

Bio – Microelectromechanical Systems

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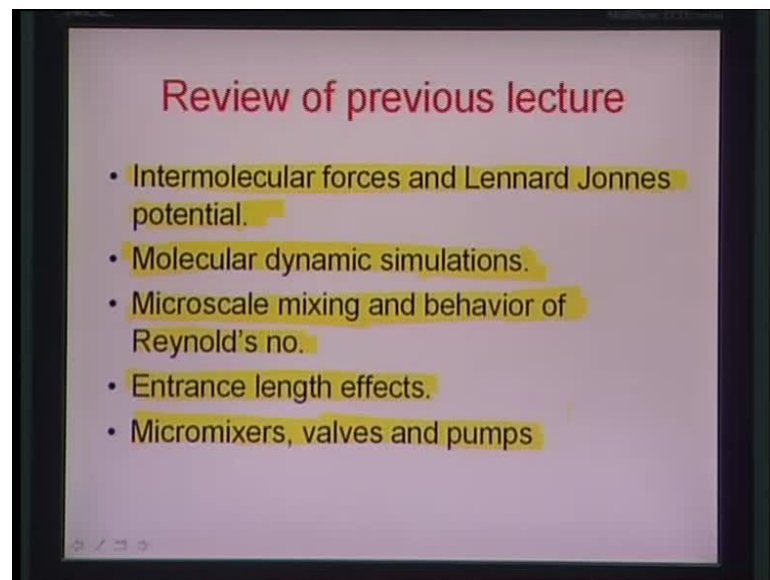
Department of Mechanical Engineering

Indian Institute of Technology, Kanpur

Module No. # 01

Lecture No. # 31

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Hello and welcome back to this thirty first lecture on Biomicroelectromechanical systems. Let us do a quick review of what we did in the last lecture. We talked about intermolecular forces, Lennard Jones potential model, and molecular systems. If you may recall, two molecular systems i and j separated by a distance r were defined, where we tried to find out what the potential is between i and j in terms of an energy scale and the ratio r by σ , where σ is the distance scale, r is the intermolecular distance or the separation between i and j . We got an expression, 4ϵ energy scale times of r by σ to the power minus 12 minus r by σ to the power minus 6, where the first term would correspond to the pair-wise repulsion between the electrons and the outer most shells of both systems and the second term would actually be weak Van der waals forces

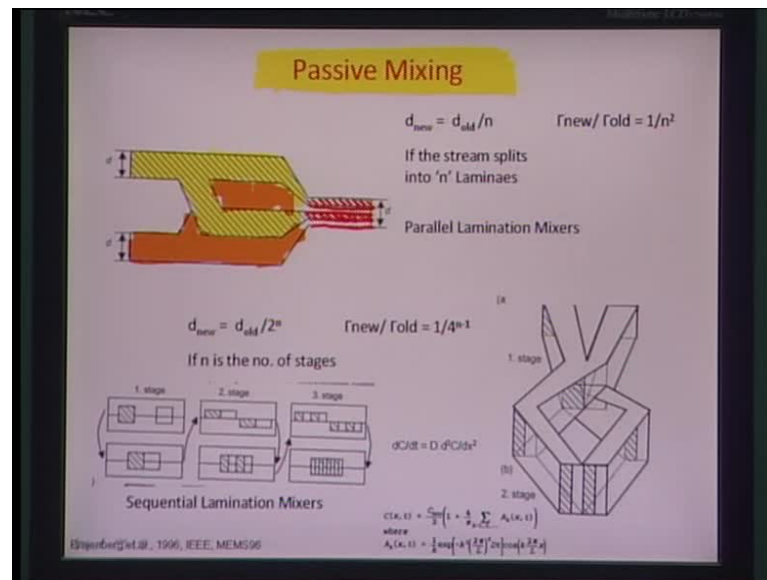
of attraction between the nucleus on another one with the electron on the other particular system.

We also tried to derive the forces between the two systems i and j where we take the negative grad of potential and this would be in terms of an attractive force and a repulsive force. We studied these in context of several diatomic gases, some other gaseous systems including CO₂ and air and then tried to determine the energy scales in terms of energy per unit temperature, energy per unit Boltzmann constant and distance of separation in nanometers. Then we also saw that if we plot such systems, we will have typically a potential well on which exactly the repulsion and attraction would be same and beyond which the attraction would predominate over some distance up till there is a reversal and the repulsion again becomes strong.

We also talked about molecular dynamic simulation which means essentially trying to simulate Newton's second law, Md^2r by dt^2 with respect to the total amount of forces based on the negative grad of the Lennard Jones potential. We also defined the cut off radius to simplify calculations or truncate calculations so that they may not just go infinitely and may have a converged solution. Wherever the continuum assumption fails, Navier-Stokes is replaced by the MD simulation models and this was followed by a brief description about micro scale mixing behaviour of Reynold's numbers and different scales and its change from laminar to the transition region to fully turbulent flow. We talked about entrance length effects particularly in case of micro channels about 60 percent by hydraulic diameter in small Reynold numbers. Then we started discussing in detail about Micromixers valves and pumps.

We like to proceed ahead on this note. I would just like to reiterate that micro scale mixing is essentially all diffusional which is based on either the interfacial area or by somehow reducing the diffusion time. When we talk about reducing diffusion time, there are some mostly architectures which would come into the category of passive mixing. Therefore, in such mixers there is no mechanical part; so, it is non mechanical and passive means that there is no energy supply essentially. So, you have to just by intelligent architecture design let the flow have reduced diffusion time. One way of doing it really is lamination and as you all know from before lamination is thin laminae of different type of fluids stacked against each other.

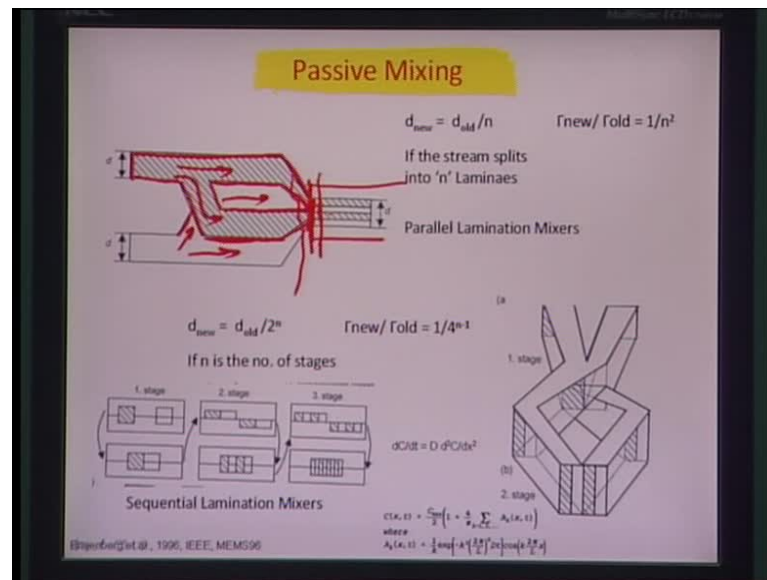
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If you have a green dye in a water sample which is moving past a channel, these are two laminae essentially and if I can somehow split these flows apart and bring them together in a manner that we can have four laminae from two, eight from four, so on and so forth. What is reducing here is the laminae size, the effective diffusion length which is between the water and the dye reduces to d by 4, d by 8 so on so forth. Let us look at this example here - you have a case where you are actually splitting apart two flows. There is this channel here which I am marking through this yellow highlighter which is filled up with dye and let me just clear this region.

This is filled up with the dye; it is also represented by the hatched area and we have this other channel which is represented by brown color which is of a different nature. This goes here; this goes into this region. In this particular region, you see there are four laminates. If you look at this region closely there are four laminates and there are two of the hatched sections and two plane areas. So, there are four laminates. Let us look at how you do that.

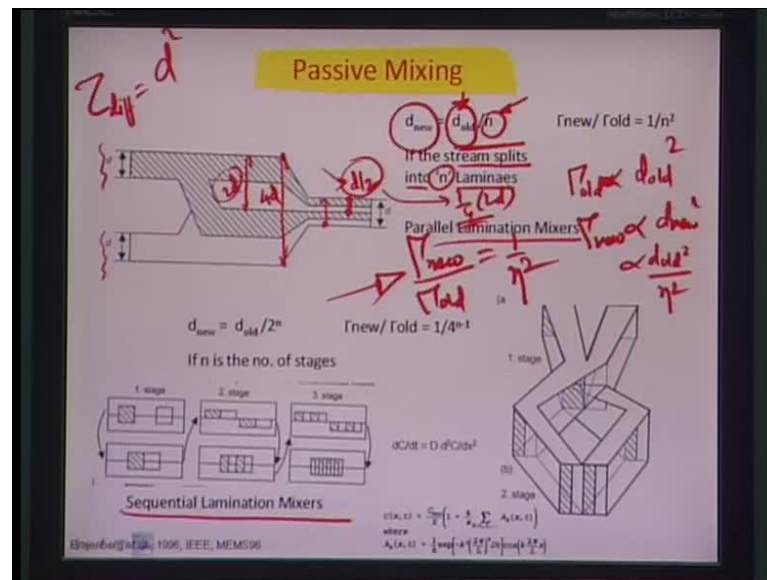
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You have two planes, you have a plane which is flowing below another; the dye here which is shown in white is at a bottom plane and the dye which is shown by the hatched is at the top plane and these two dyes flow on the top of each other up till an extent when they are able to converge into a small channel so that you make sections in a manner - let us say you have split up this top region you have split up this top region into two channels as you can see and you have similarly, split up the bottom region again into two channels- one here going this direction and another here going this direction. The top region is one here going this direction another going this direction. Now, what you do is you take all these together here and rapidly converge it into a smaller section in this particular region. So, you are converging rapidly into this smaller section here. However you are converging it in a manner that all the four channels have their dye outlets getting converged here. What essentially results is that you have - two from the bottom and two from the top made in a manner that you have an alternate one from bottom, one from top, second one from bottom, and the second one from top laid out parallel to each other. So, you are stacking the two dyes on a smaller channel.

I call it lamination. We are trying to laminate. If you remember from your composite's lectures, laminates are essentially layer by layer - so you have one layer of fluid of type one, secondly another layer of fluid of type two, then another of one another of two and you are trying to laminate the flow into pieces. So, the advantage here is many folds.

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Let us look at some of the advantages that you automatically get out of this. So, if you look at the diffusion length here, the diffusion length is d plus d that is $2d$, the diffusion length here as you are seeing - this is d and this part is d , so, it is $2d$. Basically this whole area is nothing but $4d$ and this particular diffusion length here is $2d$.

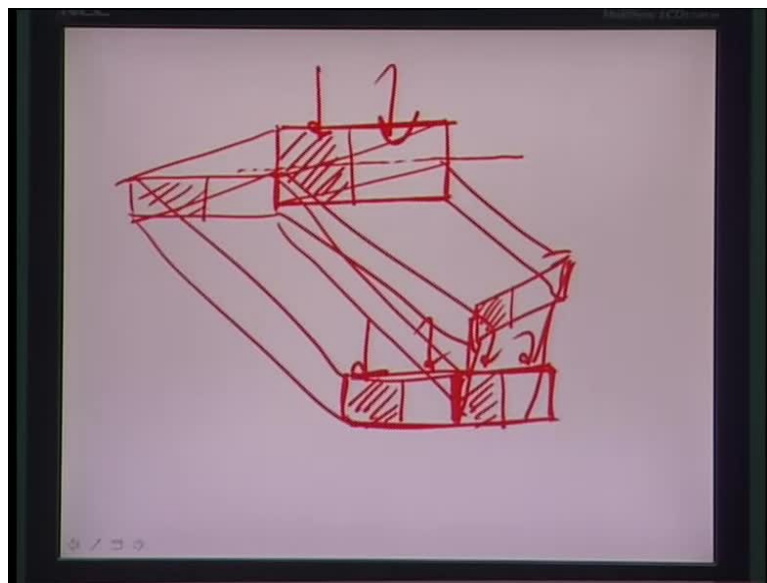
Now, the length changes to d by 2 , from $2d$ length you are changing to d by 2 . The diffusion length which was $2d$ got changed to d by 2 which is actually one-fourth of $2d$; this is one-fourth of $2d$. The diffusion length is changing as a function of laminates. If you are laminating four times, the diffusion gets reduced by the old length by the number of laminates. Similarly, if it was eight times, the diffusion would get reduced by 8 and so on. Therefore, what you are doing is - you are trying to split up the streams into n laminates where the new diffusion length is d by 2 and this is $2d$. So, it is equal to the $2d$ which is the old diffusion length divided by the number of laminates. Very interesting phenomena! What is the conclusion for all these? The time of diffusion as you already know from our previous descriptions is proportional to the square of the diffusion length. Therefore, if there is a transverse length between two flows which are flowing in a channel, the cross sectional length actually is the diffusion length in that case.

The diffusion time is proportional to the square of that particular length. If you see the time old was actually proportional to d_{old}^2 and the time t_{new} is now proportional

to a new square which is actually proportional to the old square by n^2 . Therefore, the new time is lesser than the old time by a factor of n^2 .

So, definitely it is an advantage, because the new time of diffusion reduces by a factor of the square of the number of laminates that you are packing in a small section of the channel. That is also known as a parallel lamination mixer; that is how technically it is defined because you are actually in a same plane parallelly laminating the flows into several laminates.

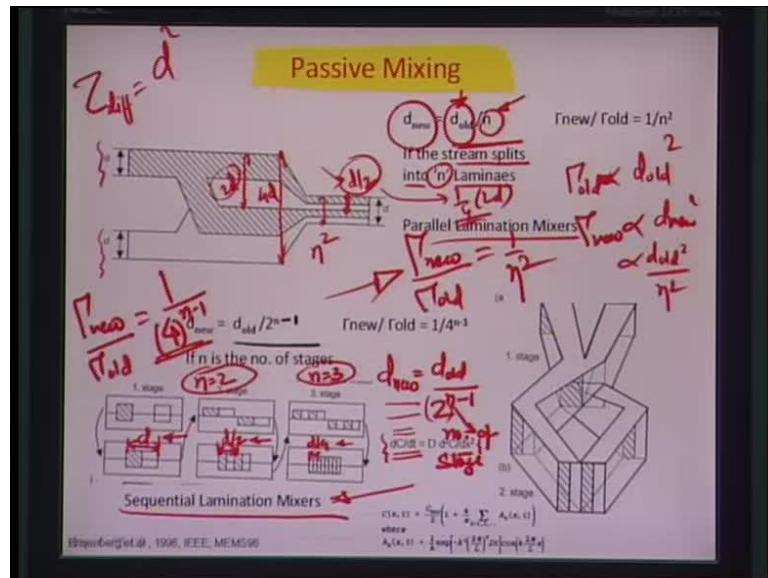
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You can actually try to introduce these flows in a manner where you can do it sequentially. If you try to split it apart n times and take it back again in a 3-dimensional manner, then you can do it sequentially. That is how sequential laminarisation mixture comes into being. As you will see, the ratios there between the time of diffusion new and the time of diffusion old would be quite different than that in the parallel lamination case. Let us see what the sequential lamination mixer would look like. Let us suppose you have two flows and these flows are parallelly flowing through a small channel like this. What you do is, you divide the section of these two flows half and half and one half - you take up the plane, another half you take down the plane - so it is now a 3-dimensional situation. Then you mix them together back into a similar plane but in a different manner. Let me just look at it in this way. Let us just look at it through a more clear illustration. Let us suppose you have a case here where you have one channel cross-

section in the center with two flowing dyes- let us say you have a hatched dye which is probably a green dye and water here. What you do is you split this flow into two parts. This part, you take up here on this particular plane.

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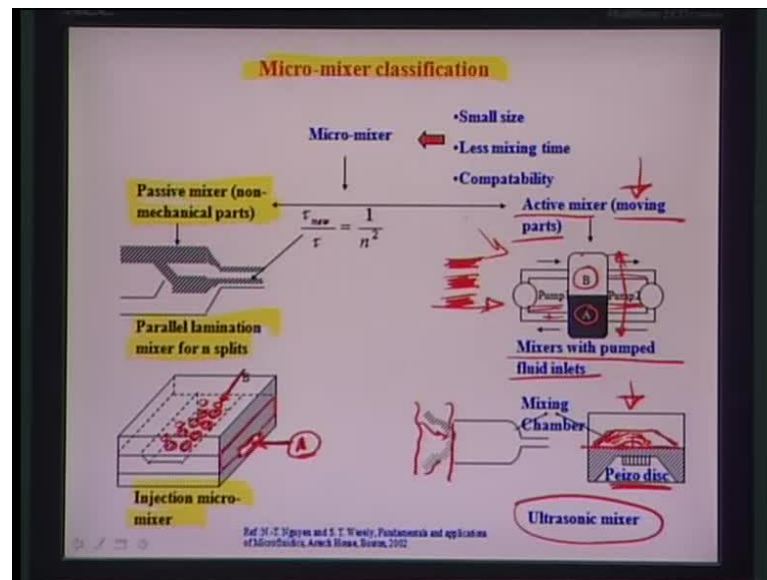


This is a different plane, so, you have half and half here and this other part you take in a different manner in a plane which is at the bottom. As I told you, take one on the top side and another on the bottom like this and here this again is a hatched and a plane dye. Then what you do is you take this back into the same plane and mix them together. So, you have partly flows coming like this and partly flows coming back from here into the same plane. You have a half hatched half water; another half hatched another half water, but now what you have done is from an initial two laminates, you have split it into four laminates and this sequence carries on. So, you have one channel which is half of the main channel which goes in the plane down; another half of the channel being projected in the plane up and then you are coming back again into the same area where you are actually laminating these half and half into four, and then four into eight, and eight into sixteen and so on so forth. This is called as sequential lamination mixer. You can see the different stages - at stage 1, you have from two which generates into four, in stage 2 -you have split it up and you are mixing it together this four.

In stage 3, you are mixing it again into eight and so on so forth. If you consider the way this lamination happens, the d_{new} is equal to the d_{old} divided by 2 to the power of n minus 1.

Let me just explain this a little bit more here so that for the common reference sake. Let us say, we are beginning with the diffusion length d between these two hatched dye and the water sample. If you see here in this particular instance after the second stage has been crossed, n is equal to two here in the second stage, the d actually becomes d by 2. The diffusion length is only related to half the length between the two hatched dyes and water sample, so, d by 2. In the third case, when n equal to 3 and the diffusion length actually further becomes d by 4. The only equation which can fit all this is that the d_{new} is actually equal to the d_{old} divided by 2 to the power of n minus 1 where n is the number of stage. If n is 2 here, d_{new} should be d_{old} by 2^{n-1} is 2^{2-1} is 2. If n is 3 here, d_{new} should become d_{old} by 4 so on so forth. It pretty much matches with the physical dimensions that we have been showing the different stages of mixing. One more interesting thing here is that of course - d_{new} and d_{old} are related by this 2 to the power n minus 1 relationship, the time new in this case should be actually 1 by 4 to the power of n minus 1 times of the time old. So, for the initial n 's, the one which earlier kind of mixers the parallel lamination mixers showed n square relationship changes to 4 to the power of n minus 1. The concept and the fabrication mechanism and the whole mechanism of laminating the flow is totally different in both the cases. This is also known as sequential lamination mixers. The idea was that if the d is smaller, the rate kinetics in the diffusion model can be represented by the temporal variation of concentration dC by dt being proportional to the second derivative of concentration with respect to x . You have this relationship dC by dt equal to diffusion constant D times of d^2C by dx^2 , where x is the diffusion cross-sectional length of the two flows.

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That is what sequential lamination mixers typically do. Micromixers, based on the way that they are able to promote mixing can be classified into passive and active mixers. Passive mixers are by intelligent design where there are absolutely no mechanical parts or no energy that is being supplied. Several different types of mixers come in this category - all the parallel and lamination mixers where either you split the flows using a parallel sequence and stack the laminates close to each other, whether by going in plane, out of plane or going in the same plane -that could be one mechanism of mixing.

Another mechanism could be injection based Micromixer. It is a very simple mechanism; as you are seeing - there are two channels. There is a channel which is pointed out by this area in the lower stack or lower layer of the stack which is called A. This is one channel. The top channel is in the second portion of the stack which is called channel B. There are small capillaries drill in this channel B so that when you flow material A, there is an oozing out of A into the channel B. You are forming micro globulets of dye A within the flow in dye B and all you are doing here is increasing the cross-sectional area so that the mass transport goes up at a certain flux level and because of the higher mass transport and more prominent diffusion, mixing happens quicker.

The other categories of mixer are active Micromixers where you supply some kind of energy by either a mechanically moving part or as we will see in some illustrations and we have been seeing before also, electrically induced mixing is carried out or magnetic

field induced mixing is carried out. Some form of energy has to be supplied to active Micromixers that is the way they are defined. Let us suppose this particular illustration is an example of mixers with pumped fluid inlets; you have fluids A and B here and this also known as a chaotic mixture. What you do is - you take two pumps here pump 1 and 2 and then pump excerpts from fluid A into the stream of B and excerpts from fluid B back into the stream of A. You are creating laminates, this is the black layer, this is again the white layer, this is the black layer, and again this is the white layer and so on so forth.

So, you are creating laminates throughout this cross-section here as you are seeing by repeated pumping and that again creates a smaller length of diffusion and the time diffusion again reduces because of smaller length of diffusion. So, that is a kind of chaotic mixer which comes into the active mixer category. The other kind of mixer is ultrasonic mixer as you are seeing here is another active mixer. Here there is a channel at the top which has two fluids flowing past each other like mentioned here- this channel here is showing the two fluids inlets of different colorations passing through somewhere in this particular channel and there's a Peizo disc at the bottom which vibrates and thus creates perturbations and deformations between these two layers so that there is a mass transport enhanced by this is Peizo effect.

Thousands of bubbles are actually generated and this principle is also known as cavitation. Whenever there is a vibrating surface which is vibrating at a very high frequency and there is a fluid in contact with the surface, there is a tendency of the fluid to lag the motion of the plates, the plate moves forward and backward much more rapidly than the fluid can actually respond. Therefore, there are almost always vacuum traps which are created and from the diffused mass of air in the liquid actually come into this trap and this result in formation of thousands of bubbles called cavitation.

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Designing a Y-Mixer

Mix Ethanol Completely with water in a parallel micromixer with 2 inlets (Y-Mixer) at Room temperature. The flow rates of both ethanol and water are $10 \mu\text{l}/\text{min}$. Determine the required length of the mixing channel if the channel cross-section has a dimension of $(100 \mu\text{m})^2$.

$E. 10 \mu\text{l}/\text{min}$
 $W. 10 \mu\text{l}/\text{min}$

The diffusion coefficient of Ethanol in water at room temp is $0.84 \times 10^{-5} \text{cm}^2/\text{sec}$.

The characteristic mixing length in the channel = width = 100

$(100 \mu\text{m})^2$

Length L

The bubbles come upwards; because they are lighter in weight, because of the Archimedes principle they would come to the top. These bubbles are responsible for all the transportation, because these bubbles would create differential pressure zones within this small interface of two mixers that would create a huge amount of mass transport based on that. That is what another kind of active mixer be categorized as is called ultrasonic mixer. The basic classifications of Micromixers are on the basis of whether they have energy supplied for mixing or did not have energy supplied for mixing. I would like to now do a small example, numerical example where we want to design a y-mixer. The task is to mix ethanol completely with water in a parallel Micromixer with 2 inlets and is a y-mixer. You have ethanol coming from one inlet and water coming from another inlet and let us say is mixing along this channel length. This mixing is totally taking place at room temperature.

So, you do not need to consider the energy equation here. The flow rates of both ethanol and water are both 10 micro liters per minute. Each 10 micro liters per minute from both sides and so we have to determine the required length l of mixing channel. If the cross-section has 100 micron square that means it is actually a 100 micron by 100 micron square section which is the cross-section of this channel. We now need to determine the length. What do we have to really consider here is the critical factor of what is the time of diffusion and with this kind of a velocity the idea is the fluid should be able to contain itself at least for a time equal to the time of diffusion for the diffusion to happen properly

before the fluid emanates from the other end of the Micromixer. These are the various inlets, inlet 1, inlet 2, and this is the outlet. The length which is traversed by the fluid is diffusion times of the velocity of the fluid and let us see what this would be.

The diffusion coefficient of ethanol in water at room temperature is 0.84 times 10 to the power of minus 5 centimeter square per second and the characteristic mixing length in the channel width, w is equal to 100 micrometers.

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The image shows a whiteboard with handwritten calculations in red ink. The first equation is $\tau_{diff} = \frac{(w)^2}{2D} = \frac{(100 \times 10^{-6})^2}{2 \times 0.84 \times 10^{-5}} = 5.95 \text{ s}$. Below this, it says 'Average velocity of the mixing liquids' and gives the formula $u = \frac{Q_{water} + Q_{ethanol}}{\text{Area of cross section}}$. The area calculation is $\frac{2 \times 10 \times 10^{-6} \times 10^{-3}}{60 \times (100 \times 10^{-6})^2} = 33.33 \times 10^{-3} \text{ m/s}$. Finally, the 'Required length for mixing' is calculated as $5.95 \times 33.33 \times 10^{-3} = 198 \text{ mm}$.

The diffusion length is the width in this case is 100 micrometers. The time of diffusion tau diff can be also be represented as square of w by twice d - this case it is 100 10 to the power minus 6 square divided by 2 times of 0.84 10 to the power minus 5 times. This is in centimeter square per second; if you want to convert this into meter square, you have to multiply with minus 4 10 to power minus 4.

This comes out to be equal to 5.95 seconds. If we consider the average velocity of flow of the mixing liquids, u in this case is equal to the flow rate of water plus flow rate of ethanol by the area of cross-section of the channel. Velocity of flow again is equal to the volume rate of flow divided by the cross-sectional area; meter per second in which the fluid move.

Both the flows are flowing at a rate of 10 micro liters per minute which means it is 2 times of 10 10 to the power minus 6 liters which is 10 to the power minus 6 into 10 to the

power minus 3 meter cube and this is per minute, so, you have divide this by 60 and then the area of cross-section is of course 100 micron square, so, 100 10 to the power minus 6 square. This comes out to be equal to flow velocity of 33.33 10 to the power of minus 3 meters per second. I would like to draw your attention to the fact that the velocity is only about 33 millimeters per second which is not very high considering the other flow rates etcetera which have seen here - that is primarily because the cross-section of the channel.

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Designing a Y-Mixer

Mix Ethanol Completely with water in a parallel micromixer with 2 inlets (Y-Mixer) at Room temperature. The flow rates of both ethanol and water are 10 $\mu\text{l}/\text{min}$. Determine the required length of the mixing channel if the channel cross-section has a dimension of $(100 \mu\text{m})^2$.

Handwritten notes:
 The diffusion coefficient of Ethanol in water at room temp is $0.84 \times 10^{-5} \text{ cm}^2/\text{sec}$.
 The characteristic mixing length in the channel = width = $100 \mu\text{m}$

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Designing of a long mixing channel

The above mixing channel is to be designed with a meander shape to save lateral device surface. If the channel structure is to be placed inside a square area, determine the dimension of the area. Determine the no. of turns.

Handwritten notes:
 $W = 100 \mu\text{m}$, $L = 198 \mu\text{m}$
 We assume that the channel walls consume the same area as the channel itself.
 The total surface area required for the mixing channel is
 $A = W L_1 + W L_2 = 2 W L$
 $2 W L + 2 W L = 4 W L$
 $= 2 W [L_1 + L_2] = 2 W L$

So, the required length for the mixing in this case is nothing but the diffusion time which is 5.95 times of the velocity of the fluid flow and that comes out to be equal to about 198 millimeters. The length of the channel here which you need is really about 200 millimeters for the flow to mix completely. That is how you do these problems of designing y-mixer. Let us go on to another numerical problem. The problem is pretty much similar, the above mixing channel whatever we found out is to be designed with a meandering shape or a nest shape just to save lateral device surface area - MEMS is all about integrating into a small platform. Let us say if this channel structure is placed inside a square area, we have to determine the dimensions of the area and also number of turns that it would take for this channel to meander in the small area.

Let us actually see this as the figure illustrated here at the bottom. Each channel has a width of w and in between the channels also there is a width of w . In between the channels is a dead zone essentially where there is a just a flat surface. The channels are engraved within this flat surface. The channels have thickness w and between one meandering channel - the inter-channel spacing is about W , W is kind of an island here in this region. We need to find out the dimensions of this particular square platform over which these channels have been integrated and also we need to determine how many turns this channel would take within this square platform. We assume the channel walls consume the same amount of area as the channel itself.

Now something interesting can be done here. Let us suppose that this dimension here all the way to here is x_1 . Similarly, you have another dimension x_2 here so on so forth. The total surface area required for this particular channel with 1 length equal to x_1 let us say from here to here and 1 length equal to x_2 from here to here. The total amount of area that this channel would need is essentially W the width times of x_1 plus the width of the wall times of another x_1 . For each of these runs, the total area is twice $w x_1$. There are several such x_1 's, x_2 's, x_3 's, so on and so forth to make the whole channel. The total amount of area is suppose there are several of these different terms inside is twice $w x_1$ - let us say they are all different lengths plus twice $w x_2$ plus twice $w x_3$ so on so forth. So, $2W$ can be taken common and then this is x_1 plus x_2 plus x_3 plus x_5 so this can be taken so on so forth as the total length of the channel L . Total area that the channel would need is two $W L$. As we know here the W , the width of the channel is given as 100 micrometers and the length we found out from the earlier equation was 198

mm for the diffusion time to be smaller or equal to the residence time and therefore $2WL$ here comes out to be equal to 3.96 times of 10 to the power of 7 micrometer square. If you assume these dimensions of this particular area to be a square because it is a square area, you have a here and a here.

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The dimension of the square area

$$a = \sqrt{A} = \sqrt{39.6 \times 10^6} = \underline{\underline{62934\text{mm}}}$$

Designing a parallel lamination mixer

The above mixer has to be redesigned with more lamination layers. In the new design, the channel length should be 1mm. In how many laminae should each stream to separated?

So, you are left with the dimensions of the square area which is root of this A where A is the total area that of the mixing channel that would occupy is equal to root over 39.6 in to 10 to the power of 6 micron square which is about 6293 micrometers. This is the area about 6.3 millimeters by 6.3 millimeters over which you can pack the serpentine or this meandering channel.

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Designing of a long mixing channel

The above mixing channel is to be designed with a meander shape to save lateral device surface. If the channel structure is to be placed inside a square area, determine the dimension of the area. Determine the no. of turns.

$W = 100 \mu\text{m}$, $L = 198 \text{ mm}$

We assume that the channel walls consume the same area as the channel itself. $L_{\text{turn}} = 400 \mu\text{m}$

The total surface area required for the mixing channel is

$$A = W L_1 + W L_2 = 2 W L$$

$$2 W L_1 + 2 W L_2 = 2 W [L_1 + L_2 + \dots] = 2 W L$$


Also there is a question about the number of turns that this channel would take and so assuming that we have each turn comprising of about 4 different vets. If you look at this particular case you can find out that in one particular turn which starts from let us say point here all the way to point here, there about four w's - w plus w plus w. So, it starts from here so it has about four w's. I will just reiterate this again w plus w plus w plus w. So, from here to here of the channel which is one complete turn essentially the total width that this channel would move is about four w and this is equal to about 400 microns. If one turn corresponds to 400 microns, the total 6293 microns would have exactly 6293 by 400 hundred turns which is equal to about 16. So, the whole channel can be meandered into 16 turn architecture with an area of about 39.6 times 10 to the power of 6 micron square and one side equal to about 6.3 millimeters.

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The dimension of the square area

$$a = \sqrt{A} = \sqrt{39.6 \times 10^6} = 6293.14 \text{ mm}$$

no. of turns = $\frac{6293}{400} = 16$



Designing a parallel lamination mixer

The above mixer has to be redesigned with more lamination layers. In the new design, the channel length should be 1mm. In how many laminae should each stream be separated?

What is important for me to tell you is that the scale essentially is very small, I mean 6 by 6 millimeter by 6 millimeter is probably close to the size of a 25 paisa coin and so in this particular area, you can always create 16 turns and make the length of the channel to be good enough or as long as close to about 198 millimeters for the diffusion time to be equal to the residence of the two floors.

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Since, the average velocity remains the same as in all mixer designs earlier & the new mixing time τ (new diffusion type)

$$\tau_{\text{new}} = \tau_{\text{old}} \times \frac{L_{\text{new}}}{L_{\text{old}}}$$

$\tau_{\text{old}} = 5.75 \text{ sec.}$ $L_{\text{old}} = 198 \text{ mm}$
 $\tau_{\text{new}} = 3 \times 10^{-6} \text{ sec.}$ $L_{\text{new}} = 1 \text{ mm}$

$$\tau_{\text{new}} = \frac{w^2}{2D\eta^2} \Rightarrow \eta = \frac{w}{\sqrt{2D\tau_{\text{new}}}}$$

$$= \frac{100 \times 10^{-6}}{\sqrt{2 \times 0.8 \times 10^{-6} \times 5.16 \times 10^{-9} \times 3.5 \times 10^9}}$$

$$= 14$$

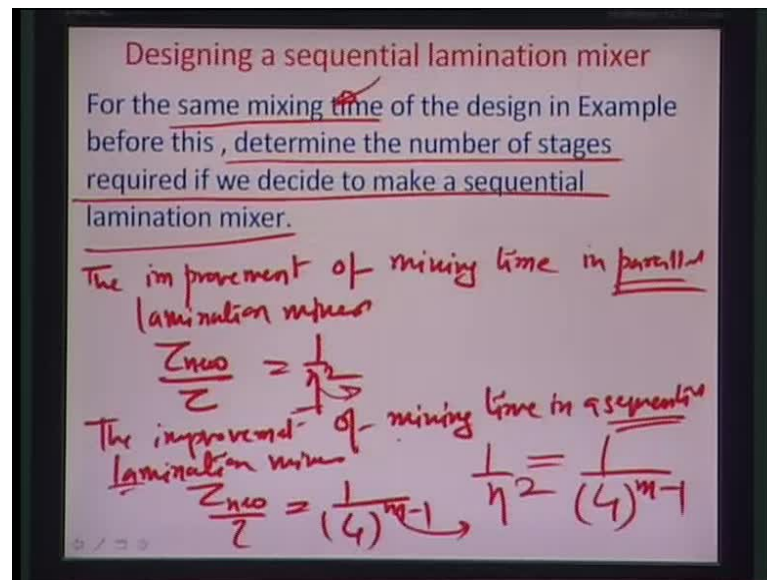
Let us do another numerical example. We want to design a parallel lamination mixer and let us say the above mixer has to be redesigned with more lamination layers. Instead of

green dye and the white dye mixing on one particular channel as a part of the y, you now want to laminate it into several different cross sections. You want to find out in the new design- the channel length has been constrained to be 1 millimeter that has been given in the question, how many laminates would be need for doing the same thing as these two laminae set was doing in a 198 millimeter long channel. This is what the question is. Let us assume that since the average velocity remains the same as in all other mixer designs earlier.

The new mixing time will be proportional to the square of the new diffusion length and therefore, t_{new} is equal to t_{old} times of L_{new} by L_{old} . L_{new} as we know is about 1 millimeter which is the requirement and L_{old} is about 198 millimeters. Let me just reiterate one thing here that this is L_{new} is essentially the length of the channel, it is not the length of the cross-section, this is not the diffusion length- this is the length of the channel. If you are assuming that the velocity of flow is same in both the cases - you have the ratio between L_{new} and τ_{new} giving the new velocity, similarly, ratio between L_{old} and τ_{old} as the old velocity- they are same.

These two are same. Therefore, it is an inverse ratio in which you can calculate the t_{new} by t_{old} . It is the same ratio; it is the ratio of different lengths L_{new} by L_{old} . We have said in the question that the L_{new} has to be about 1 millimeter when the L_{old} is about 198 millimeters as we calculated before. If we assume this t_{old} to be 5.95 seconds remember from the first example, then this t_{new} time would acquire a value 3×10^{-2} seconds. As we know that the time of new is also proportional to square of the cross-section length and inversely proportional to $2D$ and in this case there would be a number of laminates n^2 . Therefore, this n value can be derived here as w divided by twice $D \tau_{\text{new}}$. If you put the value of w as 100×10^{-6} and the diffusion coefficient to be the same as we took earlier, 0.84×10^{-5} meter square per second times of τ_{new} which is 3×10^{-2} seconds. We are left with value of n which is equal to about 14. So, there have to be 14 laminae for the microfluidic mixer to have a similar performance in 1 mm length as happens in 198 mm length will have only 1 lamina of each kind.

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That is how you solve these problems. I would like to do another numerical example for the sake of clarity. Let us suppose we want to design the same mixing time of the example before this, it is about 3×10^{-2} seconds. You have to determine the number of stages required if we decide to make a sequential laminae mixture instead of a parallel lamination mixer.

What improvements do we need to make here. The improvements of mixing time in parallel lamination mixer case are given by the equation, $t_{new} \text{ by } \tau \text{ equals } \frac{1}{n^2}$ and the improvement in time for a sequential one is given by $t_{new} \text{ by } \tau \text{ equal to } \frac{1}{4^{m-1}}$, where m is the number of stages here- mind you, this n and m are not same- n is the number of stages in the parallel lamination mixer and m is the number of stages in the sequential lamination mixer. For the similar kind of improvement what is being reiterated in the question here that for same mixing time of the design in the example before, determine the number of stages required if we decide to make a sequential instead of a parallel.

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The image shows a whiteboard with handwritten mathematical equations and a diagram. The equations are:

$$2 \ln(n) = (m-1) \ln 4$$
$$m = 1 + \frac{2 \ln(n)}{\ln(4)}$$

Next to the second equation, there is a diagram of a vertical channel with a zigzag line representing a mixing stage. An arrow points from the top of the channel down to the zigzag line, and another arrow points from the zigzag line down to the bottom of the channel. The number '14' is written above the zigzag line.

Below the equations, the text reads:

Total mixing channel length
= 1mm = 1000µm
Each stage is exactly at = 250µm
from the beginning.

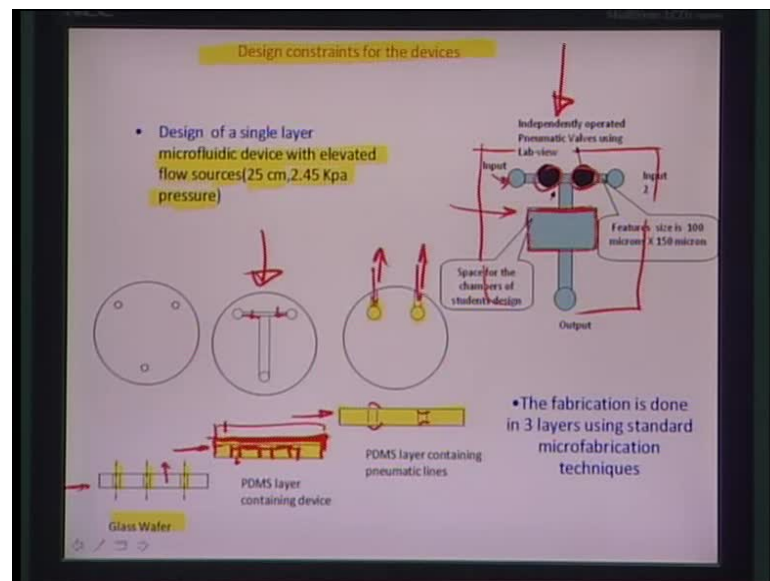
We know for that 1 by n square has to be equated to 1 by 4 to the power of m minus 1 and if we take natural log on both sides, we have twice \ln of n equals m minus 1 $\ln 4$ or m comes out to be 1 plus twice $\ln n$ by $\ln 4$. You know n is 14 which we calculated last example and therefore, the m value comes out to be equal to 5 . So, there are only about 5 stages in a parallel sequential lamination mixer which corresponds to about 14 stages in the parallel lamination mixer or 14 laminates in the parallel lamination mixture. So, a parallel of course may be a little faster because you are bringing all the 14 together assuming that you have split apart these flows seven times and you know bringing them together back again, the parallel would be a little faster than the sequential from practical stand point.

So, for a 1 mm mixing channel, each mixing stage of the new design only occupies about 200 microns or so. You have the total mixing channel length in case of the parallel lamination mixer about 1 mm or about 1000 microns and so each stage is exactly 200 microns from the beginning. That is how you categorize these kinds of mixers. In a nutshell, you probably have seen or now you can do some calculations by yourself and have a good idea about these different forms of mixers - be parallel or sequential or y-shaped meandering, any other geometry.

Let us look into some practical examples after this as to how the in the macro scale intuition can play havoc when you translate the information onto the micro scale. I would

like to look at a problem which we experimentally did back in our graduate days where we talked about white type a T-shaped mixer with the flow controlling valves on both arms of the T, so there is a case where now there are two streams which are coming on both arms of the t and mixing along the stem of the t and then you have control valves on both sides of the arms, so you can control the flow rates from both these streams one at a time and then try to investigate what happens within the stem of a T.

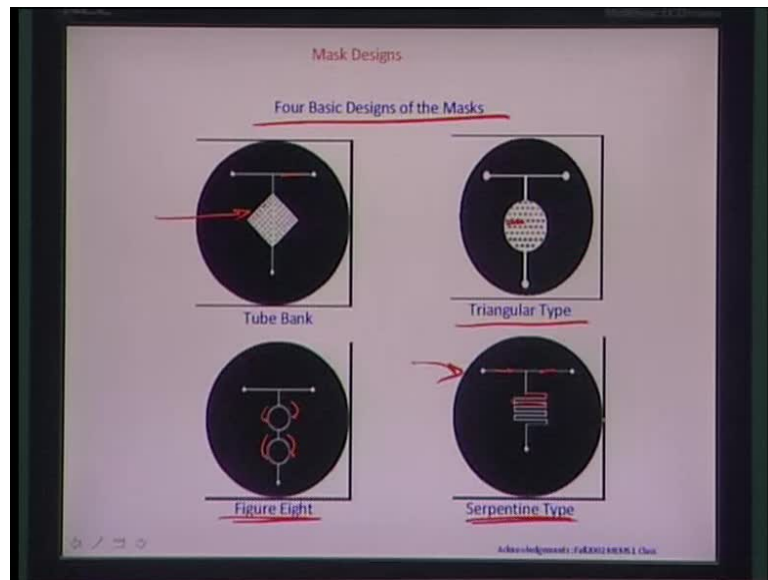
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The problem is somewhat like this as illustrated here in the figure. As you can see here, the constraint for this device is that it is actually a microfluidic device with flow sources which are placed about 25 centimeters above as like gravity driven flow and the pressure that it creates at the entries is about 2.45 kilo Pascal's. The device essentially comprise of three layers, there is a glass layer here which has these small drilled holes- cylindrical holes which can do the inlet, outlet of the fluids and there is a PDMS layer which has this T feature and micro channels in between which does the mixing action and then on the top, there is another PDMS layer with these two blister pockets. So, there is a tendency of these pockets to be filled in and out with compressed air so that you can act them as valves. There is a small opening on the top in the PDMS which kind of inflates and when it inflates, the low layer which is this particular layer here channel layer get squeezed and the valve gets closed and so the flow rate reduces all the way to about very negligible on both sides. You assemble these by putting layer one on the bottom of layer two and on the bottom of layer three and then plasma bound all these three layers together as you

can see here in this particular figure. You have a layer at the bottom here which are not shown, it is a glass layer it is a transparent layer, then layer at the top here which is again transparent but contains the channels essentially in the PDMS, the upper portion of the layer here as you can see is all PDMS- the upper portion is all PDMS.

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So, you have a channel only embedded between his glass and the PDMS surface. This part is solid PDMS and then you have another layer of PDMS which comes and sits at the top of this layer which are these blister pockets here as you can see and the blisters can be fitted in with ruptured scientific tubing about 1 by 21 gauge made up of steel and then peak and you can epoxy a manner that the air can flow in and out and thus inflate and deflate these blisters like this and this. Therefore, as they inflate, they kind of squeeze the channel which is below it here and here and this channel kind of bends and blocks the flow because of these blisters.

Now, you have to find out what happens by giving different designs here to the mixing rates. This was actually a student experiment and it was a graduate class than when they started doing this experiment they realized that whatever intuition they were carrying forth from the micro macro scale does not really translate very well to the micro scale. There were about four basic designs of structures which were investigated in this black box region; one was as you see here we call it a tube bank kind of structure. These are small perturbations, these are small dots which are kind of in the PDMS like small

cavities and we expected that when we flow these two fluids one from this end and another from this end, there should be at least some twisting and turning because of the small cavities locally and local eddies would be created. In the second case we talked about a triangular type of design where these cavities are now actually triangular in nature like this - they are small triangles. The third was a figurate kind of design - where we are splitting apart the flow and putting it back again and again splitting apart and putting it back and then we had this serpentine type of design where you had a meandering channel and the two flows coming from both ends here mixing along this particular channel. We are kind of towards the end of the lecture. What I would like to illustrate next time is how when we flow fluids across these different structures different patterns are formulated and which are the best designs that can promote rapid mixing. Surprisingly the results were very counter intuitive in this case and once we finish this then we will start with micro valves and micro-pumps and try to round it off in the next two lectures or so. Thank you!