Computer Aided Engineering Design
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Lecture - 14

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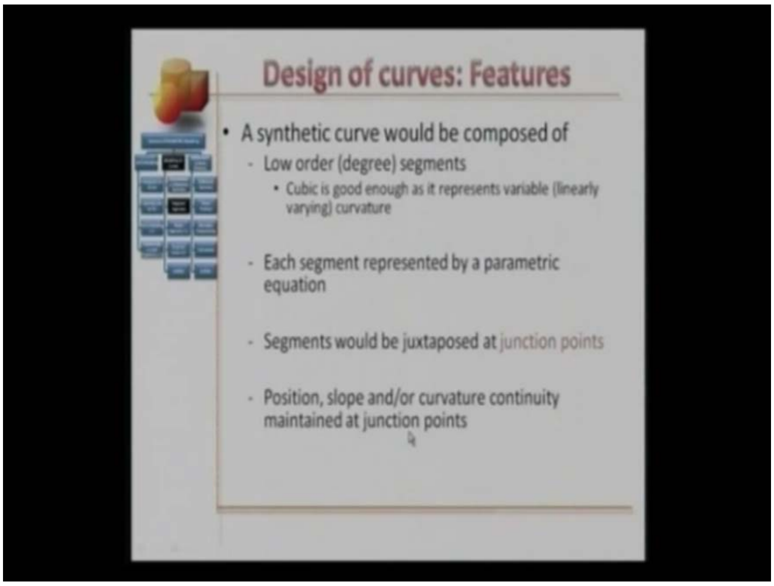

Hello, this is lecture number 14 on the Design of Cognitive Segments and Curves. Design of curves - features. A synthetic curve would be composed of lower order or lower degree segments, here a cubic segment will be good now as it would be represents variable or linearly varying curvature. That is a reason why we preferred low order or low degree segment. Because higher degree segments that have more oscillations or fluctuations which we do not want. A linear segment will not be good, because this surface constant pragmatic segment may not be adequate, because there curvature will be constant. Therefore, a cubic segment will be adequate now . We were discussed before that each segment should be represented by a parametric equation, and segments would be juxtaposed at junction points, they would be joined together at junction point, while think so you need to ensure that position slope and or curvature continuity is maintained at junction points.

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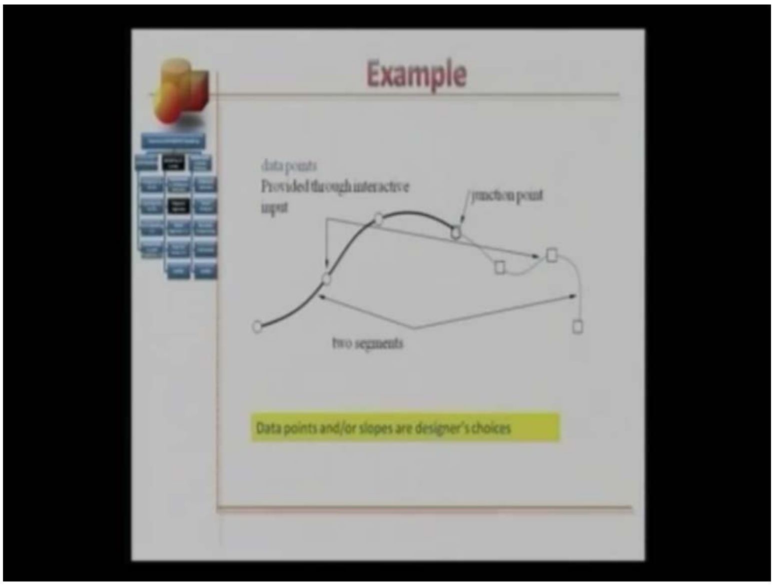

 Let us take look an example: as to what am I mean, let us we have a 7 digit points, the circles represents a different sub group of that points and rectangles represent, a different sub group. We are going to be considering these data points in different sub groups. When designing a curve segment for example: This circular data points and be interpolated using these cubic segments. you can as working these 4 data points, does not matter and this rectangular data points, will be interpolated or may be best way using a different cubic segment, the difference is shown in the colored these 2 are different segments.

These are the dare points farcified to interactive input by the user. A user may also want to call them the design points, that would rapidly defect the shape of each segment of a cure and therefore inter curve is point here, The disjunction point and common to the segment on the left, and the segment on the right, As I said before these are 2 segments that constitute this composite curve. Again data points and or slopes are essentially a designer’s choice. And it is here that need to worry about different continuity, conditions like the position continuity, slope continuity, the curvature continuity, And so on so for.

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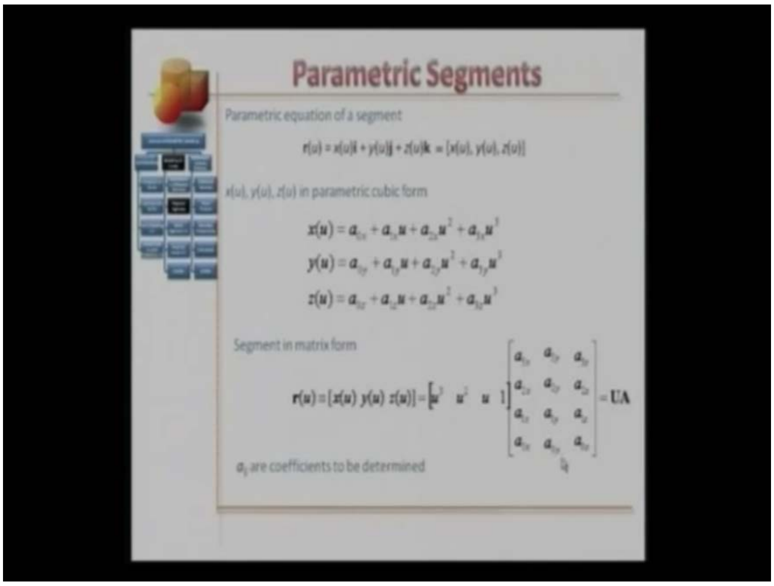

Let us talk a little bit about parametric segments a word about the difference between segments and curves, by segment I mean individual low degree segments and by curve mean entire composite curve. This is difference between segment and curves that I would try to make and throughout relations a throughout the discussion on the design of curves in general.

Parametric equation of a segment the position factor of any point in the curve as a function of parameter use that is r of u can be given by x of u times I thus y of u times chase plus z of u hence k x of u y of u and z of u are scale of function in compact they would be written as an ordered set x u y u and z u.

We might want to think about modeling x of u y of u and z of u each in parametric cubic form again cubic will be enough for us because cubic form would represent a linearly varying curvature more the more the oscillation x of u may be written as a 0 x plus a 1 x times u plus h o x and u square plus a three x and c cube a 0 x a 1 x a 2 x and a 3 x are unknown co-ordinations, that will determine later likewise y of u and z of u the 2 others function and be model in a similar manner.

Once again co-efficient a 0 y a 1 y a 2 y a 3 y and a 0 z a 1 z a 2 z and a 3 z are unknown co-efficient. In general co-efficient as I j which represents all these co-efficient12 in number need to be determine. I can write these equations in matrix form if I use this ordered rotation for r of u we have r of u equals x u y u c u and 1 by 3 vector rotation.

Which will be equal to u cube u square u and 1 this is the 4 by 4 matrix have been different degrees in u this is the co-efficient in short this matrix is represented by capital u and this co-efficient matrix is represented by capital a.

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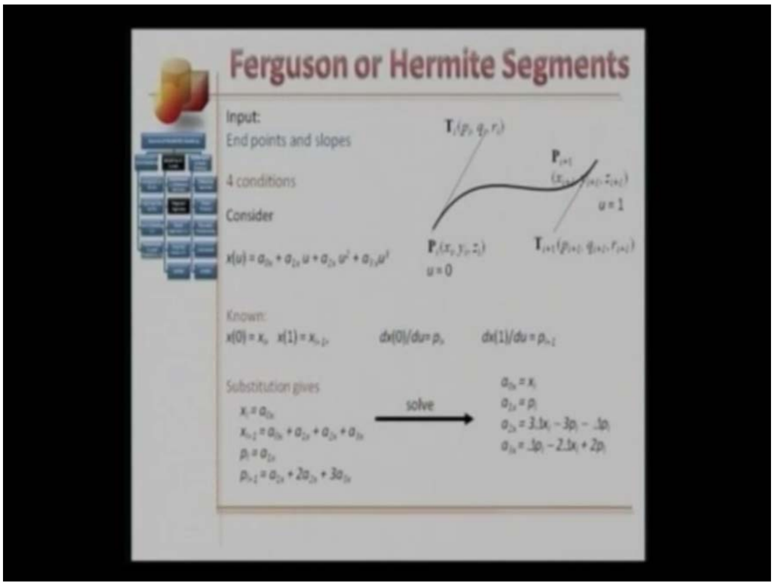

May I to Ferguson or hermit segments as they call let say a designer specifies 2 dead points p I co-ordinates x I y I and z I and p I plus 1 co-ordinates x I plus 1 y I plus 1 and z I plus 1 in addition he or she also specifies the slopes at this their points the slopes are denoted by by and by plus 1 the x y c comp1nts are p I q I r I and p I plus 1 q I plus 1 and r I plus 1, a typical Ferguson or hermit segments. Well look like that it through this rate of points and this slopes, will be to the curve at the end points at point p I the parameter value u is 0 and at point p I plus 1 u equals 1 all the point on this curve, will be a different values of u in between 0 and 1.

As I said before the input would be the end point and slope from the designer behave therefore, four conditions 2 points and 2 slopes for each of the x y and z. Now let us consider the scale of function x of u if u notice we have four unknowns here 4 conditions 4 unknown, this is solvable going further for u equals 0 x of 0 is x I x of 1 is x I plus 1 for u equals 1d x over d u evaluated at u equals 0 p I and d x over d u evaluated at u equals 1 is p I plus 1.These are the 4 conditions that can be used to determine these unknowns the rest is algebra on substitution, we get these equation x sub I equals a 0 xx sub I plus 1 equal a 0 x plus a 1 x plus a 2 x plus a 3 x p I equals a 1 x and p I plus 1 equals a 1 x plus 2 times a 2 x plus 3 times a 3 x, I can solve these 4 equations and get the 4 co-equations a 0 x equals x I a 1 x equals p I a 2 x equals 3 times delta x I minus 3 times p I minus delta p I and a 3 x equals delta p I minus 2 time delta x I plus 2 times p I. Delta x I is x I plus 1 minus x I and delta p I is equal to p I plus 1 minus p I.

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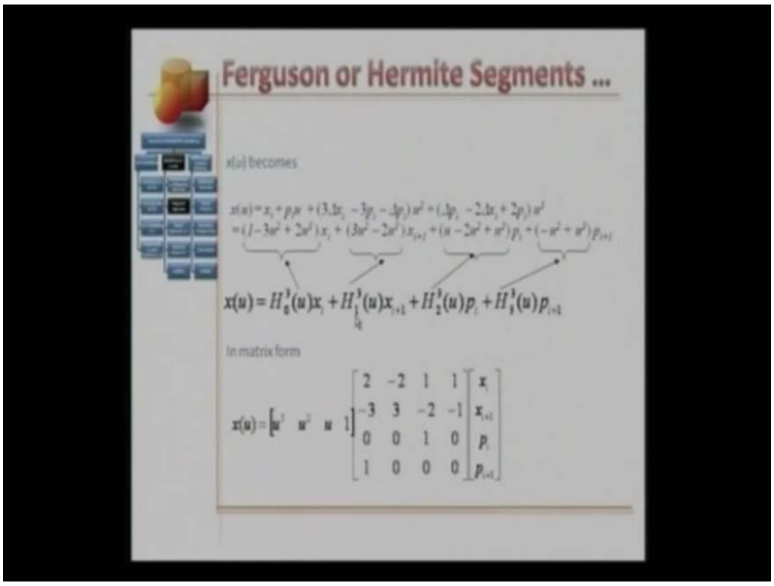

If I substitute these equations are the scale of function x of u will become x of u is equal to x I plus p I and u plus within parentheses 3 times delta x I minus 3 times minus delta p I and u square plus delta p I minus 2 times delta x I plus 2 times p I all with in parentheses and u cube. We can think of re-arranging the right hand side here so that we can prove all sum involving u and all sum involving x s and p s, the geometry that designer had specified once you regroup this the co efficient of x I is 1 minus 3 times u square plus 2 time u cube the co-efficient for x I plus 1 as 3 u square minus 3 cube that is u minus 2 u square plus u cube and that p I plus 1 its minus u square plus u cube. what can use we can represented co efficient as functions h h 0,3 u and x I plus h 1,3 of u and x I plus 1 plus h 2,3 of u and p I plus h 3,3 of u and p I plus 1 this subscripts over here they represent the index and the superscripts represents the degrees of each of the functions h 0,3 is this function here it is prebake h 1,3 the function again cubic h 2,3 is this function and h 3,3 is this function all the has happen to be cubic in u I can represent this taken in compact native form.

X of u is 1 by 4 matrixes involving u cube u square u and 1 this is 4 by 4 co-efficient matrix and if you notice these numbers come from here and this column vector represent the x co-ordinance of the 2 points and the x comp1nts of the 2 slopes and the 2 points respectively.

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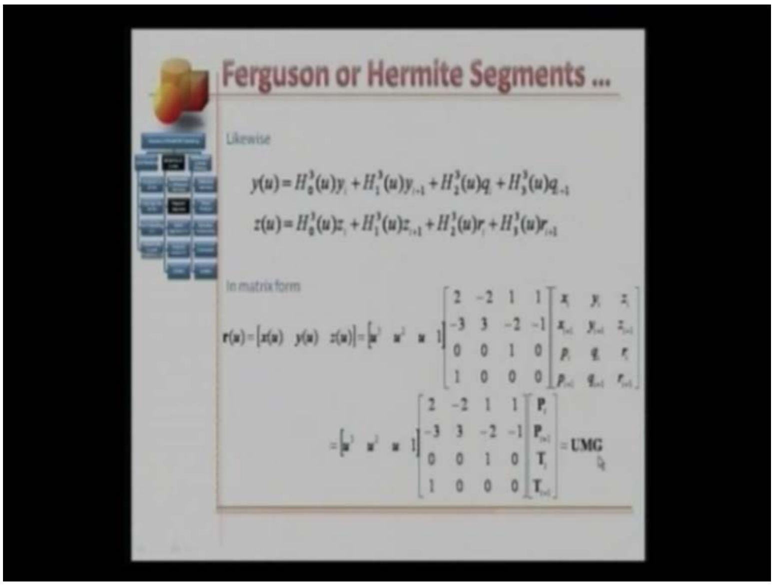

Likewise if you think about it the scale of function y u will have a very similar form, that is the hermit functions h 03 h 13 h 23 and h 33 will be identical all will the here geometry will involve the y co-ordinates and y comp1nts of those likewise for the z as well the scale of function z u will use the hermit functions and the geometry corresponding to the z direction the z co-ordinates at 2 points and the comp1nts of the z slopes in matrix form.

I can combine all the information’s for telling to the x y and its co ordinates in order form x u y u z u is r of u the position vector of a point and that will be equal to its 1 by four matrix u cube u square u and 1 the co efficient matrix will be identical in all of the cases the entries being 2 minus 2 11 minus 33 minus 2 minus 1,0,0,1,0 and 1,0,0,0and the geometry for telling to the x y and z co ordinates will be arranged column vise this is a Ferguson cubic segment for you in matrix or compact form.

If I look at this row here this row represents the position vector of p, I the second row is the position vector p I plus 1 the third row is the first slope t I and fourth row is the second slope t I plus 1, and more compact equation this matrix is represented by capital u and bold this 4 by 4 co-efficient matrix is represented by capital m and this column vector which is actually a 4 by 3 matrix represents the geometry by the design g.

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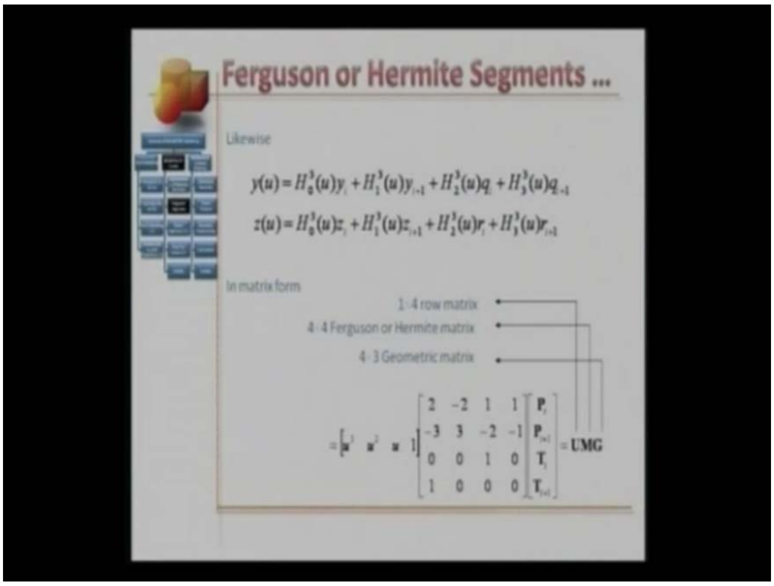

And I meant1d u is of the slides 1 by 4 m is of the slide 4 by 4 and it is called Ferguson or hermit co-efficient matrix and g is of sides 4 by 3 the geometric matrix.

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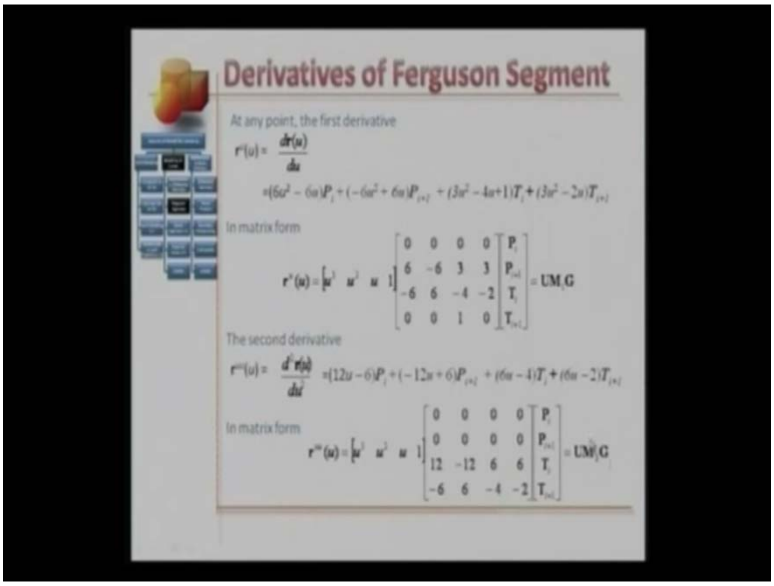

Once we know the scale of function x u y u and z u computing the derivatives of a Ferguson segment should be split forward at any point the first derivative given by r super u of u which is d r du which is 6 u squared minus 6 u and P i plus minus 6 u square plus 6 u P i plus 1 plus 3 u square minus 4 u plus 1 and T i plus 3 u square minus 2 u and T i plus 1. These are straight forward results that we get by different shading hermit function with respect to u in matrix form d r by d u and r super u is given by u cube u square u and 1 the co-efficient matrix will have the entries 0,0,0,0,6 minus 6,3,3 minus 6,6 minus four minus 2,0,0,1,0 and this is the geometry matrix that we have seen before.

Now it is that u it could radix in all these four function because of which the first row as call 0s if you compute the second derivative r super u u of u which is d 2 r over d u square matrix given by 12 u minus 6 times p I plus minus 12 u plus 6 and p I plus 1 plus within 6 u minus 4 times t I plus 6 u minus 2 times t I plus 1 notice this equation and different shade only these terms assuming that the resign information p I pi plus 1 t I.

And t I plus 1 they do not vary this u in matrix form the second derivative is given by u cube u square u and 1,1 by four matrix the co-efficient matrix will change and will have the entries 0,0,0,0 and second row again 0s the third row is 12 minus 12,6,6 and the fourth 1 is minus 6,6 minus 4 minus 2 this co-efficient matrix is represented by m sub 2 while this Poincare is represented by m sub 1 the geometry matrix g remains un part of it.

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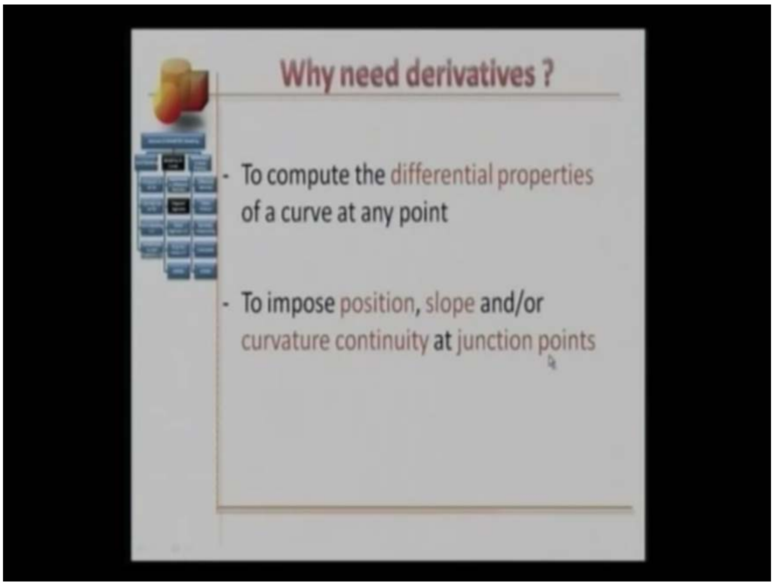

Why do we need derivatives well we want to compute the different properties of a curve at any point and in particular you like to impose position slope or curvature continuity at junction points, the points which are common between 2 for this segment.

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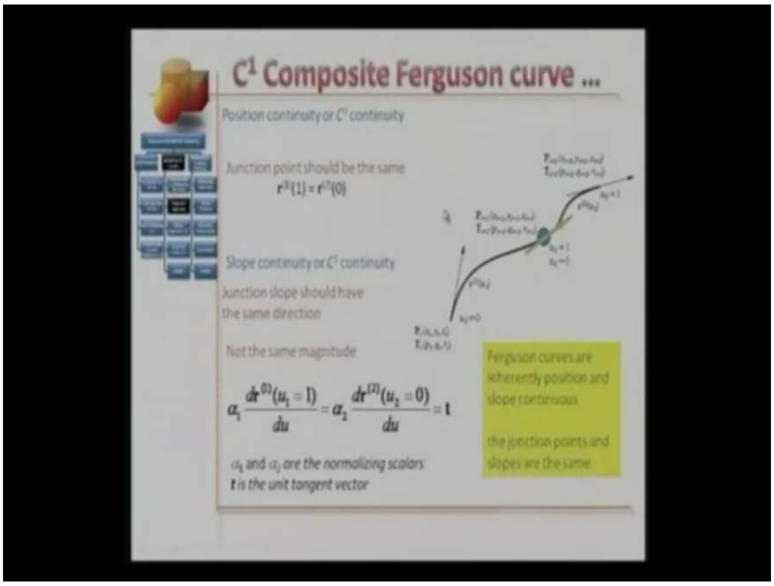

Composite Ferguson curve this would be a sequential that a position of individual Ferguson segments, there we have the first data points second data point the position vector is p I co-ordinates x I y I and z I hear the position vector p I plus 1 their co-ordinates x I plus 1 y I plus 1 z I plus 1 the slope are t I and t I plus 1 their comp1nts p I q I r I and p I plus 1 q I plus 1 and r I plus 1 these are the direction of t I and t I plus 1 respectively and this is how of Ferguson segment between the 2 points they look like, a designer chooses represent this segment y r 2 for 1 this number here denotes that this he first segment and this variable 1 represent the parameter only for this sec and know other sec u 1 equals 0 at point p I and u 1 equals 1 at point p I plus 1.

If a designer chooses this specify another point let say p I plus 2 its coordinates x I plus 2 y I plus 2 and z I plus 2 and also a slope at this point, t I plus 2 their comp1nt p I plus 2 q I plus 2 and r I plus 2. The second Ferguson segment for probably look like this. This represent the segment by r super 2 this number represents the segment this segment will have a different parameter u 2 u 2 and u 1 and are not the same for u 2 equals 0 the point on this segment p I plus 1 and for u 2 equals 1 the end point p I plus 2 now that u 1 and u 2 both are suppose to vary between value 0 and 1, in other word u 2 greater than 1 will have no varying on this portion of the composite curve and u 2 smaller than 0 will not affect the shade of this sec.

Now c 1 composite Ferguson curve or c 1 continuous composite, Ferguson let us keep this diagram here notice that this point is common between this 2 segments r 2 for 1 and r 2 for 2 for position continuity or c 0 continuity we need that the junction point between the 2 segments is the same in other words the end point forgive equals 1 for the first segment should be the same as start point for r 2 at give equal 0 this is this point right and for slope continuity or c 1 continuity.

The junction slope should have the same direction note that the 2 slopes here may or may not have the same magnitude. In other words f the first derivative apart 1 with respect to its parameter u evaluated at 1 represents the slope for this segment, here and the first derivative a part 2 evaluated and u 2 it was 0 represents the slope here for this segment then alpha 1 time d r over d u at 1 is equal to alpha 2 time d r over d u at 0 for this segment equal to a vector small t the unit tangible alpha 1 and alpha 2 are normalizing scalars and t is the unit tangent vector at the coming point of the junction it is not very difficult to absorb that Ferguson curves are in inventory position and slope continuity that intermediate points and slopes I used by 2 ambiguous Ferguson segment re emphasize this because the junction points and slopes will be the same between the 2 ambiguous or edges in sec.

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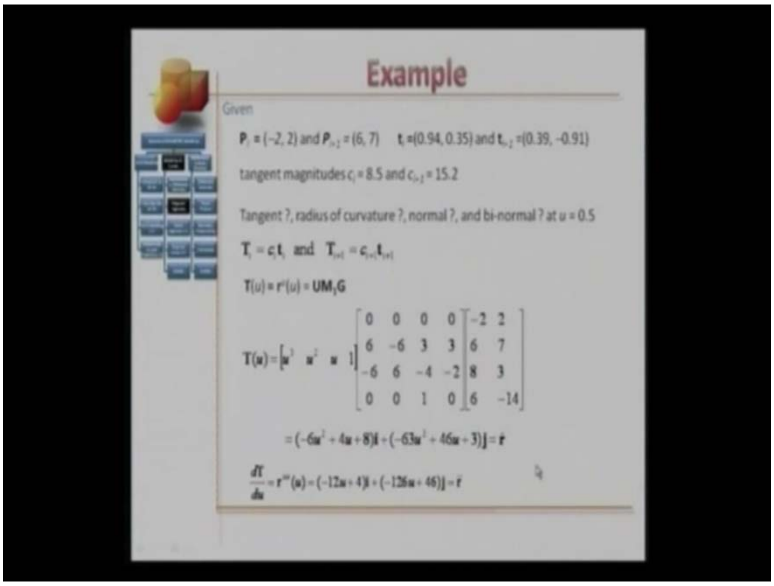

Let us consider an example: give tow data points let us see p I as minus 2,2 and p I plus 1 at 6,7 and the unit tangents by as 0 point 9,4,0 point 3,5 and t I plus 1 as 0 point 3,9 and minus 0 point 9,1.

These are the comp1nts t I and t I plus 1 about unit tangling at p I and p I plus 1 let us have the tangent magnitudes t I at p I was point was 8 point 5 and c I plus 1 as 15 point 2 at the second, recall that we are only interested in matching the tangent directions and not in the magnitude in other words these magnitude had be additional design twice for us questions. What would be the tangent the various of curvature in normal and bi normal at parameter value u equal 0 point 5 for the sec let us try to find out we have capital t I as small c I which is this value time the unit tangent t I here and capital t I plus 1 equals c I plus 1 which is fifteen point 2 times the unit tangent t I plus 1 in short form t of u is the first derivative of r it is affected u which is equal to the u matrix times the co-efficient matrix m 1 and g here other details u as u cube u squared u and 1 m 1 is this 4 by 4 co-efficient matrix and g is the geometry matrix

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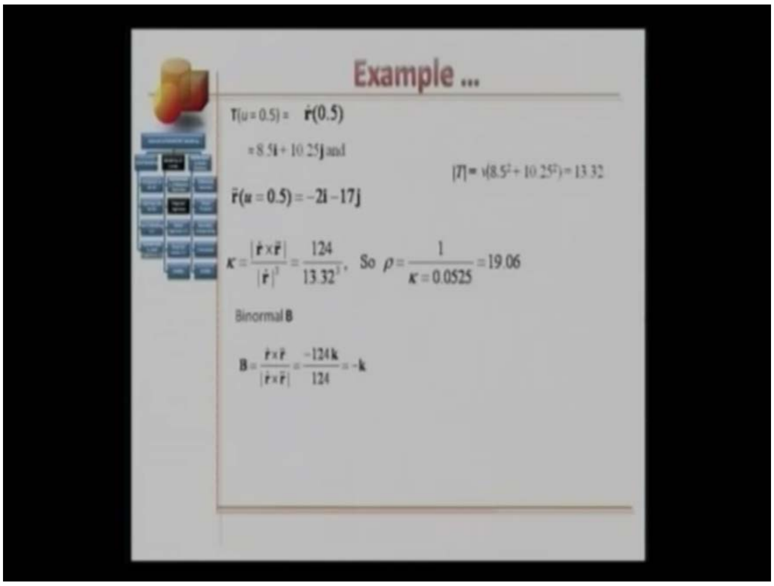

Now that minus 22 is point p I 6 and 7 is point p I plus 1, 8 and 3 would be t I time c I and 6 minus 14 will be c I plus 1 times t I plus 1 if you work hard the algebra which is not by difficult we will observe that t of u the tangent at any point on the segment. Will be of 6 u square plus 4 u plus 8 times unit factor I along the x direction plus 63 times u square plus 46 times u plus 3 times j and this is r doc if we different shade this expression again u will have r super u u it is d t over d u as minus of 12 u plus 4 times I plus minus 126 time u plus 46 times j and this is r double dot.

Evaluating t at u equals 0 point 5 will refer give us r dot 0 point 5. Which would be 8 point 5 I plus 10 point 25 times j a magnitude of t can be determine as 13 point 3,2 this is not a the unit tangent therefore, will be this factor over this magnitude likewise we can get r double dot at u equals 0 point 5 as minus 2 times I minus 17 times j.

Now for the curvature at a point from co-ordinate geometry you would know that caper given by mode of r dot cross with r double dot over mode of r dot whole cube we know what r dot is we also know what r double dot is computing the cost product is not very difficult the magnitudes come out to be 124 we know what r dot is copper is 124 over 13 point 3,2 the whole cube which would mean that the radios of curvature is 1 over copper which is nineteen point 0,6 at u equals 0 point 5, now for the bi normal b the bi normal b given by r dot cross r double dot over the models of the absolute value of the numeric recall form the previous lecture that we had expressed the in terms of the derivative of r with respect to s of the rational.

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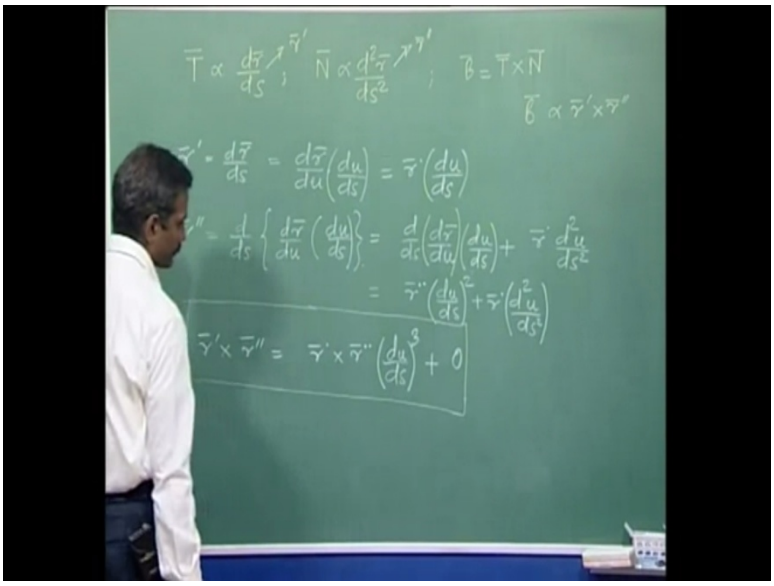

Let me explain this to on the board recall that unit tangent t had direction of d r over d s which is r prime and n is defined of d 2 r over d s square which is r double prime notice also that t and n perpendicular and a vector by normal is perpendicular to both t and n because of which b is represented as the cross product between the unit tangent and the unit term. Now let us absorb a work out how to convert information that we with respect to the a clean parameter s in terms of parameter u in other words lets work to get r primes in terms of r dots, in other words this work to get derivatives respective s convert to derivatives respect to u, now r prime is d r over d s notice that I am using the vector rotation here this is equal to d r over d u times d u over d s this part is a vector this part is a scalar I notation this is r dot times d u over d s if I differentiate this the second time the respective s I have r double prime equals d over d s of this expressions here which is d r over d u times d u over d s.

If I work hard expressions this would be equal to d over d s of d r over d u times d u over d s plus r dot times d 2 u over d s square this differential operator will first operate on this I can incorporate a use chain rule again here, I can multiply and divided by d u here so this term will become r double dot times d u over d s the whole square plus r dot time d 2 u over d s square. Now we have a prime in terms of r dot this being a scalar and also we have r double prime in term of r double dot and r dot, if you look at these 3 expressions here the bi number b will have the direction given by r prime crossed r double prime I am using here the proportional define once gets a direction. I can always divide this expression by the magnitude to get the unit vector or the unit bi num now let us see what r prime cross web r double prime is in terms of r dot and r double dot.

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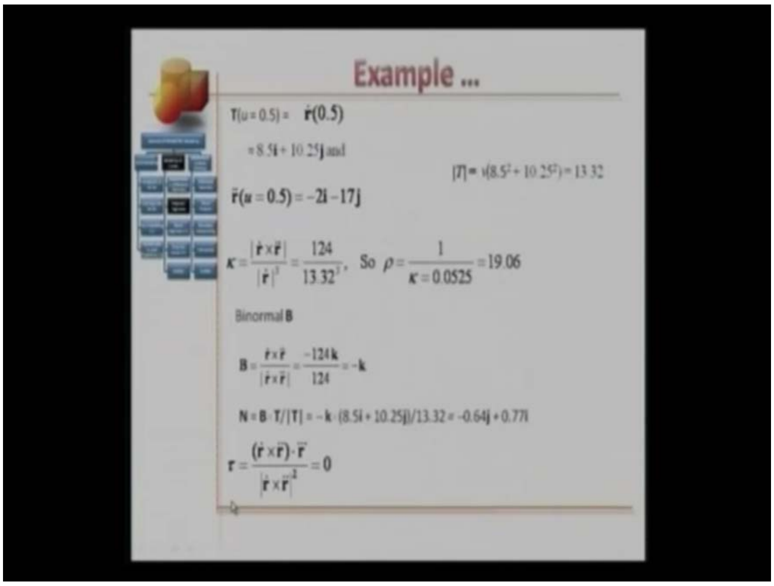

R prime crossed with r double prime is this term here crossed with this term here which will give us r dot crossed with r double dot times d u over d s the whole cube which is a scalar plus 0 notice that r dot crossed with r dot will be 0 this expression here convey to us that the direction given by r prime cross web r double prime is the same as the direction given by r dot crossed with r double dot.

Coming back the bi normal b is given by r dot cross web with r double dot over the magnitude if we work hard the algebra this is can be minus k. Which is quite straight forward because the segment that we are considering lies 1 the x y z the normal n given by b crossed web t here notice that the magnitude of this is not the unity we will have to divide t magnitude to get the unit tangent here if we work hard the algebra this is or n is given by minus of 0 point 64 times j plus 0 point 77 times I.

The tertian is given by the relation r dot crossed with r double dot this entire vector doted will are tribal dot over the absolute value of r dot crossed with r double dot the whole square it could not be difficult for us to determine that the portion of the curve is 0 physically 1 might thinker thus representing the out of plane war a local war of a curve you might want to work hard the derivation of this expression as an assignment.

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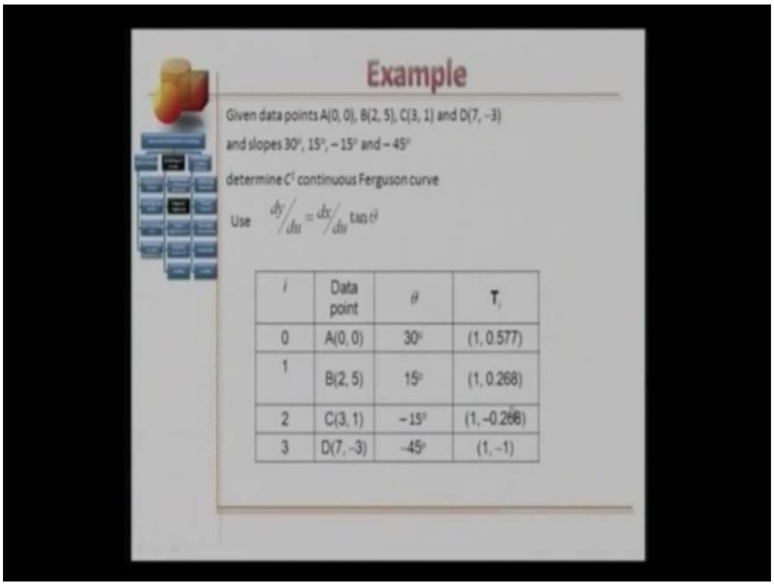

Let us consider an example: Of c 1 composite Ferguson curve they are given four data points a coordinates 00 b which 2,5 c with 3,1 and d web 7 and minus 3 and the designer of those specifies the respective 4 slopes and these design points 30,15 minus 15 and minus 45 degrees we need to determine this c 1 continuing Ferguson curve we know that d y over d x is tangent of data slope.

We can rewrite the slope equation in this form d y over d u equals d x over d u times tangent of mirror and we can assume d x over d u as 1 and then compute d y over du this table summarizes the entire geometric information provided by the designer, the coordinates of 4 data points the individual slopes notice that d x over d u the x comp1nts of slopes at all the data points are 1 the y comp1nts of these slopes are computed.

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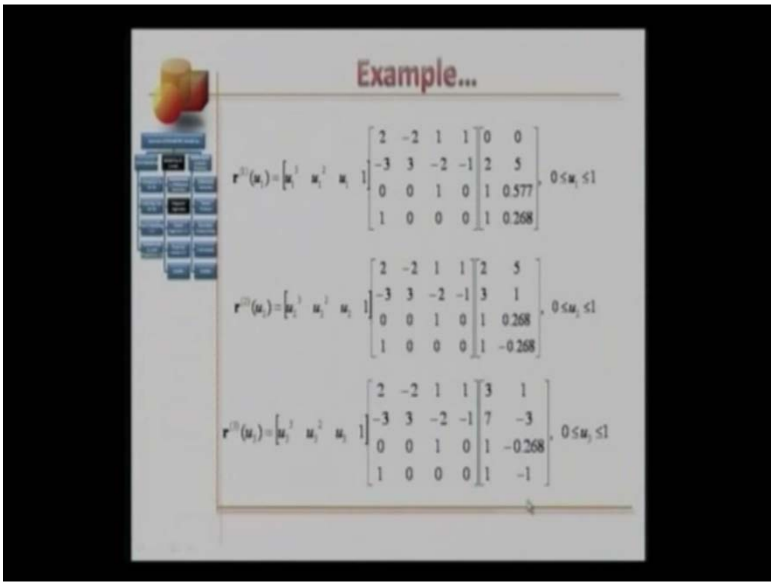

Have seen before in this lecture we can work hard the expression for the first segment, the second segment, and the third segment, this matrix here would be with respect to different parameters u 1 u 2 u and u 3 the 4 by 4 Ferguson matrix will be identical for the three cases and the geometric information will deferred this geometric matrix corresponds to the first and the second data point this point here corresponds to the second and the third and the here last 1 corresponds to the third and the fourth data point the slopes are menti1d respective.

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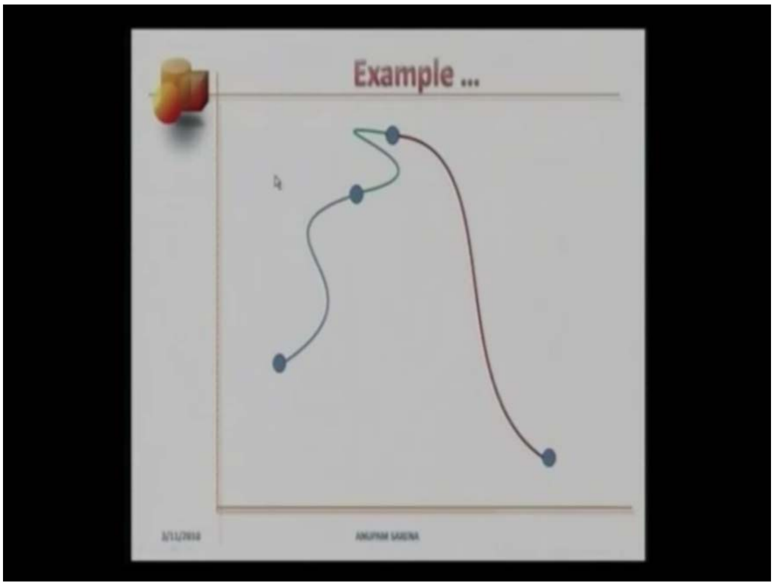

Let us absorb how the composite Ferguson curve left is the first data point is the second data point the third data point and the fourth data point this is the first Ferguson cubic segment the second Ferguson cubic segment and the third segment. There we decide relocate any 1 of these data points how do we expect to see the changes in different, Ferguson segment you were absorb that the third segment will change like so and the second segment will change like this the first segment will not change in .

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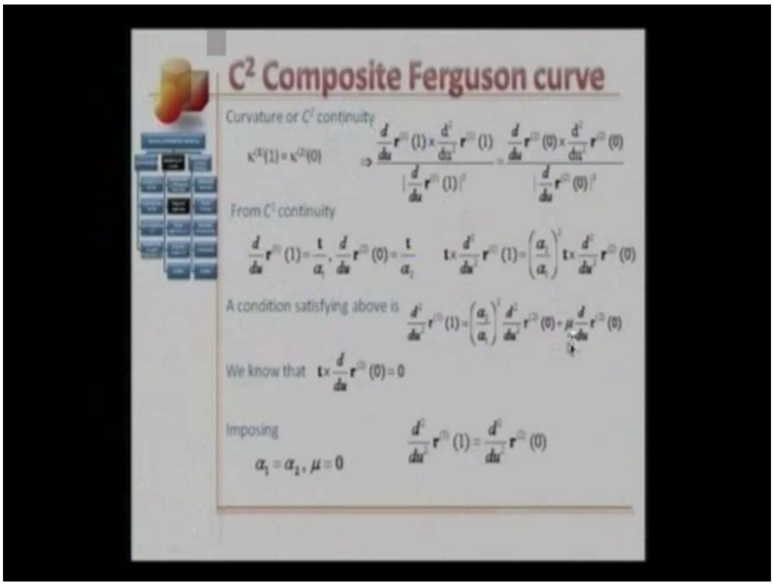

Once again this point moves the third segment changes in shade, and the second segment changes, in shade as well in general c 1 continuous Ferguson curve well exhibit local shake control. For example; if I now decide to relocate this point here only this segment and this segment will change while this segment will remain impact j.

Now for c 2 composite Ferguson curve here we consider curvature of. C 2 continuity as we call it for that at the common junction point the curvature information from this segment on the left, and the segment on the right, should be the same that is the curvatures from the left segment and the right segment to be identical.

We know that curvature given by r dot crossed with r double dot over r dot the whole cube for the first segment the curvature is d r over d u crossed with d 2 r over d u square both evaluated at the parameter value 1 over mod of d r over d u the whole cube and identical expression is to be used on the right hand side only difference is that the segment is the second segment here and the information is evaluated for parameter value is 0. We know form c 1 continuity that d r over d u for the first segment evaluated at 1 is equals the unit tangent over the corresponding normalizing scalar.

And r over d u on the second segment at u equal 0 is equal to t over alpha t the unit tangent over the corresponding normalizing scalar notice that while I am using u here I primarily mean that this parameter is different from this parameter and this is implicit throughout this lecture if you plug in this condition into this condition we will get t cross with d 2 r 1 over d u squared evaluated at 1 equals alpha 2 over alpha 1 the whole square times t crossed with d 2 r 2 over d u square evaluated at u equal 0 to reemphasize it should be set in mind by this that 2 parameters for 2 segments at different I have used u here common to above never the less above the condition that would satisfy this expression here is given by d 2 over d u square for the first segment evaluated 1 equals alpha 2 over alpha 1 whole square d 2 over d u square of the second segment at 0 plus some scalar mu times the first derivative for the second segment evaluated at 0.

Above this condition as this expression here, Let see you would know that t cross with the first derivative of the second segment at u equal 0 will be 0 this is because the direction of the unit tangent will be the same for the first segment and the second segment will be common junction point.

You can satisfy yourself plugging expression for this term here this term here will give you this term and no matter what the value of mu is t crossed with the corresponding slope will be 0 at the further imposed the normalizing scalars to be equal and mu to be 0 and the curvature continually condition simplifies to the continuity of the second derivative that is d 2 over d u square of the first segment at u equals 1 equal d 2 over d u square of the second segment at u equal 0. This is comparatively straighter unknown 0 value of mu will gave a designer and additional design through it.

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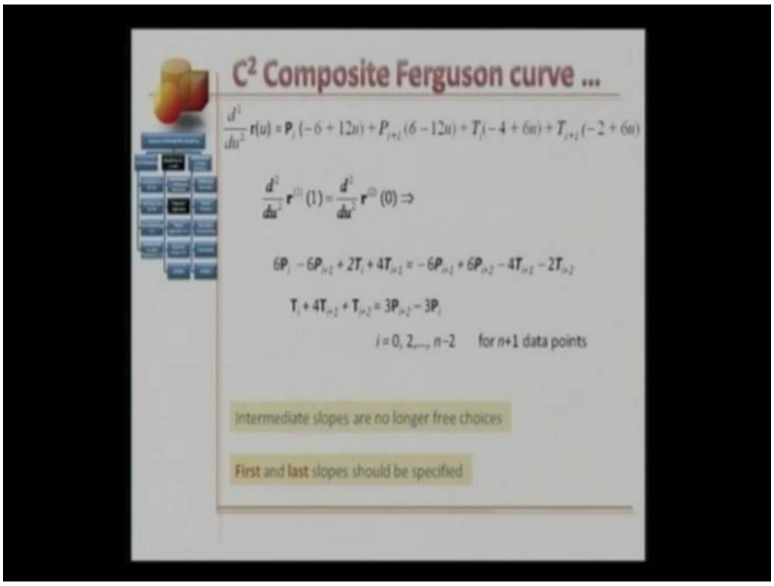

Recalling from the previous slide that the second derivatives of the 2 contiguous Ferguson segment are equal we have d 2 over d u square of r of u in general equals p I times minus 6 plus 12 u plus p I plus 1 time 6 minus 12 u plus t I times minus 4 plus 6 u plus t I plus 1 times minus 2 plus 6 u noting that the second derivative of the first segment at u equals 1 equals this second derivative of the second segment r u equal 0.

We get 6 of e I minus 6 of e I plus 11 plus 2 t I plus 4 times t I plus 1 equals minus of 6 times p I plus 1 plus 6 times p I plus 2 minus 4 t I plus 1 minus 2 t I plus 2 how did we get this equation the first segment as data points p I p I plus 1 and slopes t I and t I plus 1 for the first segment u should be 1 circular at the left hand side of the equation we have 6 times of p I minus 6 times p I plus 1 plus 2 times t I plus 4 times t I plus 1 for the second segment the data points will be p I plus 1 p I plus 2 and the slopes will be t I plus 1 and t

I plus 2 in another words p index here needs to be incremented by also 1 these co-efficiencies need to be evaluated for u equals 0.

Once d1 your right hand side 1 can re arrange these terms here to get t I plus 4 times t I plus 1 plus t I plus 2 equals 3 time p I plus 2 minus 3 times p I now this is an equation corresponding to all there is a I going from 0,1 until n minus 2 that is for n plus 1 data points this equation here or in fact s is n minus equations we late n plus 1 data points and n plus 1 slopes these n minus 1 equations ensures that intermediate slopes are no longer free choices what they need to be compute if this continuity in second derivative is to be maintained at junction point.

Since, we have n plus 1 data points and n plus 1 slopes and since we have n minus 1 equations here we have 2 free choices other slopes we can use the first and the last slopes as free choices and we can specified and all the other intermediate n minus 1 slopes will then we computed using this derivative equations, if you also closely notice this would be a linear set a linear system and this would be a try diagonal system.

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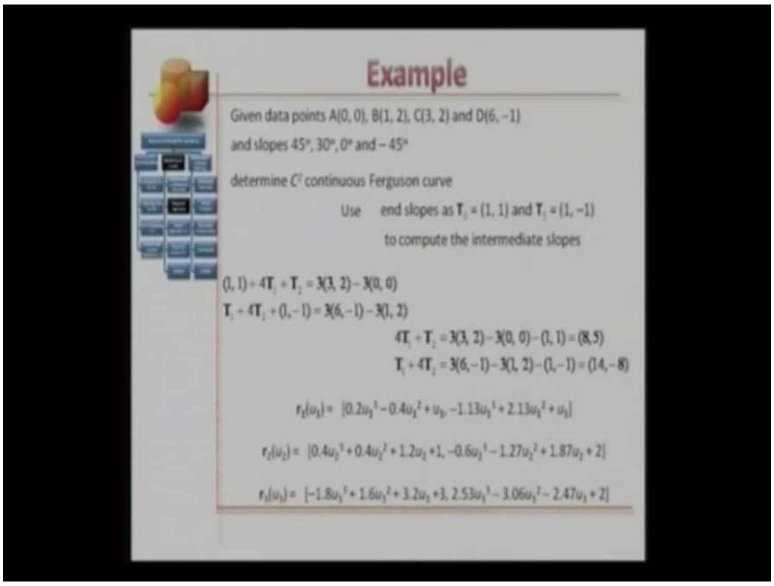

Which would be very efficient the example we have data points a the co-ordinate 00 b with 1 2 with 32 and d with 6 and minus 1 and let they have a slopes respective 45 degrees 30 degrees 0 degrees and minus 45 degrees

We need to determine the c 2 continuous Ferguson curve let use the free choices the first and the last slope as 1,1 and 1 minus 1 with that we compute the intermediate slope we have t 0 that is 1,1 plus 4 times t 1 plus t 2 equals 3 times 3,2 which is 3,2 minus 3 time p 0 that is 0,0 and t 1 plus 4 times t 2 plus t 3 which is 1 minus 1 here equals 3 times p 3 which is 6 minus 1 minus 3 times p 1 that is 1,2 note that I have written these equations in both x and y comp1nt working out the algebra it is not very difficult come here on.

And we should be able to compute this intermediate slope t 1 and t 2 once we do that the first segment r y is 0 point 2 times u 1 cube minus 0 point 4 and u 1 square plus u 1 x comp1nt the y comp1nt will be minus 1 point 1,3 u 1 cube plus 2 point 1 3 u 1 squared plus u 1 likewise the. Second segment is 0 point 4 times u 2 cube plus 0 point 4 u 2 square plus 1 point 2 and u 2 plus 1 x componenent.

And the y comp1nt is minus 0 point 6 u 2 cube minus 1 point 2,7 u 2 square plus 1 point 8,7 u 2 plus 2 the third segment can be we likewise .

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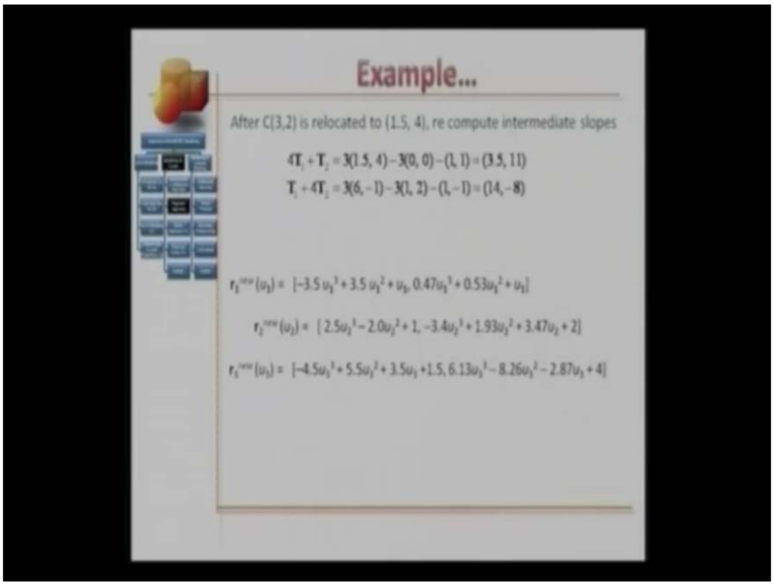

Let us look at the graphic here but, before that if we decide to relocate an intermediate point which 2 to different position 1 point 54 notice that we would need re compute all the intermediate slopes. Once again the algebra very similar this is the first segment of the recomputed composite progress in curve the second segment and the third segment and skipping the algebra, assuming that you would be able to work it out nice

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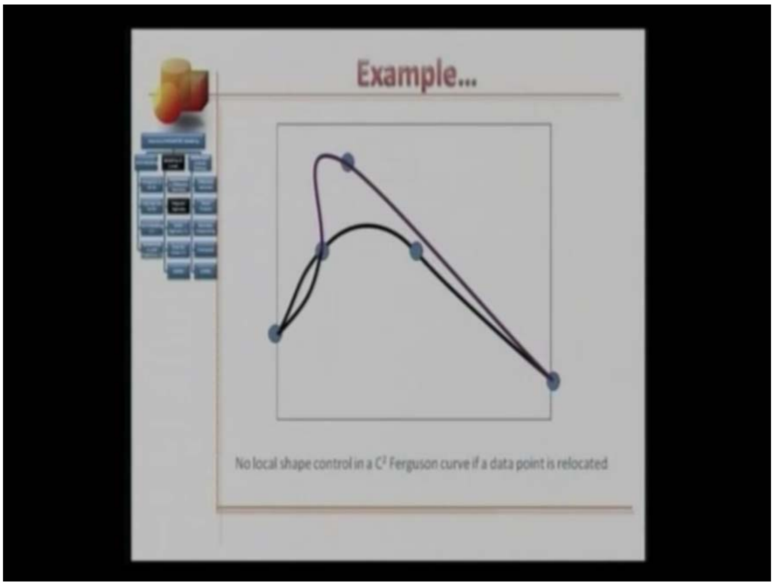

Other graphs the first 2 points and the first cubic segment the third data point second cubic segment the fourth data point the third cubic segment this is the original these 2 continents Ferguson, and I emphasis here that I will these junction points the second will be continuity if I decide to move this point when new location the first segment of the new composite curve will be this this would be the second segment and this would be the third segment this temporally location course thus re compute the intermediate slope so over here because of which shape of the impair composite curve changed in other words these two compensatory Ferguson curves do not have local shape control up.