Bio - Microelectromechanical Systems

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Module No. # 01

Lecture No. # 27

Hello and welcome back to this 27th lecture on Bio-Microelectromechanical systems. Let us do a quick preview of what we did in last class.

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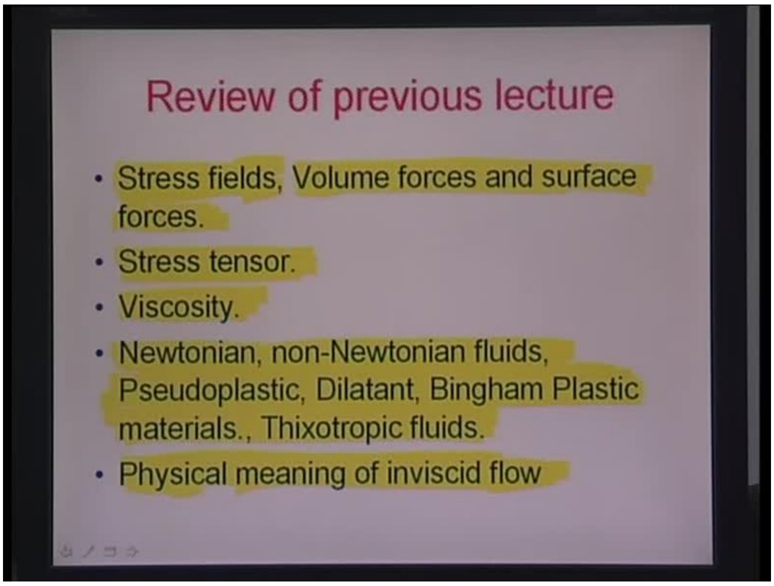

Basically, we were talking about different kind of fields like stress fields. We tried to understand what stress field is in terms of area vector and the force in the normal as well as in tangential direction. We also discussed some basic differences between volume and surface forces and some examples were kind of a coin like; for example, gravity is a big volume force; it is a body force whereas, forces relates to viscosity are more predominantly on the surface, so they are surface forces.

We talked, described the stress tensor which is essentially a matrix - a 3 by 3 matrix - where you have the principles stresses in the diagonal element and the non diagonal elements, shear stress components. We described a basic notational classification of how to represent the stress tau yx would be there is a force in the x direction and it is along an area vector pointing towards the positive y direction; that is how you call it tau yx. So, it is a stress which is shear, so area vector with this force associated is in the y direction and the force itself is in the x direction.

We saw that basic classification, we tried to derive the basic Newton's law for viscosity; Newton's law for viscous motion of fluids where in correlation was drawn out between the shear stress tau xy with the rate of deformation du by dy and we classify different fluids as Newtonian, non-Newtonian. Newtonian, wherein this stress and the velocity gradient are in directly proportional with each other, constant of proportionality being viscosity which later on got converted into kinematic viscosity because a better physical idea would be to compare the viscosity absolute values with density of the solution or the medium.

Then we talked about various different kind of non-Newtonian fluids like pseudo plastics wherein the viscosity seems to go down with the deformation rate dilatants, where the viscosity would have a reverse behavior going up with the deformation rate. Then Bingham plastics, which would essentially behave has a solid up to a certain yield stress beyond which it will follow the path of a Newtonian fluid. Then we talked about thyrotrophic material essentially, where we described about properties related to the variation or the temporal variation of viscosity with time. That means, the viscosity index eta would vary temporally with time, it will actually decrease with time.

So, then we were just about describing the differences between viscous and inviscid flows; inviscid flows again definitionally are flows where the viscosity can be treated as 0, it is normally it is really an ideal situation; it never exists in nature or there is no fluid in nature which exist with the viscosity of 0 value, but then in macro scale flows or in macroscopic flows, we can consider a region which becomes inviscid because of being away from a flat plate. We were actually describing the situation by considering what would happen when a flow of some uniform velocity passes over flat fixed plate. The proximity of the plate does not any more matter to create a velocity gradient; so those are inviscid regions of the flow.

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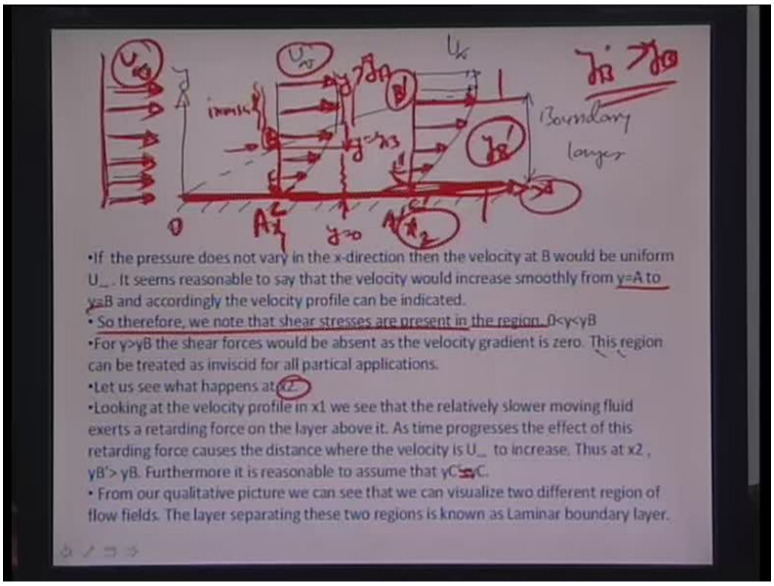

Let us go ahead and actually look at a little more of what really happens when the flow meets a flat plate. We were talking about a flow coming in the x direction with a uniform velocity U infinity as you can see here (Refer Slide Time: 04:27) and a flat plate being positioned in the o x direction as this. Then we were talking about two points A and A dash, which were represented as x 1 and x 2 on the x coordinate. Some conclusions about this process is that, if the pressure does not vary in the x direction and the velocity at B would be uniform U infinity. So, we can assume that it kind of seems reasonable to set the velocity would increase smoothly from y equal to A to y equal to B.

You have a case here (Refer Slide Time: 05:11) where there is no gradient of pressure in the x direction. Here the pressure is pretty much constant, you assume U infinity to be constant at the initial; when it is approached the plate then you consider that there is always a zone of no slip which comes into this layer, which is close proximity of the plate which actually goes all the way up to U infinity beyond a certain y. Let us say, the point where it goes to U infinity is B, so there is definitely shear stresses in the region B C; C is the point at the surface here and B is the point from which the velocity goes back to U infinity and beyond this, the flow behaves as a inviscid that is how we interpreted in the last class.

Therefore, we note that the shear stresses are present in the region 0 to y B in this particular region (Refer Slide Time: 06:14), so y equal to 0 is this plane and y equal to y

B is this plane. Essentially, for any y greater than y B in this particular case as you see shear forces are absent because the velocity is now all uniform and is rhyming very well with the initial velocity U infinity. There are no - whatsoever - shear forces in this inviscid region and the viscous forces or the shear forces are only between the y equal to y B and y equal to 0 in this particular region here.

So, we will just see what happens in x2 in this particular point. Let us look at the velocity profile in x1; we see relatively slower moving fluid exerts a retarding force on the layer above it. As time progresses, the effect of this retarding force causes the distance, where the velocity is U infinity to increase. Thus, at x2 y B dash has to be further away than this point here which is a point of contact C dash.

This is a kind of proportionation rule that as the flow enters this region and it is like let us say at point C, there is a certain velocity gradient that is established between the 0 point of 0 velocity; this B where the velocity is now U infinity but, as the flow propagates along the plate this frictional force kind of predominates. Therefore, this region here where the velocity would go back to U infinity should definitely increase because there is energy loss in forms of friction as you move from point C to C dash; C dash of course, is this new point here as you can see from this arrow.

So, if you assume this to happen, then we can think that the fluid is applying retarding force to the plates, the force goes on increasing as it goes along from 0 towards x2. Definitely the y B dash here (Refer Slide Time: 08:32) which is essentially this distance, should be greater than y B because it takes a y; because the retarding forces more at B dash, I mean C dash B dash plane, this plane.

Therefore the fully developed flow here up till y, which is y B dash should certainly be greater than the value y B. We can also reasonably assume that y C dash, so y C and y C dash are pretty much same as you can see here; the reason being that no slip zone would always be kind of closed to the surface it does not go beyond into the bulk.

So from our qualitative picture, we can see that, we can visualize this two different flows by a separating layer between them; one where there is a shear which is existing at the bottom starting from the plate all the way up to where the fully develop flow has happened that is U infinity; another which is the inviscid region, where it starts from the U infinity I mean the fully develop flow and goes into the bulk. The layer which is separating these two is also known as what we call the laminar boundary layer, it is called laminar boundary layer.

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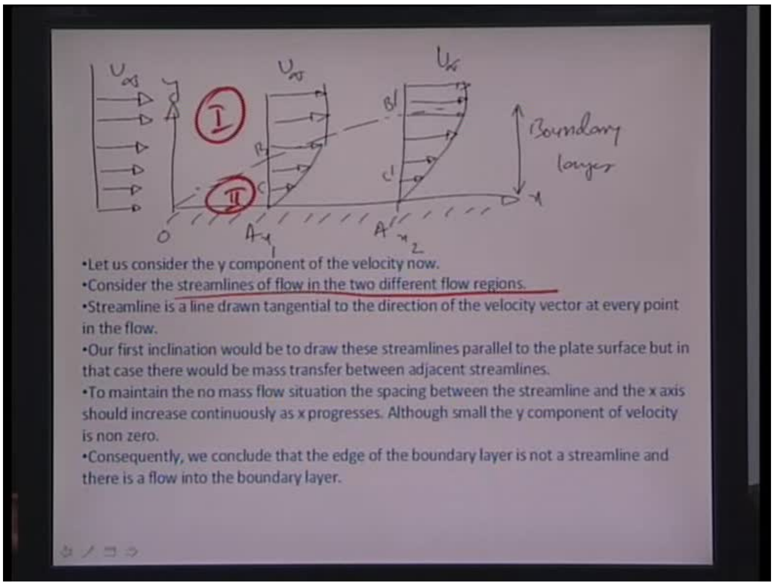

Essentially, if we consider the y component of the velocity now there is a very interesting thing which comes out that let us say, we consider the streamlines of the flow. As we know from earlier definition, what are the streamlines they are tangential vectors or line joining the tangential vectors to the direction of flow of particles, so that is how streamlines can be categorized.

Let us consider the streamlines of flow in these two different flow regions; flow region I here (Refer Slide Time: 10:40), which is the inviscid region and flow different II here, which is the viscous domain and what would be need to assume to maintain consistency?

So, our first inclination would be to draw the streamlines kind of parallel to the plate assuming that the fluids go pass the plate parallel. Now, interestingly if there are parallel streamlines generated parallel to the surface of the plates and we are saying that in one case that is a lesser amount of velocity, which increases all the way to U infinity, another case there is all U infinity.

There would not be much problem in the region I of the inviscid region but, in region II definitely there is going to be mass transfer in the y direction because, principally the amount of feed of a fluid if we want the continuity to be valid or if you want to assume the fluid is a large continuum, there cannot be any gaps in between; it can be one indivisible mass of a substance flowing over the plate. In that case, if there is a velocity gradient in this, the tendency of the lower layers parallel to the plate to reach at a slower rate at a certain point, the upper layers would move at a higher rate and try to preoccupy that point; so there is going to be a mass transfer in order to balance such a system of flow.

This situation does not really exist because as we know that they can be velocity gradient but, there cannot be any velocity in the y direction or even if there are velocities is in the y direction this continuum failure never happens within the fluid. What is needed to maintain the no mass flow kind of situation? The spacing between the streamlines difference; streamlines go on increasing their distance from the flat surface as the flow goes along.

So, the streamlines are all kind of merging out from the point where the flow is just entered along as if the flow goes along the surface the streamlines get separated by greater and greater distances. So they are not really parallelly oriented, they have different directions which go on spreading up more and more as the flow goes along the direction of the plate.

Essentially, we conclude that the edge of the boundary layer is not a streamline and it is because streamline is something across which there cannot be typically any mass transport, because tangential to the direction of the streamline the particles are all moving there velocity vectors are in the tangential direction to the streamlines. There is no inward radial flow from one streamline to another.

So the boundary layer which is the separation layer between the inviscid flow which is on the top and the viscous flow which is in the bottom is really not a streamline because there has to be a kind of mass flow to maintain the balance between the lower velocities and the fully develop flow velocity U infinity across this layer. So, consequently we conclude that the edge of the boundary layer is not streamline and then there is a flow into the boundary layer assuming the differences in the velocity across it.

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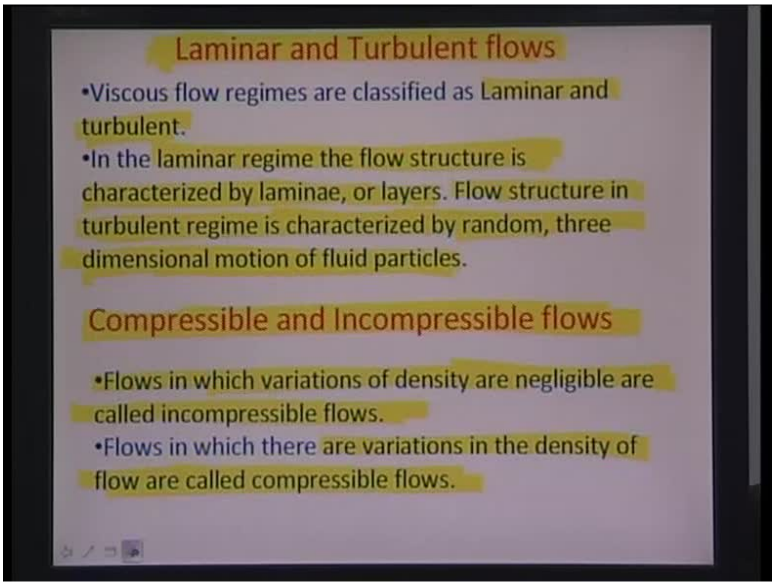

Based on some of these concepts, we can divide all the viscous flow regimes into laminar and turbulent. In laminar regime, the flow structure is characterized by laminae or layers. This is the regime where most micro flows are kind of packed up and the flow structure in turbulent a regime which is mostly a macro scale version of flow is by random three dimensional motions of fluid particles.

So we have already classified fluids as viscous and inviscid, we have categorized fluids in to laminar and turbulent, we can also categorize fluids into compressible and incompressible. Essentially the main difference is that in compressible flows there are variations of density along the fluid medium whereas, in incompressible flow we assume the density to be just a constant across the whole continuum of the fluids.

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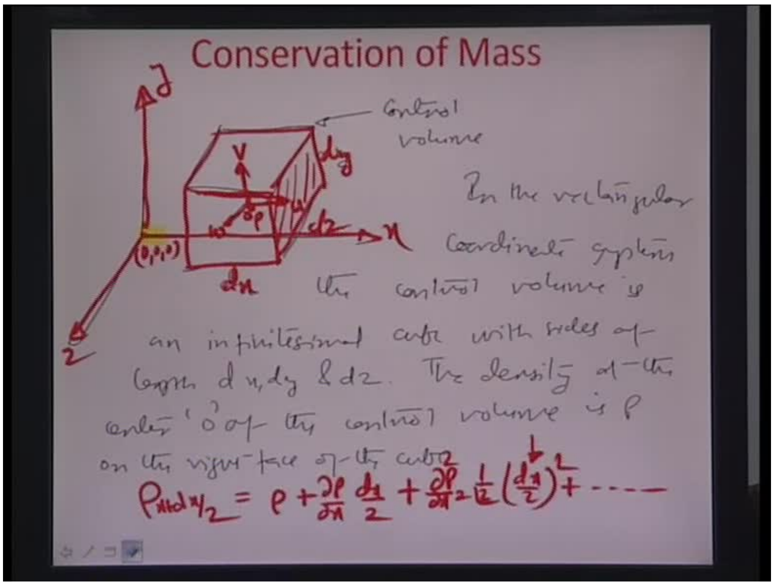

The flows in which variations of density and negligible are called incompressible and there were the variations in density are substantial they are called compressible flows. That is how you divide flows in to compressible and incompressible part from laminar and turbulent, and viscous and inviscid flows.

Let us now try to go ahead and derive the first equation of conservation of mass or what we call that the first Navier-Stokes equation. For that we need to assume again a small control volume let us say, we are trying to see the amount of mass flow into this control volume and the amount of mass flow outside this control volume, it is centered around point o. This is further like cube around this point o, a rectangle around this point o with dimensions dx dy and dz in the x, y and z directions.

What we would be looking at that if we assume that there is no creation of mass within this control volume. So whatever is inflowing into the mass is exactly the control volume and it is exactly equal to the mass that is out flowing of the control volume. So, this is also knows the continuity equation or the conservation of mass equation. Let us try to mathematically or geometrically derive this particular equation.

Let us say, we have an x, y, z direction here is rectangular co-ordinate system and we assume a control volume of rectangular shape sorry cubical shape with values dx, dy and dz dimensions (Refer Slide Time: 17:47).

We further assume this point o around which this control volume is equally spaced, symmetrically spaced. We have three components of velocity u, v and w it is a three dimensional flow, this is the origin 0, 0, 0 (Refer Slide Time: 18:22) and we are trying to investigate what happens in this point o. So the very first thing that would like to investigate is the density, given we have a density rho here at point o, what would be the density? Let us say, rho x plus dx by 2 which is this particular phase here. So, this can be expressed again as kind of Taylor approximation as rho plus d rho by dx times of dx by 2 plus d rho by dx d 2 rho by dx 2 times of 1 by 2 factorial dx by 2 square so on so forth.

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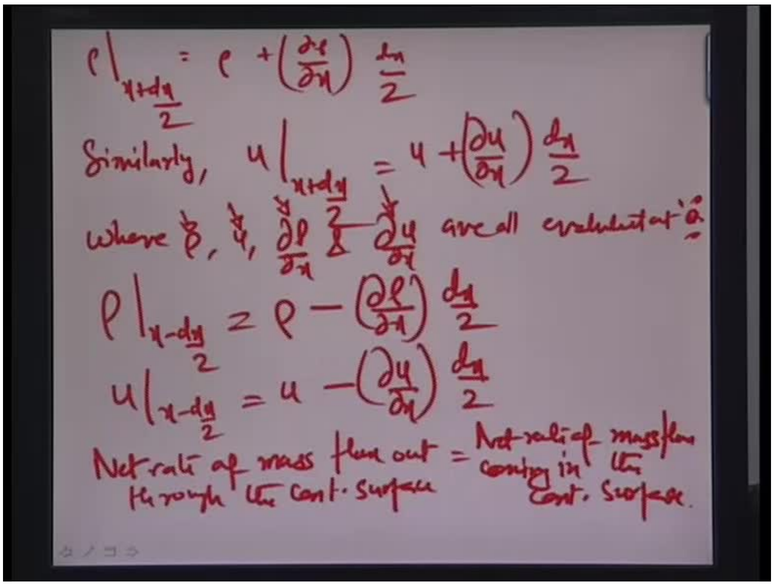

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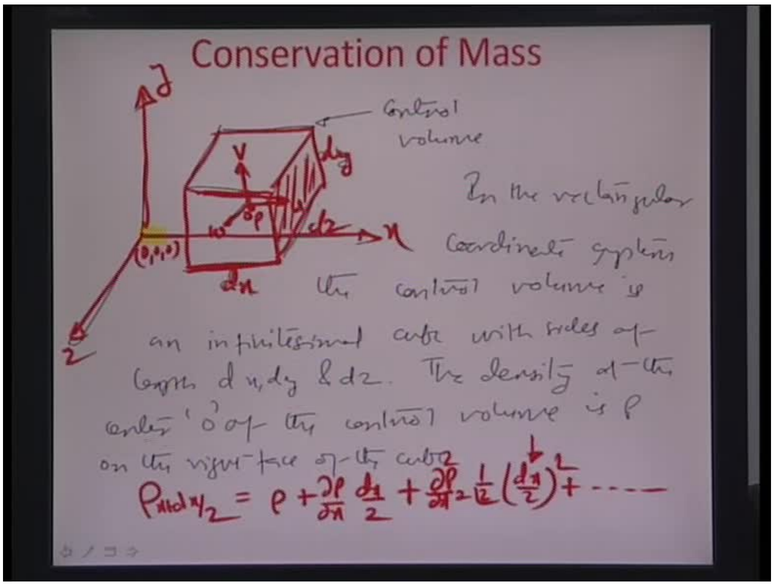

So if you assume these dx to be infinite decimally small element and neglect all the higher order terms here, the rho x plus dx by 2 really comes out. We are taking the x by 2 is because we assume this whole length to be dx and this at the centre. So, this phase (Refer Slide Time: 19:40), this shaded phase here is at distance of exactly dx by 2 from o and that is why the dx by 2 term.

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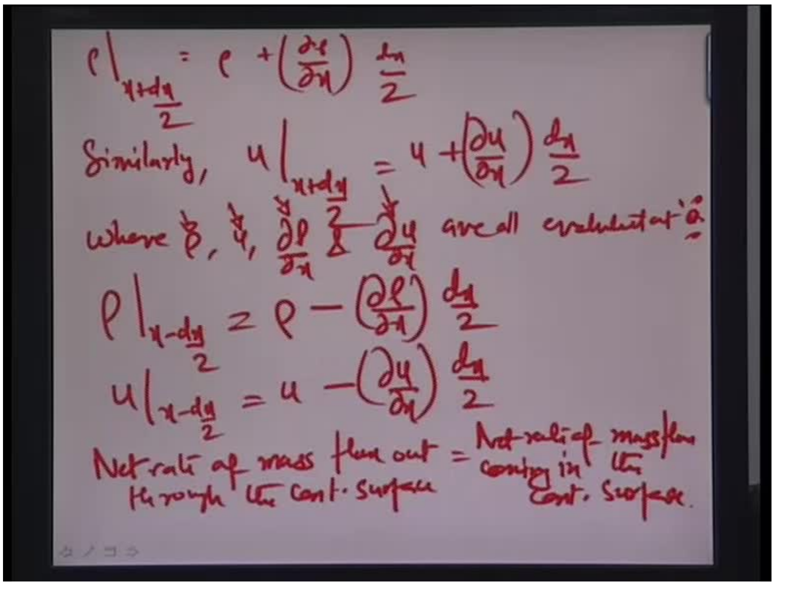

So rho at dx x plus dx by 2 is nothing but, rho plus del rho by del x times of dx by 2. Similarly, u at x plus dx by 2 is essentially equal to u plus del u by del x times of dx by 2, where rho, u, del rho by del x and del u by del x are all evaluated at o. You have to keep this in mind because we are essentially evaluating, what is happening at one of the edges based on the properties of the point o. All these values that means including the change of density with respect to x, the change of velocity with respect to x the velocity and the density must necessarily be at the point o.

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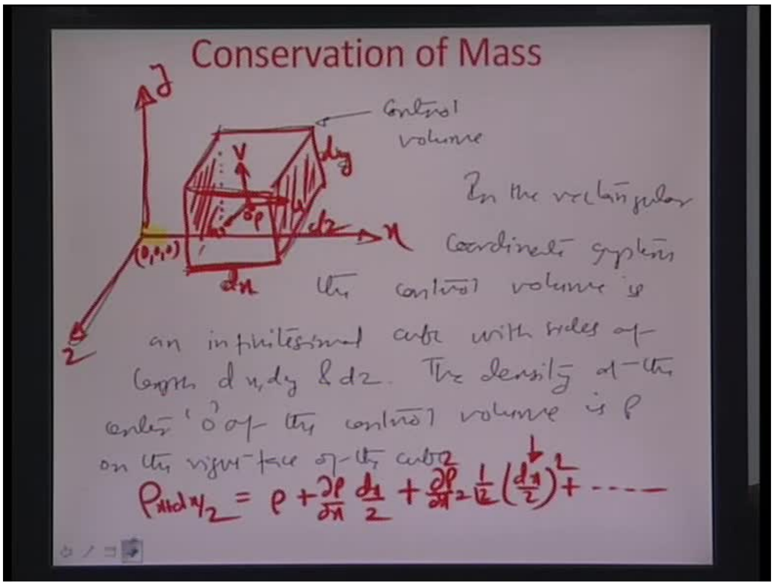

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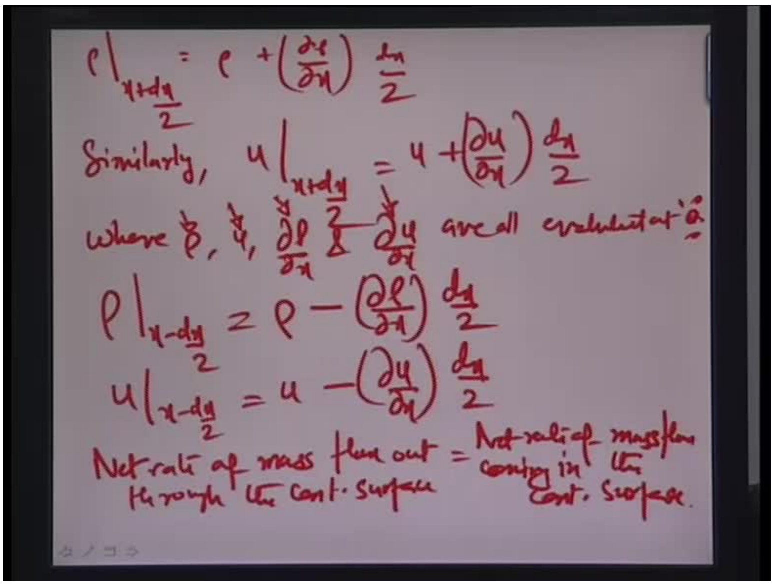

Therefore, we can write similar equations for the other phase; that is the phase on this negative side, this particular phase and here we can say rho at x minus dx by 2 is equal to rho minus d rho by dx times of dx by 2 and u at x minus dx by 2 is essentially u minus del u by del x times of dx by 2. Now, for a conservation of mass let us say, one direction or the one dimension we have to necessarily assume that the net rate of mass flux out through the control surface is essentially equal to the net rate of mass flux coming in the control surface. So it is an assumption or a supposition that we have to necessarily make here.

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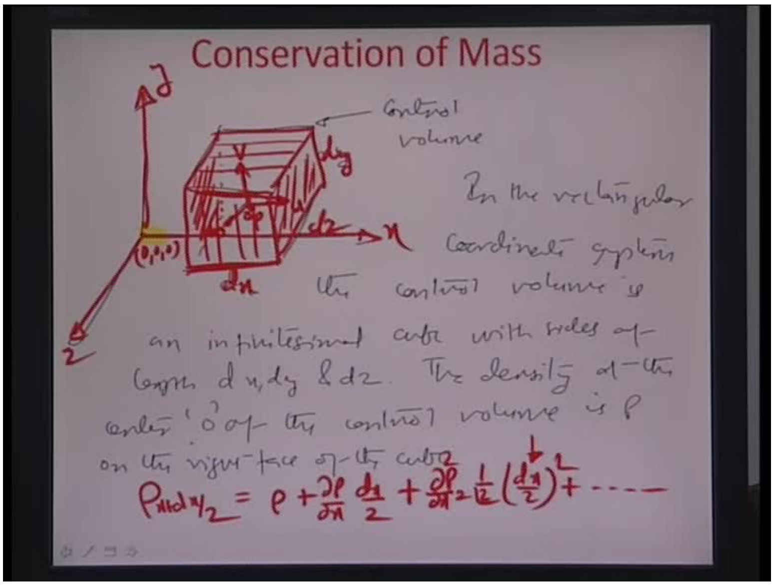

Therefore, we have to really see that there are not only these x phases but, also y phases and z phases and also phases along the minus y and minus z direction. So the whole equation can be thought of as problem, where all these different phases of inflows and outflows; we are trying to see how the fluid masses conserved in this particular case.

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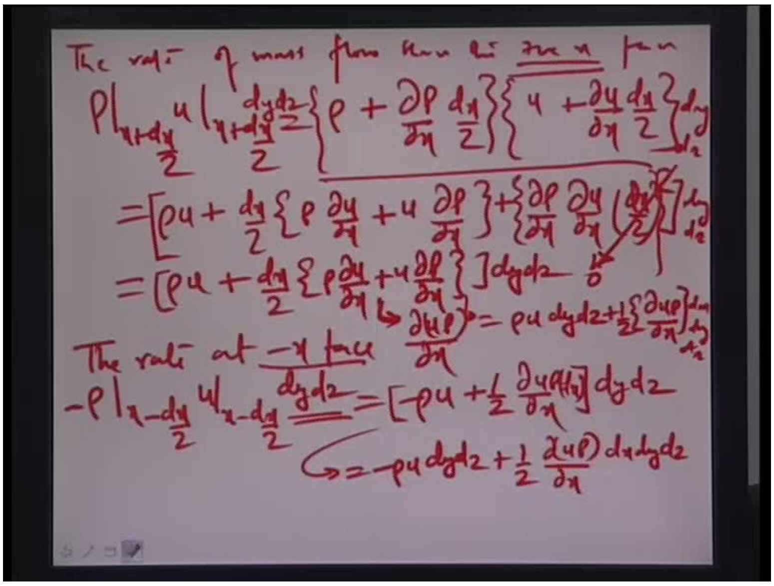

What we do here is let us evaluate the rate of mass flow - inflow and outflow at all the different phases. So, rate of mass flow through the positive x phase is the one to begin with of the control volume. We know the density times of the velocity is times of area is really the mass per second, so density times of velocity times the area of the phase is mass flow per second. Here in this case, we can write the density at x plus dx by 2 times of the velocity at x plus dx by 2, more simplified manner is rho plus del rho by del x times of dx by 2 times of u plus del u by del x times of dx by 2 and this if multiplied to the phase of the interfacial area.

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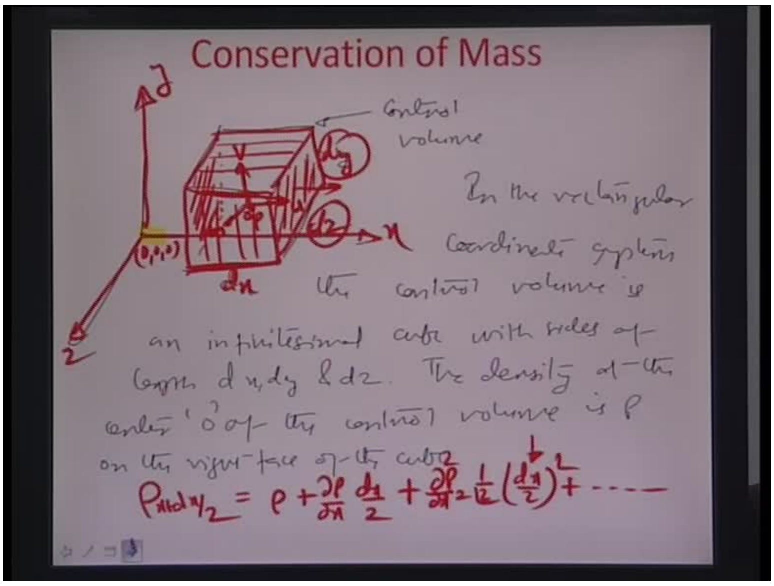

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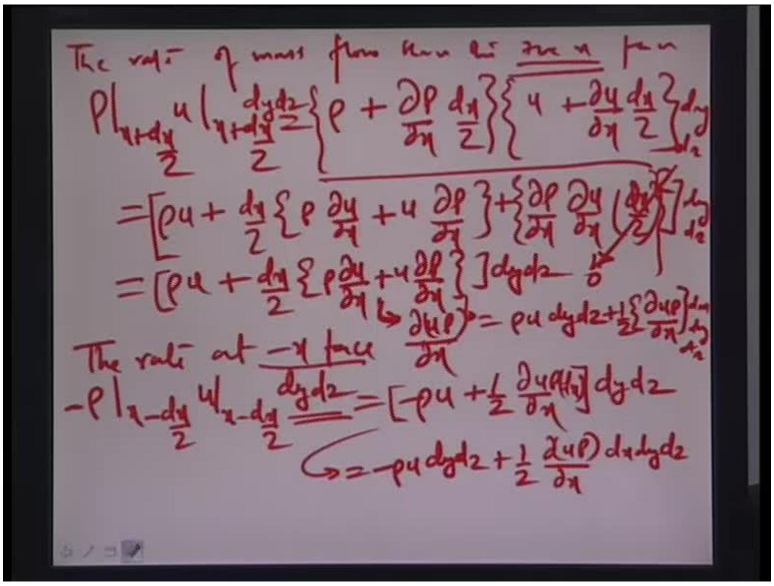

In this case as u see in case of x is nothing but, dy times of dz; so this is the area vector of this particular phase. You multiply this with the area dy dz in order to get the mass flow rate dy dz. If you solve this particular equation, you have the resolved value as rho u plus dx by 2 times of rho del u by del x plus u times of del rho by del x times of dx by 2 of course, plus you have a component here with all the del's - del p by del x, del rho by del x times of del u by del x with square of the dx by 2. Now, we assume that the component dx being very small into dy dz, the area. The component dx by 2 very small this actually can be approximated as 0 which eliminates all together this particular term here.

We are left with the equation rho u plus dx by 2 times of rho du by dx plus u del rho by del x times of dy dz and this is nothing but, del rho u by del x; this basically comes from the differentiation of product formula. So that is what the mass flow rate is really towards the positive x phase. Now let us see, what this rate would be at the negative x phase and the only difference in this case would be that the rho and the u both are evaluated at the x minus dx by 2 phase, the area vector almost remains same in magnitude dy by dx. So the area vector, although it is same in magnitude by it is actually negative in direction; it is in the exactly opposite direction points to minus x side in this case. You have to have a minus sign representing the direction of the area vector dy dz.

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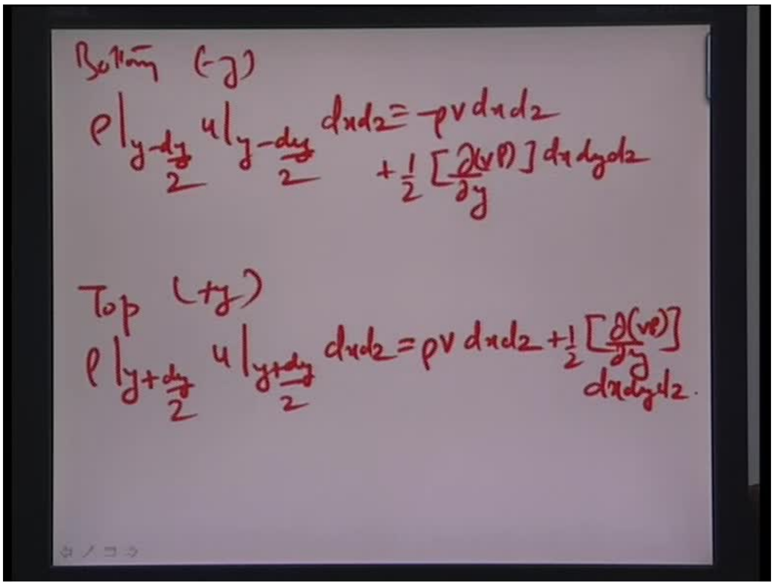

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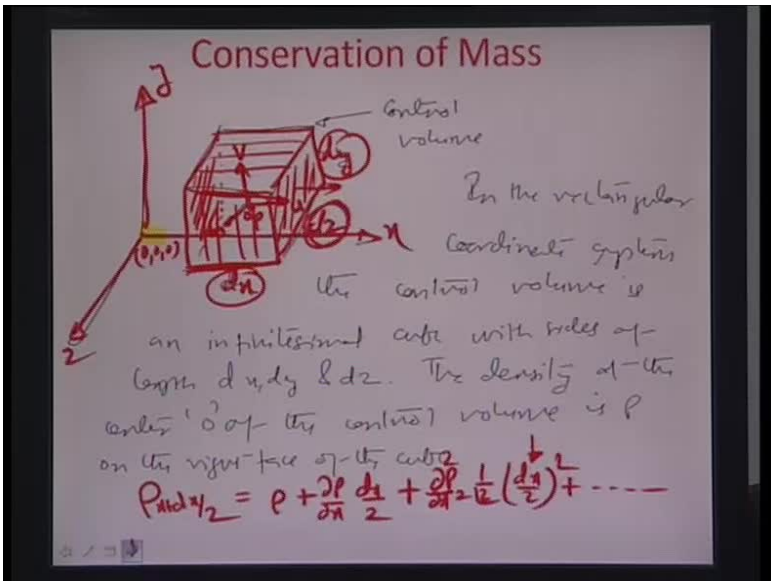

In this case, the expression can be simplified as minus rho u plus 1 by 2 times of again del u rho by del x, whole multiplied by dy dz. From these two terms, I can further simplify this equation as well as this equation (Refer Slide Time: 27:37) and write it down as rho u dy dz plus half del u rho by del x times of dx dy dz. Similarly here we write as minus rho u dy dz plus half, so there has to be a dx term here, I am sorry, so half a del up by del x times of the del x del y del z, so that is how the rated negative x phases.

Let us do the same for the positive as well as the negative y phase, so for the bottom pointing towards the negative y direction, we can represent this as rho at y minus dy by 2 times of u at y minus dy by 2 times.

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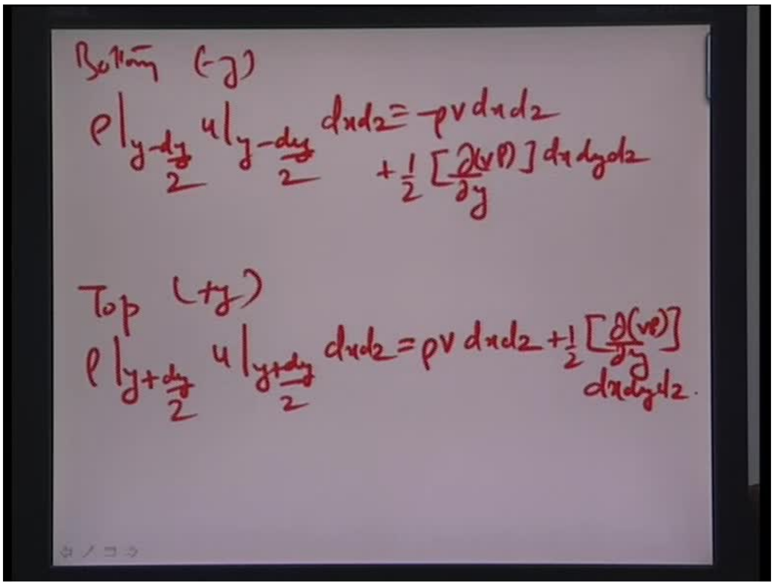

In this case, if you look at the area vector in the negative y direction it is dx times of dz. Therefore, this can be represented as rho at y minus dy by 2 times of u and y minus dy by 2 times of dx dz and this comes out to be again further simplified; in a simplified manner comes out to be minus rho v dx dz plus half times of del by del y of v rho really times of dx dy dz. For the top surface pointing towards the plus y direction, this would come out to be rho y plus dy by 2 times of u, y plus dy by 2 times of dx dz. This if simplified would come out to be rho v dx dz plus half in the differential of with respect to y of v rho times of dx dy dz.

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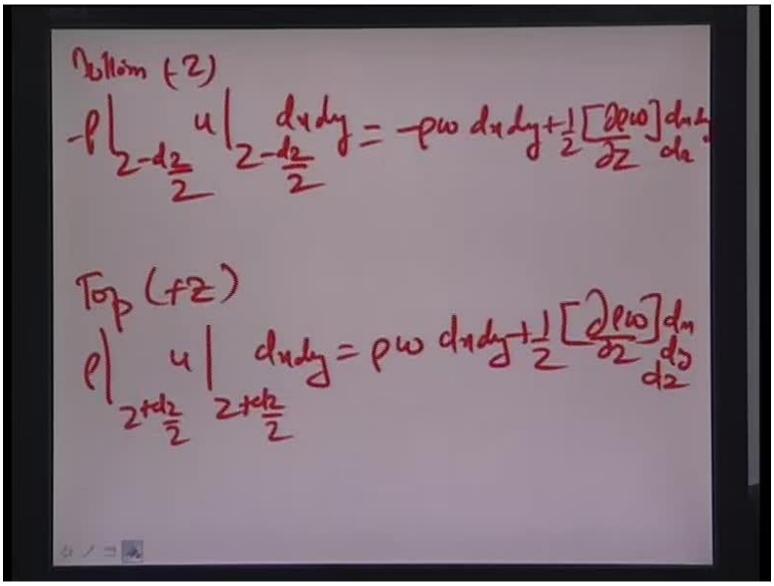

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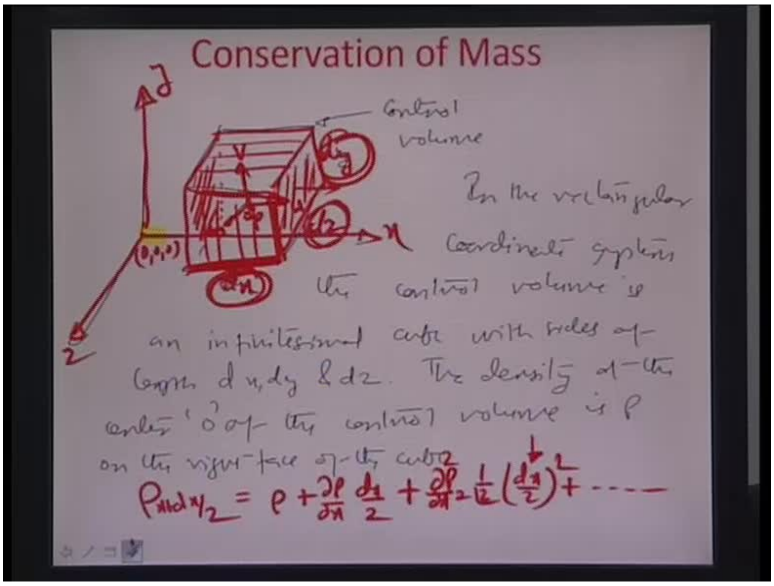

Similarly, we do the same for the phase pointing towards the minus z direction and here we can write the velocity vector to be rho at z minus dz by 2 times of u at z minus dz by 2 times of, because this is the z direction again the area which would be representing this is dx times of dy; dy is this (Refer Slide Time: 30:47) and dx is this direction, so it is this interface that is actually being represented here.

Let us write down as dx dy here; therefore, we can represent a this in a more simplified manner as rho w dy dx plus half times of d by dz times of rho w times of dx dy dz. Similarly, for the top pointing towards the plus z direction we have rho and this is minus sign, we have rho z plus dz by 2 times of u at z plus dz by 2 times of dx dy, which actually equal to rho w dy dx plus half times of del of rho w by del z times of dx dy dz.

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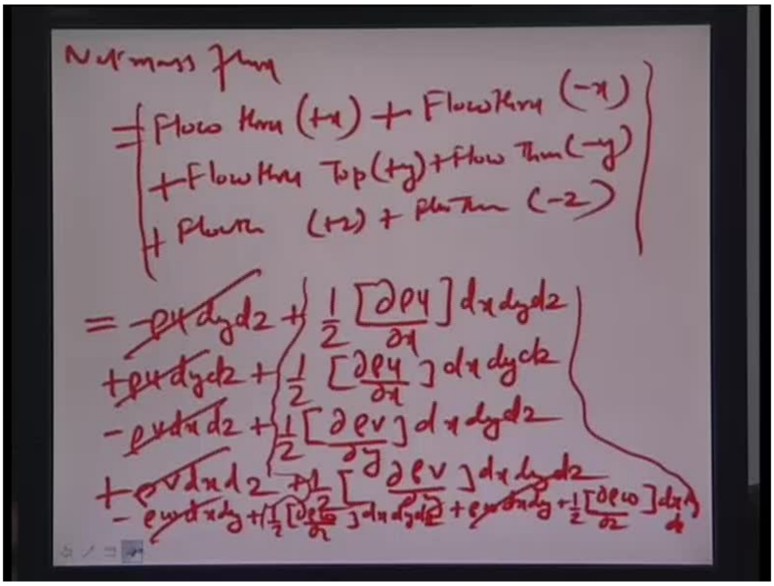

Essentially, the net mass flux then as I already talked about should be equal to the flow through plus x direction plus the flow through minus x phase plus the flow through the top plus y plus flow through minus y plus flow through plus z plus flow through minus z direction. We also assume here, that if there were making it very generic in nature. First of all, let us find out what is the summation of all these different flows. So, that comes out to be a equal to minus rho u, I am just borrowing these from the earlier expressions that we have derive minus rho u dy dz plus half of del rho u by del x times of dx dy dz plus the rho u times of dy dz plus half of del rho u by del x times of dx dy dz plus we have similar terms for the plus y minus y and plus z minus z direction; let us write them down. So, for the y direction you have minus, just erase this particular illustration here for a minute.

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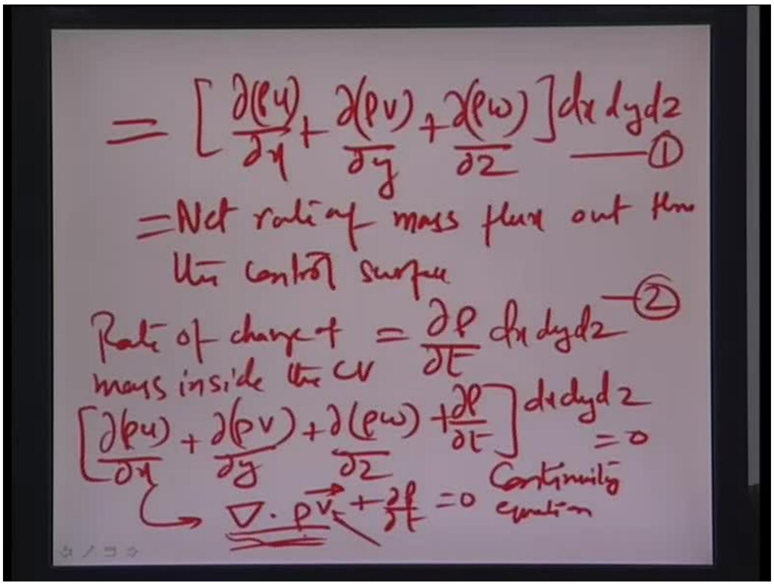

For the minus y direction we have minus rho v dx dz plus half of a del rho v by del y times of del x del y del z plus rho v dx dz plus half of del rho v by del y times of dx dy dz. Similarly for the z direction, you have rho w dx dy plus half del rho w by del z times of dx dy dz plus rho w dx dy plus half of del rho w by del z times of dx dy dz. Interestingly these first terms actually cancel each other as regards the plus x minus x, plus y minus y and plus z minus z direction, what you have left with are the second terms here, which if we sum up together we would be getting an equation which is equal to del rho u by del x plus del rho v by del y plus del rho w by del z and del x del y del z as the control volume with respect to this.

Essentially this is really the net rate of mass flux out through the control volume surface. So, I can write this as the net rate of mass flux out through the control surface. What is interesting here to point out is that the rate of change of mass inside the control volume is a function of the time that means there is a mass, which is generated or created inside the control volume. In that case, we can always write down that the rate of change of mass inside the control volume CV is equal to del rho by del t that means the rate of change of density and this can be a case for compressible flows, where there is a rate of change of density with time. The incompressible flows are of course, this dp by dt does not make any sense d rho by dt because it is 0.

We assume that the densities constant temporally, there is no variation of density with time. Times of the control volume del x del y del z, so in a more generic manner the equations 1 and 2 here (Refer Slide Time: 37:50), which have been derived if added together should give you a situation, whether it is for compressible or incompressible flows. What you can do is that the total amount of mass in this manner which either inflows or outflows, which gets generated, should be equated 0 because of the conservation of mass. So, mass cannot be created or destroyed if you are assuming continuum assumption inside such a control volume a particular fluid.

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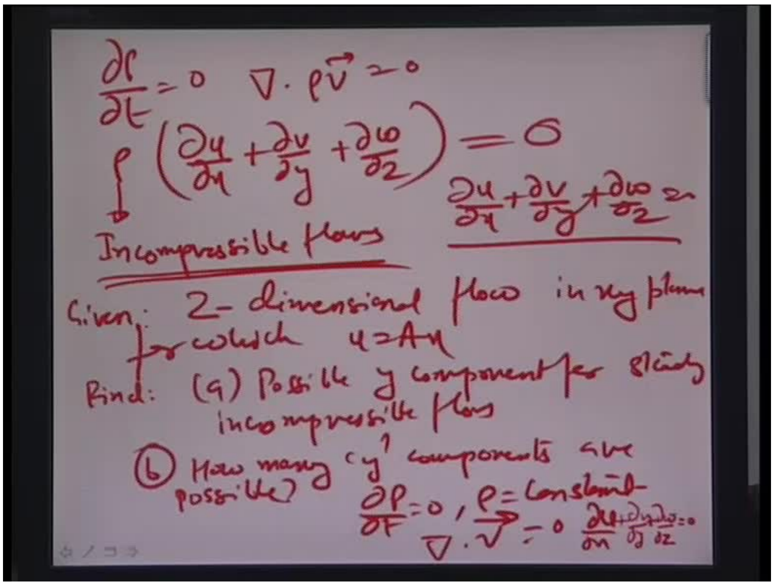

In that case, the del rho u by del x plus del rho v by del y here plus del rho w by del z plus del rho by del t whole brackets, the control volume dx dy dz should be equal to 0, which actually can be in a more a bridged manner written down as the grad vector dot rho v vector plus del rho by del t is equal to 0. So this is what the first of the Navier - Strokes equations are about continuity or conservation of mass. Typically, for a incompressible flow though what we would be left with is just this part of the term.

What will be left with is just the del dot or the grad vector dot rho v vector so incompressible flow cases when particularly del rho by the del t is equal to 0. The continuity equation really reduces to del dot rho v vector equal to 0, so that means del u by del x plus del v by del y plus del w by del z with the rho taken common out of all this a; remember, in incompressible flows this rho does not vary with time or space.

There is absolutely no variation in the density, the density either in time or space both remains same in constant; so this is the situation of incompressible flows. Therefore, this becomes equal to 0 and other words del u del x plus del v del y plus del w del z is 0 is the new form of the continuity equation particularly for incompressible flows. Let us try to understand this one example. Let us say, we have given there exists a 2-dimensional flow in xy plane for which u becomes equal to Ax. You have to find the possible y component for steady incompressible flows using the continuity equation and also how many such y components to be possible.

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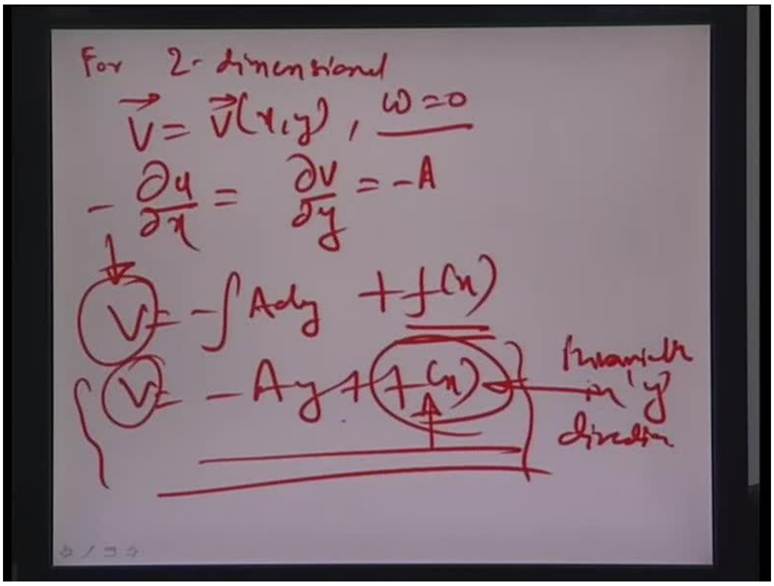

As we know, here del rho by del t 0 or rho is constant. Therefore, the whole continuity equation just changes to del cross v vector equal 0 del u by del x plus sorry del u by del x plus give me minute here plus del v by del y plus del w by del z equal to 0. Essentially this means that and Also, we have already know that the flow is 2-dimensional. So such a 2-dimensional flow v essentially should be a function of x y, right. Velocity v vector should only be a function of x and y. Therefore, there is no third component which exists or w equal to 0 in any case.

For that compressible equation or the continuity equation changes to du by dx equal to minus del v by del y; let us say, these all are equal to constant minus A. So, we can safely try to integrate and find out the value for the velocity v, so as you do that we can get v is essentially integral A dy with minus sign plus some constant. Since this v is never varying in the x direction, we can assume a function of x to be constant along the y direction.

So v essentially becomes minus Ay plus function of x, so this is essentially invariable in the y direction, it is a pure function of x. Therefore, it may be treated as a constant in this particular case it could also be a normal constant apart from that. Essentially, a possible y component for the steady incompressible flow can be expressed is minus Ay plus function of x. You can also say that because there is a function of x involved, there are many such solutions of the y component of velocity that is possible using the continuity equation.

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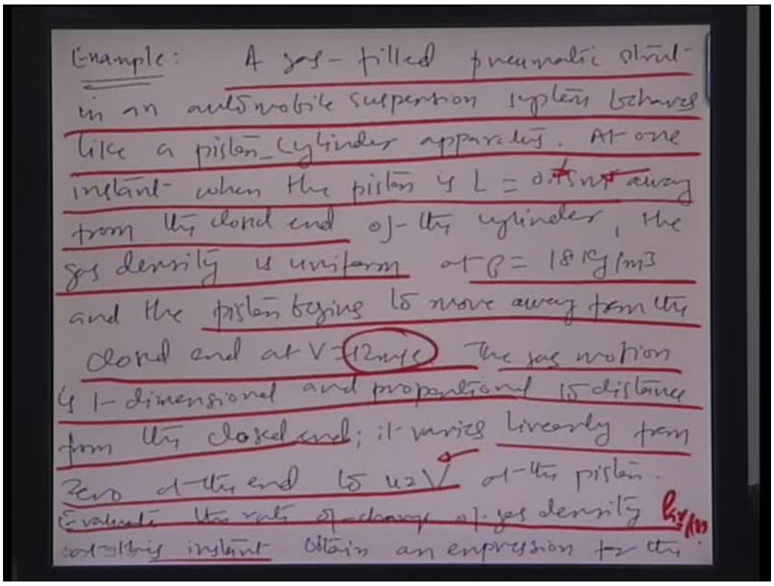

Let us also do a little bit of different kind of example related to an operating piston and certain cylinder pressure to understand the continuity equation little better. In this particular example, as you see, there is a gas filled pneumatic strut in an automobile suspension system and it behaves like a piston cylinder apparatus. The boundary conditions that are given is at one instance when the piston is L, say the total length L is equal to 0.15 meters away from the closed end. The cylinder, the gas density is uniform at rho is equal to 18 kg per meter cube and the piston begins to move away from the closed end at velocity equal to roughly about 12 meters per second. The gas motion is one dimensional in this case and proportional to the distance from the closed end.

So it varies linearly from 0 to velocity v, which means that at two ends of the closed end when the present velocity is 0 and when it goes to the length L equal to a 0.15 meter which is at one end, that means this is the way from the closed end and this is the farthest extremity at that the velocity u becomes v.

We have to evaluate the rate of change of the gas density at this particular instance particularly when the piston is at 0.15 meters from the closed end. We also need to obtain an expression for the average density is a function of time, so we need to find what rho average is in terms of rho t here.

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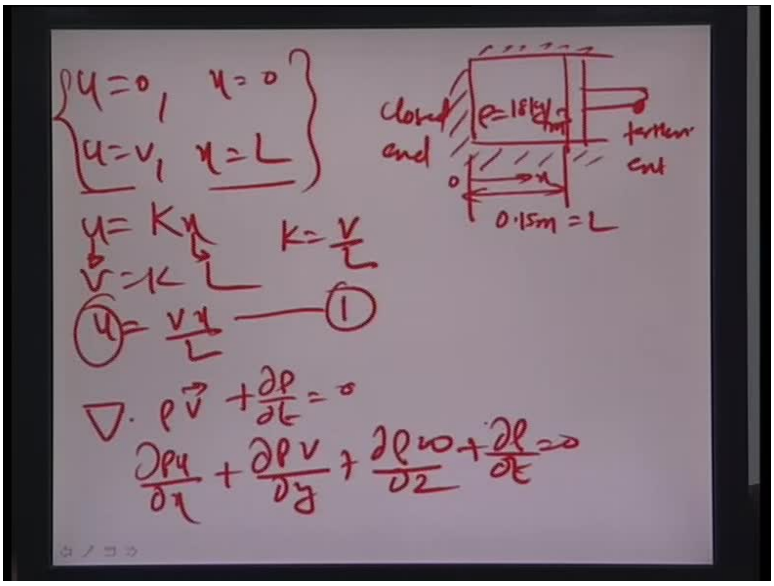

Let us actually tried to solve this using continuity equation. We have a cylinder here in the example, three fixed ends and the movable piston let us say here (Refer Slide Time: 47:47). Essentially, what has been indicated here is that the gas density within this volume is 18 kg per meter cube and this is the closed end and this is the farthest extremity that the piston can travel. Here in the question, the extremity has been given as 0.15 meters, so essentially distance here is 0.15 meters.

We also further know that the velocity is 0 let us say, we are talking about the x direction starting from 0 here all the way up to L; so velocity is 0 when x is equal to 0 and the velocity really is v when x equal to this L value. Also we further know, as it is given here that the gas motion is 1-dimensional and proportional to the distance from the closed end.

It varies linearly from 0 to ux because it is a linear variation; we can assume that the u is really equal to some constant k times of x, V becomes equal to kL because u is V as x is L essentially and k becomes v by L, so u essentially is again of vx by L. That is how u is x; x equal to 0 u is 0 and x equal to L u equal to v. So, this is what the velocity equation in terms of x. We now apply the continuity equation here, we know that by the continuity equation we have del dot rho V vector - V is the velocity vector is essentially and plus del rho by del t essentially equal to 0.

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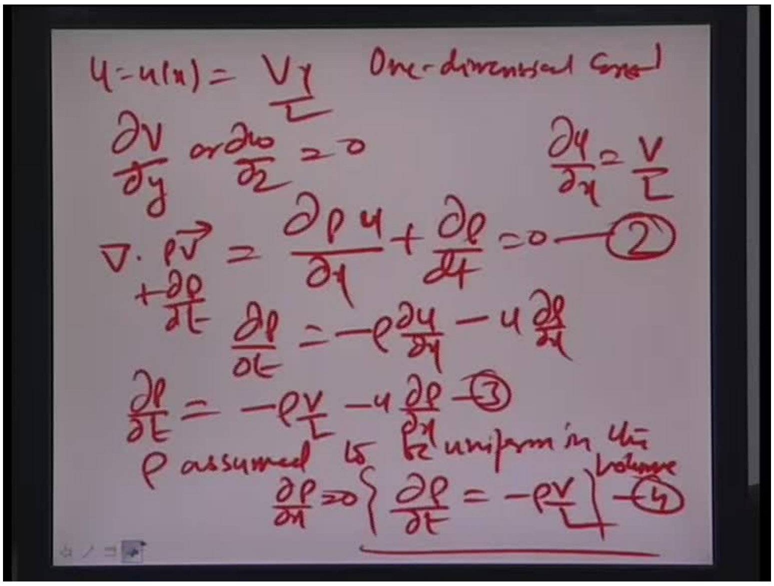

So del rho u by del x plus del rho v by del y plus del rho w by del z plus del rho by del t is equal to 0. Essentially, if we just put the value of u equal to ux, this vx by L value and with respect to this the only velocity mind you, this is only a one-dimensional case has been indicated in piston cylinder arrangement.

Therefore, dv by d0 dv by dy really or dw by dz are both 0s, so the only other value which comes out of this whole del cross rho v is essentially rho del u by del x or del rho u by del x. We know that from because it is actually a compressible flow in this particular illustration, we have this plus del t is essentially equal to 0. If we try to figure out what this value would be del rho by del t becomes equal to minus rho del u by del x minus u del rho by del x.

As we know that du by dx or dau u by dau x is essentially a constant v by L. Therefore, we have also the value del p by del t from equation 2 here becomes equal to minus rho times of V by L. So this minus rho V by L minus u del rho by del x if you look at this equation 3, so if you really look at the question or the problem statement, rho has been assumed to be uniform in the volume and not with the time t; here the rho is varying with time t is temporally varying but, it is not varying specially really.

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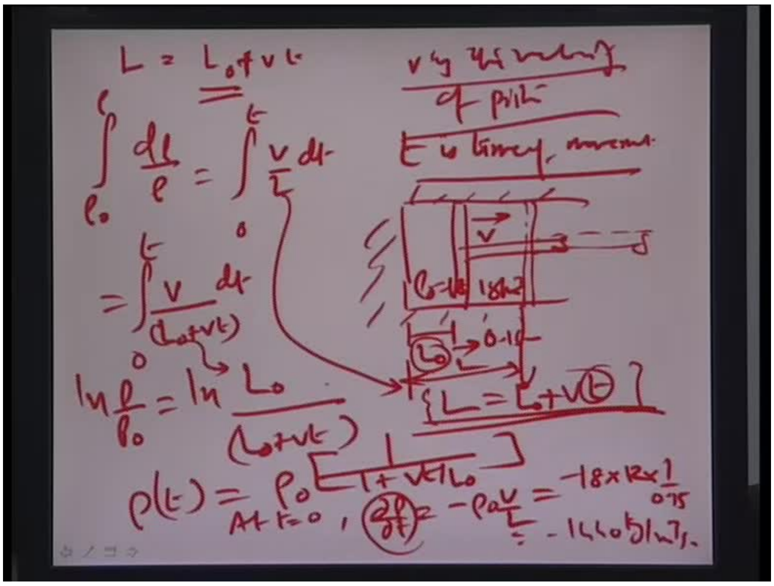

Therefore, del rho by del x because it is uniform within the volume is suppose equal to 0 and we are left with no other choice but, del rho by del t on one side equal to minus rho V by L and that is equation 4. Let us try to figure out, what an integration of this quantity would result in and what would really be the density function in terms of the velocity length etcetera. Here, we would like to illustrate that the length l really is we can assume this to be equal to the initial length L 0 that the piston is at plus v times of t where v is the velocity of the piston and t is the time of movement.

So there is some L 0 value let us say, the piston is somewhere if you see in this particular figure here (Refer Slide Time: 54:15) the piston is at some value when it moves at a certain velocity v, this value may be L 0 and we want to consider any length L which is equal to L 0 plus v times t, after time t it would be here and this is really the new length L.

That is how we define this whole length of traverse of the piston inside the cylinder. So, if we assume this to be the final length of time t assuming this to be the length in haven shown when the process started, we have integral d rho by rho where rho varies from some quantity rho 0 to some value of density rho t equals integral minus 0 to t v by L dt and essentially as you know that this L actually comes from this L 0 plus vt, so will have this as 0 to t v by L 0 plus vt dt.

Therefore, l n rho by rho naught really becomes equal to l n L naught by L naught plus vt. If you just solve this integral it is essentially and put the limit 0 and t it comes out to be L naught by L 0 plus vt. In other words, density is function of time really is equal to the density time t equal to 0 plus 1 by 1 plus vt by L 0. At time t equal to 0 therefore, as the second part of the question assumes del rho by del t the change of density, rate of change of density, would be rho 0 v by L and rho 0 being already given equal to 18 kg per meter cube when this is at length L 0 0.15 meter this particular length and it is moving to the velocity 12, the whole density variation with respect to time becomes equal to minus 1440 kg per meter cube second.

As you found out here, the continuity equation can be very easily used for this kind of compressible flow problems as well where density varies with time as well as earlier problem as you saw was that of incompressible flows. In micro scale though if you consider the flow mechanics mostly flows are treated to be incompressible and the strictly laminar in nature because all though there are twin phase flow problems of the micro scale but, the modeling becomes extremely complicated and difficult. So, we will limit ourselves mostly to the single phase flow problem in such a situation.

So, this brings us to an end of this particular lecture. We will try to cover up the second Navier-Stokes equation; that is the conservation of momentum in the next class. Thank you.