**Bio-Microeletromechanical Systems**
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**Module No. # 01**
**Lecture No. # 29**

Hello and welcome back again to this 29th lecture of Biomicroelectromechanical systems. Let us quickly review what we did in the previous lecture, we talked about some of the steps towards the revision of the first Navier-Stokes conservation of momentum equation and basically we discussed, how we can represent the acceleration of a particle at a point p in a velocity wheel v, defined or varying with respect to the position coordinates and time.

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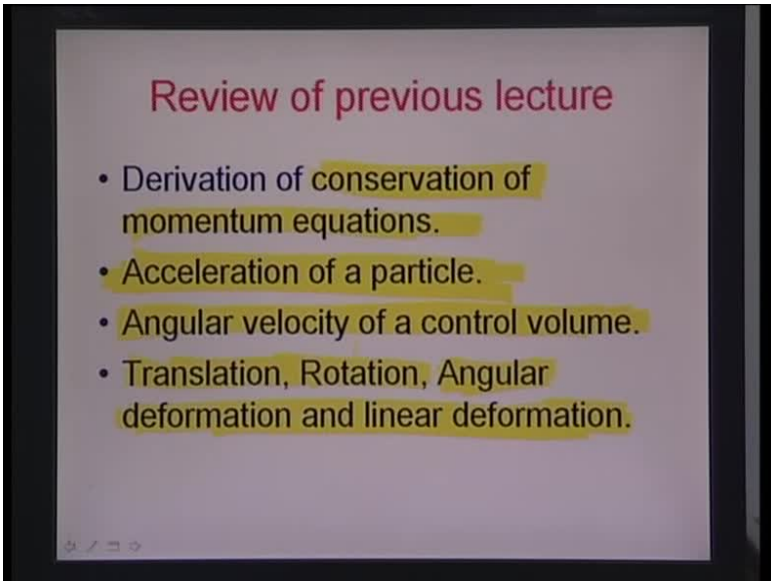


We also talked about, how angular velocity of a certain particle essentially can be related to the average velocity of both sides of a control volume. And we investigated the rotation case and the angular deformation case and found out that, they can be represented as the variation of the y velocity in the x direction and the x velocity in the y direction respectively. And we talked about these different kinds of deformations that a control volume or cubic control volume would have including translation, rotation, angular deformation and linear deformation as the control volume moves along the path of fluid in a certain medium.

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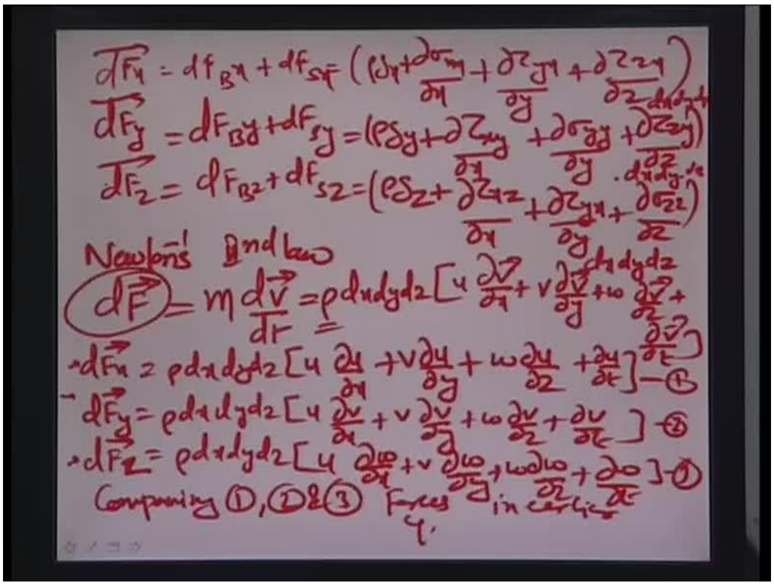

So, today, let us just go ahead and try to complete what we left unfinished in the conservation of momentum equation. We got the several force components in the x, y and z direction respectively as the equations, dF x equals dF B x which is the body force on the x direction times, the force due to stress in the x direction and we represent this as rho g x plus delta sigma xx by delta x plus delta tau yx by delta y plus delta tau zx by delta z.

Similarly, dF y and dF z respectively, dF b y plus dF sy, which is rho g y plus delta tau xy by del x plus delta sigma yy by del y plus delta tau zy by delta z. Similarly, dF z is the body force in the z direction plus d force due to stress in the z direction, which is equal to rho g z plus delta tau xz by dx plus delta tau yx by dy was delta sigma zz by dz times of dx dy dz, actually the volume element is multiplied everywhere.

So, this is also the same into dx dy dz, this is also the same times of dx dy dz respectively. So, from the Newton's second law, if you consider this control volume that really d F, the amount of force that the control volume would actually try to incorporate or face is nothing but, m the mass of the control volume or delta m times of dv by dt, where v is the velocity of the particle at a point p and it changes to vx plus dx y plus dy z plus dz at time instance t plus delta t respectively.

We did this derivation as a matter of just before we started considering the stresses in the control volume, when we talked about acceleration. Therefore, in this particular case, you can represent really this as m d velocity vector with respect to time, which is nothing but, rho dx dy dz which is the elemental volume, rho being the density; we assume rho not to vary with t or this as essentially an incompressible case.

So, times of u del v by del x plus v del v by del y plus w del v by del z plus del v by del t and if you actually, physically resolve the different components dF x dF y and dF z in this particular expression, then you are left with dF x vector is essentially rho dx dy dz times of u del u by del x plus v del u by del y plus w del u by del z plus del u by del t.

Let us call this equation 1, similarly dF y total amount of force is rho dx dy dz times of u del v by del x v del v by del y plus w del v by del z plus del v by del t, where u, v and w are basically components of velocity vector - v vector in the x, y and z direction. And similarly, dF z vector is essentially rho dx dy dz times of u del w by del x plus v del w by del y plus w del w by del z plus del w by del t. So, that is equation number 3.

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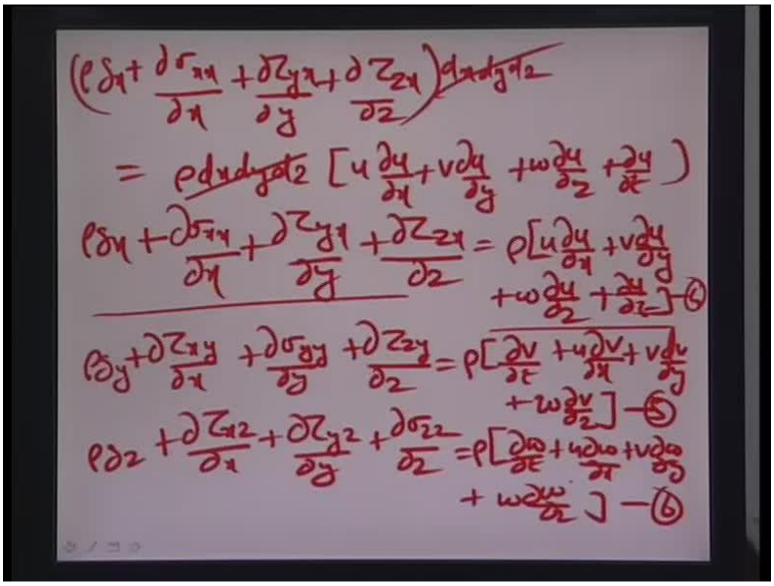

So, you have these three equations as respectively dF x dF y and dF z and we compare these to the forces obtained earlier, so comparing 1, 2 and 3 with the forces, which we got in the earlier equations, which equates the body force and the force due to the stress. So, basically, if you compare this the new set of equations, which come out because of that would be essentially rho times g x plus del sigma xx by del x plus del tau yx by del y plus del tau zx by del z times of dx dy dz essentially equal to rho dx dy dz times of u del u by del x plus v del u by del y plus w del u by del z plus del u by del t of this time variation. The time component with respect to time is a separate entity altogether as you are seeing here, as you made in the first assumption before.

So, this elemental volumes kind of cancel each other and we left with straight equations rho g x plus del sigma xx by del x plus del tau yx by del y plus tau del tau zx by del z is equal to rho times of u del u by del x plus v del u by del y plus w del u by del z plus del u by del t. So, that is what the x balance would be between the m a x; that means, the force in the x direction and the force due to the body force of the stress in the particular control volume in question. Similarly, we will do the same kind of analysis in the y direction, the z directions respectively. So, the two equations that we get as a result of it, I am just writing down. So, this let this be equation 4.

So, similarly, we have equation fifth in the y direction is comparison between all y forces as rho g y plus del tau xy by del x plus del sigma yy by del y plus del tau zy by del z equals rho times of del v by del t plus u del v by del x plus v del v by del y plus w del v by del z this equation 5. Similarly, in the z direction, we have an identical result where rho g z plus del tau xz by del x plus del tau yz by del y plus del sigma zz by del z is nothing but, rho times of del w by del t plus u del w by del x plus v del w by del y plus w del w by del z respectively. So, this is equation 6.

So, if you consider the values of the different shear stresses tau xy, tau yx similarly, tau zx and tau xz and tau yz and tau zy respectively, in terms of its respective variations of the velocity components with respect to space components, as we derived in case of rotation and angular deformation before, we will be left with a very simplified straight forward equation.

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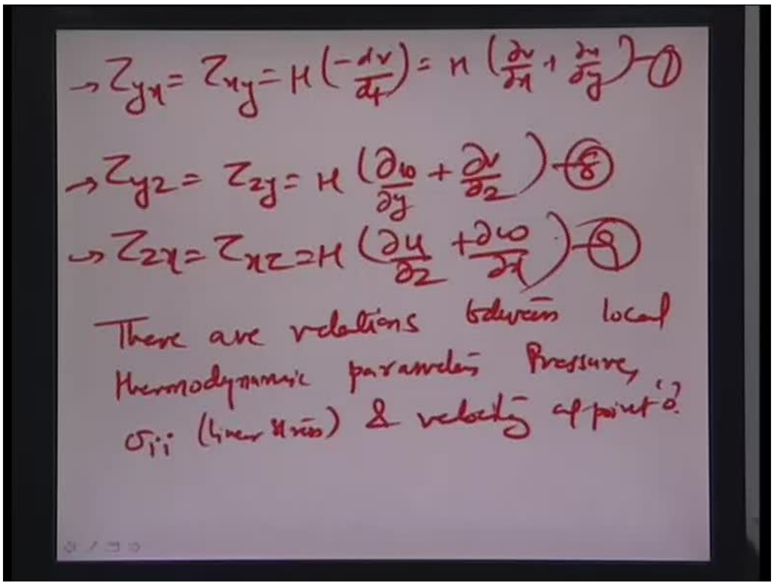


So, let us say as we have done before, tau yx and tau xy causing angular deformation can also be expressed as minus mu. Let us say d gamma by dt, which is equal to mu times of del v by del x, it was del u by del y and similarly, tau yz equal to tau zx zy is same as mu times of del w by del y plus del v by del z right. Similarly, tau zx equals tau xz and this we did actually in the last class or last lecture, how this derivation happens again du by del z plus del w by del x respectively.

There are some other approximations that we need to make here, which comes from a thermodynamic pressure, the relationship between the thermodynamic pressure, the stress components will be it shear or be it principle stress and the velocity.

So, all these three links together and I am not going to actually derive these pressure stress equation separately, it is an altogether separate topic, but I am going to assume the approximations, which are made in terms of relationships between the different stresses in the pressure etcetera and then try to put this back into the equation in question and try to figure out, what the final form of the Navier-Stokes conservation of momentum equations would look like.

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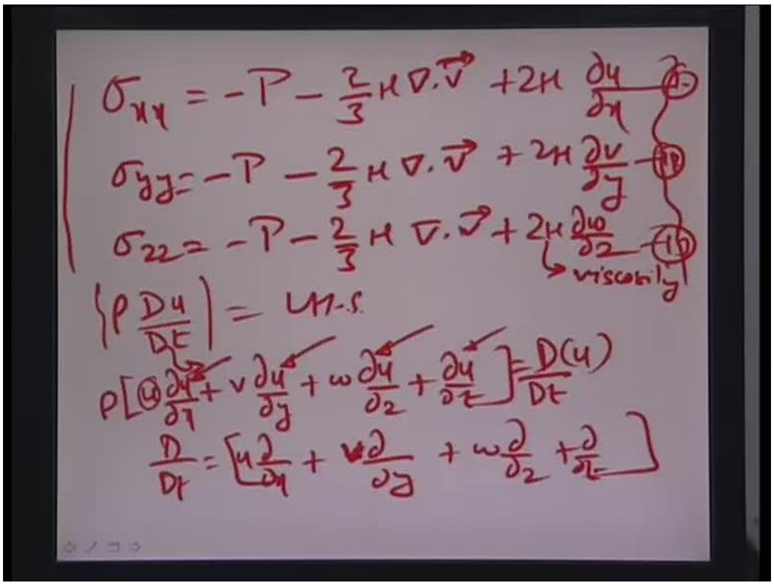


So, there are definitely relationships between local thermodynamic parameters like pressure, sigma ii and so, is linear stress and velocity at point o, around which this control volume has been indicated. If you may remember, we indicated the control volume by defining a central location o on both sides of which the control volume extents dx by, 2 dy by 2 and dz by 2 respectively with the plus and minus sign both. So, here the relationships that come based on this argument sigma xx equal to minus P, P is the thermodynamic pressure minus 2 by 3 mu del dot v v again is nothing but, u i plus v j plus w k plus twice mu del u by del x.

Similarly, have sigma yy equals minus P minus 2 by 3 mu del dot v plus twice mu del v by del y and sigma zz equals minus P minus 2 by 3 mu del dot v plus twice mu give me a minute here twice mu del w by del z mind you, this mu is essentially the viscosity relationship between the shear stress and the velocity gradient with respect to the perpendicular direction. The direction of flow and del v of course is essentially nothing but, again ratio between du by dx plus del v by del y plus del z by del x or del w by del z respectively.

So, these relationships if you assume them and we do not derive them and then put this back along with the stress vectors that we have seen before, the shear stress vectors here in equations, let us say 7, 8 and 9 we finally, get a form of the Navier-Stokes equations which really is something that under the incompressible flow conditions are assumed to be true.

So, by the final form again, so I am going to write from here by substitution of these shear stresses and the relationships between the different principle stresses, let us call these equations 10, 11 and 12 respectively. So, what was our earlier relation? Our earlier relation was between the m Dv by Dt and the forces due to the stress components which came into being and here, the relationship was really rho times of D by Dt of u. So, basically this Du by Dt here though is an operator, which we have designed in a very particular in a peculiar manner. So, as you know here the left side of the equation already was rho times of u del u by del x plus v del u by del y plus w del u by del z plus del u by del t right.

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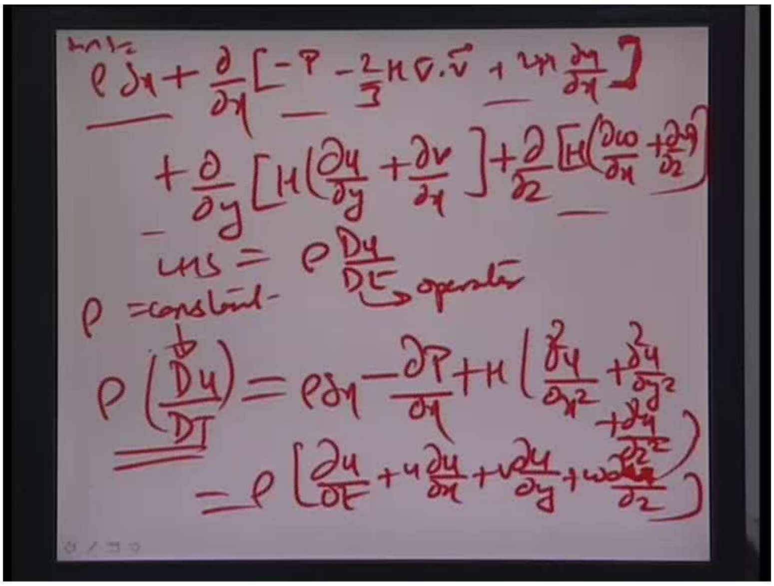

So, we take this to be the operator D by Dt of u, u being this variable here essentially, the other format the operated D by Dt is nothing but, u del by del x plus v del by del y plus w del by del z plus del by del t, where is essentially what the operators is. So, we have defined this operator in this manner. So, left side becomes rho Du by Dt and the right side of this equation as we already know, from previous examples becomes a rho g x plus del by del x of sigma xx and sigma xx as you already know, comes from the pressure equation as minus P minus 2 by 3 mu del dot v plus twice mu del u by del x that is what sigma xx is. So, d by dx is of sigma xx plus del by del y of the equation was tau x tau yx which can be represented as from the angular deformation equation, mu times the du by del u by del y del v by del x plus we had, d by d z tau z x which can again be defined from the angular deformity equation as mu times del w by del x plus del u by del z respectively.

So, this is equated in general to rho Du by Dt whereas, I told you this essentially is an operator, it is how you represent this equation. So, if you solve this whole equation here on the right hand side, this is of course, the LHS, this is the RHS of the equation. So, you are left with more particularly for incompressible flow if you assume the density is really constant, you are left with a properly more the appropriate form of equation, which is more like rho times of the operator D by Dt of u is essentially equal to rho g x minus del p by del x and plus you are left with mu times of del 2 u by del x 2 plus del 2 u by del y 2 plus del 2 u by del z 2 respectively.

So, this is the x direction really and this again as you know is nothing but, rho times of del u by del t plus u del u by del x plus v del u by del y plus w del u by del z that is what the del operator or the D operator here, D by DT really is.

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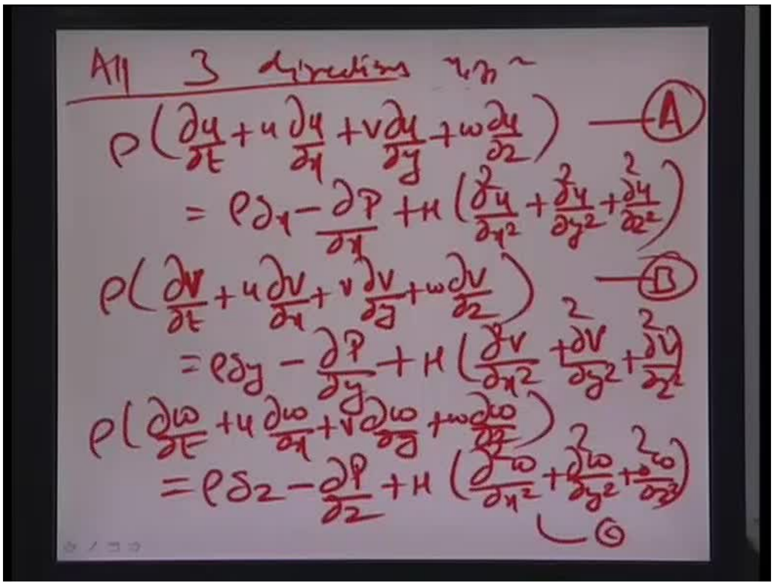


So, in a nutshell Navier-Stokes equations than in all 3 dimensions, all 3 directions x, y, z can be written as rho times of del u by del t plus u del u by del x plus v del u by del y plus w del u by del z equals rho g x minus del p by del x plus mu del 2 u by del x square plus del 2 u by del y square plus del 2 u by del z square, that is let us say equation A in the x direction. Similarly, you have rho del v by del t plus u del v by del x plus v del v by del y plus w del v by del z equals rho g y minus del p by del y now and plus you have mu times of del 2 v by del x 2 plus del 2 v by del y 2 plus del 2 v by del z 2 and similarly, this let us call as B equation in the y direction and in the C in the z direction, we call this equation C.

So, del w by del t plus u times of del w by del x plus v times of del w by del y plus w times of del w by del z is equal to rho g z minus del p by del z plus mu times of del 2 w by del x square plus del 2 w by del y square plus del 2 w by del z square respectively and this we call as equation C.

So, these are really the 3 directions of the conservation of momentum equation, Navier-Stokes second equation as you can see, I would like to further kind of try to notate the two equations that we have formulated. So, far in terms of the conservation of mass and the conservation of momentum in terms of i's and j's. So, this is a generic notation which can be used an extended to all the three dimensions, but then essentially the notational representation makes the equation much more look much more compressed. And so would essentially do a dimensional analysis on this these equations, probably in the next slide, where will see that if I can translate the scale in question, where these equations are executed to the micron level, what is going to happen to both the conservation of mass and conservation of momentum equation?

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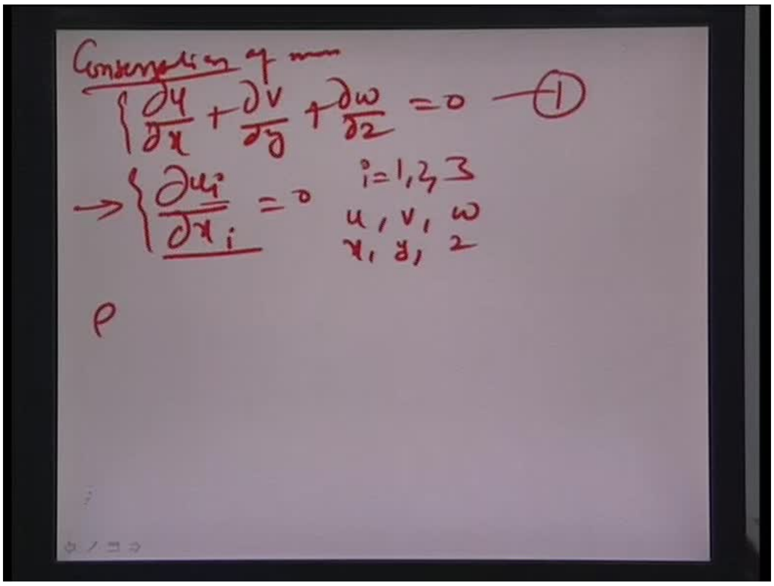

Therefore, I would like to represent these equations, this all three equations (Refer Slide Time: 22:28) by a notation and before that let us actually writes to the conservation of mass again. So, conservation of mass as here is del u by del x plus del v by del y plus del w by del z equal to 0 for an incompressible flow essentially. And therefore, this I can notate a little more appropriate manner is del u i by del x i equals 0, we assume that i is essentially or all the therefore, as you see here notionally, the i represents or i varies between 1, 2, 3 would represent u, v and w right and x, y and z. So, this is a very straightforward equation, that del u i by del x i that means, del u by del x del v by del x del w del z summation is equal to 0.

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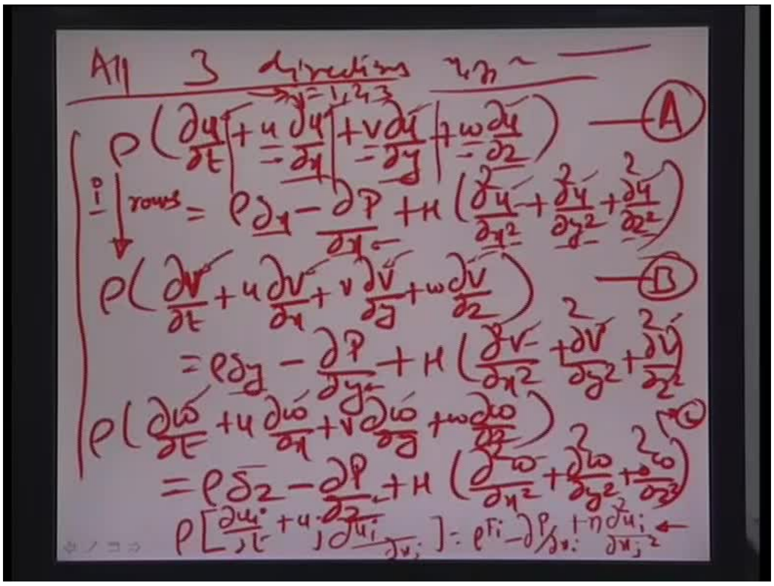


So, this is a notational representation of the conservation of mass equation. So, first Navier-Stokes equations alright, the other equation that we derive just about last slide can be notated as del; let us actually do it here and then translate back the information. So, what is this equation, really if you see here, there is a u component in all these operators in equation A, similarly there is a v component in all these operators and equation B and similarly, a w component in C and what is interesting also is that the x, y z are varying in each of these equations alright.

So, if I notate all these u's v's and w's as i that means, i varies in the direction of the rho's and j varies in the direction of the columns; as if j is varying in the direction of the columns, so I can notate this equation in a more appropriate manner as rho times of del ui by del t plus u j. Now, as you here, u, v, w are varying in the j direction. So, j varying between 1, 2 and 3 meaning this u, this v and this w is actually corresponding to the j.

So, u j times of delta u i by delta x j. So, essentially again as you see the j is varying, wherever there is a variation in the columnar direction it is j, wherever there is a variation in the rho wise manner is i, that is how you are subscripting both the variable. So, that is equal to rho times of Fi again i varies in the columnar direction, this is g x, this is g y, this is g z all these in the rho direction.

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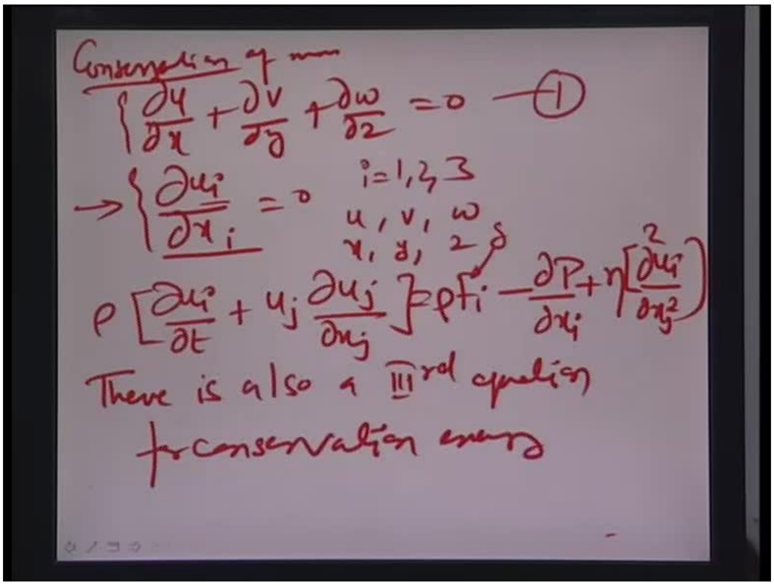


So, Fi minus del p and let me just quickly delete this right here, lets mark C here ok, this is C. So, this s del p over del x i again as you see here, in case of p in the subscript varies in a rho wise direction rho wise manner. So, that i plus eta and essentially here you have plus eta times of del 2 and you have a variation of u here, as you see u, then v, then w is in the rho wise direction. So, this is corresponding to i.

So, del 2 u i and you have in the denominator here, del x j 2 because x, y, z as you seeing here is varying more in the columnar direction. So, that is what the representational the notational representation of this particular 3 directional momentum equation of Navier-Stokes is really like. So, we can write as rho del u i by del t plus u j del u j by del x j equals rho Fi, Fi is actually a representation of the body force of g minus del pressure P by del x i plus eta the viscosity, del 2 u i by del x j 2 that is how you represent the conservation of momentum equation.

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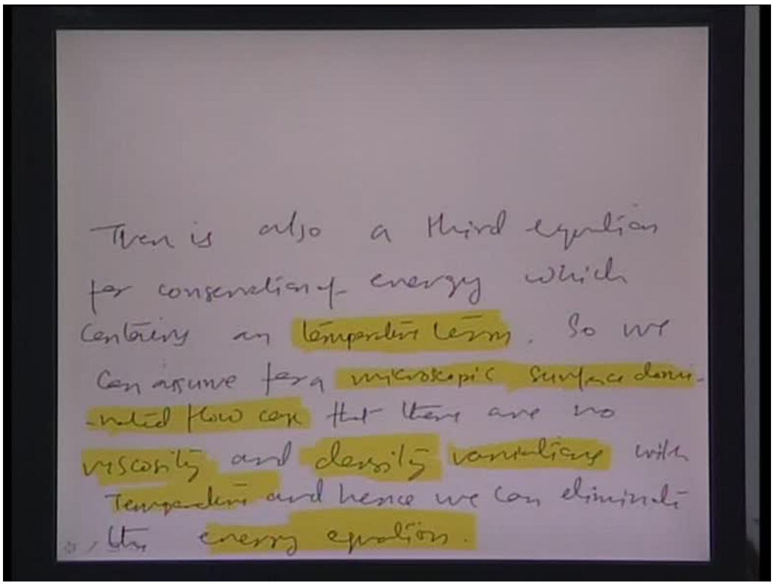


Now, there is also a third equation for conservation of energy, but essentially it contains temperature term, that is the only difference that this particular equation has and therefore, as in this particular scale, we consider in the microscopic, particularly surface domain, we our flows are mostly dominated by the prominence of the surface over the volume right. And there are effectively no not a much change in the viscosity and the density, we still as assumed continuum base properties at this particular scale, the micron scale at least. And therefore, there are no variations in these properties with temperature therefore, really the energy equation is not needed as per the micro scale flows are concerned. What I would be more worried about at this stage is that, how we can scale down the momentum equation?

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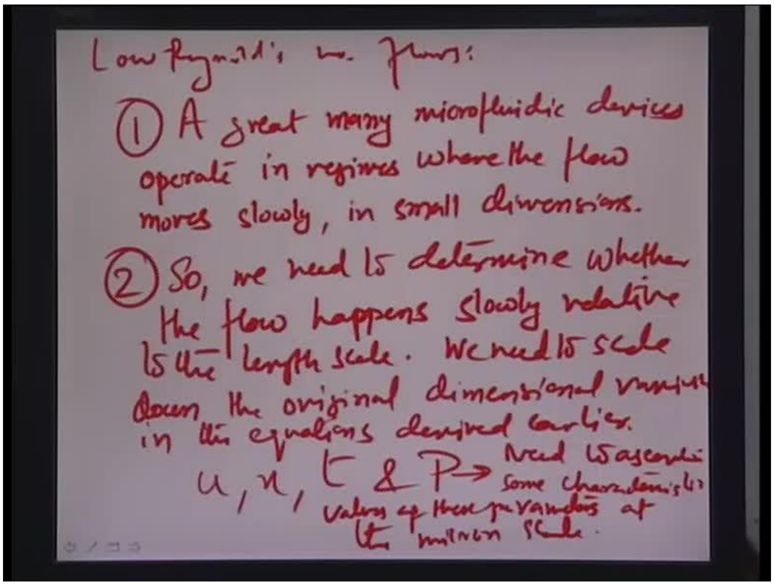


So, in this scaling equation, we have to assume the following presumptions. Number 1 is that, all flows are low Reynolds number flows. So, why we actually try to take a low Reynolds number flow is that, a great many micro fluidic devices operate in regimes, where the flow moves slowly that is number 1. Number 2 in small dimensions, that is number 2, say for instance you are talking about a very thin peace of a channel or a very thin size of the micro channel, which is defined by a photolithography. So, there the channel thickness is defined by the film thickness and the full thickness could be anywhere between let us say, 20 microns all the way to about 100 microns or so.

Now, 100 microns is effectively the diameter of a human hair. So, you can consider, what the effective volume is through which this flow would actually flow. And it is very obvious to assume that, the flow rates typically would be a few micro liters per minute; that means, the volume discharge through this thin sample is really low and so you are packing molecules in a smaller volume. And Secondly, moving them in a very slow manner which is constrained by the geometry. And therefore, most of the cases the flows are typically laminar in nature and low Reynolds number is an obvious conclusion out of all this because, Reynolds number is nothing but, rho v d by mu, where velocity v or this dimension d, length dimension d whichever is smaller, mixed overall Reynolds number very small.

So, effectively we really need to determine, whether the flow happen slowly relative to the length scale that we are considering right and therefore, we really need to scale down the original dimensional variables in the earlier two equations, the conservation of mass and the conservation of momentum. So, one of the reasons why dimensionalization, non dimensionalization is preferred at these and many other applications because, scaling down will ensure that did not have any absolute physical parameters like density, viscosity than a length scale, time scale etcetera.

So, what you instead have is a ratio and the ratio is a comparative to certain feature size or a certain parameter size, which is generally prevalent at the scale at which you are non-dimensionalizing the particular equation. So, this is a method which is used be it Lenard-Jones potential, be it micro fluidics, be it m d simulation, just to certain that you are essentially using non-dimensional variables at the scale that your experiments are all supposed to be, so the equation would be a good estimate of that particular scale when instead of a dimensional form, you use it in a non dimensionalized way compare to in comparison to parameters at the particular scale of the experiment.

So, therefore, in this particular case also, how do we do that? We first of all find out what are the variables which are effectively there in all Navier-Stokes equations. So, you have velocity u as 1 variable, space coordinate x, y, z whatever you call as this is the length variable, there is a time variable and then there is a pressure variable P. And therefore, we need to ascertain some characteristic values of these parameters at the micron scale for non-initialization.

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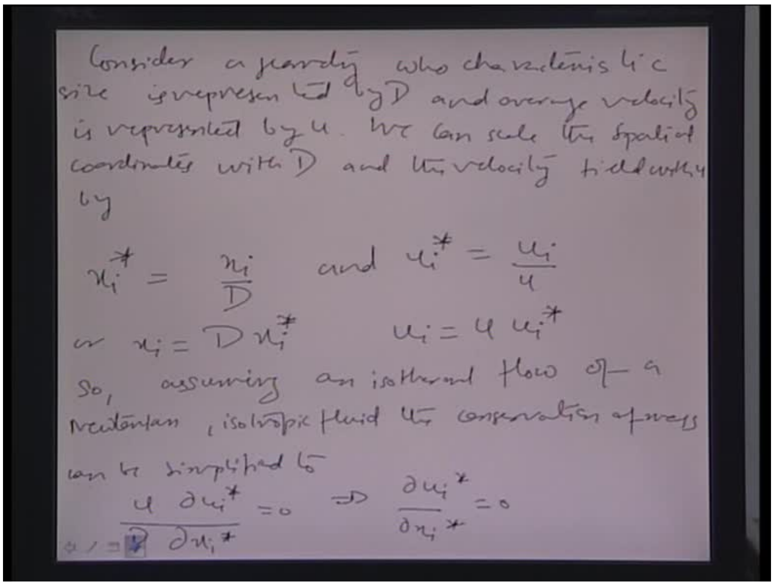


So, essentially let us consider geometry; we now consider geometry was a characteristic length is let suppose D and whose characteristics velocity is u. So, we represent everything in terms of D and u, timescale automatically follow suit and as we will see the density and the pressure etcetera will also be represented in terms of all these quantities. So, essentially that is what will be trying to scale down and we make this non dimensional numbers and call them with or notate them with the subscript star like x i star u i start, so on, so forth.

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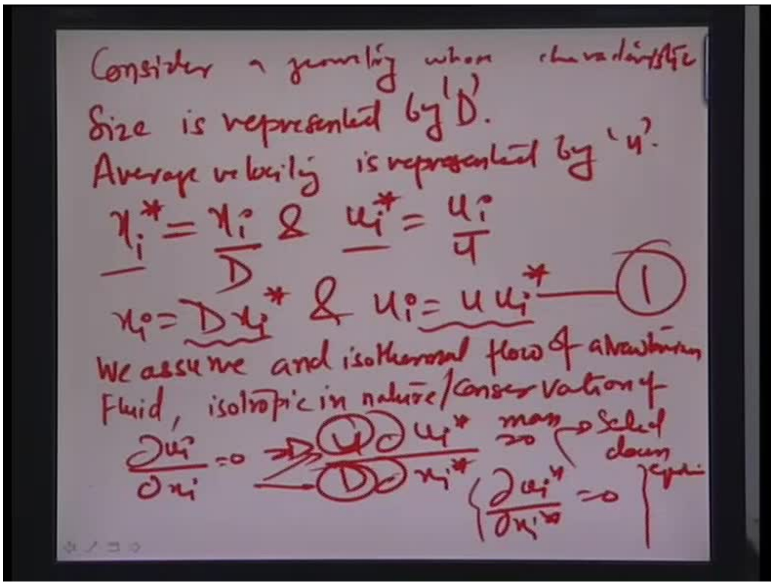

So, basically now if you look at, let us say the scaling. So, we consider geometry whose characteristic size is represented just give me a minute here is represented by of the quantity x i. So, with the quantity D, so it is represented by the quantity D. Similarly, average velocity is represented by u here. So, average velocity at that scale is represented by u. Therefore, this number x i star which is actually a dimensional non dimensional number is exactly equal to x i by D and similarly, u i star the non dimensional velocity number is equal to u i by u.

So, therefore, that is how you represent these dimensionless numbers and the idea is convert the Navier-Stokes equations both the mass as well as the momentum conservation equations into these quantities with subscript star. So, that equation is kind of a scale down model into the micro scale for applications. Now, from these two equations, we can further derive that x i definitely can be represented as D times of x i star and similarly, u i can be represented as u times of u i star. So, assuming an isothermal flow of Newtonian isotropic fluid, the conservation of mass is essentially a very simplified as D u i by Dx i is equal to 0 right.

So, we assume an isothermal flow isothermal being incompressible because there is no variation in density with temperature etcetera, all the flow is at a constant temperature. So, this is of a Newtonian fluid which means again that, the shear stress is proportional to the rate of change of velocity with respect to the cross section. So, D u by D y proportional to tau essentially, so of a Newtonian fluid essentially, which is also isotropic in nature. So, isotropic in nature means, there is no non homogeneity or inconsistency problem the mentions with the density or viscosity they are all homogeneous, there are all uniform across the whole medium.

So, we assume the three conditions. So, the conservation of mass equation than can be really represented as you saw earlier as Du i by Dx i equal to 0. So, we try to now represent or put these different quantities here which have been formulated here and let us say set of equations 1. So, we are left with u del u i star by D del x i star equal to 0; in other worlds, del u i star by del x i star equal to 0. These two being characteristic numbers representing velocity and dimensions the kind of remain constant and so they can be taken outside the differential here and therefore, del u i star by del x i star is 0. So, this is the scale down equation, scale down equation.

So, the formulation of the scale down equation in case of conservation of mass is really not very critical, it does not go unchanged just the ratio of the u i star number with respect to the Dx i star number again. However, the changes would occur when you look at the Navier-Stokes second conservation of momentum equations in significant changes would occur which can be interpreted and some of the properties essential properties of the micro scale of flows would really come, if you scaled down the second equation from Navier-Stokes. So, let us do that.

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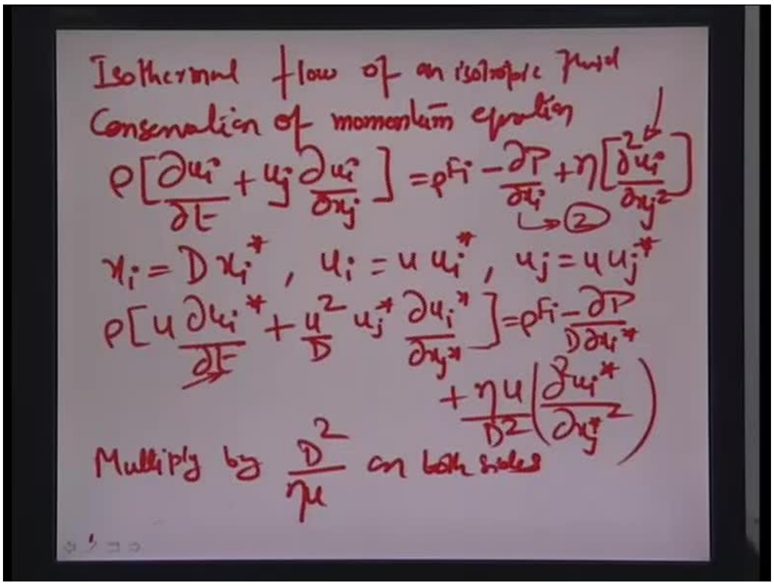


So, we assume an isothermal flow isotropic fluid. So, the conservation of so we have a so we assume an isothermal flow of homogeneous fluid, an isotropic fluid. So, the conservation of momentum equation, as we saw earlier can be represented in terms of if you just go ahead and look to the equation, the momentum equation that we derive before, it was rho del u i by del t right plus u j rather, del u i by del x j equals rho F i minus del p by del x i plus eta del 2 u i by del x j 2 that is what conservation of momentum equation was really in terms of the notational form.

Now, if you want to go ahead and substitute the different values of the non dimensional number here, we have again 2 numbers as you may just recall last slide we did this. So, x i is D x i star and all velocities whether it is u i or u j, essentially u u i star similarly, u j we just talking about a scale. So, essentially this is a representative quantity, u is a representative velocity at the particular scale of interest. So, whether it is the subscript j or i, whether it is a change in the columnar fashion or the rho fashion, the corresponding dimensioned number or non dimension number will really not change because of that. And therefore, the relationship j also holds valid have this u j as u u j star, where u j star is the dimension number in the or the variation, as j varies in the columnar manner as j is equal to 1, 2, 3 we already talking details about this notation, if you may recall, when we were trying to notate the whole set of the conservation of the momentum equations in Navier-Stokes derivation.

So, just substituting this back into the equation here, let us say this equation was equation number 2, we are left with the condition, where rho times of u del u i star by del t. We have not yet characterized or we have not yet changed the time dimension that will be doing in the next step. So, plus we call it u square by D and this should be equal to a really, u j star times of del u i star by del x j star, that is how letters can be written and is equal to rho F i again minus del pressure P by del x i star into D that is how you can calculate this plus eta and you call this, u divided by square of D as this notation represents here, it is del x j square. So, it is d square times of x j star square with the del.

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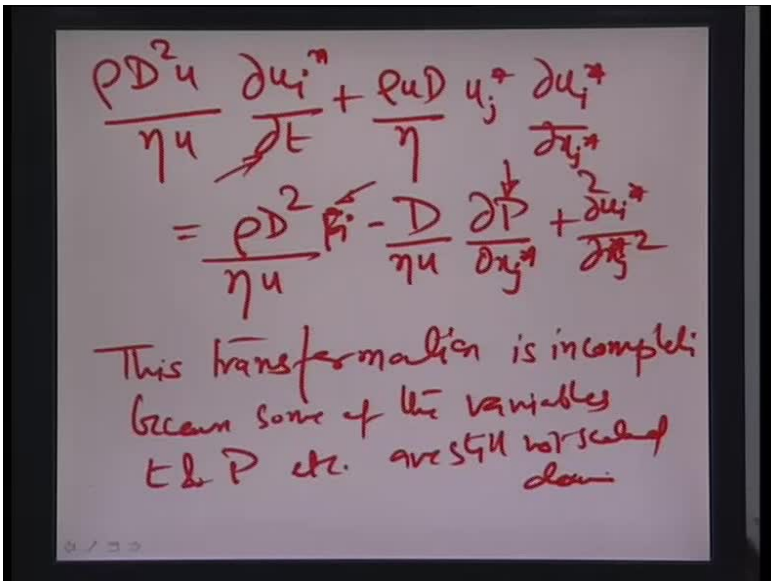


So, you have del 2 u i star here and you have del x j start square here and what comes out of the equation is eta u by D square. So, this kind of clear at the stage what this is about right and let us just do a little bit of algebraic manipulation here, we multiply this equation by D square by eta u on both sites. So, we are left with now rho D square u by eta u del u i star by del t plus rho u D by eta u j star times of del u i star by u del x j star and here, we have rho D square by eta u Fi minus del by eta u del P by del x j star plus del 2 u i star by del x j star square.

So, that is how you can write the non dimensional form of this equation, although this is not a complete non dimension form again, there are certain quantities here I will like to illustrate like t here, pressure P here or the force F here, which is still in the old domain and you have values here, absolute values of these forces and somehow we have to develop a mechanism out of whatever parameters we have now, to find out if we can really a scaled down these numbers or a scaled down these particular parameters by comparing it to a parameter of same type at that particular scale.

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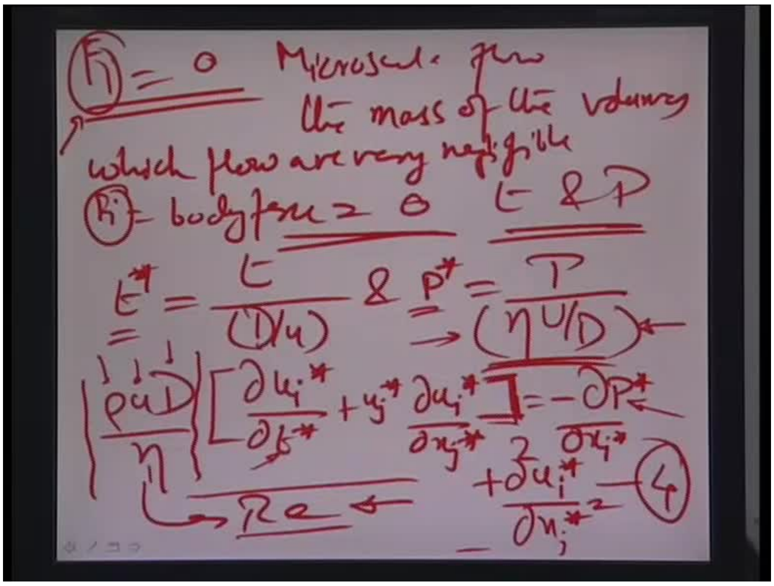

So, let us actually go ahead and transform. So, we will say that this transformation is incomplete because, some of the variables like let us say temperature, pressure, etcetera are still not scaled down right and so for that, or for doing that, we would go ahead and actually try to see how we can represent these variables. Now, for all practical purposes, the only quantity which may differ a little bit is F i, force which is related to the body force. Now, we are talking about micro scale flows, the mass of the volume which flow really a very negligible.

So, therefore, the whole business about F i of the body force is also negligibly small, we can neglected to 0. So, we need really quantities like t and P to be scaled down in order to ascertain whether we can complete a scaling down of the Navier-Stokes equation add the particular scale of reference. So, let us actually figure out what this scale would be, t start would definitely be equal to the time in this particular scale t divided by time equivalent in the scale that we are considering and we already have the corresponding scaled scaling parameters, for the micro scale of D and u, for the length and the velocity respectively.

So, D by u definitely would give an idea of what kind of timescales would be appropriate for the scaling question or for the micro scale at which this equation is being scaled down. So, therefore, we can represent t star a quantity, which is equal t by D by u. And similarly, for the pressure P star can be the ratio between P and as we know eta does not vary because eta is actually a scale independent property. It is a viscosity of the fluid and it is same across all scale whether it is micro, nano, till it goes to a level where continuum is destabilized.

But, we are talking about the micro scale, were we still the continuum holds true and so eta essentially viscosity or mu whatever you call, remains kind of fixed across all these different scales, wherever the continuum is still maintained or established. So, therefore, this pressure unit as you know is same as that of the shear stress, shear stress is nothing but, a eta u by D, the rate of change of velocity with respect to the separation distance on the perpendicular direction.

So, therefore, when we are talking about just scaled parameter, it is good to assume u by D times of eta to be the corresponding shear stress, which is required for separating such flows or layers of such flows. Therefore, this can be considered equivalent to the kind of pressure scale. So, P star again becomes P by eta u by D, now if I put all these derivations back into our equation here, which we formulated just about a minute back, we will be left with something like rho u D by eta times of del u i star by del t star plus u j star times of del u i star by del x j star minus you can call it not minus, you can finish bracket here, equals minus of del P star by del x i star plus del 2 u i star by del x j star square, let me just write this in little more clearer manner.

So, this is a del P star minus del P star by del x i star plus really a del 2 u i star divided by del x j start square. So, that is what essentially the relationship would be in a totally scaled down manner. Now, have t star here, note which is the kind of non dimensional analog of time t, when have P star here which is the non dimensional level of P star. And essentially the F i here, the body forces which was the term about towards the right inside here is neglected because we consider in micro scale flows, the volumes or the mass is involved to be too low for the gravity affect to be significantly affecting the floor.

So, therefore, F i for all practical purposes in micro scale is 0, but something very interesting is happened in this equation, let us call it equation 4. So, what is interesting here is that this term is nothing but, the Reynolds number the rho u D by n and this is kind of characteristic Reynolds number at this scale that you are considering, because u D rho and eta, rho and eta, of course do not change across the scales still the continuum is established and u and D are the scaled velocities and the length dimensions at the scale that you are questioning or concern.

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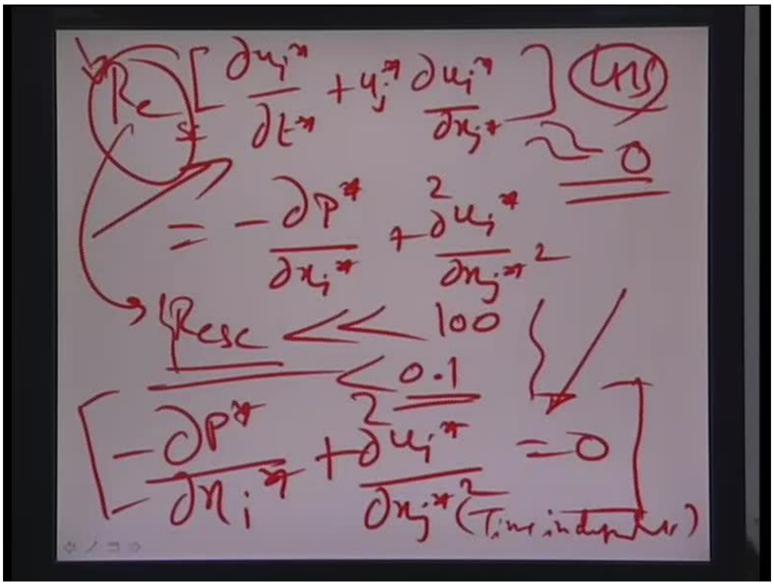

So, therefore, it can be very appropriate to assume that the Reynolds number at the particular scale, I call it R e sc times of del u i star by del t star, all these different components of the equation on the left side, del u i star by del x j star is equal to minus del p star by del x i star plus del 2 u i star by del x j star square. Now as we know that, the Reynolds number at the scales that we are looking at is really small, it is a very less than I mean, almost always less than 100 and very often less than 0.1, Reynolds number is very low.

So, therefore, the contribution coming from the LHS of this equation is kind of overshadowed by the smallness of the Reynolds number itself and therefore, LHS vanishes away, you can say that this is very negligibly small and it is 0. Therefore, the Navier-Stokes equations finally, turned around into a minus del P star by del x i star plus del 2 u i star by del x j star square is equal to 0.

So, this is very important goal that we have established here, that if you scaled down the conservation of momentum equation at the scale of the Reynolds number being very small or micro scale, you immediately find out that the equation becomes time independent. And therefore, there are certain effects and situations in the micro scale will become very prominent, where time no longer matters, I mean things like mixing etcetera if you just consider a mixing by the means of just mass transport, that mixing actually becomes insignificant at the micro scale, just because if you have 2 flows, which are timing into together onto a chip and they go side by side for a little bit and if you want to reverse them back in time, it should be able to extract the flows as it is back unmixed.

So, therefore, this is a very important conclusion out of scaling down the Reynolds number. So, we are towards the end of this particular lecture, I would like to kind of take on from here next lecture and try to show you some of the observations and conclusions that we can have from the scaling approach, which essentially starts the domain of micro fluidics and then probably go over, some of these fluid devices like mixers, valves, pumps, etcetera in little more details and try to see how they can be applied to Bio-MEMS platforms. Thank you.