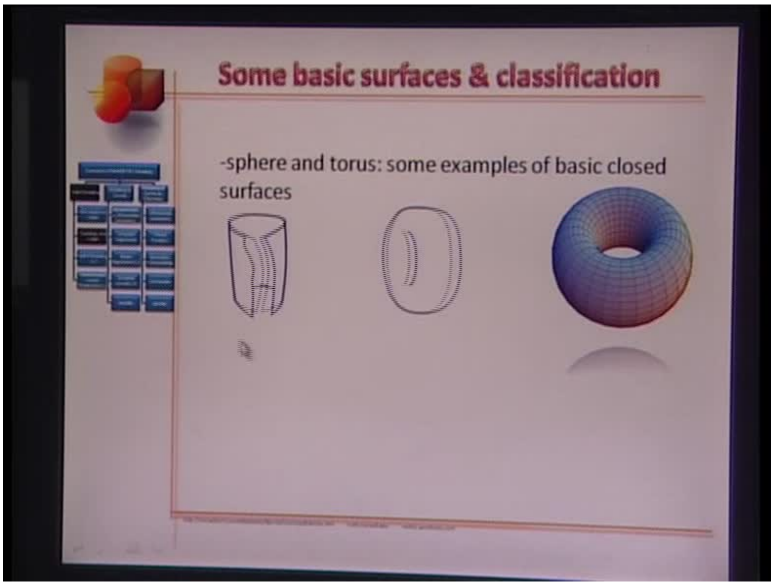

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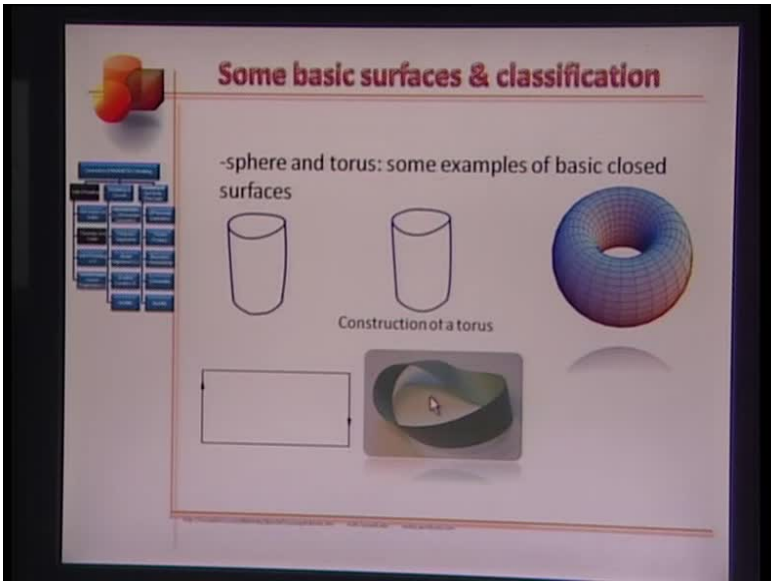

So, that this arrow aligns with this one and glue the two ends together, to get a mobious strip this is what I mean. Take this rectangular strip of paper flip this edge, and then glue the two opposite edges together to get a mobius strip. A question how many boundaries do you think a mobius strip would have, both you and I know that the number will not be 0, are you thinking 2.

Well let us find out, let me mark a point on this mobius strip. Now, what I will do is I will move my index finger and my thumb along the boundary from this point. You might going to follow very closely, my finger movements. No matter what happens my fingers are stuck to the boundary. What has happened here I started with this point started moving along the boundary and have come back to the same point, it is for me that the number of boundaries of this strip would be 1. Now, let us look at some other basic surface.

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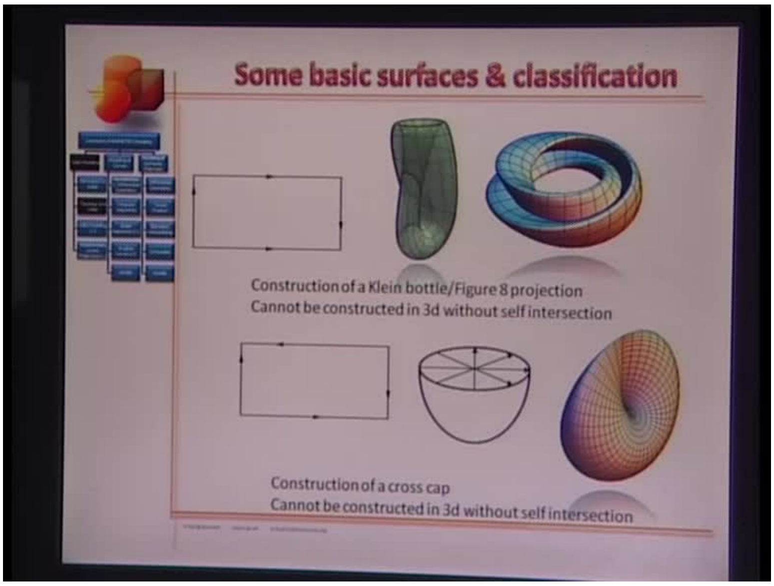

This is a klein bottle equivalently it is also a figure eight projection, klein bottle and figure eight projection. How do I get a klein bottle from a rectangular strip? Well step one glue the opposite sides together to align these arrow heads, what you would get would be an open cylinder and then take these two boundaries flip them. So, that this arrow aligns with this one and then glue these two opposite edges to get a klein bottle.

Let me show this construction using a piece of dough, this is almost a rectangular shaped piece of dough. While I am getting hungry by looking at this, let us try to execute the first step, join this and this edge together to get an open cylinder. I am going to be a little careful here. My finger seems to be gotten stuck, what you see know is like a two ended open cylinder. Now, what I will do is I will narrow down all the ends. Since, I have to flip or I have to ensure that this end, does not get glued with this, I need to make a cut on this surface.

I am going be using a cutter, this operation is dangerous do not try this unsupervised. I am going make sure I do not cut my fingers. Now, taking this part off, I am going elongate the other end, bend it and insert it through this hole. Look at the two-glued ends glued ends now, cut take again look at the initially glued ends now, they do not face each other. What I will do now is I will glue this surface to this one. I will start with the glued end first of the inserted end, I will keep joining the points this solid looks like a klein bottle not as good as the figure you see on the slide, but good now for us to understand, the construction.

Note that the klien bottle or the figure eight projection cannot be constructed in 3 D dimensions without self intersection. How do I construct a figure, eight projection is not going to be easy for me to show this in three dimensions, but I can just give you an hint we have seen that a mobius strip would have a single boundary, take 2 mobius strips and think of joining them together at the boundary, the result that you will get or the result you would expect would be a figure of eight projection.

Now, let us talk about another basic surface called a cross cap, the idea is very simple to construct it form a rectangular strip, try to join these two opposite edges so that these two arrow heads get aligned and then try to join these two edges so, that the two arrow heads get aligned. Equivalently you can do the same with a disc, take a disc get it into hemispherical form and then try to join diagonally opposite points here, the result will be a cross cap, let me demonstrate its construction.

This is almost a circular shaped dough a disc, first I will try to give it a hemispherical shape it is not very stable, but it will work for us. Now, what I will try to do is I will try to join two diagonally opposite points this, and this, this I will keep doing this to get a cross cap. Note that even a cross cap cannot be constructed in three dimensions without self intersection. We know come to the answering the main questions we asked at the start of the lecture, whether it is possible for us to make a single statement about a variety of surfaces of solids that we see around.

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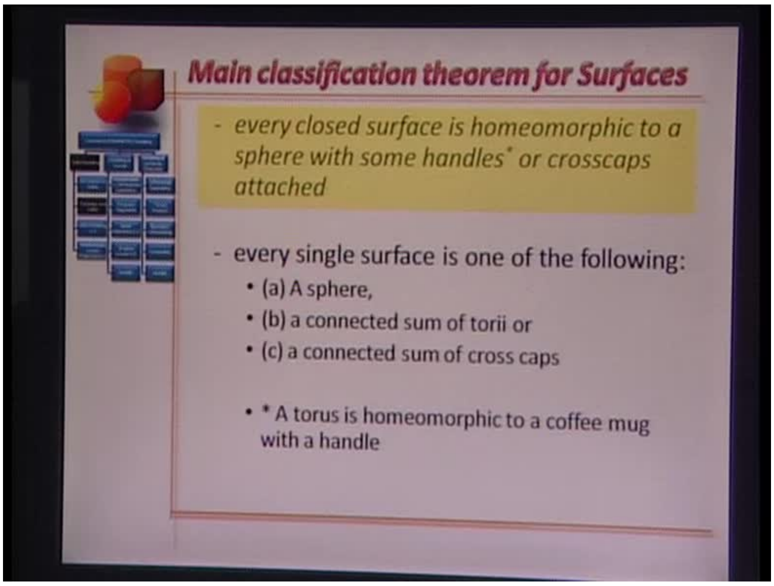

The answer lies in the main classification theorem. Every closed surface is homeomorphic to a sphere with some handles or cross caps attached. I will tell you what handles are, but this is a basic statement or a general statement that the main classification theorem states for each and every solid around us, I will demonstrate a little later.

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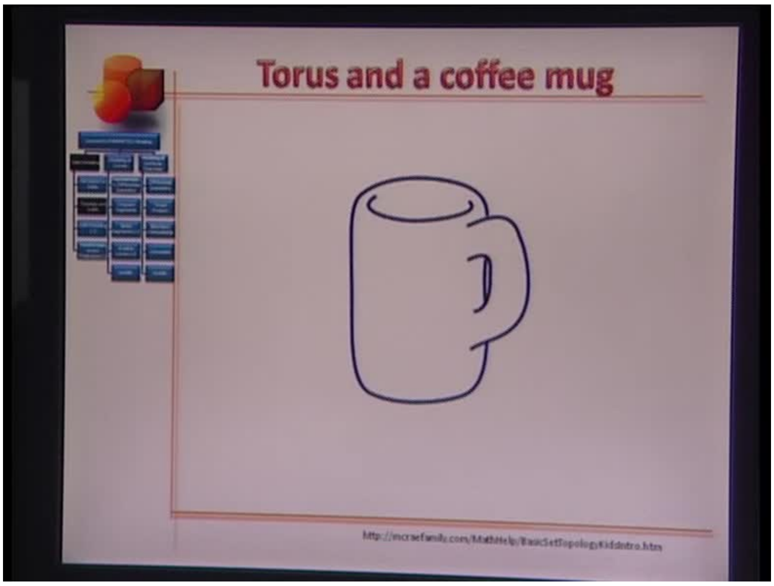

What do I mean by a handle, but first every single surface according to the main classification theorem is one of the following; it is either a sphere or it is a connected sum of torii or it is a connected sum of cross caps. In solid modeling, we are not going to be worried about self intersection in three dimensions. So, we will let go of this point c here. So, for us every single surface is either a sphere or a connected sum of torii. Now, what do I mean by a handle?

This animation demonstrates it. A torus is getting transformed into a coffee mug the term handle comes from here.