Computer Aided Engineering Design
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Lecture -18

Hello and welcome, we continue with our discussion on the design of Bezier segments and curves, this is lecture 18 on composite and rational Bezier curves. But before, we get into composite and rational Bezier curves, let us try to relate cubic Bezier curves with Cubic Ferguson segments.

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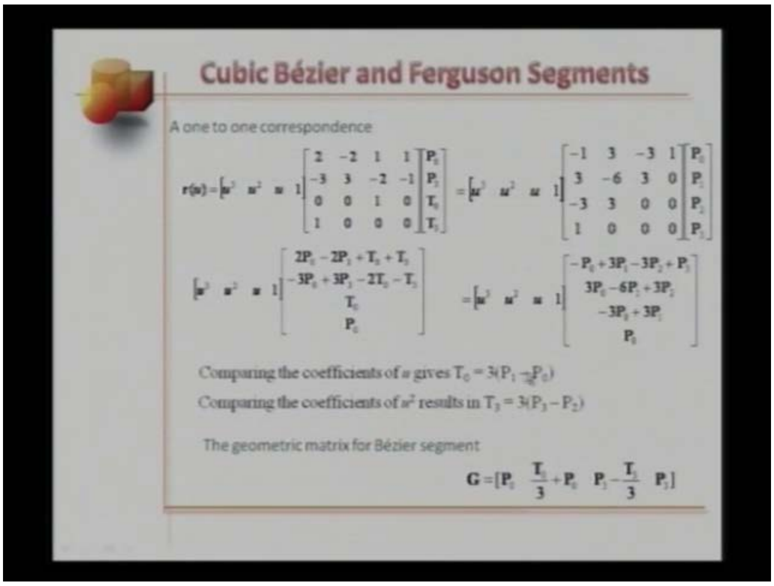

That is a one to one correspondence between, these 2 segments. For Ferguson curve the position vector is given by is 1 by 4 matrix of u cube, u square, u and 1, it Ferguson coefficient matrix, which is of size 4 by 4 and the geometric matrix here we have the first point, the second point the first slope and the second slope.

Correspondingly for a cubic Bezier curve is 1 by 4 matrix is the same at this 1 in u, this 4 by 4 matrix here is a Bezier matrix. And this 4 by 3 matrix is the geometric matrix, that takes the information from the 4 design points or control points that it specified, recall that these are Cartesian coordinates in Euclidean space. So, each P will have the coordinates x y and G accordingly, those coordinates are going to be placed column wise.

If these segments are to be the same will have to impose and equal to sign here and this could be true for all values the parameter u between 0 and 1. So, what we do here is we absorb this geometric information into this and come up with the result in 4 by 3 matrix here.

So, the first term is 2 times P sub 0 minus 2 times P sub 3 plus t 0 plus T 3, second term minus 3 P 0 plus 3 P 3 minus 2 P 0 minus T 3. The third term is T 0 that is the first slope and the fourth term is P 0 that is the first point likewise, we do the same here. We multiply these 2 matrices to get the first term as minus P sub 0 plus 3 P 1 minus 3 P 2 plus P 3 P is 0 minus 6 P 1 plus 3 P 2 minus 3 P 0 plus 3 P 1 and P 0.

And we now compare the coefficients of u u will get the first condition as the slope T 0 is equal to 3 times P 1 minus P 0, these are the first 2 design points corresponding to the Bezier segment. Comparing the coefficients of u square will give us the second slope of the Ferguson curve, which is represented by T 3 and that is equal to 3 times P 3 minus P 2.

These 2 will be the last 2 design points on the Bezier segment, the geometric matrix for Bezier segment in terms of the slopes from Ferguson segment will be given by P 0, the second term as T 0 over 3 plus P 0. The third term as P 3 minus T 3 over 3 and the fourth term as P 3, what I am trying to say here, what is possible is the interrelation and therefore, conversion of geometric matrices between Ferguson and Bezier segments given a Ferguson curve, you can always convert the slopes into the corresponding design points like here.

And on the other hand, if you have given the geometry pertaining to a Bezier segments, it is always possible to convert this geometry back into the point and slope information, that is required by a cubic Ferguson segment again using these 2 conditions.

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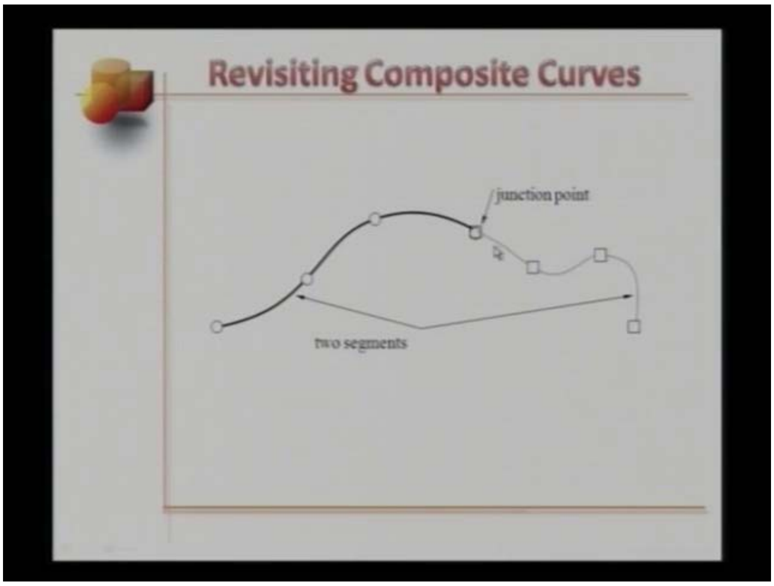

Let us now get back to our discussion on composite curves, I have been repeatedly emphasizing the concept. A composite curve is constituted by individual segments, for example, this one and this one. Each segment is a low degree polynomial may be quadratic, may be cubic, not more than that you know why this is so.

This is because, we need to minimize oscillation in a segment, these 2 segments are joint together or arranged continuously or just appose together at this junction point. So, this point is common between these two segments, when I say. So, I have inherently imposed position continuity.

If I further go ahead and impose slope continuity, I need to ensure that the slope from this segment at this point is a factor of the slope from this segment at this point. In other words the direction is the same for these 2 segments, but the magnitudes of the tangents may or may not be the same. For a C 2 continuous composite curve, I need to ensure that the curvature competition on this segment is the same as the curvature information from this segment at this point, let us try now to get in to the mathematics of composite Bezier curves.

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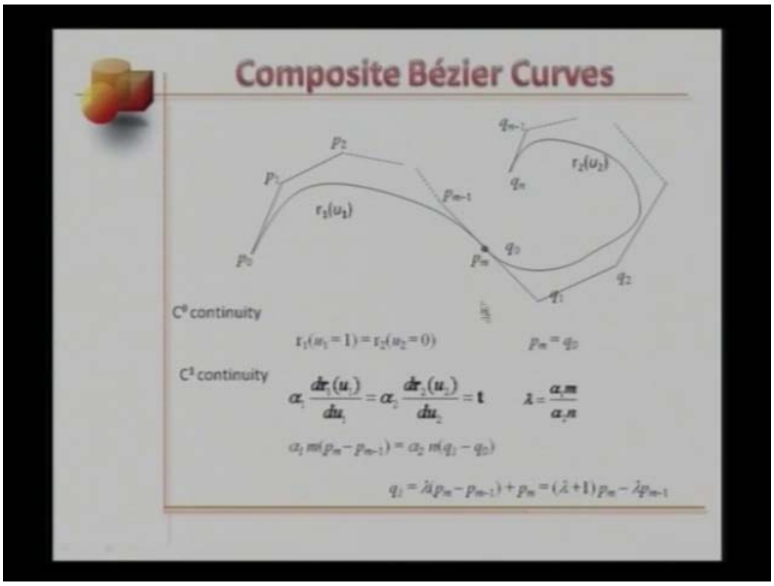

Let us start with the first Bezier segment and let us try to generalize our discussion even though, I have mentioned before that this should be a low degree polynomial let us try with a general n degree Bezier segment.

Likewise for the second segment this is the control poly line for the first Bezier segment given by data points p sub 0 p sub 1 p sub 2 up till p sub n. This would make this segment and m degree polynomial, m for mathematics, likewise this control polyline is for the second segment, it is given by data point’s q sub 0 q sub 1 q sub 2 up till q sub n, n here stands for notation.

This segment will be an n degree Bezier segment let us represent the segment r sub 1 with parameter u sub 1, u sub 1 equals 0 would mean, I am here and u sub 1 equals 1 would mean that I am here. Likewise this segment is represented by r sub 2 with parameter u 2 again u 2 equals 0 makes me go here and u 2 equals 1 makes me go here.

And of course, these parameters u 1 and u 2 are not dependent on each other they are independent in a sense of that values of u 1 would not dictate the shape of the curve of this segment and the values of u 2 will not govern the shape of this segment. Let us now try to concentrate on this neighborhood here for C 0 continuity, I would want to impose that this point here, which corresponds to r sub 1 for value of u 1 equals 1 should be equal to this point here, which corresponds to r sub 2 for u 2 equals 0.

So, p sub m and q sub 0 should be the same for C 0 continuity let us try to bring these 2 points together. Notice here that I am changing this segment of control polyline accordingly the second segment will change in the shape except. Notice here that these slopes are not the same in direction as it now, we are not talked about C 1 continuity or slope continuity as yet. Let us talk about it now C 1 continuity will have to impose that the directions of these 2 lines should be the same and not the magnitude.

This would give us additional design field, mathematically the condition is that alpha sub 1 times, the first derivative of the segment r 1 with respect to u 1. Evaluated at u 1 equals to 1 should be the same as some scale factor alpha 2 times the first derivative of segment r 2 with respect to u 2, evaluated at u 2 equals 0, alpha 1 and alpha 2 are scale factors.

Such that these 2 terms represent the unit tangent t at this point, recall from our discussion on Bezier segments as to how the first derivatives are computed. So, the first derivative for this Bezier segment will be m times, p sub m minus p sub m minus 1 at u 1 equals 1 and be retain this term alpha 1. Likewise for the second segment the first derivative is given by n times, q 1 minus q 0, q 1 minus q 0, we retain the scale factor as well.

We can use this condition to determine q 1, q 1 is given by lambda times p m minus p n minus 1 plus p n, which is equal to lambda plus 1 times p m minus lambda times p m minus 1. In terms of design possibilities, what would this condition need.

This condition will constrained q 1 remembers that as a designer you have. So, far been choosing these control points act well, if we go back to C 0 continuity q 0 here is constrained to be the same as p m. So, you lose 1 design freedom right here further, if we talk about C 1 continuity.

This condition here constrains q 1 as well and if you observe closely q 1 can only be placed along the slope right here. It cannot be placed anywhere else, if we are considering slope continuity at this junction point. If you do your algebra then, you will figure that lambda equals alpha sub 1 times m, the degree of the first Bezier segment over alpha sub 2 times n, the degree of the second Bezier segment.

What I am trying to do here is graphically show you this scenario. The control polyline for the second Bezier segment changes accordingly the shape of the second Bezier segment changes. The direction of the slope is common at this junction point and now, you would see the slope is same here.

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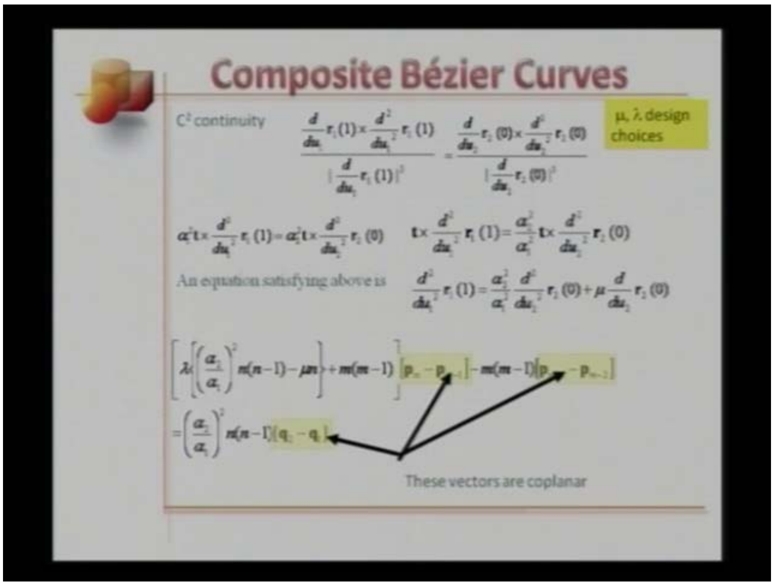

Let us go into C 2 continuity conditions, once again we have seen this before in case of C 2 continues Ferguson curves recall this relation here. The first derivative of r 1 evaluated at 1 cross with the second derivative of r 1, evaluated at 1 over. The absolute value of the first derivative r 1 at u 1 equals to 1 rise to 3, this is the expression for curvature at point u 1 equals 1 for first Bezeir segment.

Likewise the expression for curvature for the second Bezier segment evaluated at u 2 equals 0 can be similarly, computed and for C 2 continuity, these 2 expressions at v e. In the sense curvature at the junction point should unique, let us plug in the information pertaining to the first derivatives of the 2 Bezier segment in terms of the unit tangent t and the scale factors alpha.

So, this term here can be simplified to alpha 1, square times t crossed with the segment derivative of the first Bezier segment evaluated at u 1 equals 1. And this could be equal to alpha 2 square times the unit tangent at the junction point cross with the second derivative of the second Bezier segment evaluated evaluated and u 2 equals 0.

It can be further simplified to t crossed with d 2 over d u 1 square of r 1 at u 1 equals 1 equals to alpha 2 square over alpha 1 square times t. The unit tangent crossed with d 2 over d u 2 square r 2 at u 2 equals 0 an equation, that is satisfies this relation here is given by the second derivative of r 1 at u 1 equals 1, which is p equals to alpha 2 square over alpha 1 square times, the second derivative of r 2 at u 2 equals 0 plus some scalar mu times the first derivative of r 2 at u 2 equals 0.

Let us see how this relation satisfies this condition, if you plug in the right hand side in place a this term, you will get t crossed with alpha 2 square over alpha 1 square times d 2 over d u 2 square of r 2 at it equals 0, which is this term. And what happens to t cross with d r 2 over d u 2 at u 2 equals 0 well, you guess it right.

It is 0, because the direction of the unit tangent t and the first derivative of the second Bezier segment is the same, when I say so, I implicitly assume that the composite Bezier curve is a C 1 continues Bezier curve. Let us go further, we know how to compute the first derivative, the second derivative and so on. So, for of this Bezier segments.

Let us try to plug in those expressions directly, into this condition or rather this condition. So, we have m times m minus 1 times p sub m minus 2 p sub m minus 1 plus p m minus 2 and that is equal to alpha 2 over alpha 1 the whole square times n times n minus 1 times q 2 minus 2 q 1 plus q 0, which corresponds to the second derivative like here plus mu times n times q 1 minus q 0.

We, try to rearrange this relation further and in doing, so we get m times m minus 1 times p m minus p m minus 1 minus m times m minus 1 times p m minus 1 minus p m minus 2 plus alpha 2 over alpha 1 the whole square, times n, times n minus 1 minus mu n. This is the coefficient for q 1 minus q 2 and that should be equal to alpha 2 over alpha 1 the whole square times n, times n minus 1 times q 2 minus q 1, all I have done is, I have regrouped the terms.

P m minus p m minus 1, p m minus 1 minus p m minus 2, q 1 q 0 and q 2 minus q 1, if you go back to your figure, you will realize that, these are free vectors joined head to tail in divisionary of the junction 1.

We rearrange this further in terms of 3 vectors here, the first free vector is p m minus p m minus 1, the second free vector is p m minus 1 minus p m minus 2 and the third free vector p n minus 2 and the third p vector is q 2 minus q 1 notice that I am also using C 0 and C 1 continuity conditions to get this equation.

The coefficients of p m minus p m minus 1 is lambda times alpha 2 over alpha 1 the whole square times n times n minus 1 minus mu times n. Lambda multiplies this entire term here plus m times m minus 1 coefficient of p m minus 1 minus p m minus 2 is minus m times m minus 1. And the coefficients for q 2 minus q 1 is alpha 2 over alpha 1, the whole square times n, times n minus 1.

The algebra for the expressions might be looking quite complex. But, this has a physical meaning look at this free vector, this free vector and this free vector. This relation essentially, says from the design perspective, that these 3 vector or these 3 free vectors are coplanar.

They have to lie on the same plane in 3 dimensions or in Euclidian space to ensure that the composite Bezier curve is C 2 continues at the junction point. Here scalars lambda and mu are a design choices, lambda has the same expression that, we saw before and mu is any scalar that the design can choose, basically what were this condition need.

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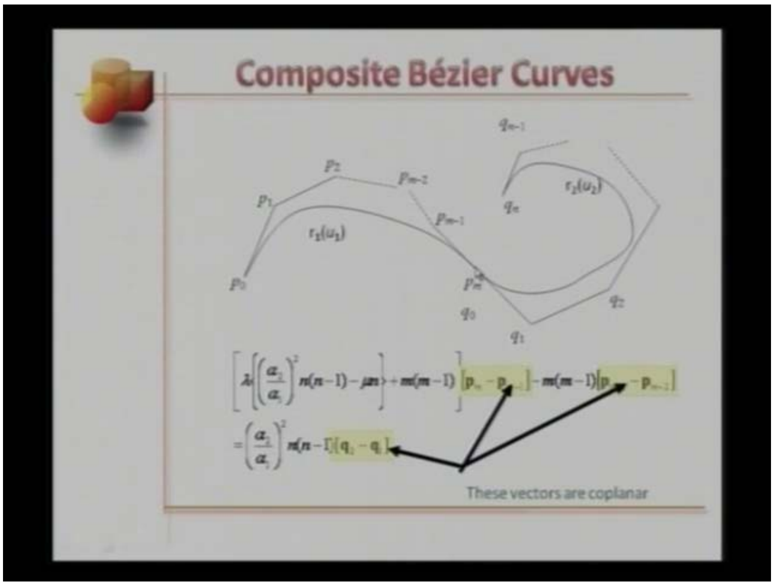

Let us go back to the figure, where a Bezier segments, where C 0 and C 1 continues and this point here p m and q 0 are the same. Now look at these 3 vectors p m minus p m minus 1, which would be this p vector here, p m minus 1 minus p m minus 2, which would be this vector here and q 2 minus q 1, which will be this.

From C 1 continuity, we already had q 1 minus q 0 as a scalar multiple of p m minus p m minus 1, the fact that these 3 vectors are coplanar would mean, that p m minus 2 p m minus 1 p m, which is the same as q 0 q 1 and q 2 are the 5 design points, that will have to lie on the same 2-dimensional plane. These 5 points will be coplanar from the design perspective what could that mean as a designer, if you had chosen these points at well q 0 is constrained to be the same as p m or C 0 continuity.

That is the first design choice that, you would use q 1 is constrained to lie along this line here, that is the partial design choice that he loose, you are still free to choose q 1 along this angle. And q 2 now is constrained to lie on the plane defined by any 2 of these 3 vectors again that is a partial lots of choice. Now for 2 dimensional composite Bezier curve this condition is automatically satisfied, which would make q 2 as a free choice on a plane.

But, for a 3 dimensional composite Bezier curve, this condition has to be satisfied for the curvature information to be unique at this junction point, we now come to rational Bezier curves.

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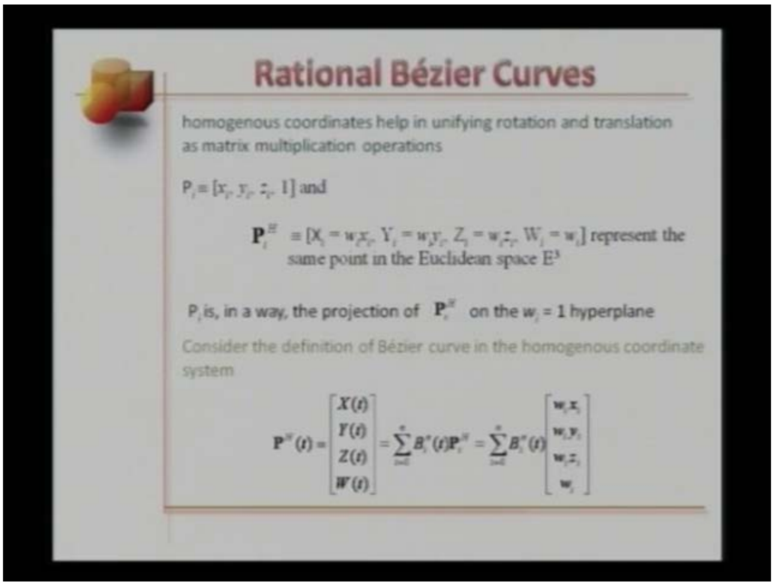

We have seen before that homogenous coordinates help in unifying rotation and translational transformations as matrix multiplication operations, we will see today as to have homogenous coordinates can also help define individual segments, more general. In this case a Bezier segment.

Recall that P i is given by x i y i z i and 1 and P ignore, this 1 here, x y and z with indices i are the cartesian coordinates of the P i. And P i super H are the homogenous coordinates, capital X i capital Y i capital Z i and capital W i capital X i is given by W i times x i y i is given by w i times small y i capital z i is w i times small z i and capital w i is small w i enough of notations. Some physical significance, now small w sub i is the weight or scale factor.

You also seen this before that P sub i super h and this P i here, P represent the same point, there is an equivalence between this representation and this representation. If you think about this in a slightly different way P sub i is the projection of P i h on the w equals 1 hyper plane, you set this point w equals 1 and you will get back, this vector here by the way both these are homogenous representations of point P i the different weights or scale factors, you must know that.

Consider the definition of a Bezier segment in the homogenous coordinate system, you have P super h as a function of parameter t with components capital X capital Y capital Z and the weight or the scale here, which also a function of t. And this is given by summation i going from 0 to n b i-th constant polynomial of degree n as a function parameter t times these design points, p sub i super h. And the Bezier segments that, we have seen before, we have been specifying only cartesian coordinates x i y i and z i.

Here, we specify the homogenous coordinates, which are given by w y times x I, w i times y I, w i times z i and w y from the design perspective notice, what is happen. We have introduced additional design field to these weights, 1 weight per design point that would make it, n plus one such ways. And we will see later how this design freedom works in our favor.

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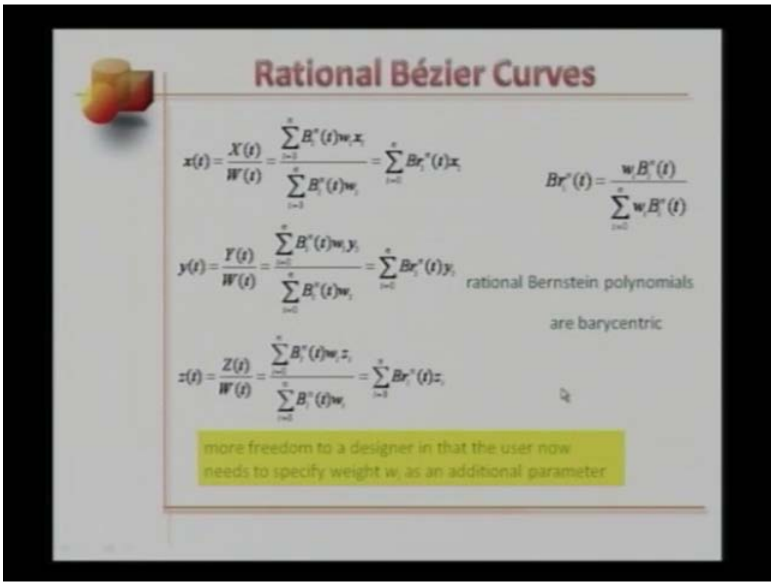

Let us continue with the math small x of t and capital X of t would represent the same point in the Cartesian space, that is what I make. Now, continuing forward small x of t is equal to capital X of t over W of t, which is equal to summation i going from 0 to n B i-th Bernstein polynomial with degree n.

which is a function of t times w i x i over summation i going from 0 to n, B sub i super n t times w i try to absorb this denominator with this term here. B i n as a function of t times w y and replace this term by B r sub i super n t times x i retain, the summation sign i going from 0 to n.

Likewise for the y coordinate Y t is capital Y over capital W, which is summation i going from 0 to n, B sub i super n times w i times y i over summation B i-th Bernstein polynomial degree n times w y. In a similar manner as above B r i n is this term here over this entire term, we will get y i here and will we retain this summation here.

Likewise, you can compute small z as a function of t the parameter as capital Z over capital W very similar expressions the only change is this z i. And this is given by summation i going from 0 to n, B r i super n as a function of t times z i, formally B r sub i super n as a function of t is given by w i B sub i super n. These are the original Bernstein polynomials with index i and degree n over summation i going from 0 to n w i and the i-th Bernstein polynomial right here.

You can think of B r sub i super n as rational Bernstein polynomials notice that like the original Bernstein polynomials. They 2 are barycentric recall again, what i mean by barycentric that these coefficients of weights associated with the Cartesian coordinates or in general the design points are all greater than 0 and it sum to 1. These are greater than 0, if we ensure all these weights are positive and we already know from before that these Bernstein polynomials are greater than 0.

The second aspect of being barycentric is that no matter, what t is the rational Bernstein polynomials will always sum to 1 and that is something that you can easily observe, try to add all these terms together, when you are done adding, what you would notice is the numerator is the same as the denominator. They would cancel out and the result will be 1 as I said before by additionally specifying the weights small w i.

We give more freedom to a designer in that he or she can now choose these weights independently and as additional parameters, just take a look at the example and see how the change in weights would affect the change in the shape of a rational Bezier curve.

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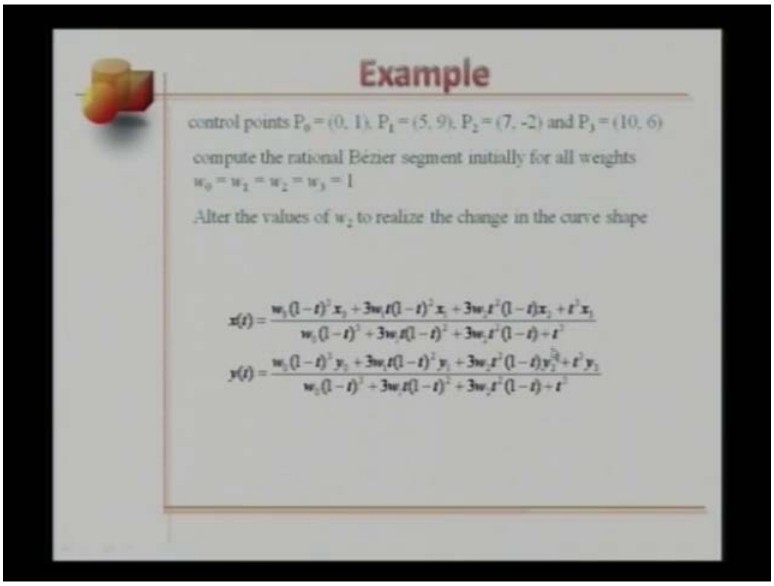

Let us start with 4 control points P 0 as 0 1, P 1 as 5 9, P 2 as 7 minus 2 and P 3 as 10 and 6. We will compute the rational Bezier segments for different weights, but to start with, we will set all weights equal to 1 again realize that each weight is associated respectively with each design point here. We will later alter the values of w 2 to realize the change in the curve shape and may be also look at what happens to this shape of a Bezier segment.

If w 1 is change well algebraically, this is a 2 dimensional Bezier segment. So, the x component is given by w 0 1 minus t, the whole cube times x 0 plus 3 w 1 t 1 minus t, the whole square times x 1 plus 3 w 2 t square 1 minus t times x 2 plus t cube x 3 over. W 0 1 minus t, the whole cube plus 3 w 1 t 1 minus t the whole square plus 3 w 2 t square 1 minus t plus t cube. This term is only slightly different than this term, here the axes do not appear, likewise for y t all you would need to do is replace x i is with y i is and that is.

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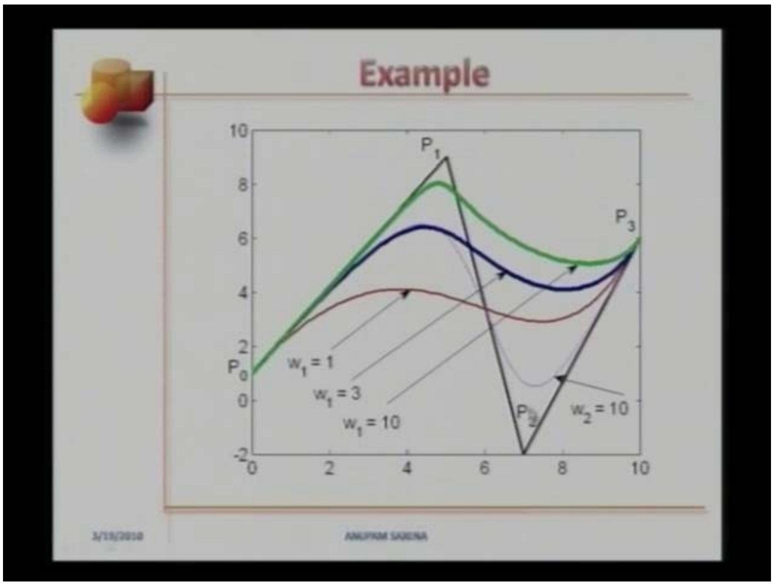

It let us look at how rational Bezier segments change in shape P 0 P 1 P 2 and P 3 are are original data points, lines in black, show the original control polyline. This curve is a cubic Bezier curve, when all the weights are set to 1, let us try to play with the weight corresponding to this data point, which is W 1. If I increase the weight W 1 to 3, I get the blue segment here observe the change in shape.

This region of the curve has moved towards this data point, in other words, if has increased the weight associated with P 1, point P 1 has attracted the curve towards itself. Let us confirm this, if I increase the weight W 1 from 3 to 10, I get this green rational curve notice, what is happened, P 1 has significantly attracted the curve towards it. Let us now see, if the same is true, if I raise any other way, now if I keep W 0 as 1 W1 as 10 W 3 as 1 and if I raise W 2 to 10.

I get this curve in margined, now P 2 has attractive the curve towards it. The point that, I am try to make here is had b consider only a Bezier segment and not rational Bezier segment with the same control polyline, I could not have change the shape of the red curve, which is are original to be Bezier segment with the additional of these ways keeping the control polyline this same, I can now play with a shape of a Bezier segment by merely changing these numbers other way. As I increase the weight corresponding to any data point, the curve would gravity towards at data features with the rational Bezier curves.

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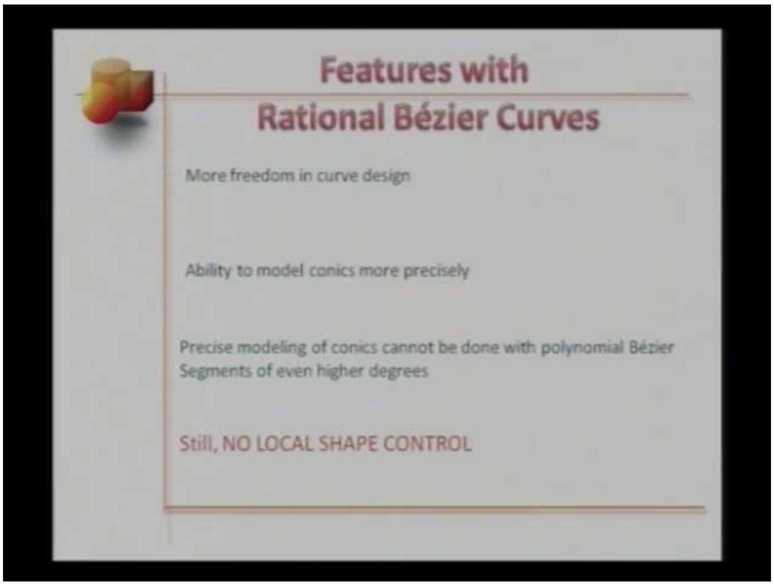

You would realized that, we have more deign freedom, when modeling rational Bezier curves. This is because of the additional weights that, we assign along with the data points or design points.

This is something that, we are going to discussing a little later, but to rational bezier segments, we acquire the ability to model conics precise. Precise modeling of conics cannot be done with polynomial Bezier segments with even higher degrees, we still have this problem of no local shape control with the rational Bezier curves. In a sense, if you move or relocate a data point 2 any other position or if you change the value of any way, the entire shape of the segment changes.

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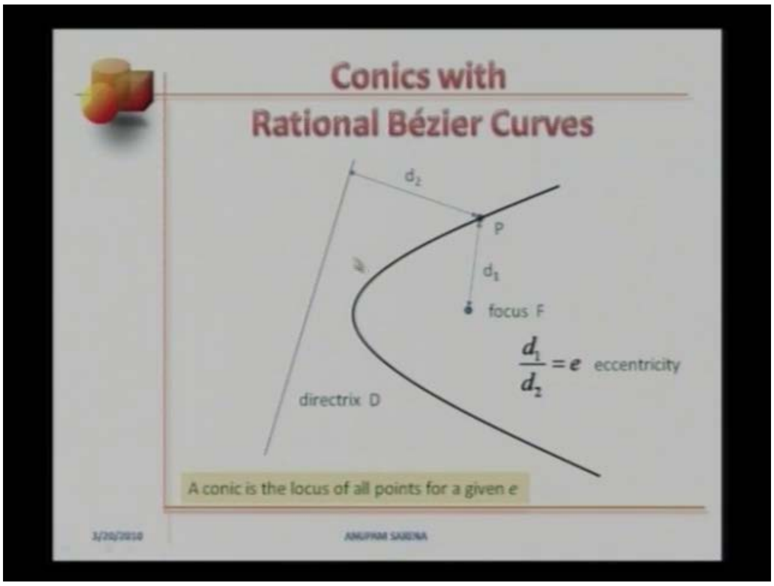

Let us discus now how to model conics with rational Bezier segments and before that let us brush up our concepts on conics. Let us say, we have a point and let us call it focus f let us assume that, we have a line, that does not pass through this point here. And let us nomenclate this line as direct x and by noted by d, let us sketch another point p, let us measure these 2 distances.

The distance between P and the focus and the perpendicular distance between P and the directrix, let us call them d 1 and d 2 respective. These distances will have ratio d 1 over d 2 equals e let us call e the eccentricity.

A conic will now be locus of all points P for a constant value of eccentricity e excel, in other words all these points on this curve will be such that the distance between any point here. And the focus over the distance between that point and the directrix has a constant ratio the eccentricity e.

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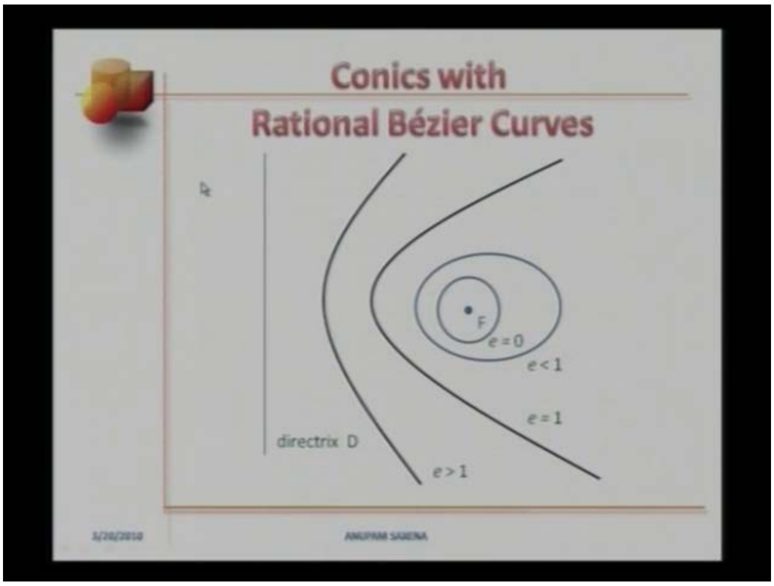

Let us look at a few cases, we have the focus f and the directrix D. This directrix again is not suppose to pass to this focus for the circle, which is a conic, we have the eccentricity value equals 0 for in l fs. The eccentricity value will be anywhere between 0 and 1 for of a parabula P is equal to 1 and for a hyperbole e is greater than 1, let us use this concept and try designing conics, using rational Bezier segments.

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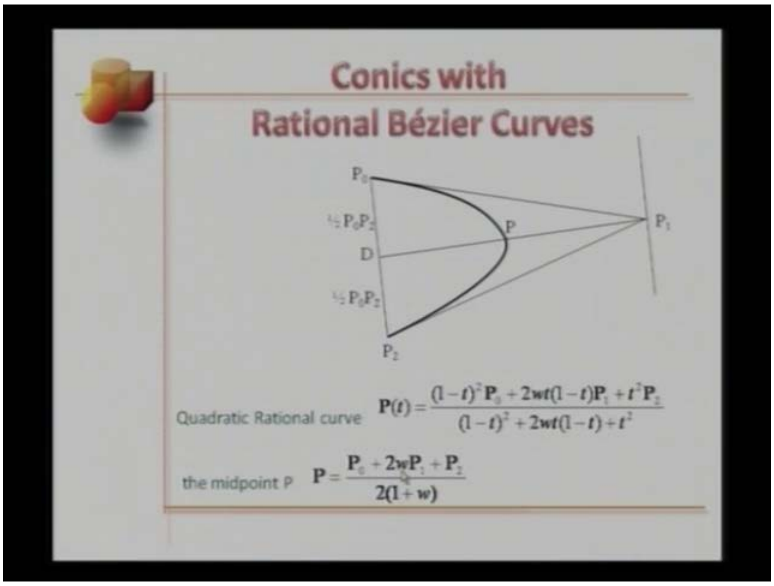

Let us consider a quadratic rational Bezier segment, define by 3 data points P 0 P 1 and P 2, let us sketch the corresponding control polyline. This would be a quadratic rational

Bezier curve, if I also assign weights w sub 0, w sub 1 and w sub 2 to these corresponding points, let us join P sub 0 and P sub 2.

Let me also draw a line parallel to P 0 P 2 here, which passes through P 1 and from P 1, let me draw a line that intersects P 0 P 2 at point D point, D is the midpoint of the segment P 0 P 2. In other words P 0, D is half of the distance P 0 P 2 and likewise D P 2 is half, the distance P 0 P 2.

We would consider D as a focus and this line here passing through P 1 as e directrix, let us take a point P on the curve and let this point correspond to the parameter value half. Now, we are going to be choosing a quadratic rational Bezier curve in a slightly different manner in sense that will have the weights corresponding to P 0 and P 2 assigned as 1 and will have the weight corresponding to P 1 assigned as w that, we need to fine.

So, this is how a quadratic rational Bezier segment a look like the position vector P, which is a function of parameter t will be 1 minus t square P 0 plus 2 w, which is the weight corresponding to P 1 times t 1 minus t times P 1 plus t square times P 2 over 1 minus t square plus 2 times w times t 1 minus t plus t square. Once again the weights corresponding to P 0 and P 2 are 1 and it is this weight w, which is associated with P 1 that, we need to determine. For midpoint P, that is for t equals half, we get P equals P sub 0 plus 2 times, w times P 1 plus P 2 over 2 times 1 plus w.

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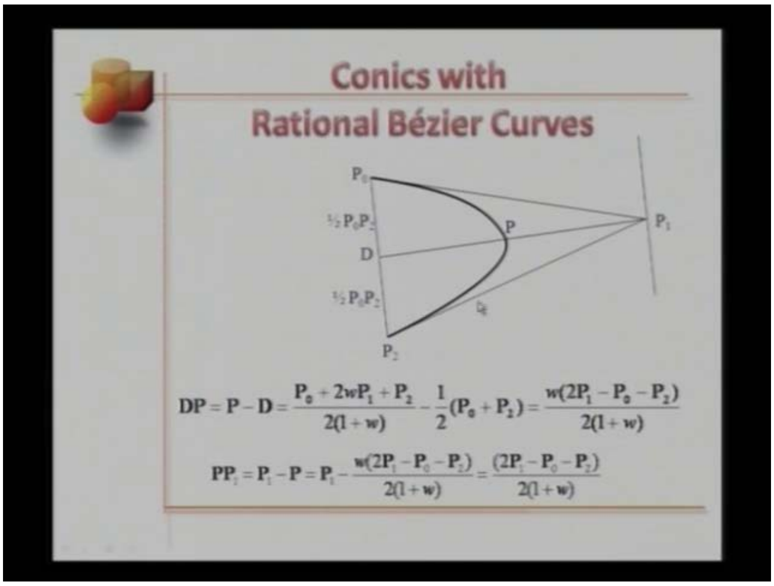

Let us compute the distance D P of course, the vector D P is given by the position vector of P minus that of D, if you work the algebra is equal to P sub 0 plus 2 times w times P 1 plus P 2 over 2 times 1 plus w minus half of P 0 plus P 2. This point D on simplification this expression becomes w times 2 P 1 minus P 0 minus P 2 over 2 times 1 plus w, now the distance P P 1 would be the magnitude in the vector P P 1 here, just P 1 minus P, which is equal to P 1 minus w times 2 P 1 minus P 0 minus P 2 over 2 times 1 plus w upon simplification, that is 2 times P 1 minus P 0 minus P 2 over 2 times 1 plus w. All we need to do is, we need to compute the magnitude of D P and that of P P 1, compute the ratio and equated to the eccentricity e.

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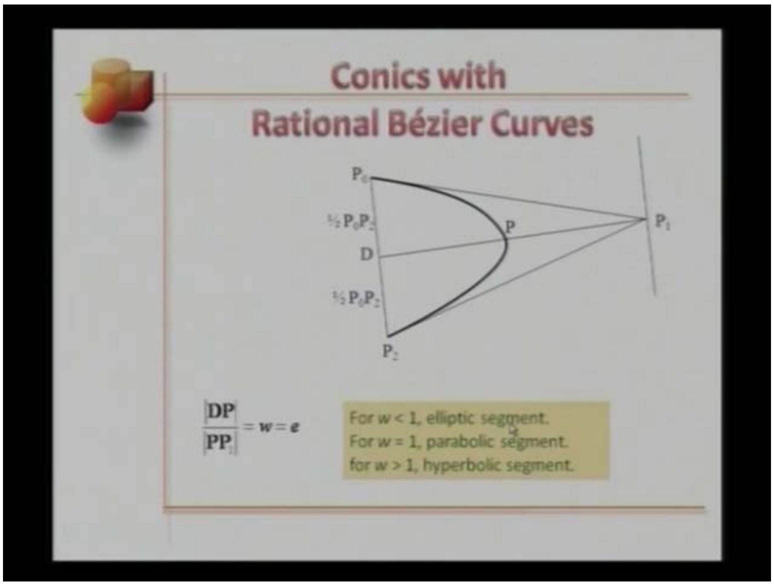

So, the absolute value of D P over the absolute value of P P 1 equals w, if we go back notice that, these terms are the same here by divide the absolute value of D P over P P 1. These terms cancel out and these terms cancel out and what is left is w here. So, absolute value of D P over absolute value of P P 1 equals w and which is the eccentricity the conics.

Really from the definition that, we have seen previously for values of w or eccentricity smaller than 1, we get an elliptic segment for w equals 1, we have a parabolic segment and for values of w greater than 1. We get a hyperbolic segment in a sense all 1 would need to do is specify P sub 0 P sub 1 and P sub 2 arbitrary and set different values of w to get different conics sections.

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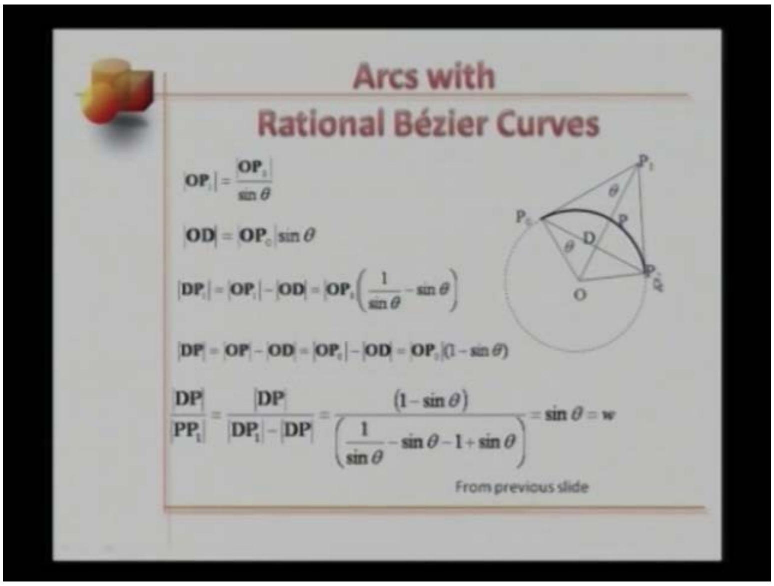

How about modeling circular arcs with rational Bezier curves, here is the figure here, you have a circle with center O and 3 points P sub 0 P and P 2 line on the circle. This curve here represents the circular r and it is model let say as a quadratic Bezier curve again defined by the control polyline P 0 P 1 P 2 P here corresponds to the parameter value t equals half.

Let us try to work the algebra to see, if we can get the circular arc. The method here is slightly deferent from what, we had discussed a bit earlier for other conic times using trigonometry, we have O P 1, this distance is equal to O P 0 over sin theta this angle. Notice that P 0 P 1 is a tangent to the circle and serve this angle is 90, now O D like here is O P 0 sin theta.

Let us take problem D P 1, which is this distance is here is equal to O P 1 minus O D, which is equal to this term here minus term here, taking O P common out. Rather than magnitude of O P 0 common out, you remains in terms are 1 over sin theta minus sin theta half of D P, this distance here. D P is given by O P minus O D O P is the same as the radius of the circle, which is O P 0 and O D is what, we had obtained before radius of the circle times the sin theta on simplification, the absolute value of D P is O P 0 times 1 minus sin theta.

Now, we have this distance D P here, we are assuming that D would act as the focus, the absolute value of D P over P P 1 is equal to the modules D P over mod of D P 1 minus mod of D P D P1 minus D P. D P is given by the radius times 1 minus sin theta and D P 1 minus D P is given by the radius times 1 over sin theta minus sin theta minus 1 plus sin theta. The radius terms get cancel out and this expression gets simplify to sin theta, which is the eccentricity and from previous slide.

That is equal to the weight corresponding to P 1 in other words, if we assign the weight w 0 and weight w 2 as 1, we need to assign weight w to P 1, such that w is equal to sin theta. The theta is this angle, which is also the same as the sine, this for ensure at this point P corresponding to t equals half, lies on the circle defined by t points P sub 0 P and P 2.

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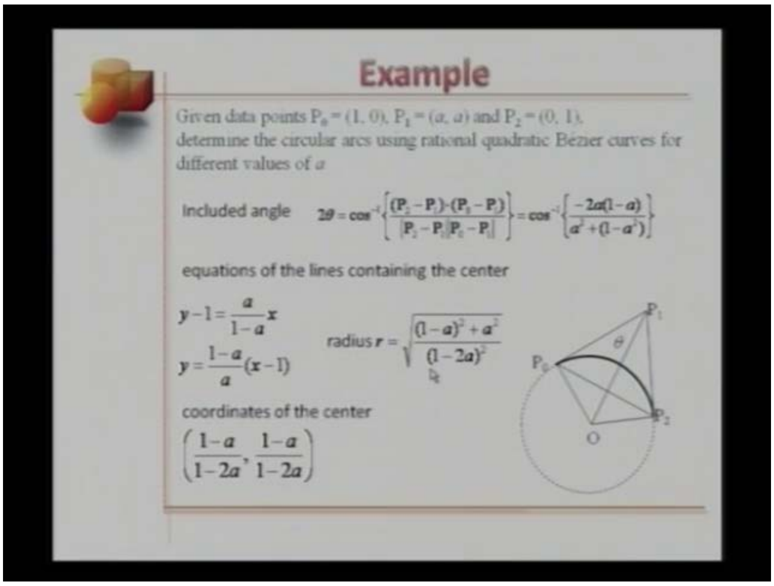

Let try to solve this example in a slightly different way. So, given data points P sub 0 as 1 0 P sub 1 as a a and P sub 2 as 0 1, we will determine the circular r using rational quadratic Bezier curve for different values away.

First let us talk about the included angle theta, sense these 2 triangles are congruent, this angle is the same as this angle. So, 2 theta is the inverts cosine of the dot product of these 2 vectors, which is P 2 minus P 1 dotted with P 0 minus P 1 over the absolute values of these 2 vectors.

We can work the algebra out and we will find that 2 theta is equal to invert cosine of minus 2 a times 1 minus a over a square plus 1 minus a square, we can use this angle directly to compute. This quadratic rational Bezier segment defecting an arc of a circle all we need to do is assign the weight corresponding to P 1 as sin of theta, which is this angle or this angle alternatively.

What we can do is, we can compute the equations of the lines containing the center O 1 of the line is given by y minus 1 equals a over 1 minus a times x and the second 1 is given by 1 minus a over a times x minus 1. All we need to do is either find the equations of these 2 lines and compute the intersection point here or may be find the equation of this line and this line compute the intersection point how would, you get the equation of this line.

Notice that O P 0 is perpendicular to P 0 P 1 and you would already know the equation of this line, because you know the coordinates of P 0, which are 1 0 and P 1, which are a a the slope of O P 0 is minus of 1 over the slope of P 0 P 1.

You should be able to work of the algebra anyhow, the coordinates of the center are given by 1 minus a over 1 minus 2 a. This is the x coordinate 1 minus a over 1 minus 2 a is the y coordinate and there are radius of this arc is given by square root of 1 minus a square plus a square over 1 minus 2 a the whole square.

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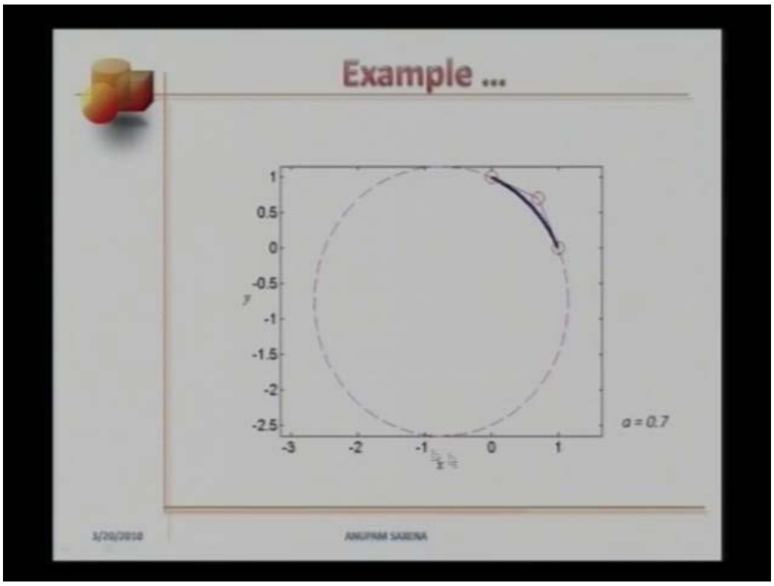

This is an example for a equals 0.7 the blue segment here is the quadratic Bezier curve, which is lying, precisely over this dashed match into circle.