Bio – Microelectomechanical Systems

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Module No. # 01

Lecture No. # 26

Hello! Welcome back to this 26th lecture on Biomicroelectromechanical Systems. Today, let us first quickly look into brief review of the previous lecture.

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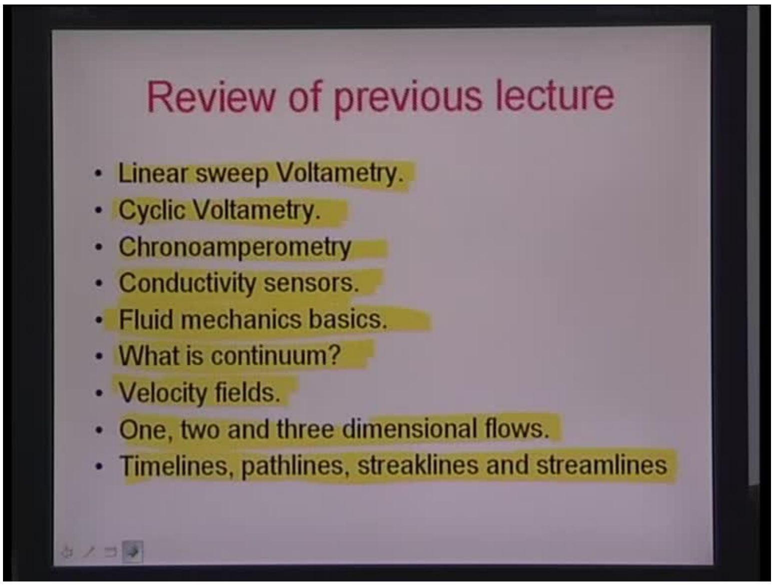

We started with understanding the basic voltammetric mechanisms like linear sweep and cyclic voltammetry; again voltammetry is the technique of measurement of reduction or oxidation potentials of various electrochemical species with rapid voltage scan. The measurements are made between current and voltage and basically, get a peek which shows whether the electrons have been released or suddenly absorbed at a certain potential corresponding to the oxidation and reduction potential of the species. Then comparison to standards, kind of lets us just know what the species are or in what concentration they are present.

We also talked briefly about chronoamperometry, where we discussed the application of a square wave going to a certain peak potential which would oxidize or reduce a species and then try to understand the kinetics of decay of the current as we go temporally. So, various species would have a different rate of oxidation or reduction. In other words, various species would have a different rate of release of electrons or absorption of electrons, which would make chronoamperometric measurements comparable and would let us draw inferences from the current versus time plot in such situations.

So, we also talked about conductivity sensors. Conductivity essentially means the inverse of resistivity and the increase or decrease in ions of one kind of a particular species, would definitely lead to the increase or decrease in the conductivity. You can use this measurement technique by assembling together; what you know as a Wheatstone bridge.

Then, we started on a new area of some basics in fluid mechanics it is very important for me to mention here that, because we will be studying some fundamental problems in microfluidics; we need to understand these basics. So, essentially we covered about what really a fluid is by definition we talked about how it would deform on a shear force being applied to the system and how it compares with the solid, in a similar kind of situation. We try to understand, what really a continuum is or when it breaks down particularly at a level when you know the dimensions the spatial dimensions of the control volume kind of rhyme with the mean free path of the different molecules. Then, we get differences in properties like velocity, density etcetera with time and that is where the continuum breaks down.

We described in details about velocity fields; we talked about one two dimensional, one two and three dimensional flows respectively. Then, we also tried to categorize these, very important ways and means of geometrically representing flows by means of timelines, path lines, streak lines and streamlines.

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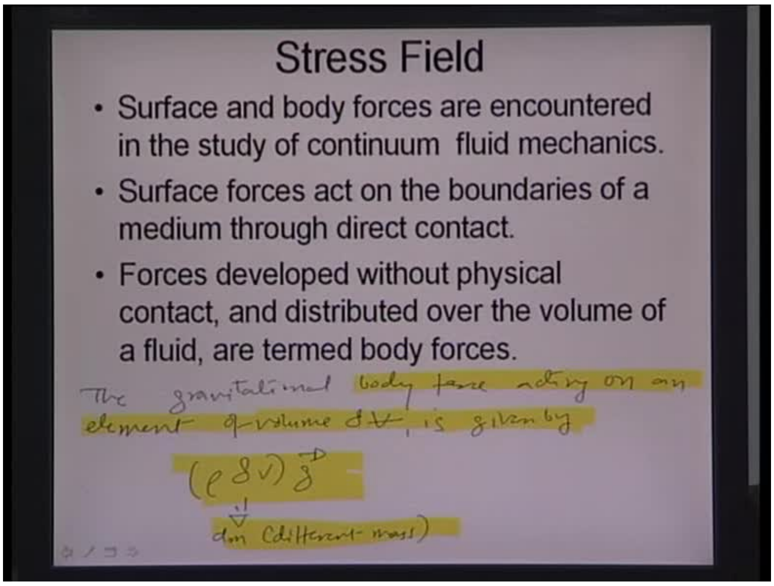

So, we will kind of start from here and then go to the next agenda today; which is stress fields. Essentially, you know if you look at really what a stress is; we all know that stresses are force per unit area from basic definition and so typically in a continuum fluid mechanics, we have a surface or volume or body forces encountered at different points of the fluids. So, if we consider a control volume somewhere in the fluid, because of the flow motion and because of this viscosity forces between the different layers, there is a tendency of these forces to affect the surface or the volume as such of that volume element or control volume element.

So, the surface forces act on the boundaries of a medium through direct contact; which means that, let us say if you consider the fluid is a bulk and it is flowing through a pipe; then, the borderline between the pipe and the fluid is, where the surface forces are directly acting and there is an impact of these surface force through into the bulk of the fluid and also forces develop without physical contact and distributed over the whole volume of the fluids are termed as body forces. So, essentially it is a volume force that we are referring to.

Like for example, gravitational force acting on a fluid element is essentially a fluid volume element - is essentially a body force. So, what that is essentially? It is rho times of v; v is the control volume times of g, rho is the density, v is the volume. So that is making it equal to the mass of a certain control volume and then you have a gravity factor g acting. So, this is a body force; it is uniformly felt over the whole volume of the control element that we are kind of figuring out.

So, here as you can see (Refer Slide Time: 05:54), the body force acting on an element of volume delta v is also given by rho delta v g and essentially, this is nothing but the differential mass, here rho delta v times of acceleration due to gravity and so it is felt within the volume.

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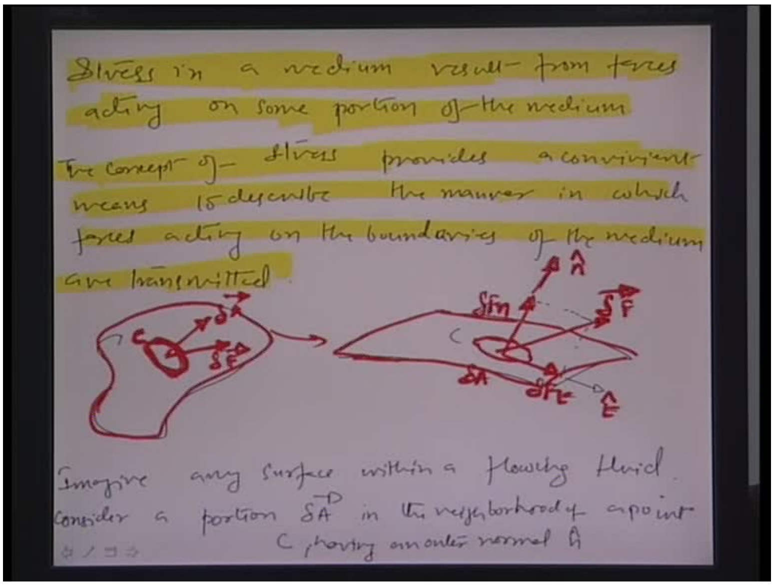

Let us actually figure out, what stress really is in terms of such a control element. It is important or pertinent in fluid mechanics to understand fluid as an assembly of control volume by control volume and therefore, if there is one representative control volume in such situations, it kind of generically represents the properties related to the flow in general like velocity, acceleration, density and so on so forth.

So, the stress in a medium results from forces acting on some portion of the medium. So, definitely there has to be a relative force between this element in consideration and the medium in which this element is for us and to understand essence of stress. So, there is a relative force between the control volume and its surrounding medium. The concept of stress provides a convenient means to describe the manner, in which forces acting on boundaries of the medium are transmitted like. Let us say for instance, consider a control volume here as in this example (Refer Slide Time: 07:28), like one sees this is a control volume and this control volume is essentially close to let us say some point C in space and we consider a small area delta A, which is adjacent to this point C inside this control volume it is a regular shaped control volume.

Now, any area as we know from vector geometry can also be represented by a direction perpendicular to the area in question. So, if we are considering this small area element as illustrated here (Refer Slide Time: 08:00) and let us say the value of this area is delta A, we can represent this area by a unit normal vector pointing away from this area and perpendicular to the area and we call it delta A vector, as in this particular case.

Now, let us suppose that there is a force delta F vector, which is acting on this area vector delta A. So, it is at a certain angle in respect to delta A, but then, there is a force delta F vector acting on this area vector delta A. So, if we imagine any surface within the flowing fluid, this surface let us say is a part of this whole control volume as has been illustrated here (Refer Slide Time: 08:48) and we also assume that this delta F, which is at a certain angle with the area vector can be resolved into a normal component; which is in the direction of this normal vector. Let us say this is the normal component delta Fn, direction of the normal vector again and one which is kind of tangential to the area of interest here close to this point C and we call this F tangential as delta F tangential. So, essentially we are kind of resolving this delta vector into delta Fn that means delta F in the normal direction to the area and another component delta Ft, delta F tangential to the direction of the area. The value here for this normal vector is also delta A and this is the tangential direction represented as t cap this is the normal direction represented as n cap.

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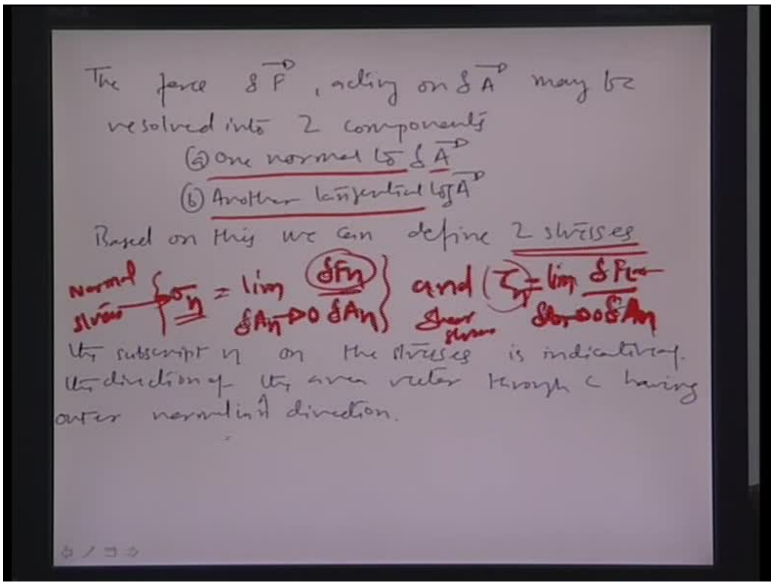

Now, if you want to really see the kind of stresses because of these two components on this area vector a, it would be represented as a normal stress and a stress which is tangential, because there are only two components of the forces the normal force and the tangential force. So, based on this we can define the two different stresses as one in the direction of the normal vector which is also represented as sigma n or the normal stress or the principle stress and it can be defined as the limit of the area element delta An approaching 0 delta Fn, which is the normal component of the force vector delta F by delta An and the shear force (Refer Slide Time: 10:44), this is known as the principle stress or the normal stress. So, this is the normal stress and the other component which is parallel to the area can be represented as tau n limit delta An tends to 0 delta Ft it is the tangential force by delta An. So, the area vector still does not unmodified, it still remains the same. So, we have a stress due to the normal force parallel to the area vector in a stress due to the tangential force perpendicular to the area vector causing the normal stress and the shear stresses.

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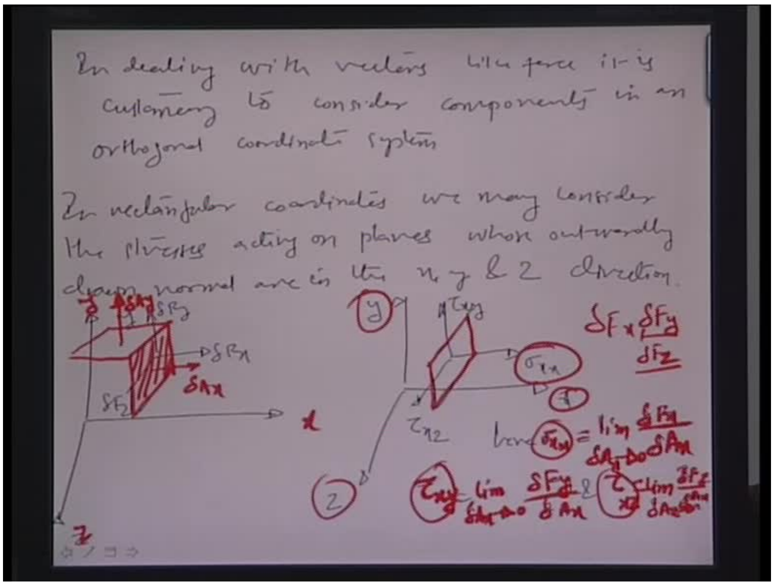

So, this essentially is how you define normal and shear in such a certain situation. Normally, it is kind of customary to consider the vectors or the components of these force vectors in orthogonal coordinate system which essentially means. So, in rectangular coordinates, we may consider the stresses acting on planes whose outwardly drawn normal are in the x, y and z direction. Essentially, if this is a plane that we are talking about in the rectangular coordinate system xyz, we consider the stresses acting on planes whose outwardly drawn normal are in the x, y and z direction. So, one of the planes is essentially whose outwardly normal here (Refer Slide Time: 12:20) is drawn in the x direction; I am representing it by this red line here, let us call this as some area vector delta Ax. Another would be similarly in the y direction, which is probably an element like this (Refer Slide Time: 12:38) which is orthogonal to this Ax element and it is called delta Ay and similarly delta Az.

Now, if we want to represent the stress vectors here (Refer Slide Time: 12:50), let us say, only on this particular plane on this Ax vector here. So, let us suppose this is the plane, which is drawn separately here and so you have again components of the force in a rectangular coordinate system well resolved into all the 3 coordinates x, y and z. Let us suppose the force along the axis Fx delta along the y is delta Fy and along z is delta Fz. So here (Refer Slide Time: 13:21) as you are seeing, there is one principal component; let us say sigma xx and 2 shear stresses based on the resolution of the force in the y and the z direction respectively. So, delta - the principle stress here sigma xx is essentially equal to limit of delta Ax tending to 0 delta Fx by delta Ax.

The other two components; we represent this as tau x y, which means the shear due to a force in the y direction, the second term here applied to the area vector Ax. So, the second term of this is the force direction and the first term is the area direction. So, tau is the shear applied due to a force in the y direction by an area vector in the x direction also represented as delta Fy by delta Ax, delta Ax tending to 0; this is the limit of delta Ax and similarly this again is a representation, where you are considering tau of xz meaning the shear stress due to a force in the z direction on an area pointing towards the x direction, the second term is the direction of the force in this subscript here (Refer Slide Time: 14:52) and the first term is the direction of the area vector. This is just purely notational and it is needed for kind of understanding the different components of the principle and shear stress, when this plane changes between let us say a plane pointing to the x direction or plane pointing to the y direction and the plane pointing to a z direction in a rectangular coordinate system.

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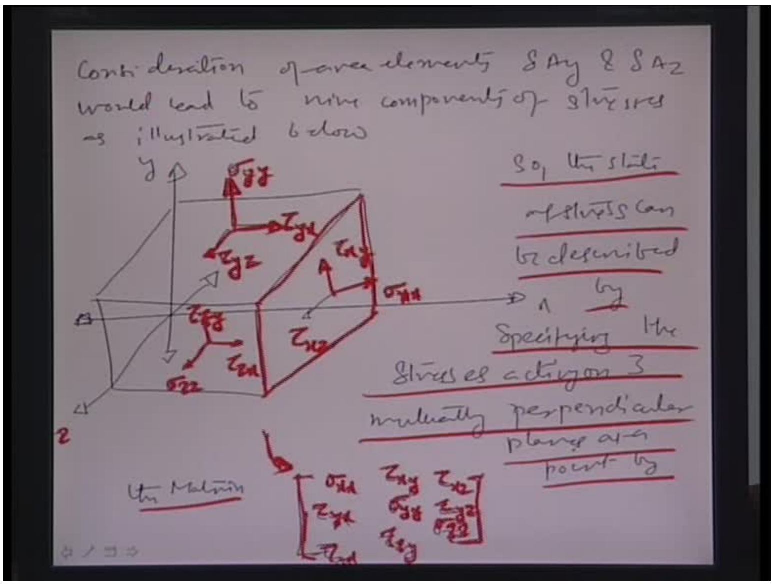

Really, if we look at all this together, as I pointed out before or illustrated before, there are 3 such planes in the orthogonal coordinates as you can see here (Refer Slide Time: 15:26) - There is a plane in the x direction, right? A plane pointing towards x direction; if I really make a control volume, as I described earlier, all fluid mechanics is really about constructing a control volume. So, let us say we make a cube as an element which represented of control volume. In this cube, you have a face facing x direction or face facing the minus x direction and similarly a face facing the y and minus y and z and minus z directions respectively. So, along all these faces you will have shear stresses and at least two components per face and you will also have principle stresses one component. Therefore, if you look at all this in totality, what are the number of stresses which exist?

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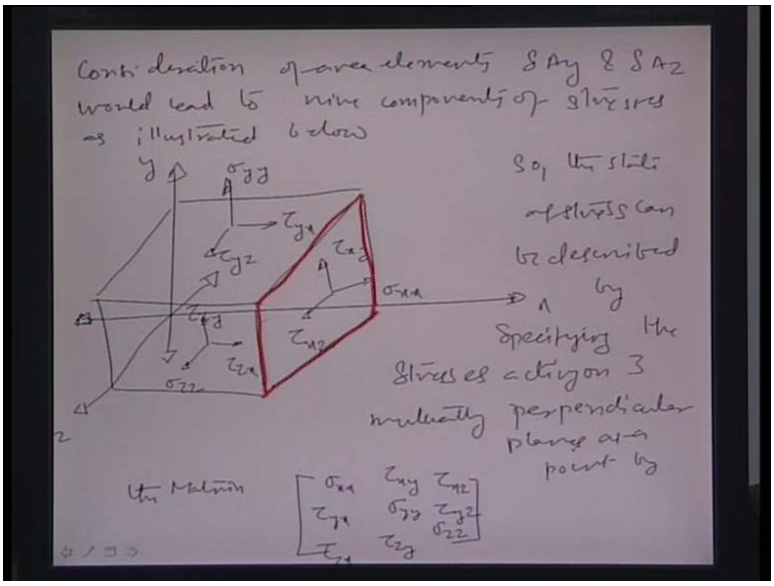

So here, let us say in the positive x direction, if the normal vector of the plane points to the positive x direction, you have sigma xx which is essentially the Fx that means the delta Fx or the force in the x direction divided by the area whose vector points towards the x direction that is delta Ax. Similarly, you have tau xy; As I have defined earlier or tau xz if you are looking at the y face that means the face where the area vector points to the positive y direction, you have again sigma yy in this direction and then you have the shear stress because of the force in the z direction applied to an area vector delta Ay pointing in the positive y direction and the shear because of a force in the x direction, a component of the force of which is a kind of resolved in the x direction divided by the area which is again the area of the face which is having a vector pointing towards the y direction so that is what tau y x is.

Similarly, you have a similar combination on the third face here (Refer Slide Time: 17:21) pointing in the positive z direction, the area vector points toward the positive z direction, where you have sigma zz and two other shear stress components tau zx and tau zy. This is very clearly illustrated here; how you can notationally express these different stresses in such a fluid element, these stresses are acting together on the fluid element as fluids go all way around and casted and there are stresses which can be shear based there are stresses which can be in the normal direction or principle shear principal stresses. This whole combination is what we have to evaluate dynamically to consider the behavior of such an element with time. That also led us to define certain equations of motion of this fluid element as it goes along considering the kinematics and dynamics which we also know as the Navier-Stokes Equation. So, probably in the next lecture, I would also be trying to derive some of these equations.

There are principally 3 such equations - equation of conservation of mass, conservation of momentum and conservation of energy. So, here if you really put all these stresses together in a matrix form, you can really define a matrix which is also known as the stress matrix, where you have the diagonal elements which are principal stresses sigma in the x, y and z direction respectively. Sigma xx, sigma yy and sigma zz and the non-diagonal elements here, really represent the different shear stresses have been illustrated before how these shear stresses come by with a certain notation. So, this is tau zy this is tau yz so on so forth.

So, the state of stress can really be then described by specifying the stresses acting on the 3 mutually perpendicular planes of a rectangular coordinate system in a orthogonal system at any particular point by this stress matrix. This is also known as the stress tensor of this particular fluid element.

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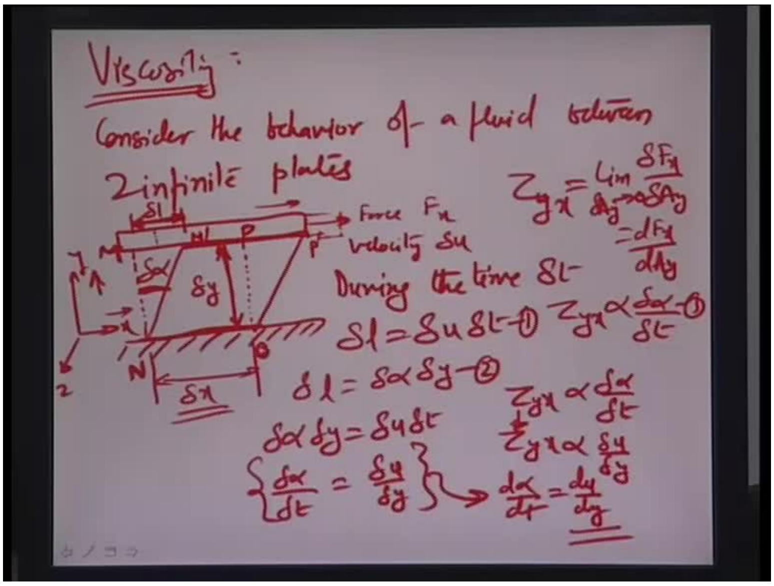

Now, I would like to kind of go ahead and evaluate the very first and very important property of a fluid which is viscosity. So, what really is viscosity; so let us see how we can understand this concept of viscosity.

Let us say, we consider the behavior of a fluid between 2 infinite plates; let us suppose, we have an infinite now these plates are essentially infinite in the z direction; they are in towards into this particular tree. So, there is a plate here and then there is a fixed support at the bottom and if you recall, we have done this back in lectures related to finding out the parabolic velocity profile how a moving plate would influence fluid column by shearing it as the plate moves ahead with respect to a fixed boundary.

So here, let us say at time instance t, we have a fluid which is static and having a boundary like this and let us say, we apply a force on this particular upper plate to an extent Fx, because of which the plate moves with a velocity delta u. So, delta u illustrated here and let us actually see that, if we try to move this with the force of x at a rate delta u; what happens after t plus delta t. Of course, this plate here would move forward and let us say the new position of this plate is formulated somewhere here, because of that movement. So that, it moves in total or in totality by some finite distance here.

So, what will you expect would happen to the fluid column? The fluid column would actually try to get sheared like this(Refer Slide Time: 22:19). As you know, that is how fluid is defined that, if you apply a force in this kind of a situation the fluid will just simply go or deform plastically and not come back, as it happens in solids normally. So, in fluid it would just go plastically and stay there and if you apply a little more force it will again bend and keep on shearing as you proceed along.

Let us assume that, we have been able to successfully move this fluid layer by a total amount of distance delta l. So here, one of the elements here (Refer Slide Time: 23:04) as we can see, let us mark it as M N O P and this moves to its new position M dash N O P dash. So, as you may be already aware, this particular layer here at the bottom is static, because the lower plate is fixed in nature and so there is zone of no slip, which is formulated and as you go ahead in the y direction, you have a velocity gradient which comes up, because of this and there are these layers which are kind of shear shearing off or sliding over each other as the fluid deforms from the position M N O P M dash N O P dash M N O P, here this is M N O P to M dash N O P dash.

Let us also assume that, the distance between the 2 plates - they are parallel plates and the distance between the 2 plates is delta y and essentially the total amount of length that this fluid element possess as at the very outset is delta x. That does not change much although, the shape changes from rectangular into more like a parallelogram, because of the shear that the fluid layer would have with respect to the zone of no slip close to the surface N O.

During the time delta t, the amount of distance that has been moved delta l, can also be represented as delta u times delta t. Essentially, the shear stress here (Refer Slide Time: 25:16) Tyx that means the stress due to the force along the x direction on the area vector pointing towards the positive y direction; let us suppose we have a right-handed rectangular coordinate system xyz are the different directions. So, the area vector pointing to the y direction is really in this particular direction here and the force is in the x direction so that is what would come along this particular plane M P or M dash P dash, whatever you may call. Therefore, Tyx is defined as limit of delta Ay turning to zero delta Fx by delta Ay or dFx by dAy whatever you may call. Essentially, as we know that from the Young’s law from the Hook’s law tau xy is also proportional to the rate of angular deformation.

So let us assume that, this angle change here (Refer Slide Time: 26:50), because of the component moving from or the fluid element moving from M N O P to M dash N O P dash is delta alpha and this delta happens in delta t time. So, tau x tau yx is definitely proportional to delta alpha by delta t. So, the rate of change of angle, that is what hook’s law defines shear stress.

So, if we consider all these factors together, we are left with another very interesting observation that delta l, which is actually this particular elemental change in the length or the displacement by which the layer M P moves to the new position M dash P dash as the plate moves ahead is also given by delta l times a delta alpha times of delta y, because this is essentially can be considered in very small situations as same as the length of the arc that this radius delta y would execute as it would moves from position M to M dash. Essentially, what we are talking about here is the length of the arc delta l by virtue of the fluid element moving from position M P to M dash P dash; the element moves by an angle delta alpha. So, delta alpha time’s radius delta y here would define what this delta l is. Let us call this equation 1, this equation 2 and this equation 3; if you actually correlate equation 1 and 2, you have a situation where delta alpha delta y becomes equal to delta u delta t and therefore, delta alpha by delta t also becomes equal to delta u by delta y.

Now, as we know that the shear force tau yx is really proportional to delta alpha delta t. So, we can easily say that tau xy is also proportional to delta u by delta y; taking limits here we can get a situation, where d alpha by dt is equal to du by dy and this is what the velocity gradient is. Thus, the fluid element when subjected to a shear stress tau yx experiences a rate of deformation given by really du by dy as can be seen in this illustration here.

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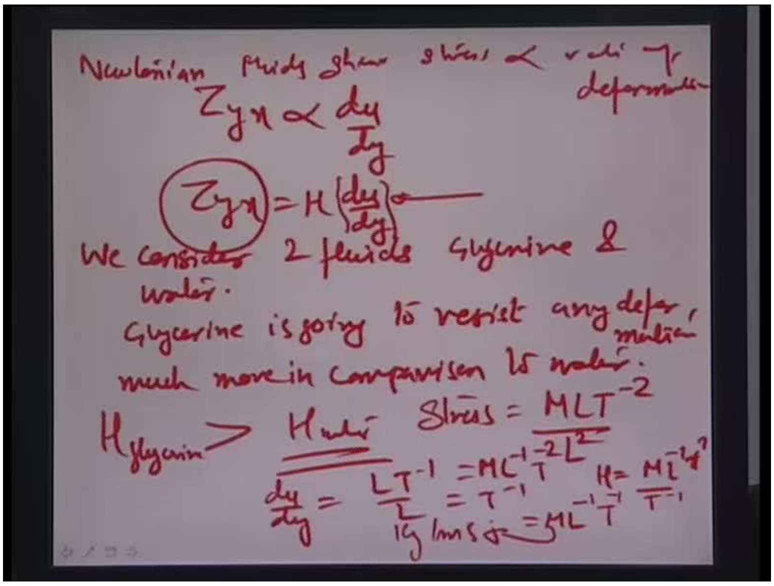

So, at least fluids in which this proportionality between shear stress and rate of deformation exists are known as “Newtonian Fluids” as we all know. So, let us define this again here that in Newtonian Fluids and we will see in just about a minute what happens in non-Newtonian case, how that is different in this particular illustration. So, in Newtonian Fluids the shear stress is also directly proportional to the rate of deformation. As we have illustrated here before, I just forgot to mention that this proportionality only holds valid for Newtonian Fluids that is how fluids are defined.

Therefore, in such a situation, we have tau yx is proportional to du by dy and the constant of proportionality in this case is also known as viscosity of the medium mu. So, what really viscosity physically means is that, let us say if we consider 2 different fluids say glycerin and water. So, we consider 2 fluids say glycerin and water; definitely glycerin is going to resist, as we all know by a natural experience, the glycerin is going to resist any deformation much more in comparison to water. So, this is definitely because glycerin is much more viscous or in other words the mu for glycerin is much higher than mu for water; which means that amount of shear stress that was needed for a certain velocity gradient to be created; that means you talking about movement of interlayers there are 2 layers which are moving with respect to each other. So, this gradient du by dy for a certain finite gradient to be created, we need much more shear stress or much more force or effort in glycerin because mu value is higher in comparison to water. So, that is the essence is what viscosity is all about.

Dimensionally, again if you investigate what viscosity is really, that stress essentially is force per unit area; so we can represent that as MLT minus 2 L square by L square. So, that is ML minus 1 T minus 2 and du by dy; here if you look at really has LT minus 1 by L which has dimensions of T minus 1 and therefore, mu would have units ML minus 1 T minus 2 by T minus 1 or ML minus 1 T minus 1 so ML minus 1 T minus 1 (Refer Slide Time: 32:50). Therefore, the unit of viscosity is k g per meter second that is what viscosity is defined.

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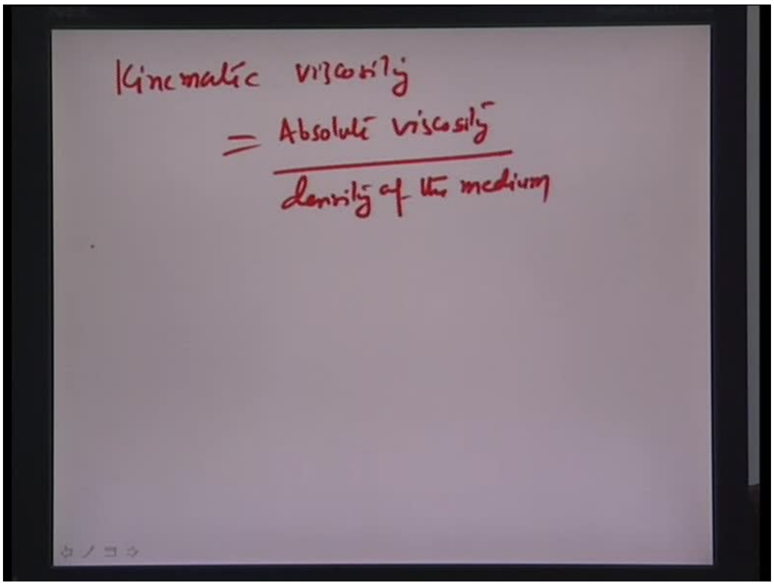

In fluid mechanics, we seldom use these units of viscosity; we rather express viscosity as a ratio between the absolute value of the viscosity and the density, we also know that better as kinematic viscosity. Therefore, we can also write here that kinematic viscosity, the new term which is normally used very often in fluid mechanics and it is very obvious, because there may be substances where density is higher and same is the viscosity. So, what really matters, if there is substance which is very diluted in nature, it would normally I mean by intuition, we can say that it would have a lower viscosity value. So, what is important to consider in physical sense really the ratio between the viscosity and the density that gives you a better perspective of the fluid medium that you are investigating.

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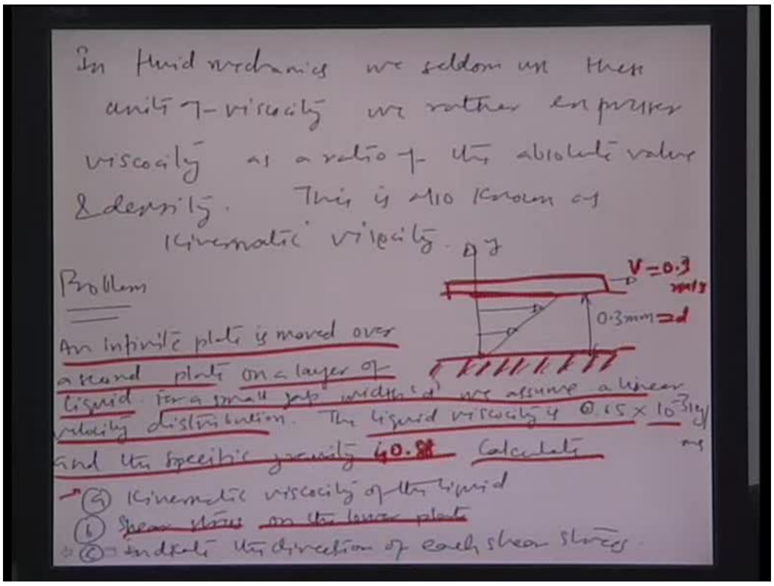

So, kinematic viscosity here is equal to the absolute value of the viscosity divided by the density of the medium. So, I would like to go ahead and do an example problem, as you can illustrate here that; there is an infinite plate as I just showed and is moved over a second plate which is fixed on layer of liquid. So, this essentially is the plate; the semi-infinite that means it is infinite in z direction plate and it is moved over this fix plate here and for a small gap of width d which is equal to 0.3 mm as you can see here. We assume a linear velocity distribution, therefore the velocity varies from 0 here point of no slip to all the way up to about v equal to 0.3 meters per second; which is the maximum velocity of the plate; the fluid here adjacent to this plate would move at the same velocity because there is another zone of no slip here and so there is a relative velocity between the point at the top here and the point at the fixed plate surface at the bottom.

So, the liquid viscosity which is used here in this case is 0.65 10 to the power minus 3 kg per meter second and the specific gravity is 0.88. So, specific gravity as we all know is basically how many times the density of water is the density of a particular fluid. So, it is a comparison the ratio comparison between the density of a fluid to density of water at standard conditions.

So, you have to calculate this case the kinematic viscosity of the liquid and also have to find out what is the shear stress which is generated in this process. So, we have to find out the shear stress on particularly on the lower plate and you have to indicate the directions of each of these shear stresses. So, let us solves this problem to understand about the viscosity.

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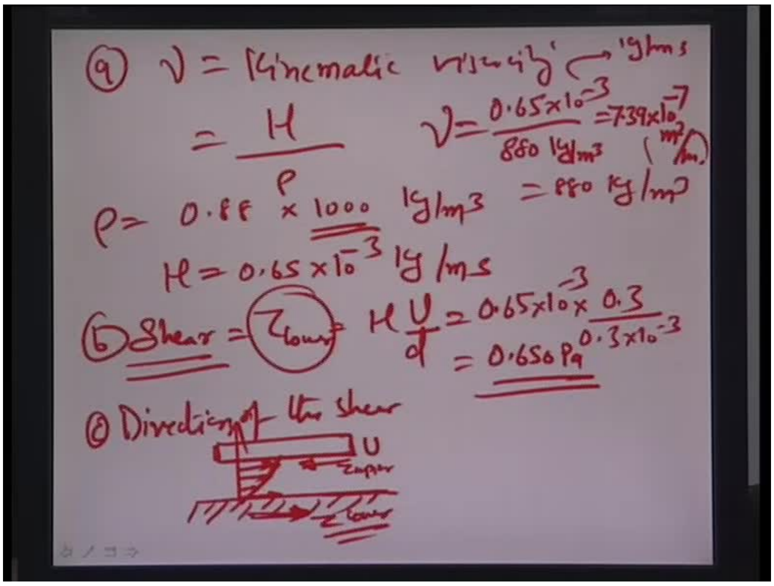

So, the first question is, what really is the kinematic viscosity? here kinematic viscosity we call it or we represent it by the symbol Nu. This is really the absolute value of viscosity per unit density; density in this case as we know is 0.88 times of 1000 kg per meter cube, which is the specific density of water at standard conditions. So, it is 880 kg per meter cube and viscosity from our earlier this thing question is given to be 0.65 times 10 to the power of minus 3 kg per meter second. So, Nu here would be 0.65 10 to the power minus 3 by 880 which is equal to 7.39 10 to the power of minus 7 and the units in this case is 10 to the power of minus 7. So, 7.39 times 10 to the power of minus 7 and the units in this case, as you can see this unit here (Refer Slide Time: 37:56) is kg per meter second, this unit being kg per meter cube and we are left with meter square per second. That is what the units of kinematic viscosity.

So, the second part of the question says, what is the shear stress in the lower plate? So, the shear stress here can be represented as tau again on the lower is mu viscosity times of u by d, u is essentially 0.3 meter per second and d has dimension 0.3 mm. So here, the total stress would be the viscosity 0.65 10 to the power of minus 3 times of the total velocity here 0.3 divided by the distance which is 0.3 10 to the power of minus 3 meters. So, it is essentially comes out to be 0.650 Pascal’s or newton per meter square; that is how you define the shear force on the lower plate. About the direction of the shear force, if you look at really the plate combination you have this as the upper plate, this is the moving fluid and this is the fix plate in the bottom side and you have this velocity vector here going from some finite value u to all the way to 0. So, you can consider that if this element is moving along with the upper plate, it would exert a force which is in the reverse direction. It is a reaction force that it would exert on this plate, as if it tries to get the plate back into its normal position. So, that is what the upper direction would be simultaneously you are trying to deform the fluid element. So, it is giving a pressure to this fluid in the other direction, here I mean to more towards the movement direction here on the lower plate, because it would have been better if this plate would have been able to carry this through along with it, but since it is not carrying it therefore, force that is being felt on this due to this resisted layer at the junction here is actually towards the direction of motion the upper plate.

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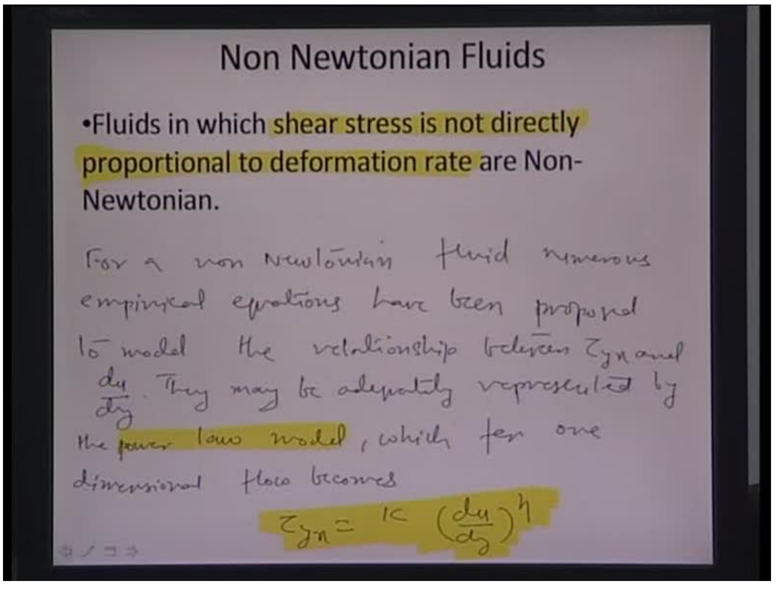

Now, once we have done Newtonian Fluids; let us actually look into the next very interesting topic of what really Non-Newtonian Fluids are. So, essentially it is again based on the relationship between shear stress on the velocity gradient. In Non-Newtonian Fluids, just contrary to what the Newtonian Fluids would show the shear stress is really not directly proportional to the deformation rate. So, essentially for such fluids there are numerous empirical equations which have been proposed to model, one of them being the power law model for describing such fluids and here as you see the shear stress yx is really proportional to instead of du by dy to the power one; it is proportional to some n term here, where n can be either more than 1 or less than 1, depending on what it is, the fluid would vary in its properties or in physical properties etcetera.

So, there are different aspects like there is a different cases for different values of n, for which these equation would signify a different property altogether of such a fluid. So, let us look them look at the case by case.

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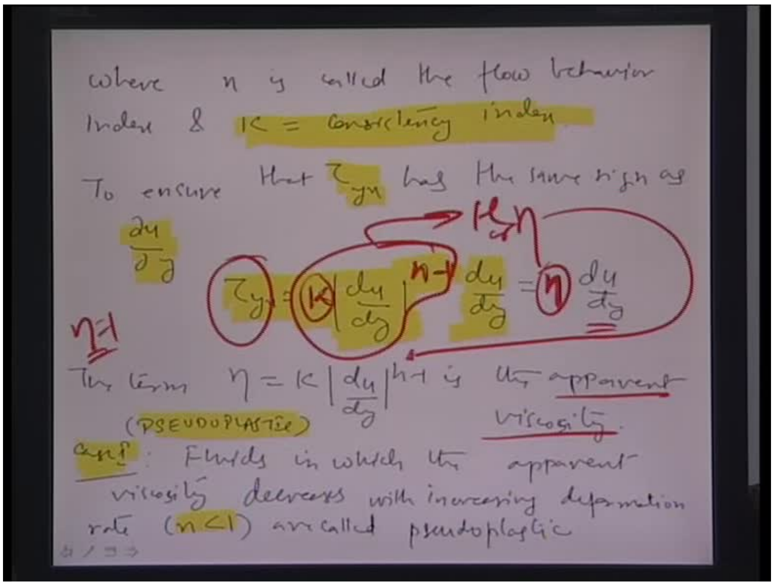

So, what your k here in this particular equation is also known as the consistency index. You can remodify this equation slightly to make it Tyx equal to k times of mod du by dy n minus 1 times of du by dy. This ensures that, the tau has the same sign as du by dx and essentially this k times of du by dy mod to the power of n minus 1. This (Refer Slide Time: 42:10) can be represented as the viscosity mu or eta, whatever you may call. So, in this case Tyx is becoming is equal to a viscosity eta value varies with respect to du by dy to the power n minus 1 with a proportionality constant equal to consistency index k and T tau yx is then related to du by dy by eta du by dy.

So, the eta here in this particular expression is also known as the apparent viscosity. This is really not the real viscosity; so, if your n minus 1 if your n is 0 if your n is 1 really in that case the eta comes out to be constant, which is the case of Newtonian Fluids with time and if it is more or less than one, there would be different the properties associated with that fluid. Let us look at the case, where this rate is or this n value is less than 1. So, such fluids are also known as pseudo-plastic materials. Here, the apparent viscosity because n is less than 1 would decrease with increasing deformation, if you look at this particular equation here n being less than 1 means that, this du by dy mod to the power n minus 1 would be essentially a negative quantity. The exponential here would be or the power here would be or the indice would be negative in nature. Therefore, any increase in du by dy would essentially mean a decrease in the in the viscosity value.

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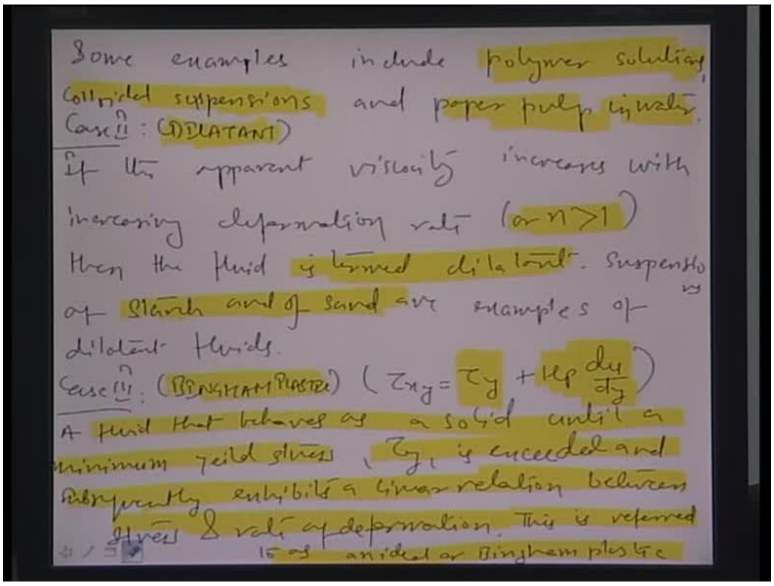

Similarly, if n is more than 1, in that case the fluids would be categorized as a dilatants. What definitionally that means is that, apparent viscosity would increase with increasing deformation rate. So, if n is more than 1 then the quotient n minus 1 of du by dy mod which we just saw in the slide back would be positive and because of that indice being positive with an increase in du by dy or d alpha by dt the shear stress tau yx would increase because of that. So, viscosity mu would increase because of that, viscosity being k times of mod du by dy minus 1.So, such fluids are known as dilatants. Some examples in case of the first earlier case of pseudo-plastics can be a polymer solution, which means that with an increase in the velocity gradient that means, if you make or stir the polymer more and more the viscosity value kind of decreases, because of this stirring action. Some other suspensions could be colloidal suspensions or paper pulp actually mixed in water, where if you move it more and stir it more viscosity decreases because of that stirring action. On the other hand, there may be these dilatant fluids like starch solution or sand, probably where the more and more stirring action would ensure that there is a greater packing between the different grains, which would cause the viscosity to go up. So, if the du by dy is more in this case, n being greater than 1; then the viscosity mu would go up because of increasing du by dy.

So, that is what a dilatant would be; there is another case however, which is related to really the way that shear stress would vary and how or where up to where which point it would be a solid and then change state. So, it is essentially kind of material where there is a certain shear stress the properties more like a solid above that cut off shear stress, the fluid would behave in a Newtonian manner, So, such fluids are also known as Bingham plastics. Here (Refer Slide Time: 46:21), the basic equation to represent tau xy would be in terms of some kind of intercept values tau y, up to which the fluid behaves as if it were just a normal solid beyond which it would also have this mu p du by dy component, which is related to how a fluid really looks like. This fluid behaves as a solid, until a minimum yield stresses is attained let us say tau Y and then after it is exceeded it start subsequently exhibiting a linear relationship between stress and rate of deformation, which is same as the Newtonian Fluid. So, this is referred to as an ideal or Bingham plastic.

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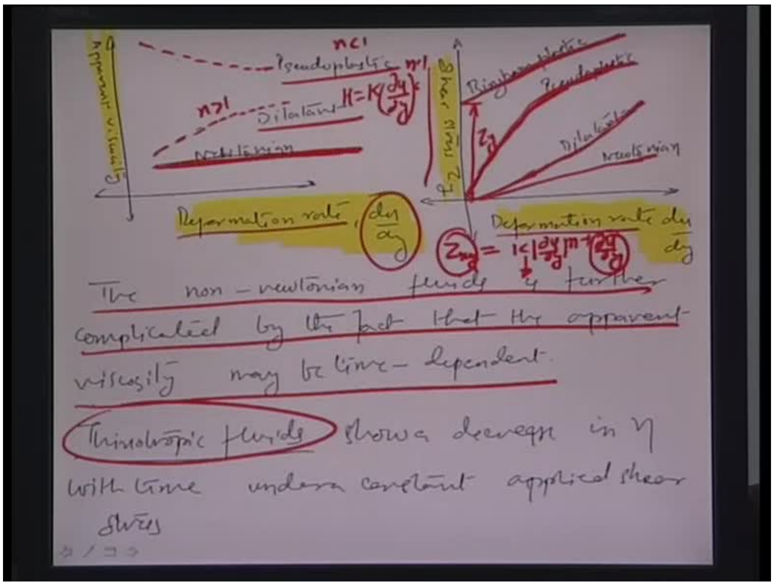

Let us actually see, what these some of these would look like on a scale of shear stress versus viscosity or shear stress versus the deformation rate du by dy. So, if you really try to draw pseudo plastic dilatant Newtonian kind of fluid on a scale of apparent viscosity versus deformation rate du by dy as can be seen here, apparently the Newtonian Fluid is one where this would be a constant parallel to the x axis, which indicates that there is a constant apparent viscosity irrespective of whatever the du by dy is or whatever the velocity gradient is and the case of pseudo plastic, as we know it is a material where if the du by dy increases, because n being less than 1 the apparent viscosity should come down because of that the index being negative if you may remember. So, this is essentially what a pseudo plastic would behave like so if deformation increases apparent viscosity comes down and for a dilatant it is opposite behavior. So, if as the deformation rate increases in that case the apparent viscosity goes up. So, that is what a dilatant essentially would mean; so this is a pseudo plastic where the viscosity apparent viscosity falls down with deformation rate dilatant where it goes up with deformation rate and Newtonian Fluid where the viscosity actually is constant with the increase in deformation rate.

So, if you have similar kind of materials or elements plotted on a scale of shear stress tau y versus deformation rate . So, the Bingham plastic can be accommodated here as you see here the Bingham plastic really definitionally is something which would be acting like a solid up to a certain yield stress tau y. So, this is the yield stress the intercept tow y after which it would start behaving as if it were a Newtonian Fluid. So here, in this range the deformation rate is really proportional to the shear stress after this intercept stress a zeal stress has been crossed over.

For a pseudo plastic material, with an increase in the deformation rate of course because, as you see here the apparent viscosity kind of goes down with increase in deformation. Initially, there is an increase in the shear stress up to a point after which it kind of again starts becoming a kind of asymptotic to a certain value. So for a dilatant as you see, the behavior is just opposite ways that means you know it kind of increasingly goes on adding up the shear stress and one of the reasons why this these pseudo plastics and dilatant behave in this manner that if you may remember for a pseudo plastic the mu the viscosity is really equal to the consistency index times of du by dy to the power of n minus 1 times of the and so where for a pseudo plastic as you know the n is less than 1 and for a dilatant it is more than 1. So, in one case you are seeing the viscosity is going up right and continuously and in other case the apparent viscosity is coming down, but as you plot the shear stress, the shear stress tau xy really would be proportional or it will be equal to this k times of the du by dy mod to the power of n minus 1 times of du by dy; which means that if there is an increase in shear stress tau xy because of an increase in du by dy in both the cases, but as the du by dy increases in case one that means in case of pseudo plastic the viscosity comes down with time. Therefore, there is an instance or there is a cut off deformation rate beyond which, the viscosity factor starts outweighing really lessening of the viscosity is kind of outweighs the increase in the du by dy. Therefore, it kind of stabilizes to a certain value and then falls down beyond it and in case of a dilatant it is the opposite effect because there is an add on and therefore the du by dy to the power of n minus 1 component kind of starts dominating after a while and it further increases the shear stress value.

In case of a Newtonian Fluid, though as the viscosity is constant would express; we would expect a linear behavior between the shear stress tau y and the deformation rate du by dy. So in Non-Newtonian Fluids, situation is further complicated by the fact that the apparent viscosity maybe time dependent, some of these fluids are also known as “Thixotropic fluids” , where it would show typically a decrease in in the viscosity value with time under a constant applied shear stress.

Thixotropic fluids may pose a situation where with time you may feel that just temporally the viscosity changes, I mean decreases, after some maybe with or without deformation; sometimes if it is with deformation the viscosity is changing it may be classified as a rather a pseudo plastic fluid, but if suppose you just keep something like, let us say glass and beyond a certain things you see beyond a certain time it kind of deforms and shears out and slowly the viscosity decreases with time so that that can be categorized as a Thixotropic fluid.

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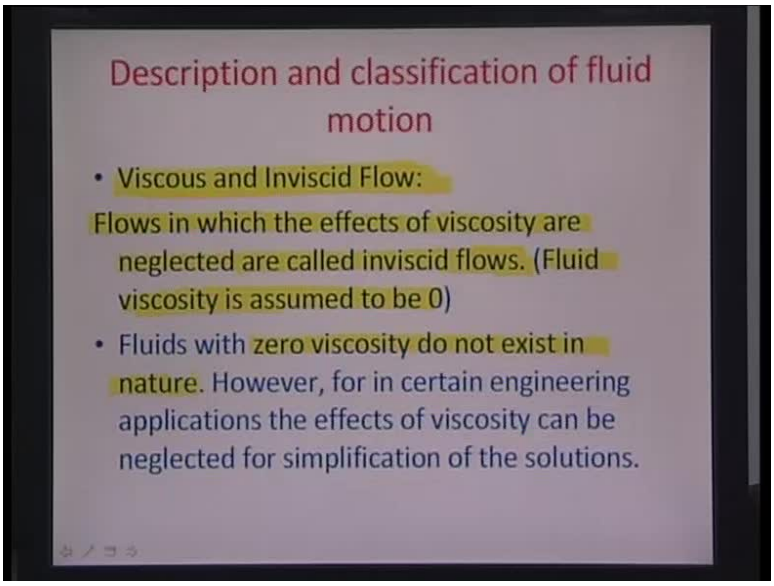

So, basically in a nutshell, you can describe fluid flow to be either Viscous or Inviscid. These concepts are very important at this stage, as I again would like to reiterate that because in case of micro scale flows, typically all fluids so basically the whole idea is that fluid flow can be really divided into Viscous and Inviscid domains. Again, I would like to reiterate that these concepts are very important for this particularly micro scale flows, because essentially all micro scale flows have very prominent viscous forces and effects, which makes this flow behave flows behave totally differently than the macro scale counterparts. So, intuitively whatever you think about would normally happened to a set of fluids in macro scale can really not be translated to the micron size scale or micron scale transport.

Effectively, you can categorize Viscous and Inviscid flows essentially as flows in which the effects of viscosity are either felt or neglected. Once, in which it is neglected is known as Inviscid. So, viscosity is assumed there to be typically 0. This is really not a real world situation but as I will illustrate in just in little bit, how the viscosity can be taken as 0 especially in macro scale whenever there is let us say a fluid layer which is approaching a fixed plate, we might have a zone or a domain where we can treat the viscosity safely as 0. So, it is more an approximation than ideal situation.

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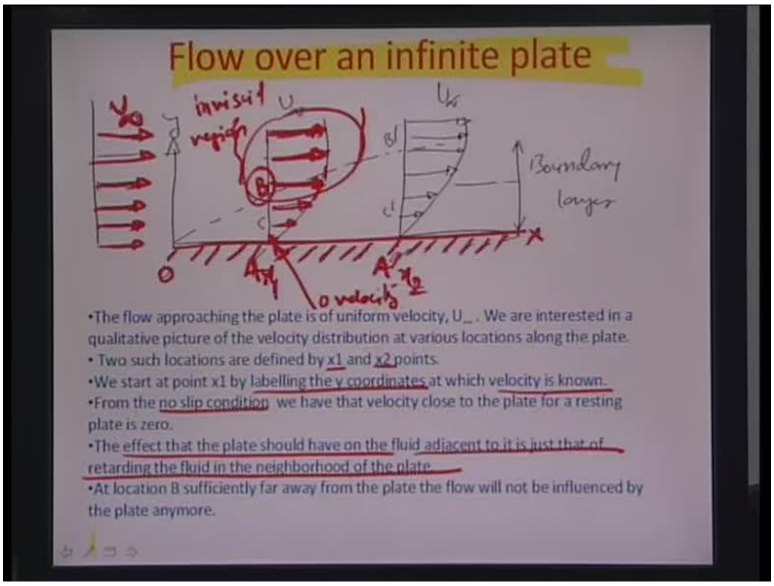

Normally, although they do not exist in nature, I mean with 0 viscosity particularly, however in certain engineering applications, the viscosity can be small enough to be neglected one such application is flow over an infinite plane as you can look at in this particular illustrations.

Suppose, you have this fixed plane here, which is represented by this surface o x and this also in the x direction of o x. So here as you see, the flow approaching the plate is of uniform velocity, let us say U infinity. So, there is a certain flow which is approaching which has a velocity U infinity. So, the floor when it approaches, we are first probably interested in getting a true picture, a qualitative picture of what would happen to the flow when it starts just about entering the zone where there is a fixed plate at the bottom.

Let us say, we have two locations along this plate x1 and x2 at points A and A dash respectively. Where we are trying to investigate, what kind of behavior will be expected so we have x1 x2 and we start let us say point x1 here by labeling the y coordinates at which the velocity is known and then ultimately plotting the velocity as a function of or in the y direction you are plotting the x velocity magnitude as it moves from x1 all the way to let us say the point B.

So, as we know that very close to the plate, we have a no slip condition or a no slip zone where typically the velocity 0 as indicated in this particular region. So, this is a case of no or 0 velocity or no slip in this particular region and what really would be the effect of fluids which are close to this particular point; so there the effect that the plate should have on fluid adjacent to it is just that of a retarding the fluid in the neighborhood of the plate. So, it has viscous forces. Now at a location B, which is sufficiently far away from the plate the flow will never be influenced by this particular no slip layer, because the velocity has already attained the certain value v infinity beyond that. So this particular region, we can actually kind of approximate as inviscid region where the viscous effects are not felt. So, velocity here irrespective of the fact that the plate is close by, has already attained the U infinity magnitude and are all same.

So, that is what an inviscid flow would typically look like in a physical situation. I would like to continue a little more of the discussion in probably the next lecture; we are kind of closing onto the time here. So next topic that I would illustrate would define these things in a little manner and try to develop a physical understanding as to how the flow develops or what is the layer which separates from the fully developed flow from the developing flow. So, I will do that analysis in the next lecture. Thank you!