Computer Aided Engineering Design
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Lecture - 29

Good morning and welcome again. After having discussed B spline basis function at length, we now will start our discussions on B spline segments and curves. This is lecture number 29. Let us first start with the definition of B spline segments.

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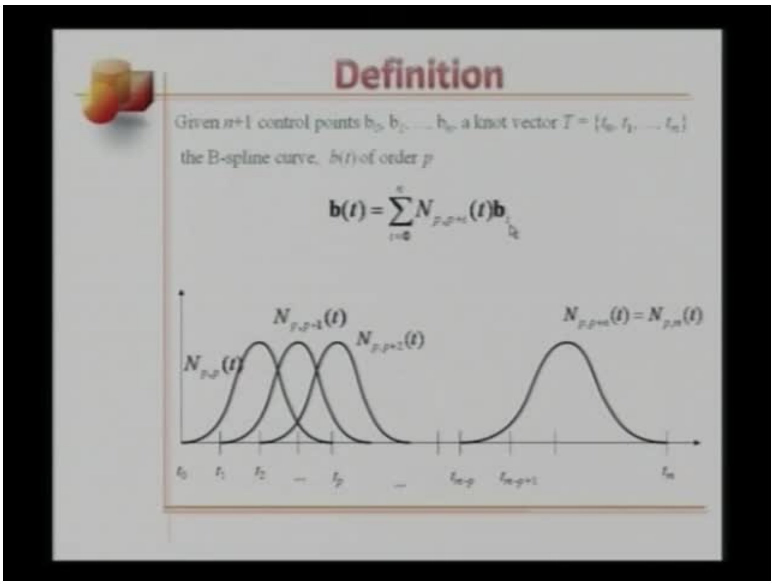

Given N plus 1 control points B 0, B 1 up till B N and also a knot vector capital T as T 0 T 1 up till T N. A B spline segment or curve b of t of order p is defined as b of t is equal to summation the index i going from 0 to N the normalize B spline basis function N sub p p plus i sub t times b i. If you notice this is a very similar constriction as we have seen in case of B zero segments. There we had (( )) polynomials, here we had B spline basis functions acting as weights for each design points that by user specified. Let us look at some graphics, so we have much of knots t 0, t 1, t 2 up till let us say t p we continue. t m minus p t m minus p plus 1 up till t m.

We absorb this naught vector going from t 0 to t n, t 0 to t m. The first basis B spline functions will be n p p corresponding to i equal 0, that would standing over denotes and t 0 to t p. The second one will be standing over t 1 to t p plus 1 and will be normal plated as n p p plus 1 of t. Likewise third one will be named, n p p plus 2 of t. We keep on continuing until we plot the last basis function of which the last knot is t n and the first knot is t m minus p. This is of course, n p p plus n of t, which is the same as n p n t. Note the last knot over here t m, this p plus 1 index corresponds to i equal n. If you notice with each of these B spline basis functions or which a design point t i will be associated.

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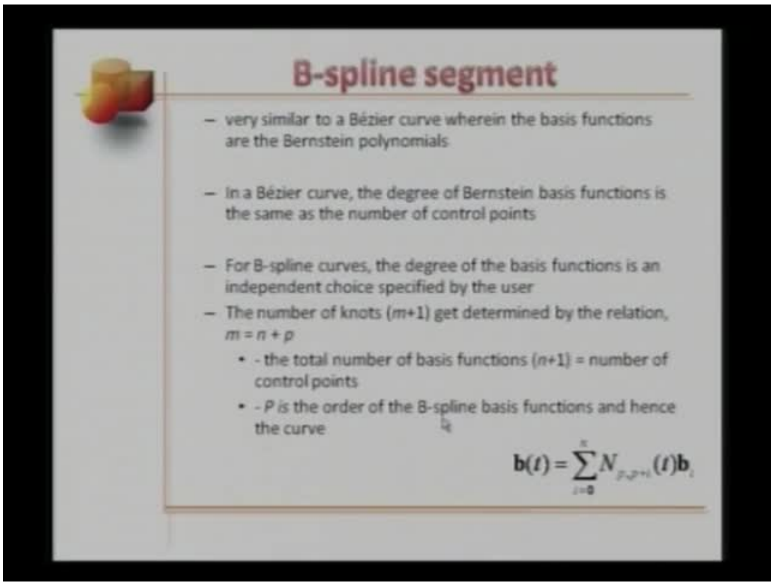

Now, a B spline segment is very similar to a Bezier curve or a segment wherein the basis functions are the Bernstein polynomials, I mention this before. This is the definition of a B spline curve or a segment. In a Bezier curve the degree of the Bernstein basis functions this is the same as the number of control points here. However, the degree or the order of the B spline basis function p is independent of the number of control points. In the sense p is not dependent on n.

For B spline curves the degree of the basis functions is an independent choice. Therefore, specified by the user, the number of knots which is m plus 1, if we recall not vector goes from p 0 of t m gets determined by the relation m equals n plus p p is the order of the B spline basis functions. n plus 1 is the number of design points specified by the user. Accordingly the number of knots get determined, this is what I just said the total number of basis functions n plus 1 is equal to the number of control points and p is should be small p is the order of basis B spline functions and hence the curve.

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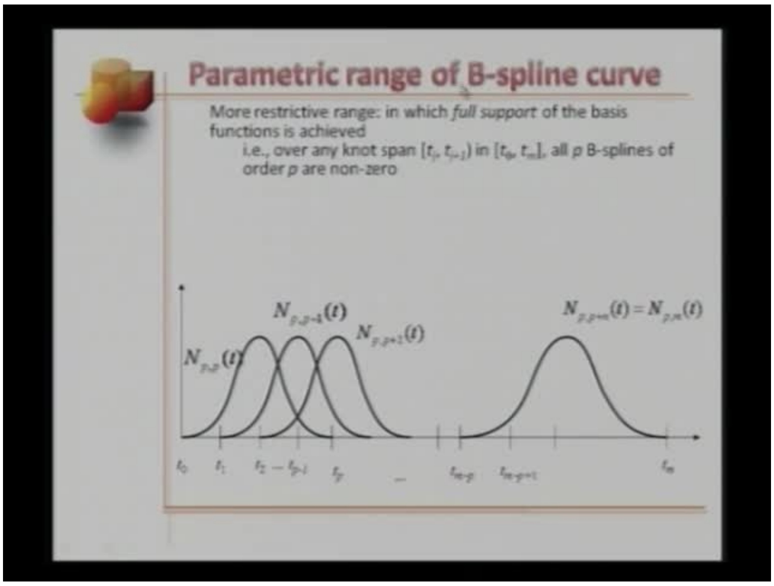

Let us now discussed the parametric range of the B spline curve. This is the figure I have borrowed from two slides before these B spline basis functions N p p, N p plus 1, N p plus 2 up till N p plus N, how many are these? Of course, N plus 1. Now, of B spline curve definition is valid for all values of t n minus infinity 2 plus infinity. Let us take a look at any of these B spline basis functions, N p p plus 2 for example. Now, for values of t smaller than t 2 N p p plus 2 0 and for values of t greater than this knot here, again N p p plus 2 will be 0.

So, an effect N p p plus of 2 of t is valid for all values of t in between minus infinity, and plus infinity. The same as the case for all these basis functions and therefore, since B spline curve is a waited linear combination of these functions, it will be valid for the entire range of t. Clearly b of t is spline segment will be equal to 0 for values of t smaller than t 0 and for values of t greater than t m. So, the range of t in between the values t 0 and t m seems reasonable. Let us look at a slickly mode respective range, that would be the 1 in which full support of the basis functions is achieved.

Now, what I mean by full support? Over any knots span t j t j plus 1 in t sub zero t sub m all p B splines of order p are non zero. If that is the case, then we say we have full support of the B spline basis functions.

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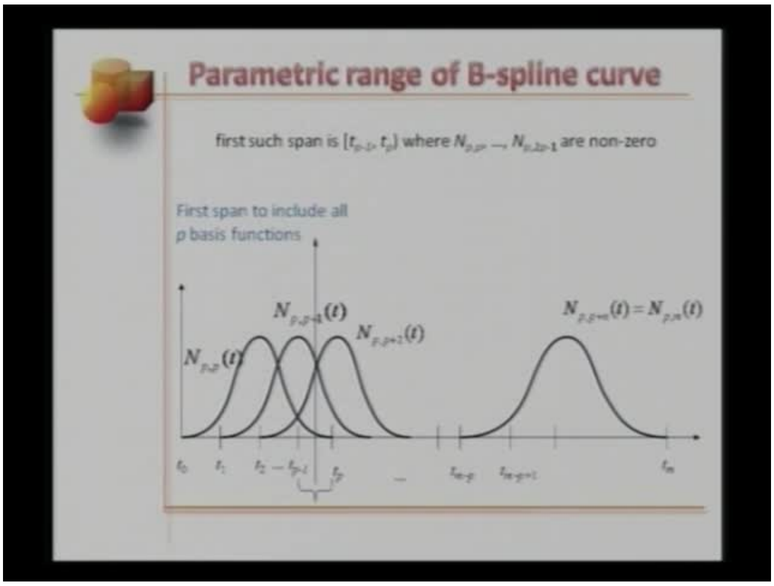

The first such span would be t p minus 1 t p, which is this 1 where N p p N p p plus 1 up till N p 2 p minus 1 are non zero. These are p B spline basis functions, which are non zero over this interval here. So, this would be the span to include all p basis functions.

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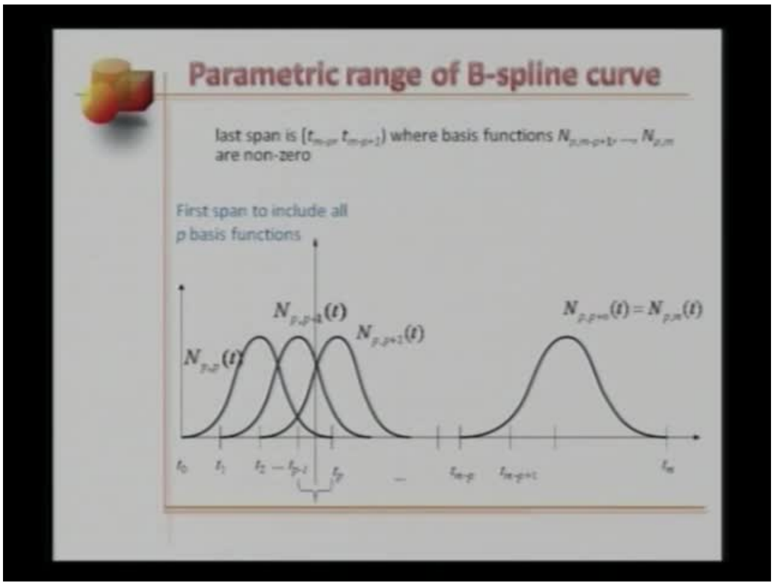

Likewise the last span to include all p basis functions will be t m minus p, t m minus p plus 1, which is right here. Over this non span N p m minus p plus 1 N p m minus p plus 2 and so on up till N p n will be non zero. So, this is the last span to include all p basis functions.

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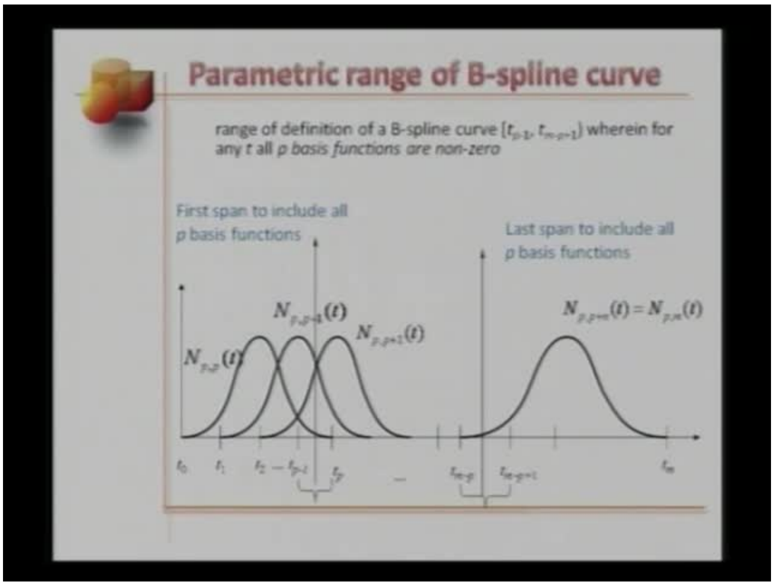

So, the range of the definition of a B spline segment will be given by until t m minus p plus 1 wherein for any t for any value of t all p basis functions are non zero. That means for values of t in between t 2 t m minus p plus 1 over this interval will be able to achieve full support of B spline basis functions. This would be the parametric range of B spline curve.

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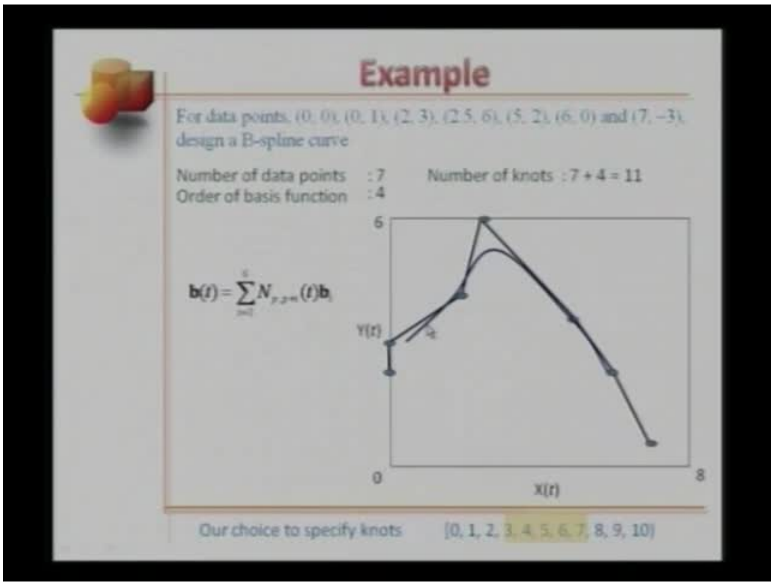

For data points 0 0, 0 1, 2 2, 2.5 6, 5 2, 6 0 and 7 minus 3. Let us design a B spline segment, we had the number of data points as 1 2 3 4 5 6 7, let us design an order 4 B spline curve. We had to specified the order here as an independent choice correspondingly. The number of knots will be the number of data points, which is 7 plus the order of the curve, which is 4. That will be equal to 11. Data's of to the user which specify the values of knots, t 0 up till t n. Here we choose to work with uniform not span t 0 is 0, t 1 is 1 and so on up till t 10 which is 10.

These are clearly 11 knots. Now, this knots span here that starts from t 3 equals 3 and n act 7 equal 7 is the range of full support over this knots span for any value of t. We will have at least 4 or a 4 B spline basis functions to be non zero. We have seen this definition of B spline segment B of t equals i going from 0 6 N p p plus i of t times b i. That you note these b sub i is ordered, it is a vector. Here p equals 4 and this value 6 corresponds to the number of data points 7. Let us look at the graphics now. We plugged these data points and we associated control polynomial. Now, what you see here in blue is a B spline curve for values of t in between three and 7.

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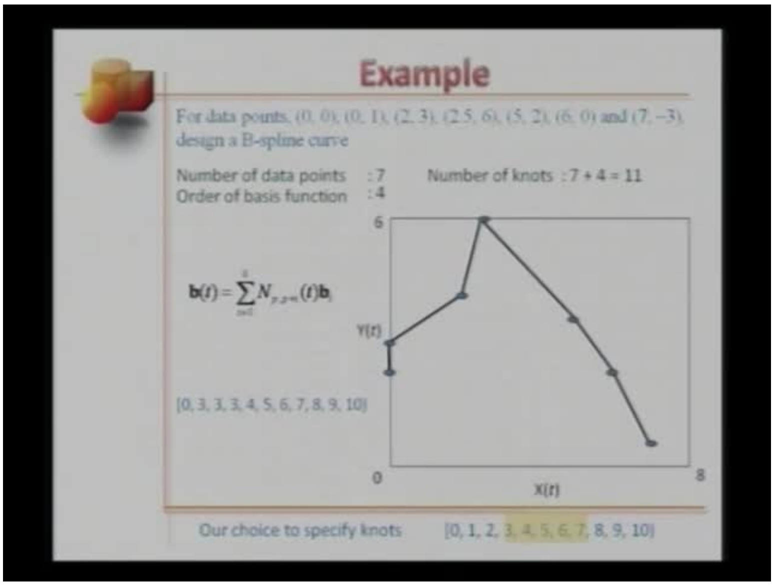

Now, let us experimental little bit. What we do now is, we change the knot vector slightly. We replays this knot and this knot by 3 in a sense. We increase the multiplicity of t 3 2 3. Let see what happens?

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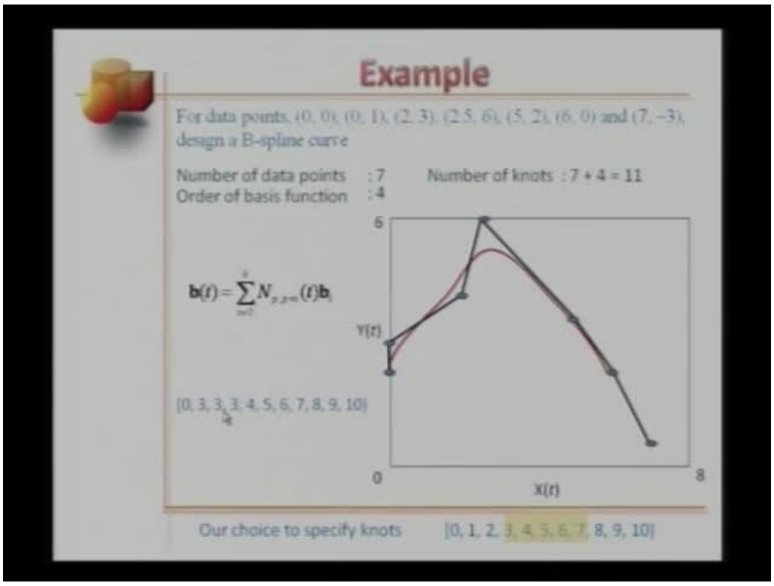

The B spline in red, now passes through the first control point all the design. Let us experiment further. Now, if we replace these knots t 8 and t 9, so that t 7 becomes a multiple knot of multiplicity 3, what happens then?

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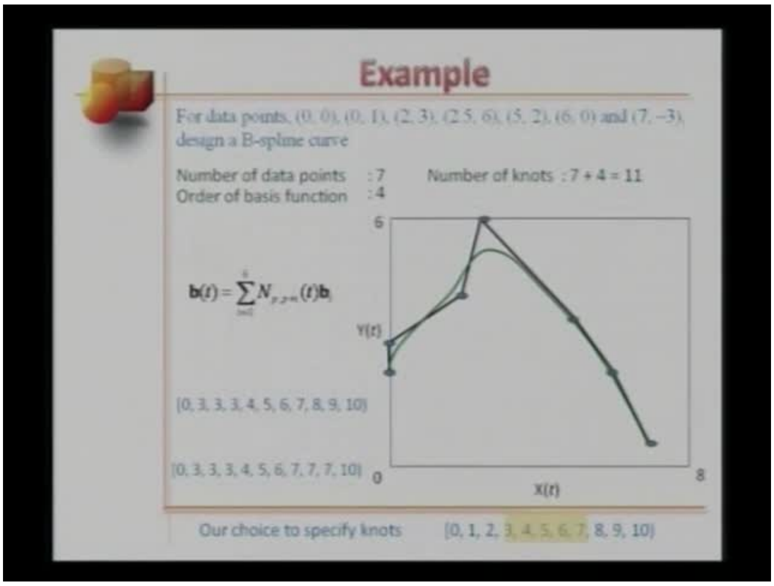

Your resultant B spline curve now passes through the last design point transfer. So, passes through the first design point as well as the last design point. What if I now make the first knot here of multiplicity 4 and the last knot as well of multiplicity 4? So, we had the first knot repeated 4 times and we have the last knot as well repeated 4 times, we do not see any change in the shade of the curve it passes through the first point ends either last point. It happens to be the same as the one in the previous case. Let us look at some properties of B spline segments.

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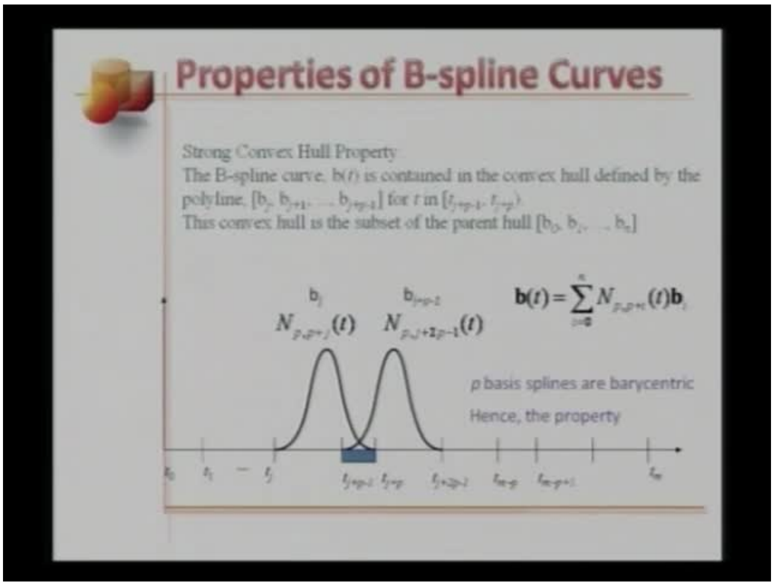

Now, these properties of B spline segments as in the case of B 0 segments will be dependent very much on the properties of the respective B spline basis functions the first 1 a B spline curve is a piecewise curve with each component an order p segment. This is expected because each of the B spline basis function by itself is piecewise curve. Each piece is an order p segment the quality m equals n plus p must be satisfied. In a sense the number of knots has to depend on the number of design points as well as the order of the B spline curve, strong (( )) sub property.

The B spline curve b of t is contained in the convex solve defined by the polyline b j b j plus 1 b j plus 2 until b j plus p minus 1 for value of t in t j plus p minus 1 t j plus p. In a sense over each knots span t j plus p minus 1 t j plus p apart of the B spline curve will lie within the convex hull defined by the corresponding p is end points. This convex hull will be a sub set of the parent hull defined by b 0 b 1 until b n, in a sense all there is end points. Let us look at the graphics now. T his is the interval of concern t j plus p minus 1 2 t j plus p this B spline basis function is N p p plus j the last knot is t j plus p.

This one here as n p j plus 2 p minus 1 of t. Now, note that this basis function ends at t j plus p while this one N p j plus 2 p minus one of t starts at t j plus p minus 1. These are as we know p basis functions, which will be non zero over this knots span of course, this is the definition of B spline curve, we are working with now. Try to figure the design point associated with this basis function and the design point associated with this basis function absorb that b of i is associated with N p p plus i. So, with N p p plus j of t the design point b j will be associated and with N p j plus 2 p minus 1 the design point j plus p minus 1 will be associated this 1 is right here.

So, the curve the B spline curve b of t for values of t for values t j plus p minus 1 and t j plus p will be line within the convex hull defined by these p control points b j until b j plus p minus 1. You know why this happens. This happens because of the local pare centric property corresponding to these p basis functions as I mentioned b j is associated with N p p plus j and b j p minus 1 is associated with this basis function. We have seen this before that this p B spline basis functions are locally horizontal as suppose to global barycentricity, which is exhibited by the Beirnzstein polynomial. Now, this property has consequences, which will investigate. Now, we continue with strong convex sun property.

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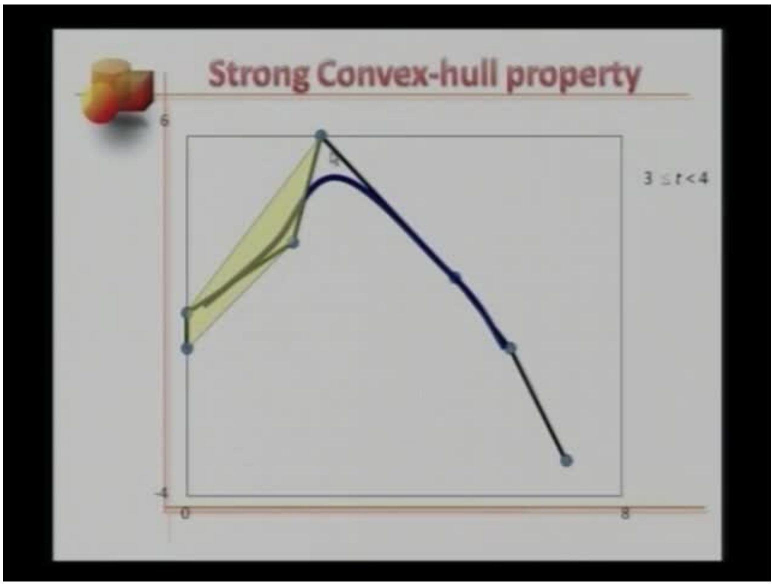

Let us look at B spline curve that we have seen previous example. We had the corresponding data points and the control polynomial shown in black. Now, this curve here corresponds to the original not vector p 0 t 0 t 1 p 1 until t 10 as 10. There no multiple knots in there. For values of t into the 3 of 4 a part of the curve will be in closed within this convex hull, defined by the first four design points.

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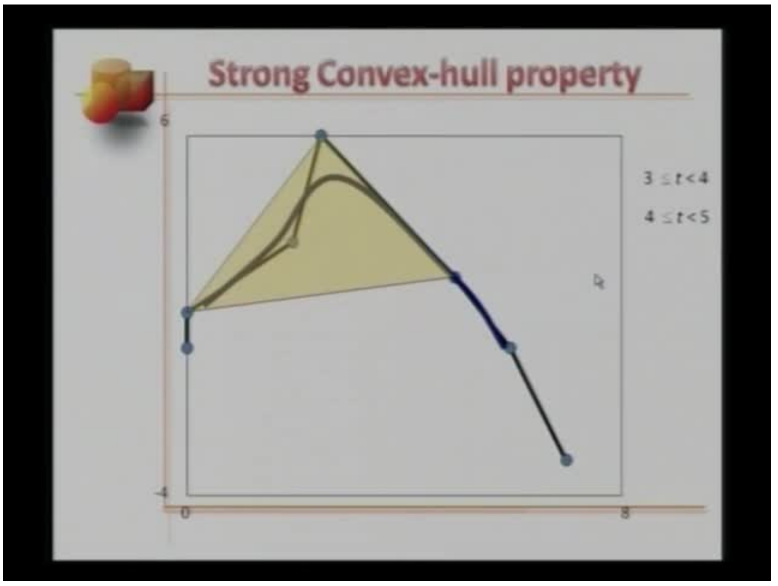

For value of t in between 4 and 5, the second correspond apart of the curve will be now in closed within the convex hull defended by b 1, b 2, b 3 and b 4.

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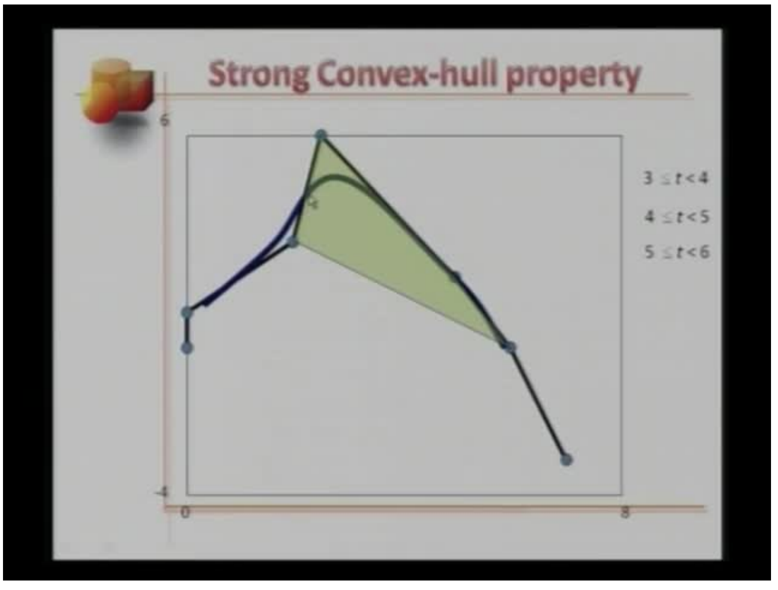

For t in the third knots span a little segment of the B spline curve will be within this convex hull, defined by the third set of four design points.

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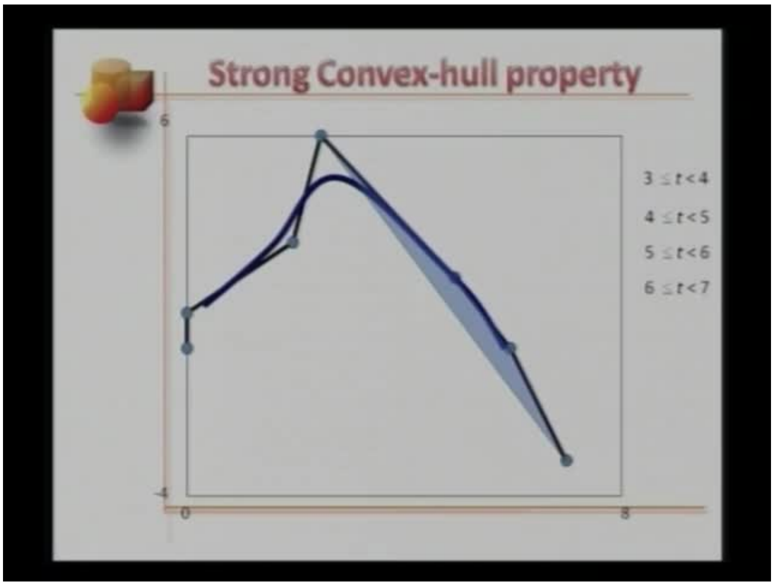

Finally, t in the last knots span the convex hull will be given by the last four design points. If you noticed the values of t are in the knots span corresponding to the full support, which for these values of t where we have all 4 B spline basis functions non zero for any t.

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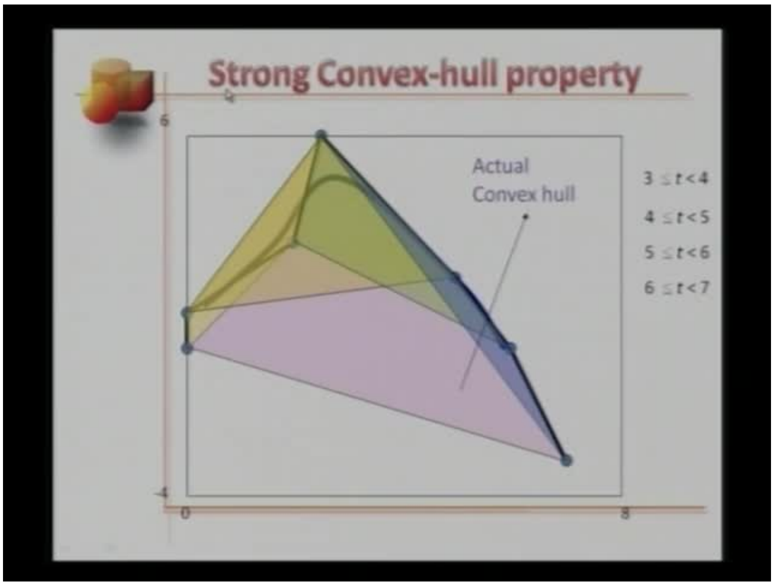

Now, if you look at the overall convex hull for the parent convex hull defined by all the design points, it looks like this. Let us super force the previous convex hulls that we have seen. Now, you would note that if you combine all these convex hulls together the area formed will be smaller than the area of the actual convex hull or the parent convex hull. What is this main junction this would mean that are B spline curve will be kissing the control poil line more proximally.

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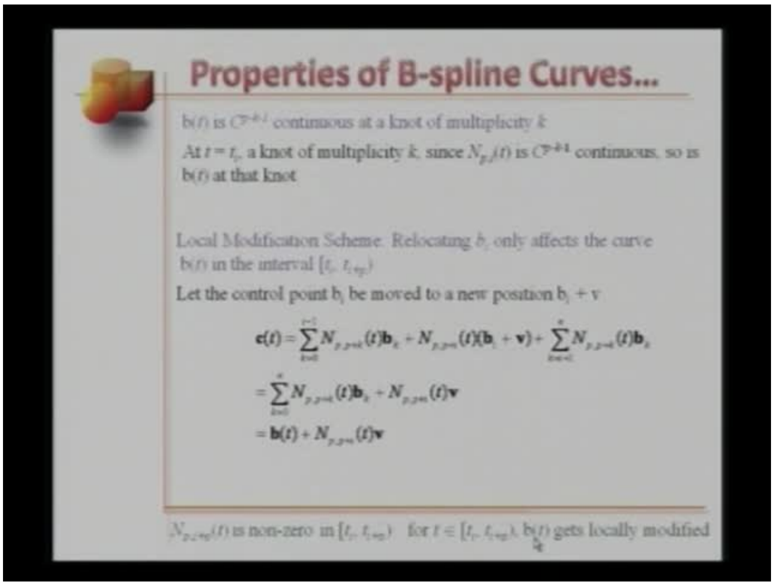

Compare to a b 0 sect, which is why the mention that this is the strong convex hull property of B spline curve b of t is c p minus k minus 1 continuous at a knot of multiplicity k. This is expected; why because each individual B spline basis function exhibits the same property. Since, b of t is a linear combination of those basis functions, it has to have the same property at t equals t i, which is a not of multiplicity k. Since, N p i of t is c p minus k minus 1 continuous, so is b t at that. Note now that is the next one local modification scheme relocating p i a design point only affects the curve b t in the interval t i t i plus p.

This is again a result of the local Barycentric property demonstrated by the B spline basis functions. Let in the case of basis segments, let us do the following analysis. Let the control point b i the move to a new position b i plus v. So, the new B spline curve c of t is given by the index k going from 0 to i minus 1 N p p plus k of t times b of k plus N p p plus i of t p i plus v. Now, this is the new location of the control point v i plus k going form i plus 1 to N N p p plus k of t times p k.

Now, let us extract this term out and add to this and this to give us summation k going from 0 to N N p p plus k of t times p k plus N p p plus i of t times v. This is our original

B spline curve with additional term N p p plus i of t times v. Now, absorb for what values of t with this term v non zero? Clearly N p plus i stands over the first knot as t i p plus i minus v the last knot as t i plus v. So, for values of t in between t i and t i plus v this term here will be non zero. This is the interval, so in a sense the shape of the curve will be locally influenced only for values of t n. This interval otherwise this shade of the B spline curve will remain unchanged. N p i plus p of t is non zero N t i t i plus p for t belonging to this interval t i t i plus p b of t are B spline curve will get locally modified.

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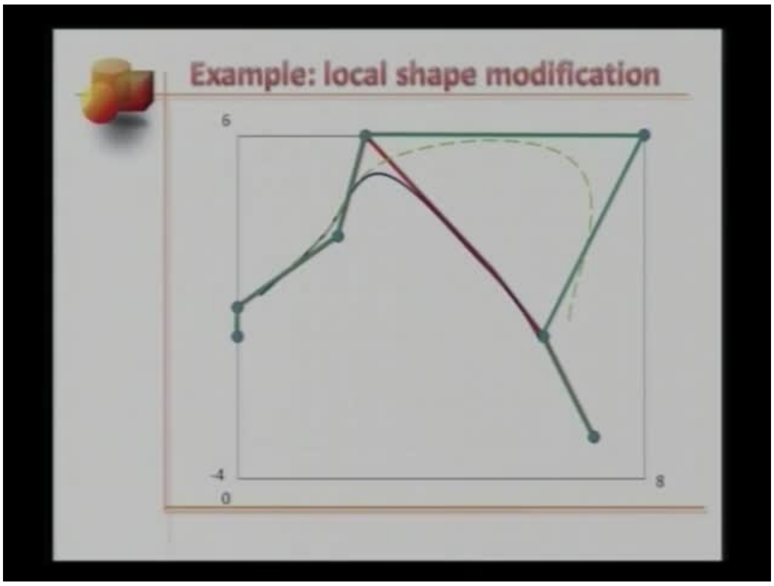

Let us work with the same example are design points the control are in line in red and blue here is the B spline curve corresponding to the uniform knots span with all simple knots. If you recall the knots span get used was from 0 to 10. Let us relocate this point from here to here, so that control poli line changes a little in green here. What will happen you think to the corresponding B spline curve? If you notice the new one in dashed green gets locally modifying shade only the portion corresponding to this part. Let us modified where as this portion here remains unchanged in shade. Now, this is the feature that we do not see in two cases; one for Ferguson curves and the second the Beizer but, now with local Barycentric property associated with B spline basis functions, we can in curve modifying the shape curve locally.

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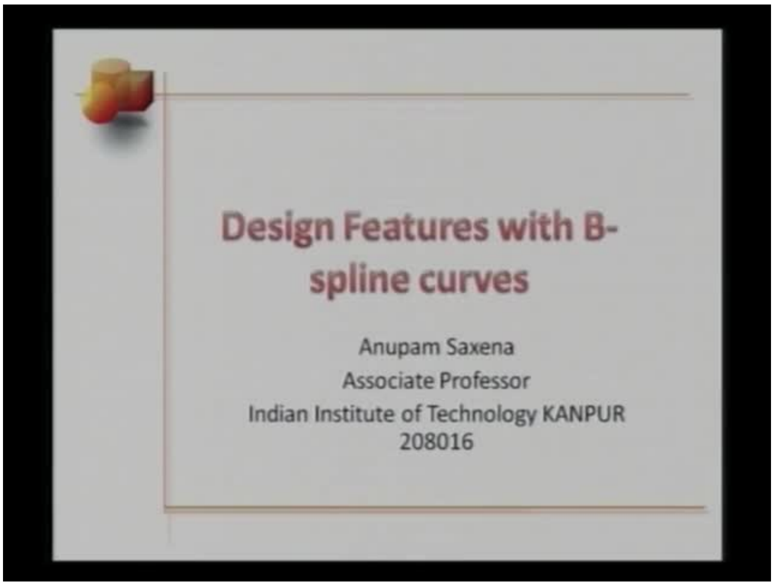

Let us now implement whatever we have learnt so far to try to understand, how to design B spline curves?

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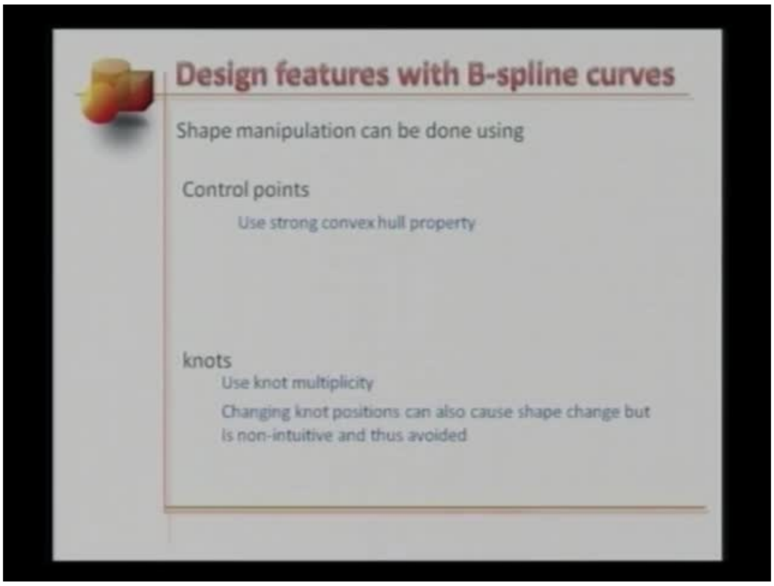

Design features with B spline curves. Now, shape manipulation of B spline curves can be done using either control points or using the knots. When working with control points we can use the strong convex hull property. When working with knots, we can use knot multiplicity. We can also use the position of knots that is not a good idea changing the position of knots can call shape change, but it is not incurable.

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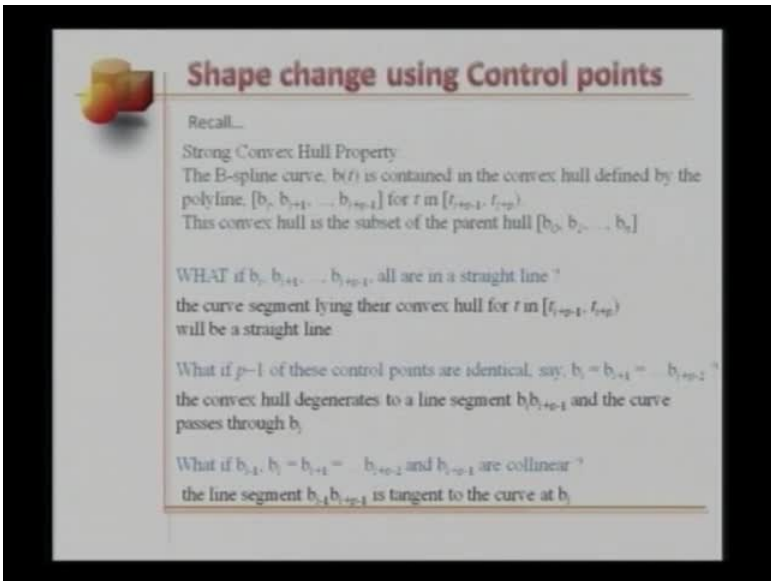

Therefore, it is a wide. Now, for the shape change using the control points, recall this strong convex sub property the B spline curve b of t is contained in the convex hull defined by the polyline b j b j plus 1 b j plus 2 and so on until b j plus 3 minus 1. For value of t in the interval t j plus p minus 1 to t j plus p. Of course, this convex hull is the subset of the parent hull given by b 0, b 1, b 2, b n these are the design points that you may has specified.

Now, the question, what if b i b i plus 1 b i plus 2 until b i p minus 1 are all n a straight line, do not get confused. All we have done here this is we have changed the index. So, the question again what if these p control points are in a straight line? Remember p is the order of the B spline basis functions. Therefore, the B spline curve well you would realizes that, the curve segment lying in the convex hull for t and t i plus p minus 1 t act as p will be a straight line. Clearly when the convex hull d generates to a straight line, so would apart of the curve within that also.

The second question, what if p minus 1 of these control points are identical? let say b i is equal to b i plus 1 is equal to b i plus 2 and so on and so as b i plus p minus 2 the only one is not identical. So, only one is not identical amongst these design points. I would noticed that the convex hull d generates to a straight line b i b i plus 3 minus 1 and the curve now passes through b i. Now, what if b i minus 1 b i equals b i plus 1 equals b i plus p minus 2 and b i plus p minus 1 are collinear? I try to understand the difference between these two phases, here we have set that p minus 1 of the control points or design points are the same.

Now, those design points a prior point b i minus 1, this point is prior to b i. Then the last point in this set b i plus p minus 1, what if there are all line in a straight line? The line segment b i minus 1 b i plus p minus 1 is now tangent to the curve at b i. Now, all these questions are easily answered once we understand the strong convex hull property. These are the three answers that a designer would want to keep in mind, when designing B spline curves.

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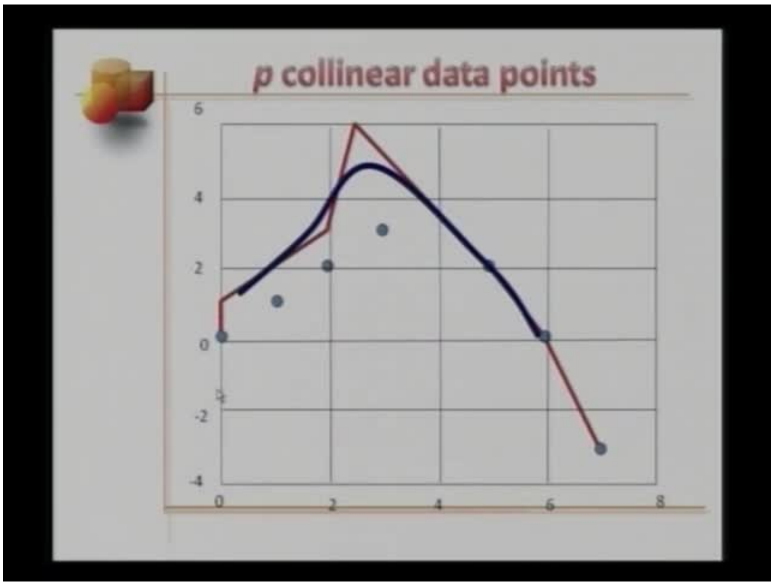

Let see few cases for the first case we have p collinear data points. We work with the same example that we have been working. So far well, this is our B spline curve corresponding to uniform not vector p 0 to p 10 as 0 to 10 respectively. What I have done is I have moved this point over here. I have moved this point over here and I have moved this point over here, so that the first four design points now lie on a straight line. How do you think the curve would change any shade? This is a new control p collinear.

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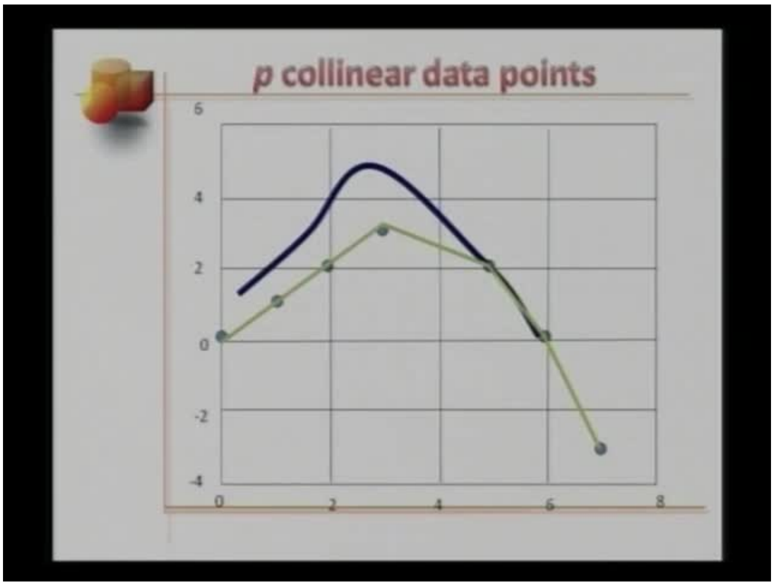

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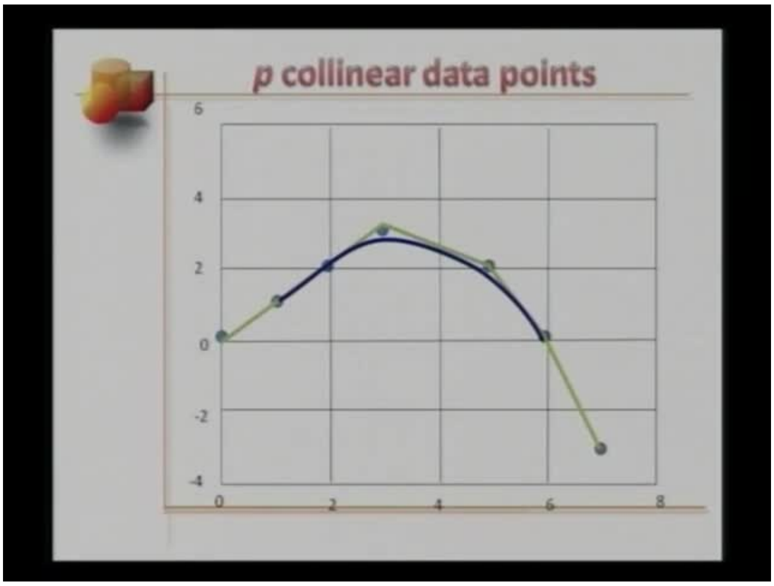

This one would be the new B spline curve. You would realize that a part of it forms a straight segment. This is because the convex hull defined by these four points de generates into a straight line.

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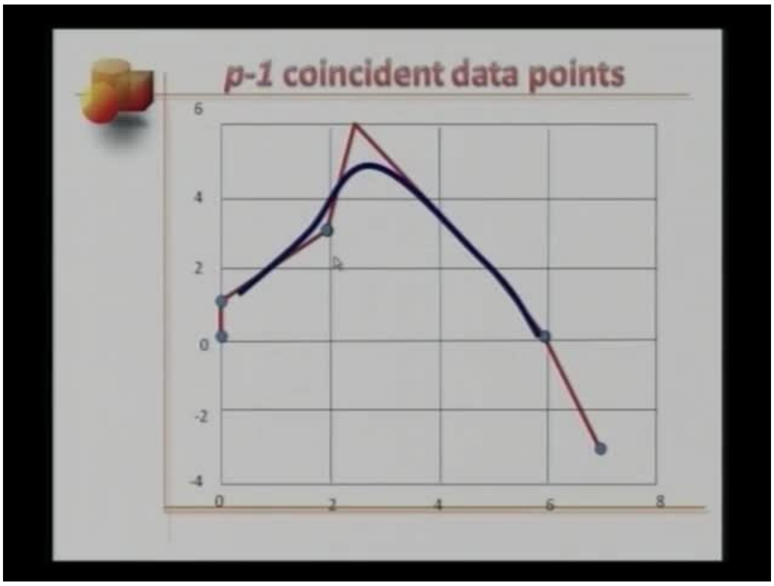

Now, the second case where p minus 1 data points or design points are co incident, that is there are the same. Let us look at the same example here this was the original B spline curve i move this point to coincide with this design point. Here this one again to coincide with these design points; so remember here this is an order 4 B spline curve p is 4 for as p minus 1 is 3. So, we have three design points which are coincidental what happens in this case.

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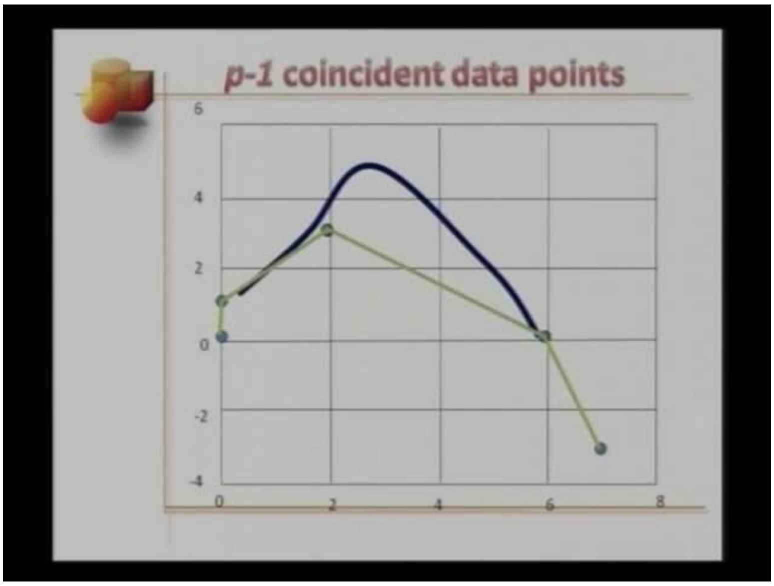

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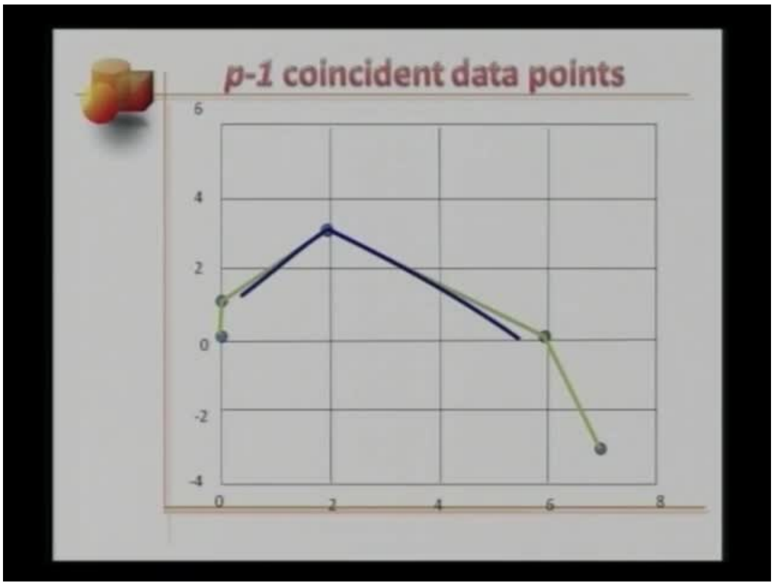

Now, the poliline in green is a new control poliline. This is a new B spline curve. You would notice now, there it passes through this point, which is a multiple design point.

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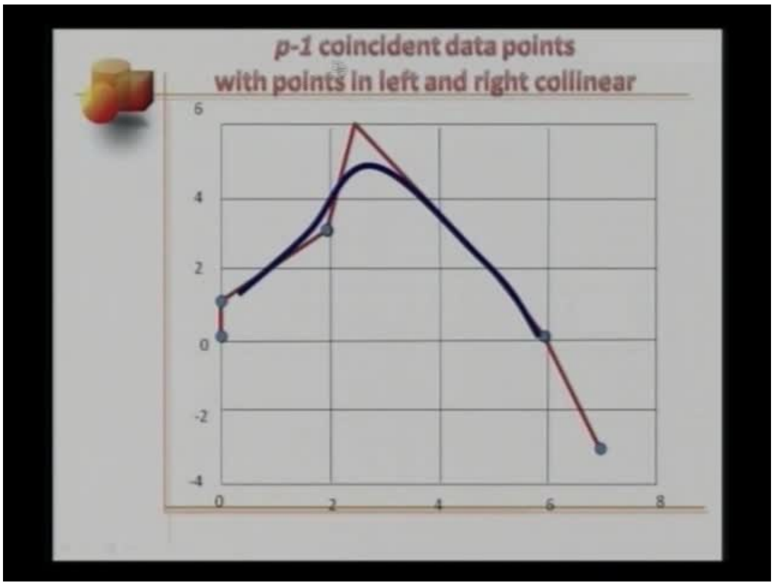

The third case, let we have p minus 1 inner case 3 co incident design points or data points with points on left and right collinear. You work with a same example, we merge 3 intermediate design points together plus take care of this condition here.

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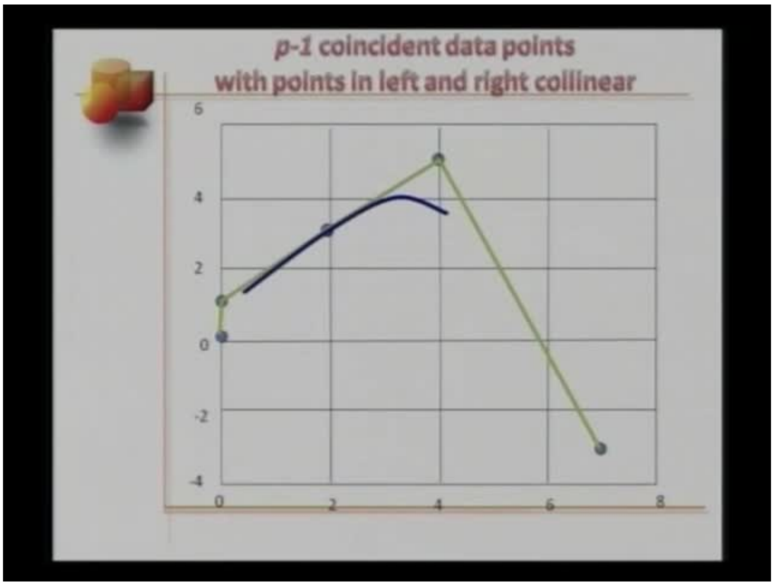

We move the next point to lie along this line. Once again we move this point here. So, that this 3 points collinear with this being a multiple design point. This is a new control poli linear here. This is the new shade of our B spline curve. Notice, that not only this a curve passing through this multiple design point is also tangent, rather tangent to the control polynomial clamping using data points. I just absorb, what is happening here? I can make a B spline curve pass through a design point, if I raise the multiplicity of that design point (( )) we reviews the same concept here.

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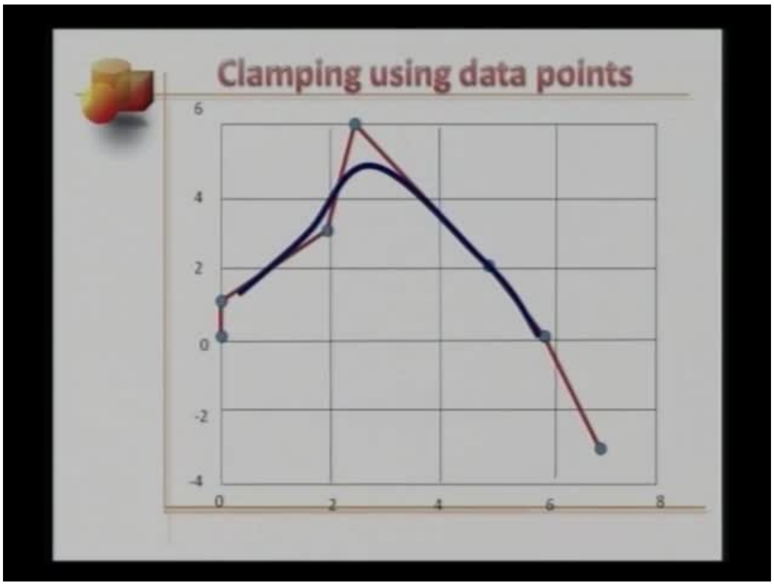

To clamp of B spline curve at the two end points we are working with this example throughout.

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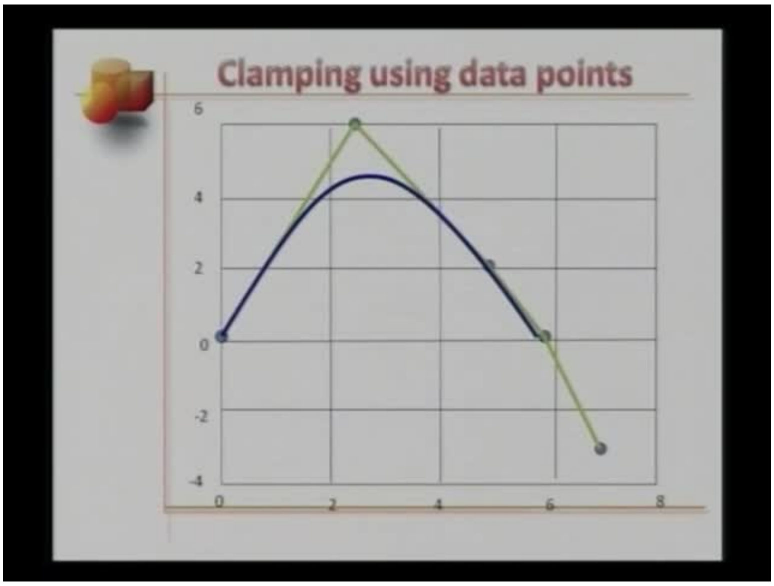

A move the second design point to coincide with the first one, the third design point again to coincide with the first one; so this is my new control poliline in green and this is the shape of the resultant B spline curve. Clearly, it passes through the first one. What else the multiplicity of this design point? Let us go back and figure this how? Two design points coinciding three design points coinciding the multiplicity of this control point is 3 in that case.

So, remember the working with p minus 1 multiple design points because of this that are new B spline curve passes through the first design point. Let us experiment further and let us merge last three control points together. So, as to raise the multiplicity of this design points by 3 as well this is our poliline and the curve in blue is the new B spline curve. You would not be surprise that this passes through the last design point as well.

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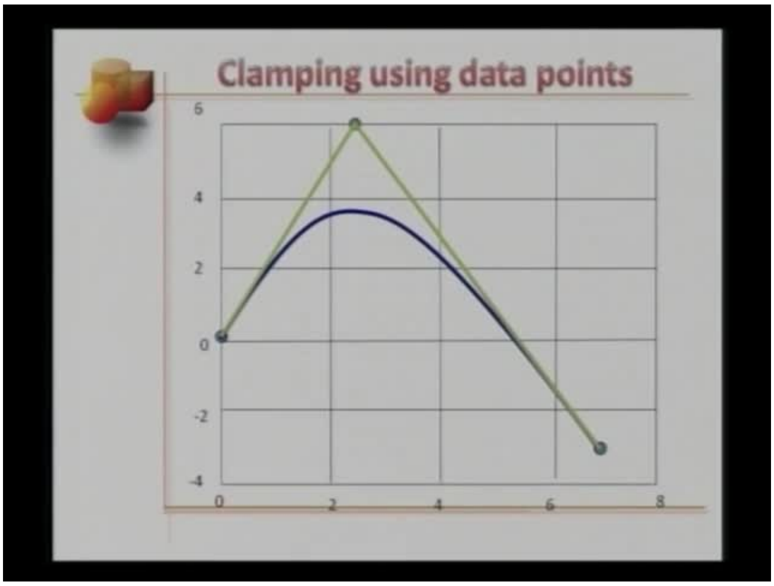

How about designing closed B spline curves using control points. So far we have been working with open curve shapes in that these curve shapes were not forming loops. Now, let us design B spline curves to farm the desired loops.

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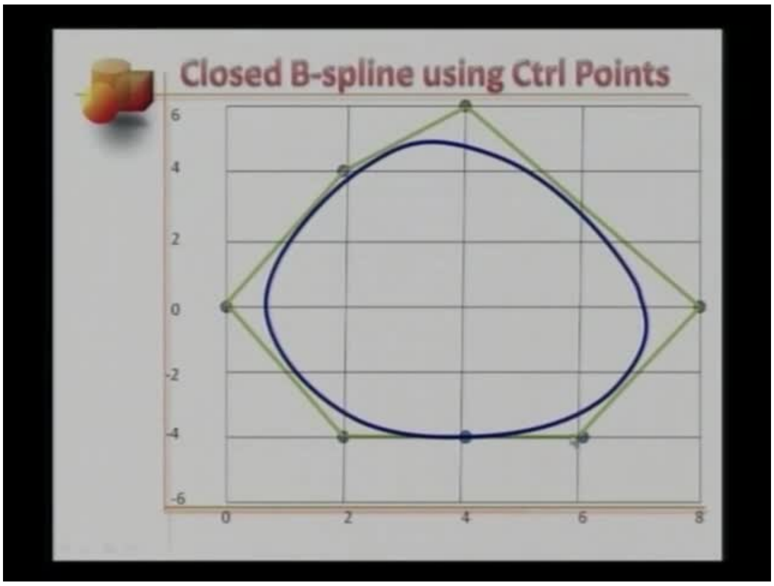

Of course, they are going to be using control points. Let me move one of my data points from here to here another one; so that this point here as multiplicity 2. Now, this point as multiplicity 3. The next design point goes to a new location, this is another one still another one we keep on going this. Now, watch carefully, what is happening? I generate a new designed point to coincide to the first one, which already had multiplicity 3 second; one going there and third one going there. So, affectively what is happening? Now, the first design point had multiplicity 3, and now the last design point also has the same multiplicity.

This is our control poliline, which forms a close polygon and this is the resulting B spline curve. It is because of the first design point and the second design point both having multiplicity of 3, that the curve passes through this point. Now, the question, why is it that the curve is tangent through this line? If it is, let us go back to the previous example once again. This design point had multiplicity 3. So with this, now if you bring this one here to coincide with this one here, imagine that we had another design point somewhere here may be.

So, that you could have made 1, 2 and this one here collinear, then you would have expected that the curve not only would had have passed through this design point, it would also had been tangent through this control poliline. Something very similar, it is happening in the next example. Right here, you would need a little bit of practice to understand this a little bit.

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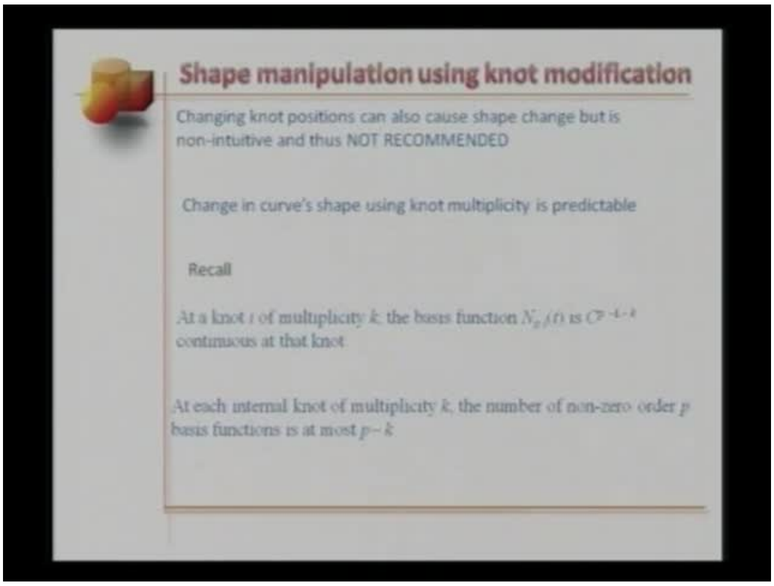

I would encourage you to get some let us move forward now and discuss shape manipulation using knot modification. Well this is something that I have mentioned before that changing knot positions can also cause shape change, but is non intuitive and therefore, it is not recommended. Rather change in curves shape if you use multiple knots is predictable. Recall at a knot i of multiplicity k the basis function N p i of t is c p minus 1 minus k continuous at that knot. Whenever we are discussing B spline curves, you should keep in mind basic definition.

Of B spline curve b the basis function or any other curve is always c p minus 2 continuous, if it is of order p of degree p minus 1. For simple knots that is for k equals 1 t p minus 1 minus 1 t p minus 2 continuity condition is satisfied for a B spline curve. Now, here that is another one at each internal knot of multiplicity k, the number of non zero order p basis functions as at most p minus k. By this time you should be able to figure this property out. All you need to do is sketch the respective B spline basis functions.

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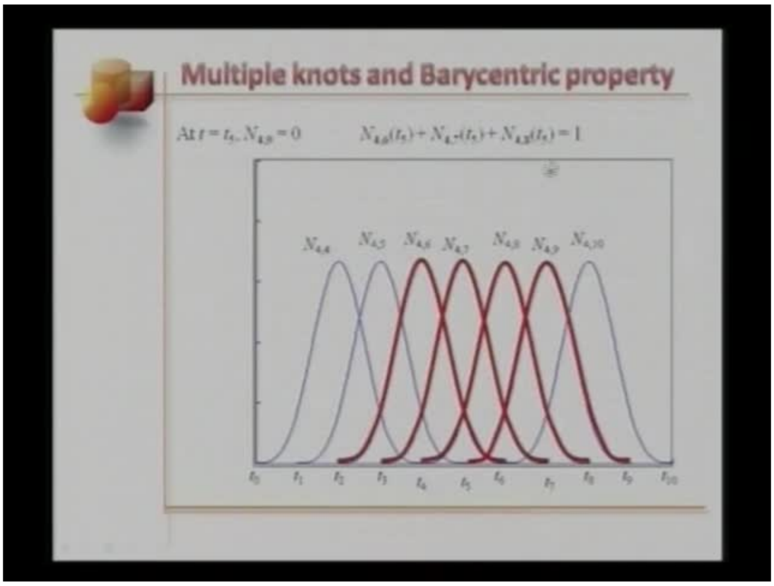

Let us see what we have with multiple knots Barycentric property. Let us say we have a set of knots going from t 0 until t 10, all simple knots. Correspondingly we have 1, 2, 3, 4, 5, 6, and 7 order 4 B spline basis functions. How would you name for sample this one? This would be N 4 4. Let us concentrate on these red shape functions for now. The B spline basis functions are named N 4 4, N 4 5, N 4 6, N 4 7 until N 4 10, you would know now how to do so.

So, the right once here are N 4 6, N 4 7, N 4 8 and N 4 9. Now, for value of t equals t 5 it is right here N 4 9 this basis function is 0. Now, our interval of concern is t 5 t 6. Let us concentrate on this interval for now. We have 4 B spline basis functions, which are non zero over this interval and because of local Barycentricity, all these four functions will sum to 1. At t 5 there is a t equals t 5 N 4 9 will be equal to 0, right here. So, one of the B spline basis functions will get dropped from the summation. N 4 6 at t 5, N 4 7 at t 5 and N 4 8 at t 5, they will now sum to 1.

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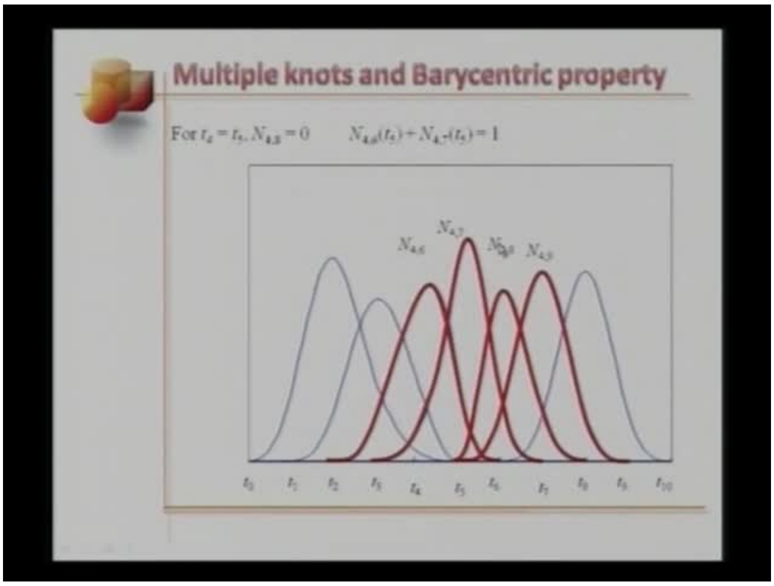

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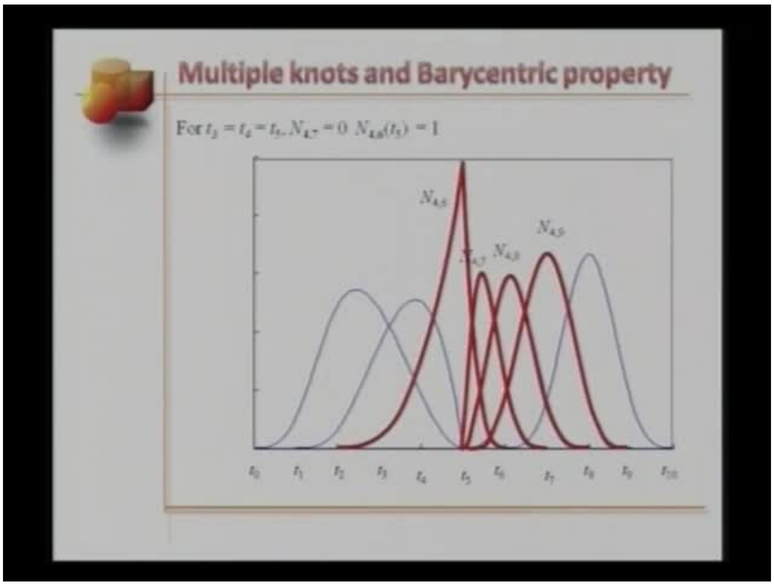

Let us deal further. A similar case, but now we have t 4 equals t 5 because of which N 4 8 will be equal to 0, that would make two of the basis splines sum to 1 N 4 6 at t 5 and N

4 7 at t 5. So, observe how knot multiplicity is helping us manipulate the local Barycentricity properties. These are of course, N 4 6, N 4 7, N 4 8 and N 4 9.

Let us go deeper now. Now, we have N 4 6, N 4 7, N 4 8 and N 4 9, shade slightly differently for different cases. Now, we have t 3 equals t 4 equals t 5. This result in N 4 7 equals 0. This would mean that at t equals t 5 only N 4 6 will be equal to 1. So, what is just happened? Let me go back, what we did was we try to raise the multiplicity of this knot over here, t 5. We started bringing the knots on the left towards t 5 1 by one1. So, at one time we had t 4 equals t 5 and the another time we had t 3 equals t 5 and so on.

We absorbed what happened to this summation of over here. What happened local Barycentricity property? So, for simple knot N 4 9 is equal to 0. So, we had three basis functions, sum into 1 when t 5 has multiplicity 2, that is when t 4 is brought t 5 from the left. The other basis function gets dropped out N 4 8 would be equal to 0. Consequently 2 of the basis functions now will sum to 1. We raise the multiplicity of t 5 by 3 by bringing t 3 also to precisely lay over t 5. This would result in N 4 7 to be equal to 0 because of which only N 4 6 at t 5 equals 1. I would this help?

Now, you got absorb that the control point associated with N 4 6 of t is b 2. So, this six index here is p plus i if you recall. p is the order of the basis functions that we using to construct a B spline curves, because of this local Barycentricity property now, you would expect that are explained curve would pass through b 2, for this example.

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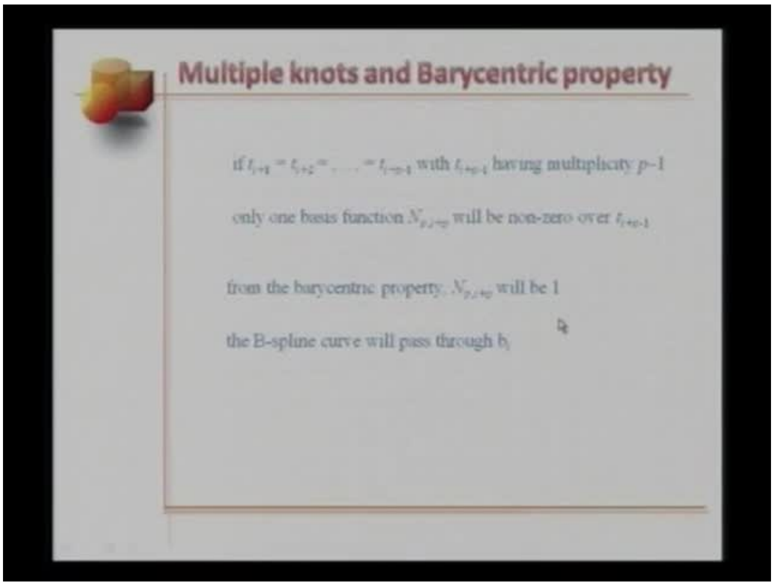

In general if t i plus 1 equals t i plus 2 equals up till t i plus p minus 1 with t i plus p minus 1 having multiplicity p minus 1. Only one basis function N p i plus p will be non 0 over t i plus p minus 1 rather at t i plus p minus 1. Of course, barycentric property N p i plus p will be 1, which would mean that the B spline curve will pass through the design point b i.

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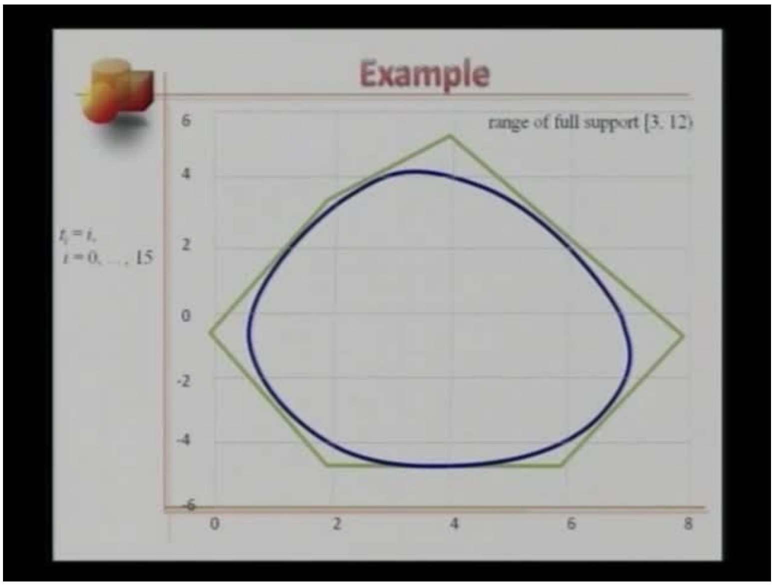

Let us now consider an example where we had designed a closed B spline curve. Recall, how we have specified our design points? So, we had three design points preciously lying over each other. We start the fourth one, the fifth, sixth, seventh, eighth, ninth and three design points coinciding with each other at the end. So, in a sense we had 12 design points. I would want you to do with exercise with me, so let us come back so we had 12 design points. The order of this closed B spline curve is 4. So, we should have 12 plus 4, which is s16 knots in all.

So, this closed spline is the result when we specified t i equals i for i going from 0 to 15. So, if you notice these are 16 equally spaced knots. You would guess by now, what the range of the full support s for values of t in between 3 and 12. Now, this time with this knots span, I would want to you to sketch all the B spline basis functions that have been used to design this closed B spline curve. You might want to take a moment to do that.

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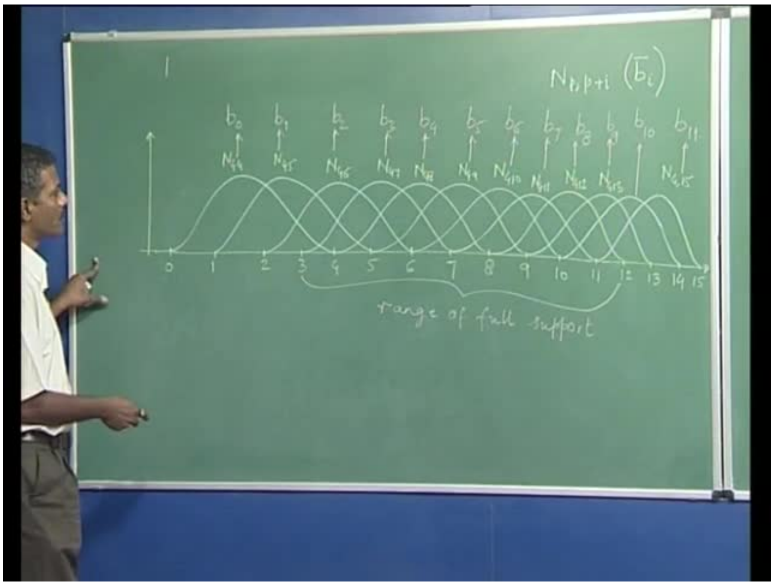

While you were busy sketching your B spline basis functions, I was sketching mine. So, this is the knots span t 0 up till t 15 uniform knots span with all simple knots at this time. This says the range of a support from 3 to 12 and these are my individual B spline basis functions N 4 4, N 4 5, N 4 6, N 4 7, N 4 8, N 4 9, N 4 10, N 4 11, N 4 12, N 4 13, N 4 14 and N 4 15. Now, remember an element in the summation when we define our B spline curve N p p plus i times b i.

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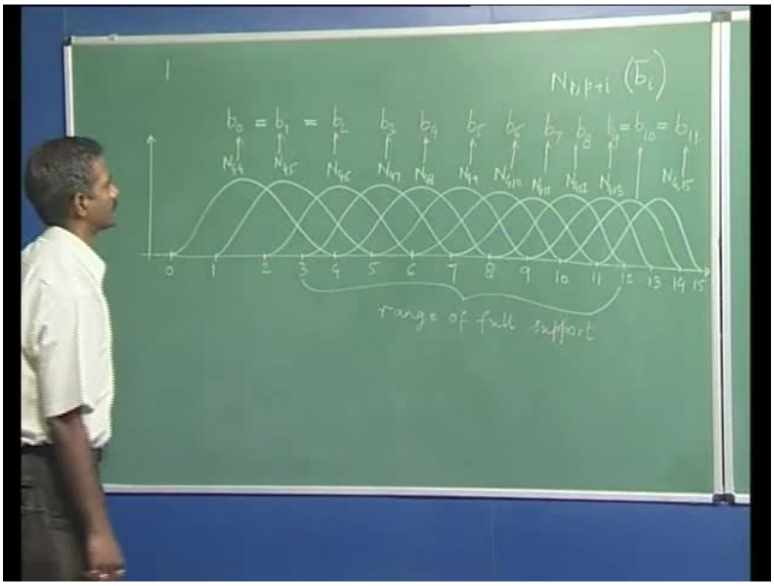

Try to associate data point b i with the corresponding B spline basis function for our case b is 4. So, b 0 is associated with 4 4, N 4 4, b 1 with N 4 5, b 2 with N 4 6, b 3 with N 4 7, b 4 with N 4 8 and so on and in the last we have b 11 affiliated with N 4 15. from b 0 to b 11 we have 12 design points, of those 12 design points 3 of the first one are coincident here and 3 of the last one are co incident at the same sight. Though this is a general figure for our example b 0 equals to b 1 equals to b 2 and b 9 equals b 10 equals b 11. Three of the last and three of the first design points are co incident.

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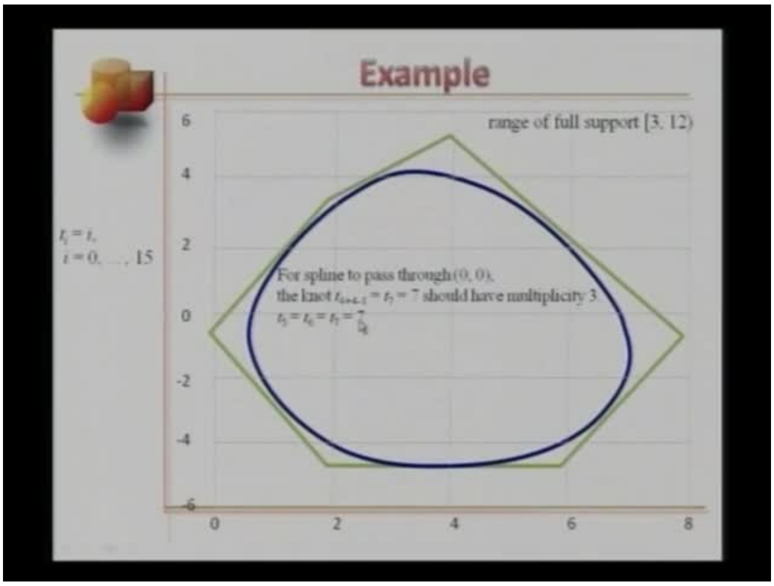

Now, if I would want to half my B spline curve pass through this design point 0 0, which is 3 plus 4 the fifth design point here. The knot t 4 plus four minus 1, which is t 7 is equal to 7, should have multiplicity 3. That means the knot values t 5, t 6 and t 7 they should same as 7. So my fifth data point or (( )) point here and I would want my B spline curve to pass through b 4. That could mean I have to make N 4 8 equal to 1. I will have to do that by raising knot multiplicity.

Let us concentrate on this knot span t 7, t 8 or which N 4 8, N 4 9, N 4 10 and N 4 11 will be non zero, because of the local valid intercity property, these four basic functions will sum to 1. Now, t equals to t 7, N 4 1 automatically gets drop down, its value is 0 here. So, t equals to t 7 N 4 8, N 4 9 and N 4 10 these three is sum to 1. If I bring t 6 to t 7 n for 10 becomes 0. Further if I bring t 5 to t 7 N 4 9 also becomes 0, making N 4 8 equals 1 and t equals to 7. This is when our B spline curve will pass through the data point b 4, which is the fifth data curve in our case.

Once we do this, this is how a new closed B spline curve looks like. Now, try to work this thing yourself for this closed spline to pass through this data point here, 8 0 this one. The knot t 7 plus 4 minus 1 which is t of 10, which is equal to 10 should have multiplicity 3. There is t 8 equals t 9 equals t 10 equals 10. Using the similar exercise that we did on board, you might gone to conform this. So, notice that this is fifth data point 6, 7 and eight data point this corresponds to b seven I let you work on detail, I just give you a final result. Note that this result this not have the knot t 7 with multiplicity 3. This is a different case.