Computer Aided Engineering Design
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Lecture - 11

Hi and welcome to lecture 11 on Computer Aided Engineering Design. This lecture is about Viewing of Solids. Why to be need viewing?

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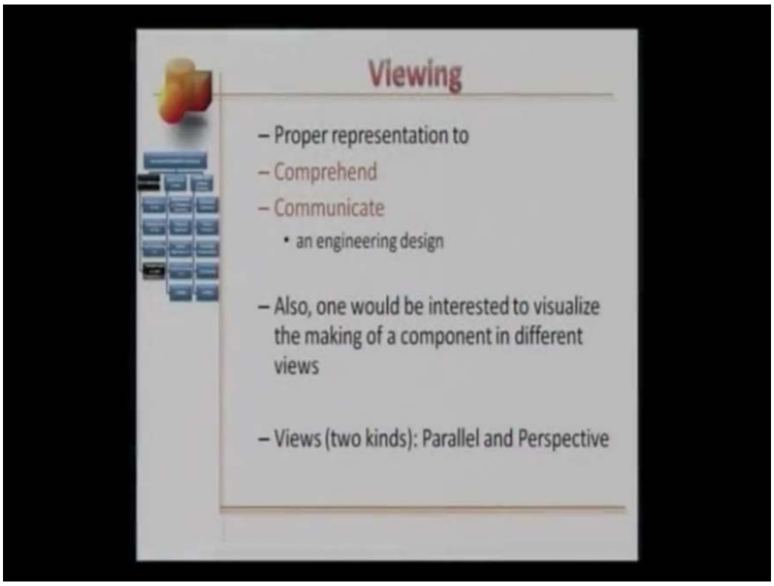

We are looking for a proper representation to comprehend solid, and to communicate and an engineering design. Also we would be interested in visualizing the making of a component in different views. Let me explain, what I meant. Imagine I am trying to design of very complex component represented by this pluck. What you see would be this portion, in the works space. You would not have any idea about this vertex which is hidden; say you would want to change the features around this vertex here, which is back side is block. What is you would want to do?

Is you would want to rotate or reposition or re orient is component appropriately; so that, this vertex becomes visibility. Once this portion is visible, we can perform any desirable feature modification. To be able to do that you are viewing this component in different ways; again in the process you are doing nothing but reoriented. In the work space however, you will be seeing only the two dimensional picture of this component. We talk about different earthquake so viewing in this lecture. Views are two kinds; parallel and perspective. Let us take the perspective usefulness.

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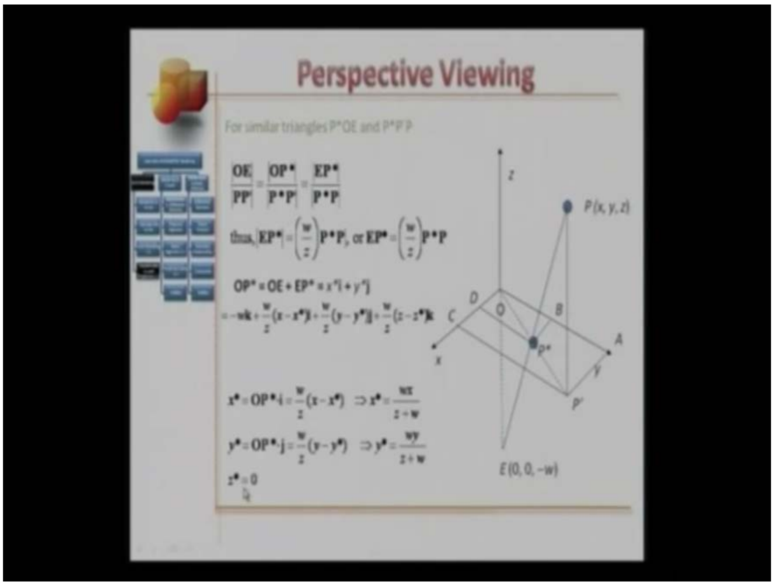

Let us start to three-dimensional cartilages space. The x is marked here and so is the origin. Considered any point P is coordinates x y and z. Let us drop a perpendicular or projection line on the x pipe line, the projector in this pipe line at point p start. On this point, let us draw rejections parallel to the x and y axis, let us also normal plate to the points of intersection as A and C.

Let us go down along the negative C direction and plates are I point are ourselves at distance w from 0; so E is are I position with coordinates 0, 0 and minus w. Next we join the points E P. The essential idea is to view to prospective of point P on the x pipeline has been by the I E. In other words, we would like to know, how this point P star is positioned on the x pipeline. P star is the perspective merge of point P. Let us continue with construction; we draw to projectors along a parallel to the x and y axis. And the normal plate to the points, the intersecting points P and D. Next we join the points O, P star and P prime. It is the construction process that could make these three points collinear; using this construction defined the triangles P star, O, E and P star, P prime, P are similar triangles.

Including that, we see that the length ratios are equals. In other words O E which is this vertical height over PP prime, which is this distance; the z projection of point P is equal to OP star which is this distance over P star P prime and that is this distance here. This also equal to the modulus of E P star of the distance E P star, this distance here over P star P; which is that is this distance. Note, that distance O E is W and distance P P prime is z. So, using this component of equation and this component of equation, we rewrite E P star as w over z, P star P; w over z, P star P. The basic equation relates only the distances are module I of E P star and P star P. But note from the figure that E P star is collinear with P star P; they have the same direction. With that said, I can relax the module of science here.

And I can rewrite equation as E P star the vector equals w over z, which is scalar time P star P, another wet; in other words the vector E P star is a scalar multiple of vector P star P. Like I said before we are interested in the co ordinance of P star the perspective merge of P. Using vector algebra, we can say that O P star equals O E plus E P star; this is using the triangle law. O P star equals O E plus E P star. O E would be minus w k, and we see that later. O P star equals x star i, x star i; i is a unique vector along the X direction plus y stat j, y star j; j is the unique vector along the Y direction. We have E P star here, that we can substitute in this expression. And we can see that O P star is minus w k, which is O E plus w over z x minus x star times i plus w over z, y minus y star times j plus w over z, z minus z star times k.

These last three expressions come from E P star. The x co ordinate x star will be corresponding to the i x components in this equation. In other word x star using vector algebra is O P star dotted with the unique vector along the x direction I. Once we do that x star equals w over z, x minus x star. I can use this equation rearrange it and get minus x star as w times x over z plus w. Likewise y star equals O P star started with j, in other words y star is j component in this equation, which is w over z, times y, minus y star. Once again I can rearrange this equation we get y star as w times y over z w and we see that z star is zero.

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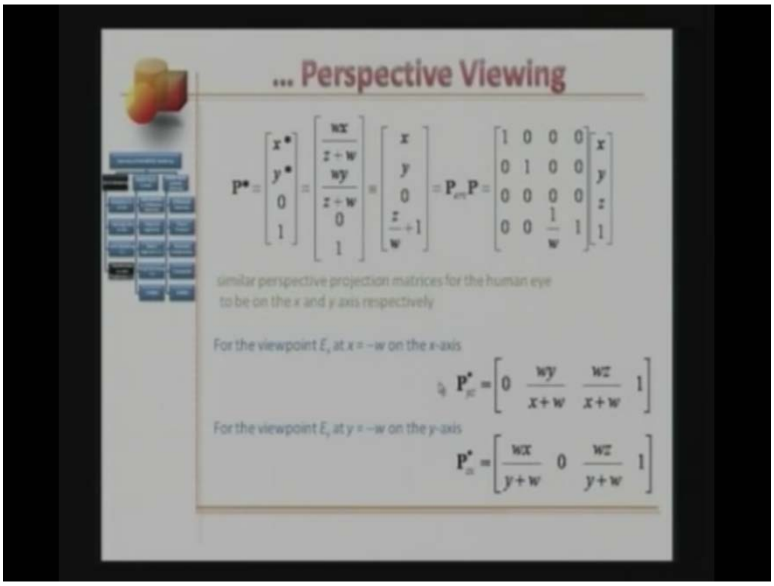

Continuing with perspective viewing, I can rearrange the expressions for x star and y star along the column vector in homogeneous coordinate's representation. x star, y star, 0 and 1. x star is w x over z plus w, y star is w times y over z plus w, 0 and 1. Again in the previous lecture we saw equivalence in homogeneous coordinates; to repeat if I multiply this column by any scale, it could not change the Cartesian coordinates of the point. So if I multiply this column vector by z plus w of w the factor, z plus w over w is in words of this factor here upon multiplication; I get x star here y star in this second row, 0 in third row and in the fourth row this will be z plus w entire thing over w or z over w plus 1.

I can represent this column vector in this from. I can extract the four by four matrix from the column vector and separate the expression over here, in terms of this four by four matrix here and the original column vector x, y, z and 1. Notice how one over w appears in the fourth by third entry fourth row and the third column. And the rigid body transformations we have seen that these three entries will all have 0 values. If we recall this is because of the few vectors that we have used derived the generic form of rigid body transformations; in perspective viewing however three entries can be non 0. P e r s is the short form of this four by four perspective matrix. Science the I position is placed along the negative z direction; this non zero entry here correspondence to that. W is a distance from the origin.

Similar perspective projection matrices for the human eye to be on the x and y axis respectively can be obtain; for the view point E x along the negative x axis at distance w from the origin. We have the perspective image of point P written in this form; 0, w y over x plus w, w z over x plus w and 1. Up course there will be a transport here because this flow P homogeneous column vector. Notice from the perspective plane will be the y is a plane; once again I can break this column vector into multiplication operation it mean, four by four matrixes and the original column.

For the view point to be along the negative y axis at distance w on the origin, we have P star the perspective image of point P on this z x plane as; w x over y plus w, 0, w z over y plus w and 1. Once again there should be a transport here I have written this thing in the row form and I can decompose this column vector into a matrix multiplication operation very similar to this. Will have four by four matrix P multiplying the original column vector; in case my view point or my eye is along the negative x direction if perspective image will be on the y z plane, and incase my eye point is along the negative y direction, my perspective plane will be the z x plane.

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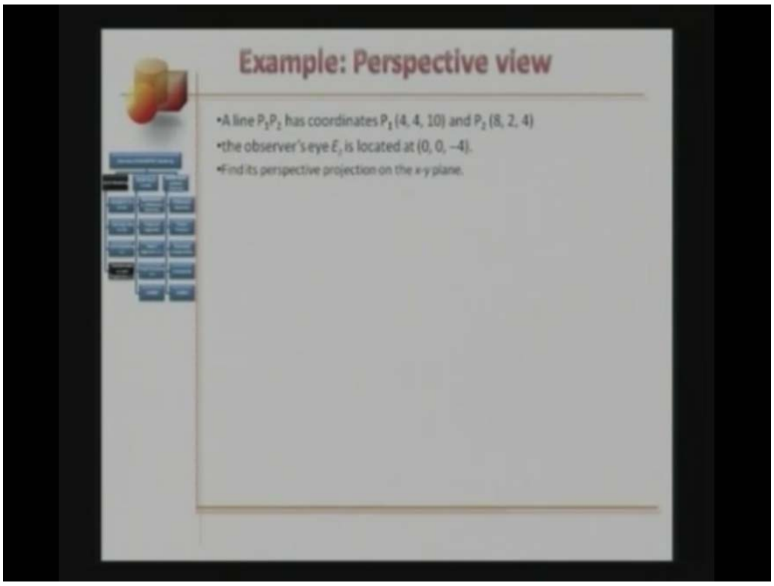

Let us take a look an example on perspective view. A line P 1, P 2 has coordinates P 1 as 4, 4, 10; and p 2 has 8, 2, 4. The observer's eye is located along the negative z direction at 0, 0, minus 4. We are interested to find the perspective projection of the line E 1, E 2 on the x y plane. And a sense, we are interested in finding perspective images of all points on the line E 1, E 2 the image on the x y plane.

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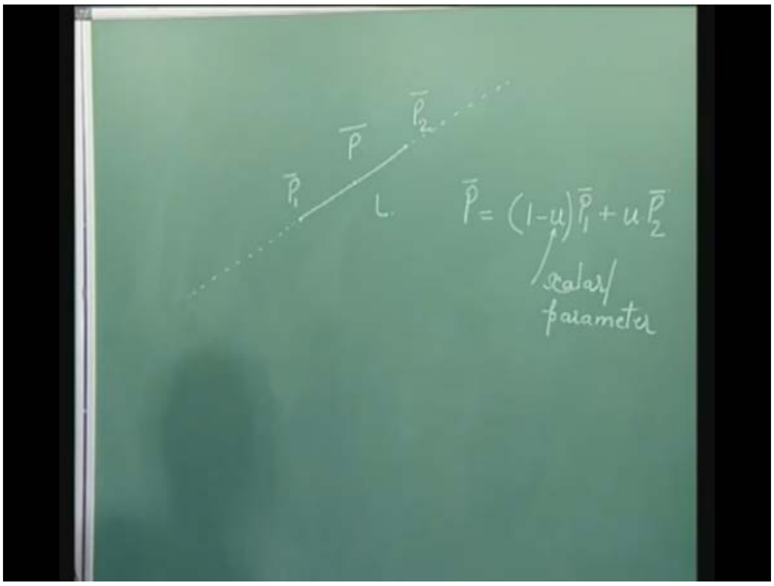

Let us say we have some position vector P 1 and another point P 2. These are position vectors and we have a lined joining these points and is L. Any point P line on this line can be expressed in the parametric form. That is P (( )) 1 minus u this is a scalar here, times P 1 vector plus u times P 2 vector, u as a mention before is a scalar or parameter. Note that mean u equal to 0 P equals P 1, so corresponding to u as 0, we get this point and corresponding to u as 1, this scalar 0, this scalar 1. So for u equal 1, we get this point P 2, P as P 2. For values of u smaller than 0, P would be lined somewhere here and profile use of u greater than 1. P will be lined somewhere here. In a sense this relations describes point P as linear combination of position vectors P 1 and P. We are going to be witnessing this relation, but few times later on this course.

As I explain right now on the board, any point P on a line defined by position vectors P 1 and P 2 can be expressed as a linear combination of two scalars. In a sense, P equal 1 minus u times P 1 plus u and P 2; for P 2 lie in between points P 1 and P 2. The value of parameter should be in between 0 and 1 including both 0 and 1. The five substitute different components of P 1 of here 4, 4 and 10; and different components of P 2 from here 8, 2, 4.

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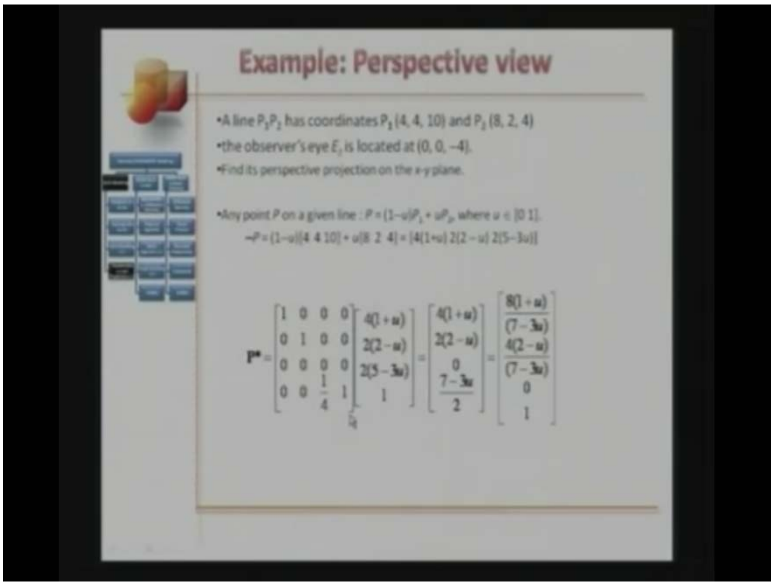

And then right key component from after performing multiplication and after also performing additional. I get the x component of P as fourth times 1 plus u, the view the y component as view two times 2 minus u, and this z component as two times 5 minus three u. With the components of the known we are ready determine with perspective image of point P which is not difficult task? We have seen the perspective matrix before, 4 by 4 perspective matrix; we also known that eye is positioned along the negative z direction at distance 4.

So, this scalar here perspective scalar will be 1 over 4, all the entries here will be 1 and this 4 by 4 perspective matrix will multiplied different component of P. Express as homogeneous for effective, after performing the multiplication you see that P star has P components fourth time 1 plus u in the x, 2 times u minus u in the y 0 in the z and 7 minus 3 u over 2, in place of 1. Using equivalence in homogenous co ordinance, I can scale this entire vector by 2 over 7 minus 3 u. Once I view that the x coordinates is 8 times 1 plus u over 7 minus 3 u, the y coordinates is 4 times u minus u over 7 minus 3 u, z co ordinates remains zero and this value becomes 1. I use this column vector and block all the points for values of u between 0 and 1.

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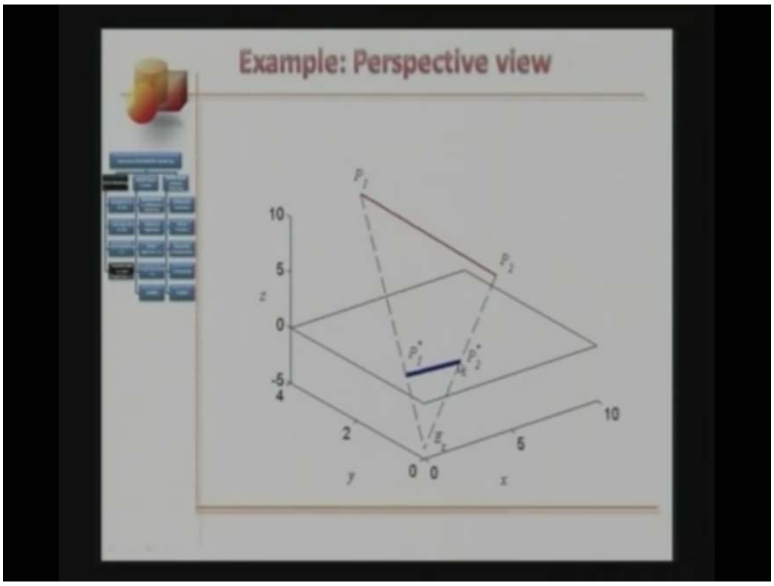

We have this figure; in the line is red, shows the original line joining P 1 and P 2. And the line blue shows the perspective image of this line in red on the x pipeline, is z is the position of the eye along the negative z direction. Notice that projections are not parallel; there converging to the eye from points P 1 and P 2. Wherever these projections inter sect the x pipe line, you would find corresponding image P 1 here and the P 2 here. This is how the perspective image might be thought of in constructed geometric parallel projections.

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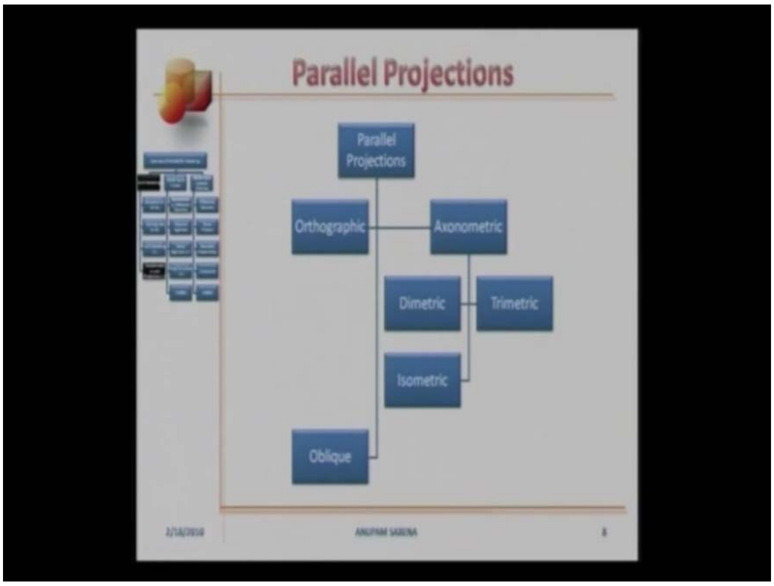

Parallel projections are three kinds: Orthographic, Axonometric and Oblique. In your first year drawing classes you must have learnt about Orthographic projections and particular case axonometric projections the Isometric projections. Well there are two other kinds of axonometric projections they are, diametric projections and Trimetric projections.

Let me explain to you, how to get orthographic projections in the first time. Consider that this is the object and we would want to capture different views of this object on three different places; one, two and three; they are all perpendicular feature. Now I will place the object in between of you and this plane. So you looking of the object from the front, imagine that parallel projections are emanating from you here hitting the object and imagine that the object is transparent. And that is allowing all the projections you pass through itself and hit on this plane. So for you this plane will capture the front view of this side.

Let me write this term. F represents the front view, next consider dark you are looking at the top of the object, you are looking at the top view. Once again imagine this object is transparent, the parallel projections are emanating from you. There are hitting this object, the object is allowing those projections through itself and projections are hitting on this plane. In other words this plane will capture the tpo view of this object.

Let me write this time again. Will have the top view obviously, finally the side view; let me rotating entire setup, now considered that you looking at this object from this side, from its left hand side. Again parallel projections are endangering from you, that hitting this object. And these projections are being allowed by the object. Projections call on this plane. So this plane will impact have this view, of the left hand side view obviously; (( )) now let me unfold these things. (( )) Observe how the three views have oriented themselves. The top plane, the top plane will show the front view; the bottom plane will show the top view and plane to the right the front view here, will show left hand side view obviously. This is what you know as the first angel projection. For the third angel projection, the process is very similar; two things are different.

Now, again parallel projections are coming from you, they are heating this object and instead of this object allowing those projections to pass through it. That is now open and projections get reflected back, now if we imagine plane in between you and the object those projection organ of fall on this plane. Similar would be the case, when you consider the top view and when you consider the side view. For the third angel orthographic projection, this plane here will be the front view, this plane here will be the top view and this plane here will be the left hand side. The orientations of these planes will be slightly different, if you considered my hands to be those planes for the third angel projections. This would be my front view, this would be my top view and plane here will be my right hand side. In other words these things get re oriented slightly different for the third angel projection.

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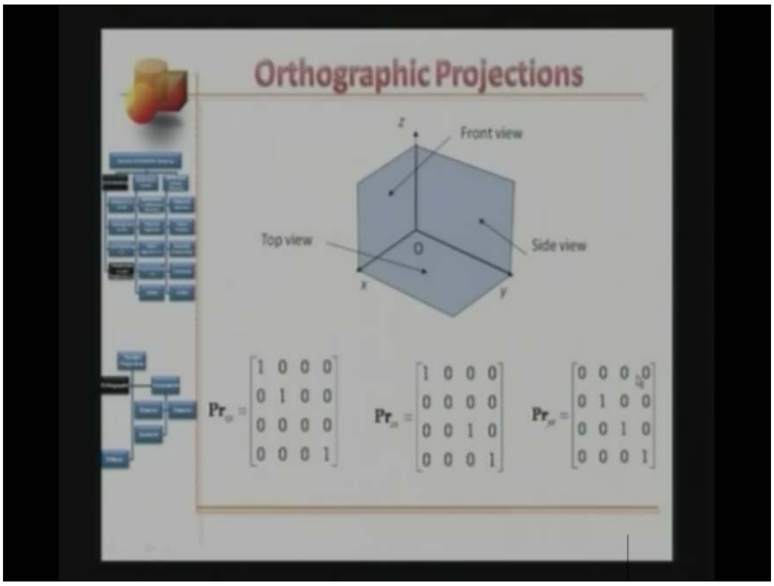

Now this explanation is the first angel projection. Displaying here the x pipeline it will be the top view. Imagine there be objectives here and we looking all the object from the top. The corresponding four by four orthographic projection effects P r sub x y will be given by 1, 0, 0, 0; 0, 1, 0, 0; all zeros in third row, notice that is z coordinate will be 0 and all entries over here 0 and last entry being one. Imagine now, that the objectives here and you looking this object from this side, along the negative y axis. This plane here will be the front view. The corresponding projection matrix P r sub z x will be given by 1, 0, 0, 0; 0, 0, 0, 0; notice now that all the y coordinates are 0. 0, 0, 1, 0; 0, 0, 0, 1 and finally this plane here will be the side view. The corresponding projection matrix Pr sub y z will be given by all zeros in the first row, again the x coordinates of all the points would be 0 and then for the second row 0, 1, 0, 0; 0, 0, 1, 0; and 0, 0, 0, 1. Likewise similar matrixes can be extracted derive for the third angel projection.

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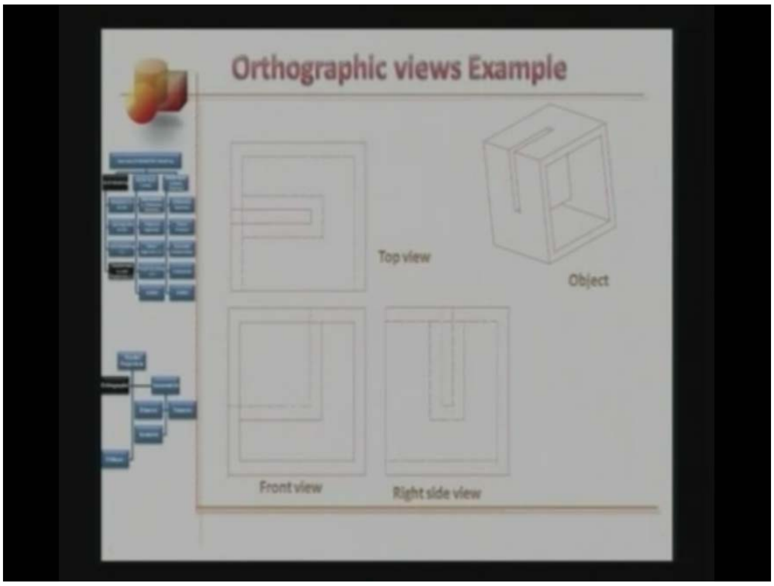

Let us considered very simple example, there this is an object and you would to have orthographic projections; the orthographic views in the third angel scheme. This would show the front view of the object. This would show top view of the object. And this would be the right hand side view of the object. You must have been told in your first year drawing classless that would be third angel projections. The front view is always below the top view and in the first angel projection that is the top view which is below the front view.

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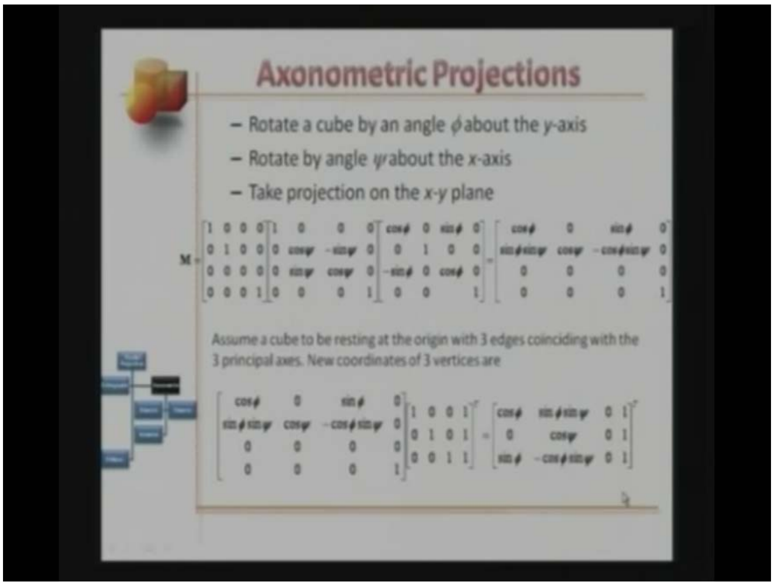

Next axonometric projections, as I mentioned earlier eyes matrix projections happened to be special case of an axonometric projection. The idea is quite simple, may be have not you and the origin of the Cartesian space coincides with 1 of the vertices of this view. All lengths are equal to 1, in a sense this is a unit cube and so the coordinates of this point will be 1, 0, 0; the coordinates of this will be 0, 1, 0; and for the this point here will be 0, 0, 1. The idea is quite straight forward when taking axonometric projections.

You would want to rotate an object represented by this cube about any two axes. Here rotate this cube about y axis, and about the x axis. Now the angels of the rotation can be arbitrary. Here, we have the angels as phi and psi. Specifically in this example, we rotate this cube about the y axis, counter clock wise by an angle phi; and then we rotate that result about the x axis, by an angle psi. This is again in the counter clock wise direction and then we take the projection of the result on the x y plane. The plane form y 2 orthogonal axis about which this object was originally rotate. What would be the resulting transformational matrix? Gets they closed.

The transformation will be given by matrix m, 4 by 4 matrix insides; the first matrix will be rotational about the y axis. You are now familiar with these terms and have to construct these rotational matrixes; 1, co sin of phi, minus sin phi, sin of phi and co sin of phi. Rotational matrix or rotational about the y axis, next is rotational about x axis; 1, co sin of psi, minus sin of psi, co sin of psi all the other elements remains 0 except for this 4 by 4 entry, this is 1.

And this could be projection matrix, for projection on the x pipe line for which these coordinates will be 0. We can multiply this matrix together and get this result 4 by 4 matrix the dieseline; co sin of phi, 0; sin of phi, 0; sin of phi sin of psi co sin of psi, minus co sin phi times sin psi 0; all entries as 0 in third row and then 0, 0, 0, 1. We already set that be assume that cube is resting are the origin with three edges of the cube coinciding with the three principal axes.

What the new coordinates of three vertices would be? Well we can use transformation. We can use this result here. So, the 3 multiply the coordinates three vertices namely 1, 0, 0, 0, 1; 0, 0, 0, 1; the transport and the express these coordinates in homogenous coordinate system. If we multiply this matrix, by this transformation we get this result;

co sin of phi, sin of phi times sin of psi, 0, 1; 0, co sin of psi, 0, 1; sin of phi, minus co sin phi times sin of psi, 0, 1. These are the new coordinates of these points.

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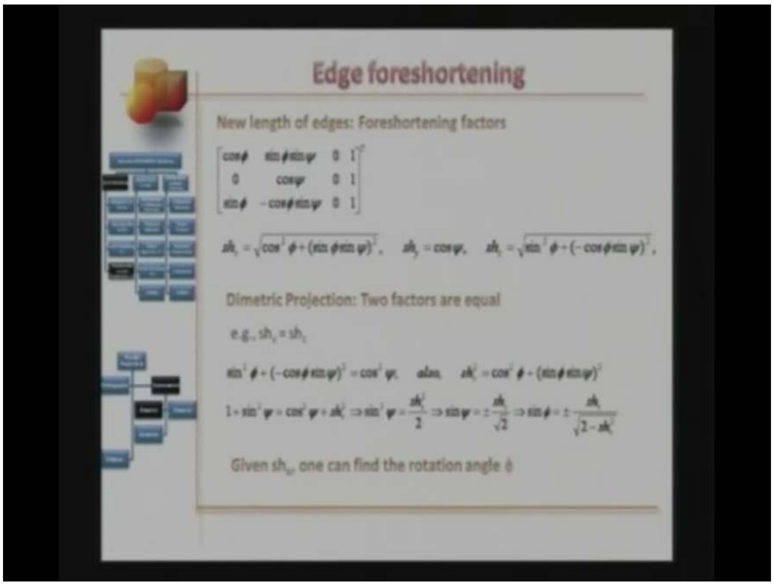

Well, what would now be the lengths of those three edges? Let see we discuss this in edge foreshortening. The new length of those edges will be; co sin for phi plus sing for phi times sin for psi plus 0 squares, we now that from coordinate geometry. And sense the original length of this edge was 1 foreshortening factor s h x will be new length over the original length; which is 1, so the new length. Correspondingly the change in the length along the y direction will be s h sub y, which would be given by co sin psi. The factor change in length along the z direction will be given by s h sub z which will be equal to sin square y plus co sin of phi time sin psi plus 0 square, which does not appear. So these three are the new length of the edges these of the corresponding foreshortening factors along the three principle direction.

Now in a diametric projection two of these factors will be equal. Noting this thing from the other side, two of these factors will not be equal. Let us take an example; when the foreshortening factor along the y and along the z directions, s h sub y and s h sub z are equal. This implies that co sine squared psi is equal to sine squared phi plus co sine squared phi times sine squared psi, which is distant. We can squared this equation s h sub x squared equals co sine squared phi plus sine squared phi times sine squared psi. What we can do now if set this equation and adds to this equation.

In other words we are adding these two terms together, and we are adding these two terms together, with some simplification. Your result is given by 1 plus sine squared psi equals cosine squared psi plus s h sub x squared. This equation is expressed in terms of free foreshortening factor, s h x. Remember that we make the foreshortening factors s h y equals s h z. This result and we used to determine the angle psi in terms of the shortening factor along the x direction.

That is, sine squared psi equals s h x squared over 2 or sine psi equals plus or minus s h x over square root of 2. Once we know sine psi, we can determine sine phi as plus minus s h x over under root of two minus s h sub x square. What we observed is, if we consider one of the foreshortening factors are the free choice that is would be consider s h x as a free choice. We can determine both rotation angles phi and psi.

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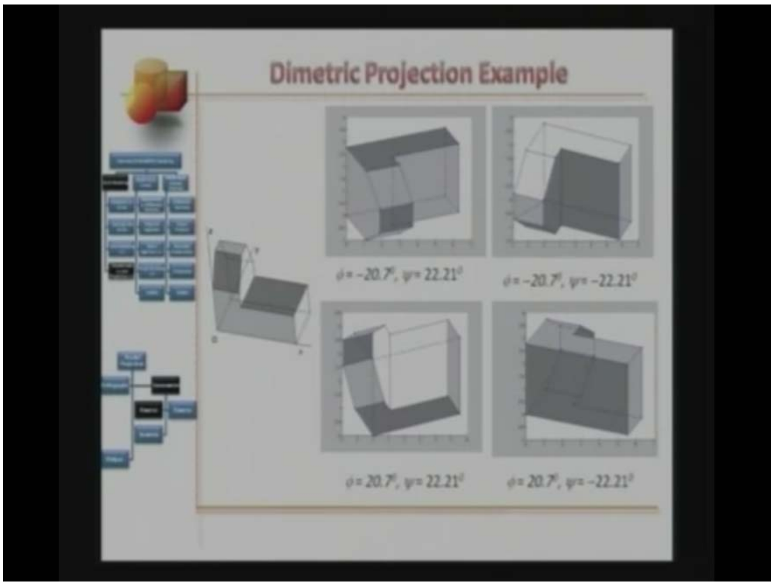

Here is an example of a diametric projection. You would like to take projections of this object. This is a view for phi equals minus 20.7 degrees and psi equals 22.21 degrees. Another view for phi equals minus 20.7degrees and for psi equals minus 22.21. The third one for phi equals 21.7 and for psi equals 22. 21 and the fourth phi as 20.7 and psi as minus of 22.21. Now these angles are for a specific value of s h x.

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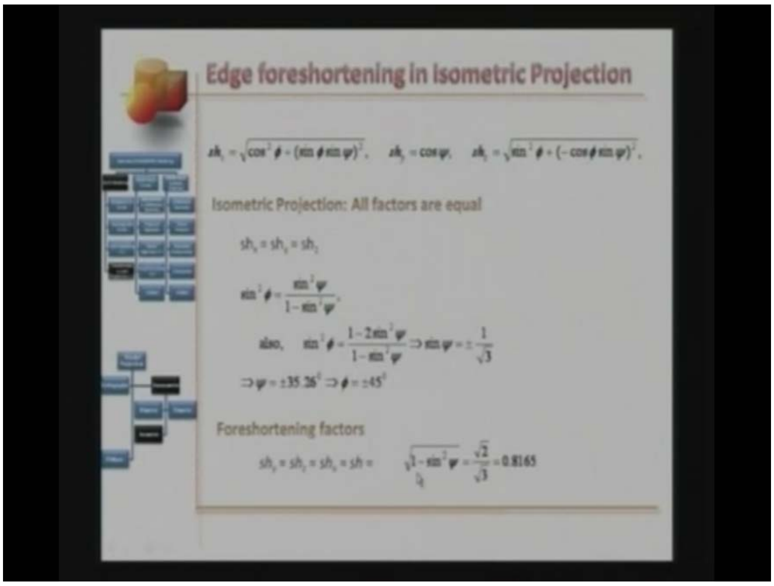

In the isometric projection, all factors are equal. That is factors along the x y and z directions s h sub x, s h sub y and s h sub z they are equal. We can work out the mathematics using the three relations and this condition. And we find that the angles psi and phi again specific values. Sine psi becomes equal to plus or minus 1over under root of 3, which is plus or minus 35.26 degrees; and phi assumes the value of plus minus 45 degree. If we consider these foreshortening factors they are all equal to under root of 1 minus sine squared psi, which is equal to under root of 2 over under root of 3 which is 0.8165.

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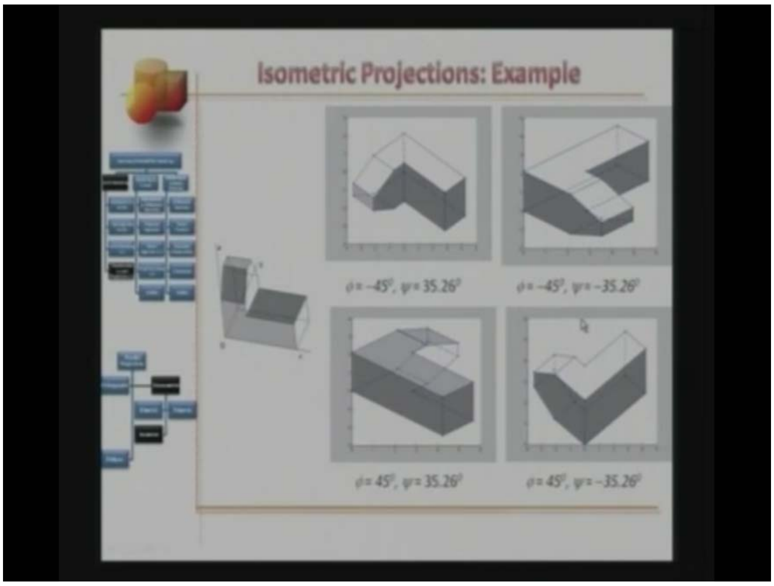

An example, with arithmetic projection these are four different views, corresponding to four different sets of angles phi and psi. Trimetric projection here none of the shortening factors are equal. In other words angles phi and psi can we choosing freely, this one is a view when phi equals 30 degrees and psi equals 45 degrees. We now come to oblique projections.

When I was young before I went into formal training, I use to draw a cube like this. I am sure many of you would be drawing in the cube in the similar fashion. When I took my first course on engineering drawing, they introduce me a skin to draw a cube by this. We all know that this is closed to an isometric view or the cube. And then I started to scratching my hair, was I sketching the cube like here it up only comes out that I worked out and in fact this is an oblique view or the cube.

Oblique projections, parallel projectors are inclined at an angle to the plane of projection. Faces parallel to the plane of projection in this case the x y plane are not foreshortened. Angles between the edges of the parallel faces are also preserved. Notice this two faces on this cube, these lengths are not foreshortened and the internal angles also remain as 90 degrees.

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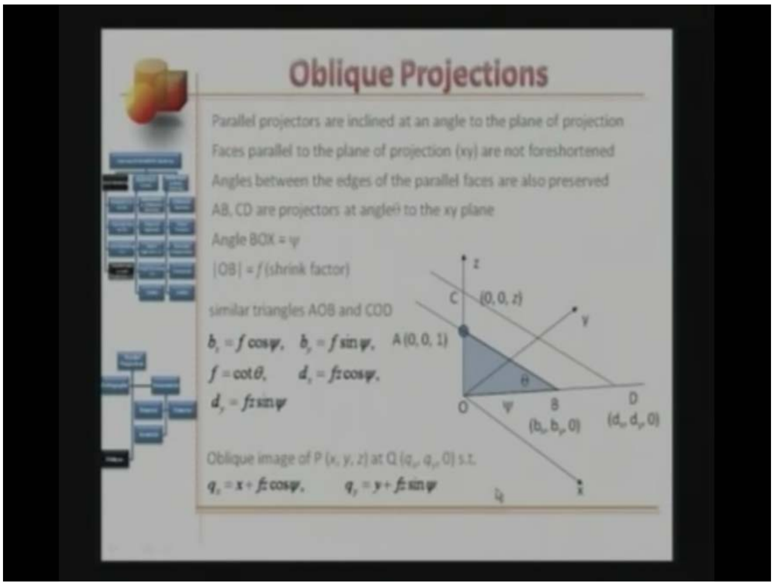

Let us work on some mathematics on oblique projections. Say we have a point A, at this means 1 from the origin along the z axis; and say we have an inclined projection passing through this point and hitting the x y plane. This projector is hitting the x y plane at point

B with coordinates b sub x, b sub y and of course the 0. Let us take another inclined projector parallel to the previous one. This projector passes through this point C with coordinates 0, 0, z.

The projector takes the x y plane at point D with coordinates d sub x, d sub y and 0. Once again A B and C D are projectors at angle theta to the x y plane, this is the angle. Let me shade the triangle A O B. Let me also specify the angle B O X as psi that is B O X as angle psi. Let me say that the distance O B is equal to f; let me call it a shrink factor. This distance O B is which is, the shrink factor; it is not difficult to note that triangles A O B and C O D are similar.

We can use some trigonometry to determine different coordinates b sub x, b sub y, d sub x and d sub y. Now b sub x is f times co sine psi, this distance is b sub x, which is the projection of O B on the x axis, then of O B is it. Likewise b sub y is f sine psi, b sub y is this distance projection of O B on the y axis. Now f equals cot theta, where we consider triangle A O B 1 over f is tan of theta and so we get this result. The x is the projection of O D on the x axis and the distance O D and be determine using this similar triangle properties. d x therefore, is f z co sine psi. Really the distance O D is f times z, clockwise d y is f times z times sine of psi.

Now imagine that we would want to get or obtain the oblique image of any point P with coordinates x y and z. And let that oblique image, we given by point q with coordinates q x, q y and 0. All we need to do is appropriately move point C to co inside the point P and accordingly these coordinates d x, d y; we get change q x, q y with minor manipulation. q x will be equal to x plus f times z co sine of psi, note that this is d x. And q y equals y plus f times z sine of psi, note that this is d y. In a sense we add x to d x to get q x and we add y to d y to get d y.

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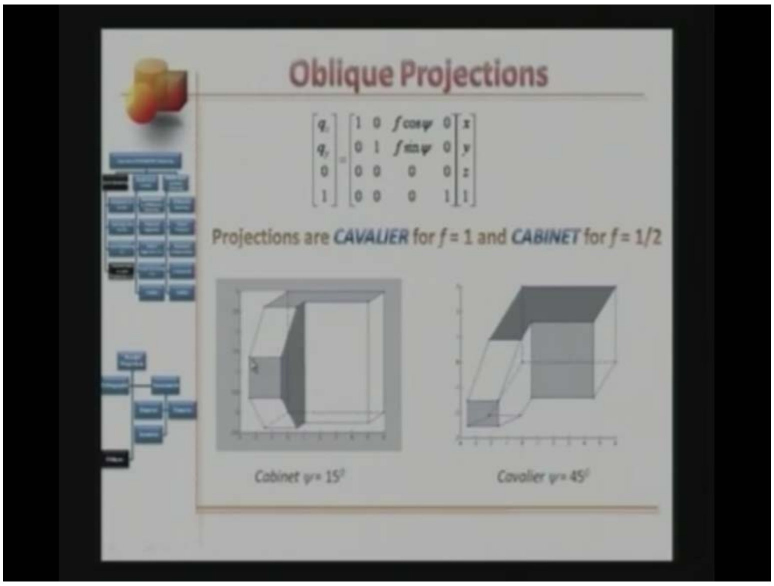

The corresponding transformation matrix on oblique projections, will then given by this 4 by 4 matrices. The entries 1, 0, f times cosine psi, 0; 0, 1, f times sine psi, 0; all zeros in third row because z is 0. For the projection and three entries are 0 on the fourth row the last entry being 1. These were represent points in Cartesian space and these were represents the coordinates of the corresponding oblique projections on the x y plane. Oblique projections are call Cavalier for f equals 1 and Cabinet for f equals 0.5. These are the two examples, this is the Cabinet projection for psi equals 15 degrees, and this is a Cavalier projection for psi equals 45 degrees. Notice that rectangular features which are parallel to the plane of projection of reserve, both in length as well as the internal angles.