Bio-Microelectromechanical Systems

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Module No. # 01

Lecture No. # 25

Hello, welcome back to this 25th lecture on bio microelectromechanical systems. We will just quickly start with a brief review of what had been done in the last lecture.

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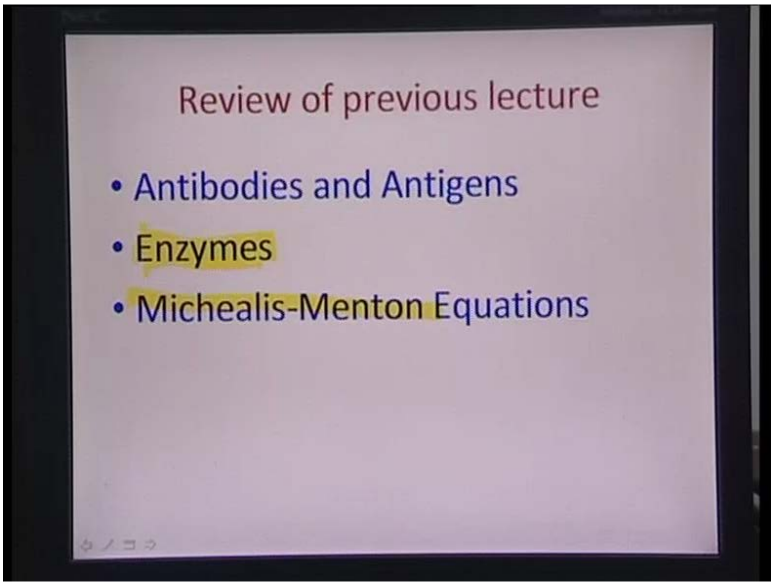

Review of previous lecture

- Antibodies and Antigens
- Enzymes
- Michealis-Menton Equations

We talked about antibodies, antigens and their kinetics of binding. We also mentioned about enzymes definitions and what they would do, and then we talked little bit about these wonderful set of rate kinetics equations called Michaealis-Menton equations for studying the properties of how an enzyme binds to a substrate producing an intermediate compound and then again breaks down into the product and the enzyme comes out as itself, essentially, how it catalyzes any process. We also talked a bit about the binding kinetics of antigens and antibodies and the way they would behave to changes in the ambient.

Today, we will try to address some issues. In last lecture, we also saw about some of the various ways and means of immobilization mechanisms of the biological entities onto the sensor surfaces side; in detail about adsorption phenomena also classified it into chemical and physical adsorptions. We also studied about micro encapsulation, about covalent bonding or direct covalent binding or the biological moiety onto the surface of interest. We also talked about cross linking mechanisms and entrapments within gels or gel mattresses.

Why all the things are required is because, you have an entity and it causes an electrochemical change or a response. We had at the very outset defined that most of the sensing mechanisms which are commercially available as on data electrochemical. Therefore, with the mobilization of such biological entities onto the sensor surface, the obvious question which comes into this is, - how to measure the electronic response?

Therefore, a very important technique that we had touched upon earlier and I would like to detail it little bit today is Voltammetry. We had seen in detail, what potentiometry would mean in this. Essentially, it is the measurement of voltage with concentration at 0 current. In this case, in voltammetry is a plot between the current and voltage. It finds out redox reactions and tries to ascertain species by measuring its reduction and oxidation potential. So, let us look at this in little more detail, what voltammetry would involve typically?

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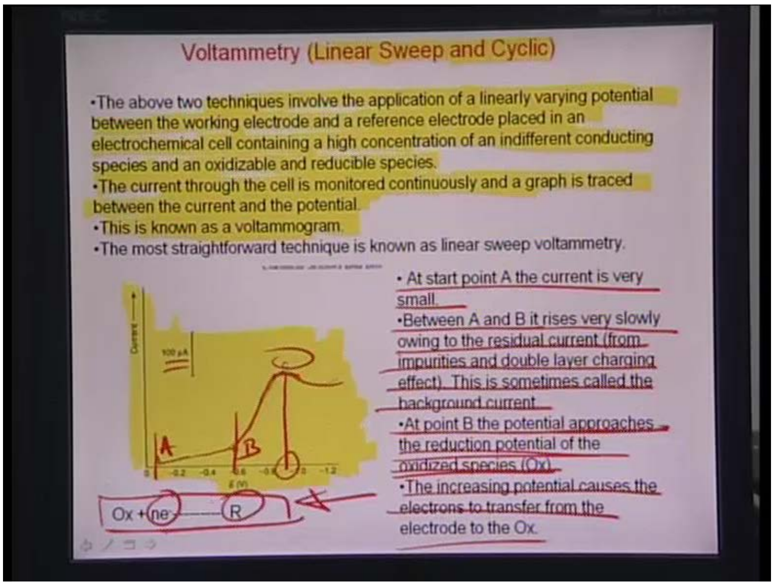

Voltammetry (Linear Sweep and Cyclic)

- The above two techniques involve the application of a linearly varying potential between the working electrode and a reference electrode placed in an electrochemical cell containing a high concentration of an indifferent conducting species and an oxidizable and reducible species.
- The current through the cell is monitored continuously and a graph is traced between the current and the potential.
- This is known as a voltammogram.
- The most straightforward technique is known as linear sweep voltammetry.

- At start point A the current is very small.
- Between A and B it rises very slowly owing to the residual current (from impurities and double layer charging effect). This is sometimes called the background current.
- At point B the potential approaches the reduction potential of the oxidized species (Ox).
- The increasing potential causes the electrons to transfer from the electrode to the Ox.

$Ox + ne^- \longrightarrow R$

There are 2 different kinds of voltammetric techniques: linear sweep and cyclic. Essentially, both these techniques involve the application of a linearly varying potential between the working electrode and the reference electrode placed in an electrochemical cell and in a manner that this cell actually contains the reduction oxidation species with a high concentration of indifferent species, which does not have any interfering ions with a particular electrode. So, the obvious thing to assume here is that as you pump in electrons inside the cell, there is a tendency of both the reduction and oxidation mechanisms to take place in the couple to formulate.

However, if you could have an electronic response of the system by measuring the current at different voltages, what you would see is a peak. The peak is more because, as a species is essentially oxidized and it suddenly liberates electrons. There is a tendency of all these electrons to shoot up one and all at a single potential point. Therefore, there is a huge rise in current at a certain potential and that is also known as the Oxidation potential of the particular species. So, species can be identified by its oxidation potential, the potential of the cell and to which the species get oxidized may vary as the oxidized agent or the agent which is getting oxidized varies in concentration as well as in type or nature.

Therefore, in such a case the current monitoring is very important through the cell and continuously a graph is traced between the current and the potential. We know this (Refer Slide Time: 04:38) is a voltammogram. Here, as you can see is nothing but a voltammogram. As you can see here, this measurement indicates the potential E v on the x axis along with the current i on the x and the y axis. As you can see the curve here (Refer Slide Time: 05:00) is really something like an irregular shaped curve starting from the point A and there is a linear rise and then there is a sudden exponential rise, where the reduction potential happens, the oxidation potential happens and there is a shootout or burst of electrons, which increases the current. Therefore, this particular value here would also ascertain what species number 1 and what concentration of the species, which is getting oxidized number 2.So, these can be obviously concluded.

Let us look at the various parts of the curve - the start part of this curve between points A and B as you can see here (Refer Slide Time: 05:38) the current essentially is very small, because it is just an Ohmic response with the voltage. As the voltage increases, there is an increase in current V is equal to I R and from the point B there is the potential suddenly approaches the reduction potential of the oxidized species. Therefore, there is a sudden shootout or burst of electrons, which increases the current. So, the increasing potential causes the electrons to transfer from the electrode to the oxidant Ox and so there is an increase in current, of course it is a reverse of electrons. If there is electron depletion, there will be an increase in current because of that. Here, the reaction which takes place is oxidation; Ox happens by combining with n electrons to formulate the reduction R in the process.

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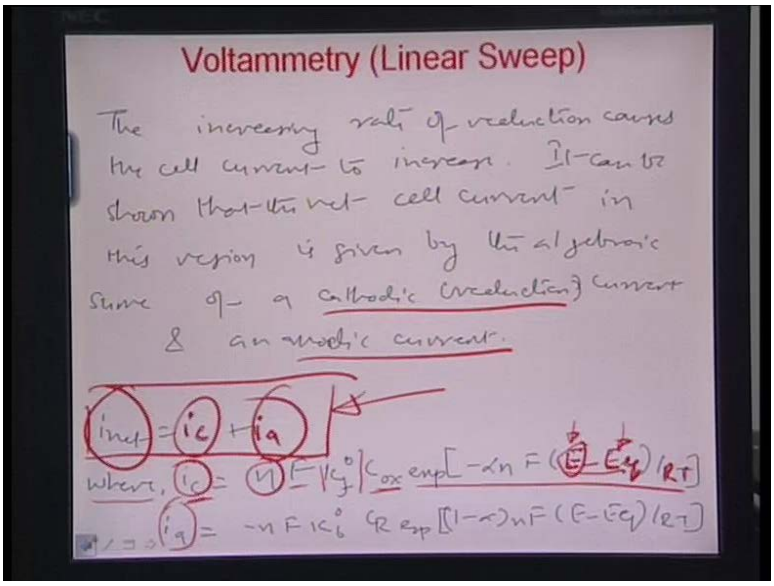

Voltammetry (Linear Sweep)

The increasing rate of reduction causes the cell current to increase. It can be shown that the net cell current in this region is given by the algebraic sum of a cathodic (reduction) current & an anodic current.

$$i_{net} = i_c + i_a$$

where, $i_c = nFk_f^0 C_{ox} \exp[-\alpha nF(E - E_q)/RT]$

$i_a = -nFk_b^0 C_R \exp[(1-\alpha)nF(E-E_q)/RT]$

As you see, some of the important conclusions here that the increasing rate of reduction causes the cell current to increase that are conclusion number 1. You can also be shown that the net current in the cell in this region is given by the algebraic sum of the cathodic and the anodic current and let us say i net here as you can see in this particular illustration here (Refer Slide Time: 07:04) is given by i c plus i a, where i c is the cathodic current due to the reduction and i a is the anodic current. Although, the derivation of how i c and i a comes is beyond the scope of this course, I would like to just illustrate that the current i c can also be measured as proportional to the number of the electrons transferred n the Faraday constant the equilibrium rate of the forward reaction to take place the concentration of the oxidant. Then, you have this exponential term here exponential to the power minus alpha n F E minus E q by R T, E q is the equilibrium potential and E is the potential at a particular point, T is temperature in Kelvin and R is the Rydberg’s constant.

Here, the whole goal is that, as the E would increase you can see that there is automatically an increase in the cathodic current and simultaneously there is a fall in the anodic current. Therefore, the overall E net would really depend on position of E with respect to the E equilibrium, where this potential E of the current time point is located in comparison to the equilibrium potential of the particular reaction.

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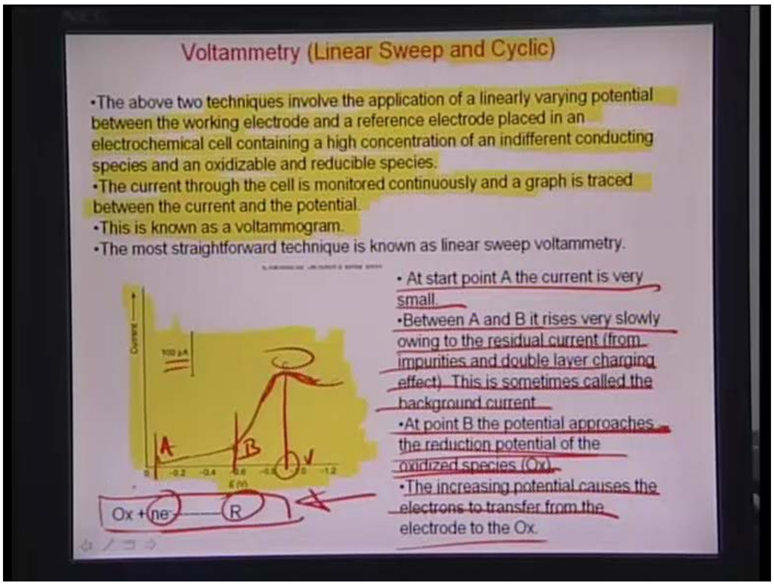

Voltammetry (Linear Sweep and Cyclic)

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So, that is how you can ascertain the various the cathodic and anodic currents in a cyclic voltammogram. But, the whole idea is to be able to ascertain, what is the voltage v at which, this reduction status getting attained the species oxidized species Ox is suddenly accepting electrons and getting reduced. So, there is an increase in current because of that current, again is the loss of electrons, in the conventional sense. So, what would typically happen, if we take the EMF on the reverse side? That means towards the oxidant side, the reverse process should take place. Therefore, there should always be a tendency of the reduced species to get re oxidized and the current to go down, because there is a sudden burst or emission of electrons or availability of electrons. Therefore, this really is a set of 2 peaks and you can back and forth alter the potential into making it more reduction potential or oxidation potential. Correspondingly, plot a hysteresis curve because of that and that is typically what you call as Cyclic voltammetry. Cyclic, because you have species getting oxidized and simultaneously, a species getting reduced on the same system by just application of EMF and then you can characterize this by measuring the current all along and getting an increase as the species get reduced and simultaneous decrease as the species get oxidized.

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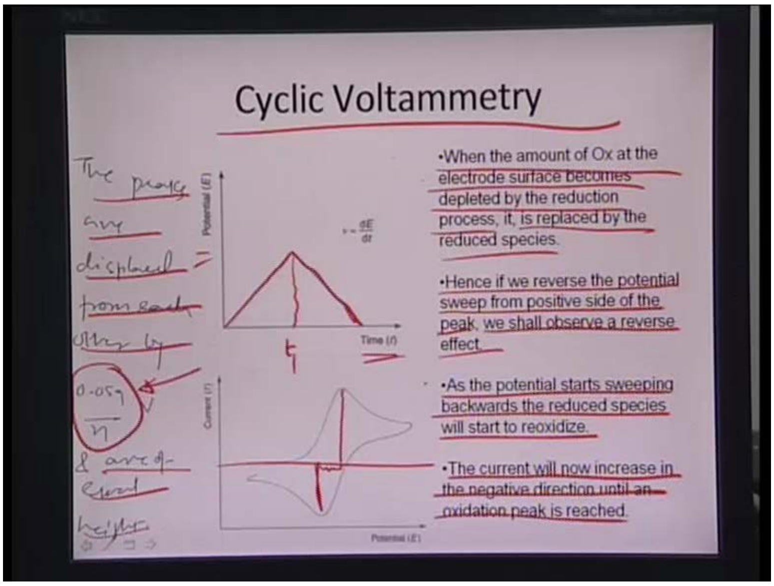

Cyclic Voltammetry

- When the amount of Ox at the electrode surface becomes depleted by the reduction process, it is replaced by the reduced species.
- Hence if we reverse the potential sweep from positive side of the peak, we shall observe a reverse effect.
- As the potential starts sweeping backwards the reduced species will start to reoxidize.
- The current will now increase in the negative direction until an oxidation peak is reached.

The peaks are displaced from each other by $\frac{0.059}{n}$ V & are of equal height

As you can see here, this illustrates what the cyclic voltammetry process really is. So, you get a potential or you give a potential with time as a triangular pulse, where you have an increased potential up to the time point let us say t 1 here and then beyond t 1 you have a reduction in the potential or the potential in the reverse direction. As you can see here, on the forward side you have an oxidized species suddenly getting reduced and losing electrons to the electrodes. Therefore, resulting in an increased current and reduced species getting oxidized here with a burst of electrons, which get a reduced current.

So, the amount of Ox at the electrode surface becomes depleted by the reduction process and is replaced by the reduced species in a nutshell. If you reverse the potential sweep from the positive side to the negative of the peak, we shall observe an exactly reverse effect that is an oxidation of the reduced species.

As the potential starts sweeping backwards, the reduced species will start to reoxidize again and the current will now increase in the negative direction until an oxidation peak is reached. What is more interesting here to observe is that, in an n electron transfer process, these 2 peaks are shifted exactly by 1 Nernstian slope 0.059 by n volts. They are typically of equal heights, because that indicates the equilibrium of concentration of the reduction and oxidation species inside the redox couple the redox reaction.

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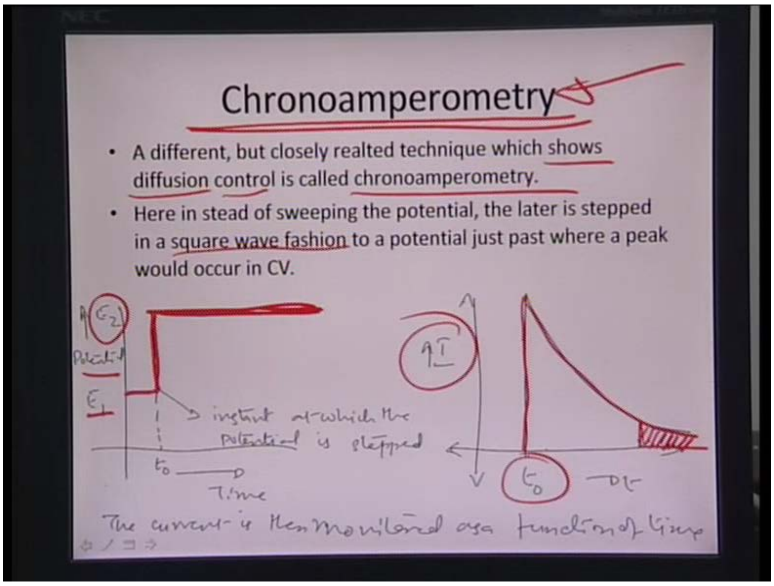

Chronoamperometry

- A different, but closely realted technique which shows diffusion control is called chronoamperometry.
- Here in stead of sweeping the potential, the later is stepped in a square wave fashion to a potential just past where a peak would occur in CV.

The current is then monitored as a function of time

So, that is kind of all about cyclic voltammetry that one should know. Another very interesting technique apart from what we had done before is Potentiometry and Cyclic voltammetry is Chronoamperometry. As the name suggests, chrono indicates time and amperometry is the measurement of current. We are actually trying to measure the current by giving a sudden potential and creating an oxidation process to happen or a reduction process to happen. So, you give a square pulse here, as you are seeing here (Refer Slide Time: 12:08) the potential E in this particular figure, a time instance t 0 goes from E 1 E 2, there is a huge step or a square pulse that has been given thereby continuing the pulse at E 2 or the state to be at E 2. What an essentially chronoamperometry show is, the diffusion control of such a case, where there is a square wave suddenly applied to a redox. So, if you measure the current with time, as you see here from time t0, there is a slow decrease in the current over time. If you monitor this, you can predict the type, nature and behavior of the redox species by monitoring how fast the current will decay with respect to time at a particular square pulse from E 1 to E 2 is generated.

The advantage here is that different slopes, different time aspects of decay of different species would be typically characteristic of the species type or nature and concentration and therefore can be used as a good technique to find out what is the active concentration of a certain species, which is electrochemically active in such a redox system or a redox couple.

So, we have by and large covered all the electrochemistry techniques; so far starting from potentiometry to cyclic voltammetry to chronoamperometry.

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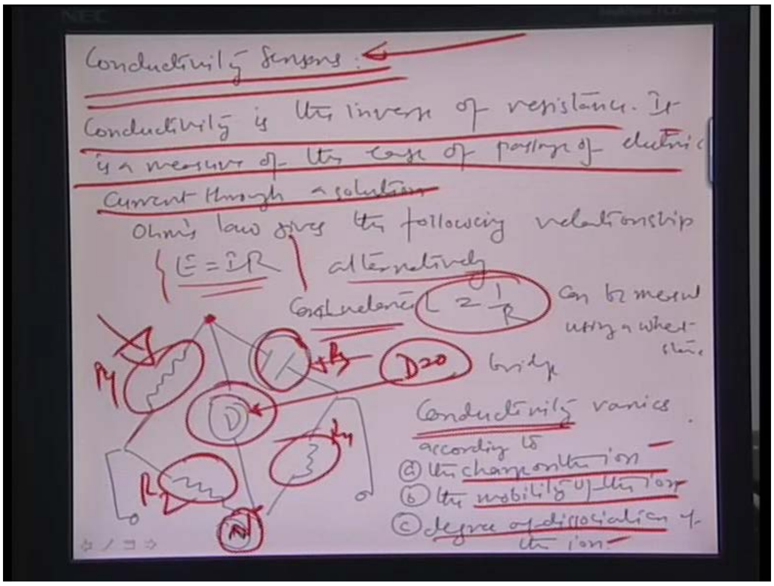

Conductivity Sensors:

Conductivity is the inverse of resistance. It is a measure of the ease of passage of electric current through a solution

Ohm's law gives the following relationship

$\{E = IR\}$ alternatively

Conductance $L = \frac{1}{R}$ can be measured using a Wheatstone bridge

Conductivity varies according to

(a) the charge on the ion

(b) the mobility of the ion

(c) degree of dissociation of the ion

We now shift to briefly to another very interesting area, which is “Conductivity based sensors”. So, conductivity is the inverse of resistance, it is a measure of the ease of the passage of electric current through a solution. If you see all the solutions, they can obey Ohm’s law. Alternatively, the conductance relation would be a kind of E by L, where 1 by R is equal to the L. So, EL is equal to I, can be what the Ohm’s law can be written as in terms of conductivity.

Now, what conductivity does is that as is obvious, it would have changing behavior with the change in the ion concentration of the other particular medium. It would also depend on the mobility of the ion. If ion is mobile, the conductivity would be more the flow of currents through that kind of a medium would be much higher and then it would also depend on the degree of dissociation of a particular ion, especially in a redox system.

So, these are some of the factors, where the conductivity, typically you can get a Wheatstone bridge kind of configuration As you can see here, (Refer Slide Time: 14:52) there are resistances; which are known in nature and then there is a meter at the centre here and this is the flow cell, where the conductance typically has to be measured. The idea is that these four combinations can be varied in a manner that R 1 by R 2, typically becomes equal to R 3 by R 4 and that is an instance by the principle of Wheatstone bridge, typically the D here would show 0, there is no current here in between these 2 terminals at a fixed potential. Therefore, you can find out the conductance of such a medium or you can also apply an AC field here as you can see here and do similar calculations of an AC Wheatstone bridge to find out what the conductivity of this cell is.

But whatever it is the conductivity sensors essentially are used for measuring, the charge of a particular ion, the mobility of the ion and also the degree of dissociation of the ion and such electrochemical information found out by this technique is very useful for sensing or measurement.

We are kind of done now, with this whole electrochemistry business and also seen some of the interesting behaviors of surfaces or surface potentials. Now, what I would like to explore is a little bit of fluidics as applied to the micron scale and for that we have to do some basic theorizing of fluid mechanics and understand some of the basic concepts.

So, what I am going to do now is to dedicate kind of the remaining part of this lecture to more towards understanding of some basic fluidics and then go into active microfluidics, where we can design pumps valves, mixers, PCR reactors so on so forth. Let us look at this a little more closely now.

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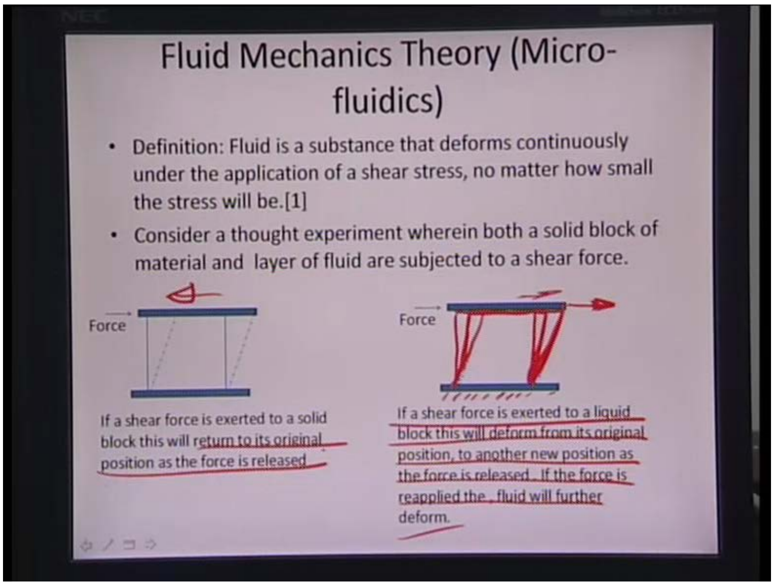

Fluid Mechanics Theory (Micro-fluidics)

- Definition: Fluid is a substance that deforms continuously under the application of a shear stress, no matter how small the stress will be.[1]
- Consider a thought experiment wherein both a solid block of material and layer of fluid are subjected to a shear force.

If a shear force is exerted to a solid block this will return to its original position as the force is released

If a shear force is exerted to a liquid block this will deform from its original position, to another new position as the force is released . If the force is reapplied the . fluid will further deform.

So, to start with microfluidics, as we know is the movement of transport of fluids at the microscopic lens scale but what really fluidics is, first coming from the term fluid. So, what is a fluid? A fluid is a substance that deforms continuously under the application of the shear stress. So, what happens, suppose I take this particular solid object and try to hold it here at the base and try to deform it from the top. So, if I deform it by holding it continuously at the base, it would kind of get sheared or it would at least try to get deformed in its shape, the immediately after we release the load it should come back automatically or elastically, if it has not crossed it is elastic domain, back into its normal shape. However, in case of fluid is the reverse kind of effect.

Whenever you have, let us say an experiment as we can illustrate here again (Refer Slide Time: 17:50) wherein you have fluid layer as you can see here, held between a mobile plate on the top and a fixed plate at the bottom and then you try to move this fluid is a plug along with this plate by moving the plate forward in the forward direction. As you can see here, the fluid would kind of shear up without returning back. So, there is some molecular deformation something where this fluid will tend to remain in a sheared condition. If you apply some more force, it will sheer to a different condition here. There is continuous shear process which is happening because of movement of the upper plate with respect to the lower plate.

So, I can summarize it by saying that, if a shear force is exerted in this case to a liquid block this will deform from its original position to another new position. As the force is released, this new position will be retained that is very important. The force is reapplied; the fluid will further deform along the same direction as that the force. However, in case of solid, it is the other way round the moment the force is released the block would tend to come exactly in the opposite direction and try to stabilize into its own shape. It will return to its original position, as the force is released in a particular solid. So that is essentially, what a fluid is it deforms on applying shear stress.

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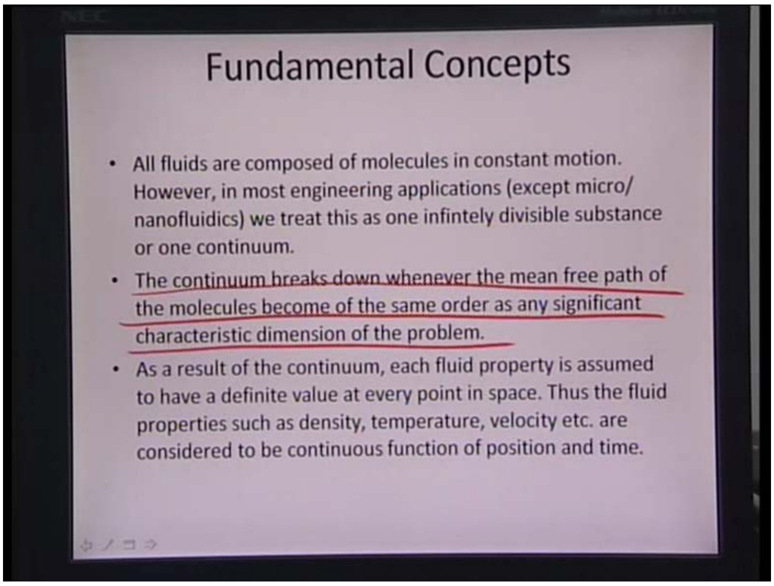

Fundamental Concepts

- All fluids are composed of molecules in constant motion. However, in most engineering applications (except micro/nanofluidics) we treat this as one infintely divisible substance or one continuum.
- The continuum breaks down whenever the mean free path of the molecules become of the same order as any significant characteristic dimension of the problem.
- As a result of the continuum, each fluid property is assumed to have a definite value at every point in space. Thus the fluid properties such as density, temperature, velocity etc. are considered to be continuous function of position and time.

What is important to know that at a more fundamental molecular level? In a fluid, the molecules are not so firmly bound, they are having lesser forces and they are in constant motion with respect to one another. Whatever it is, we still treat the fluid as 1 continuum body, even if the molecules are moving around with respect to another, we still treat as one big continuum, especially in the scales in which we can do bio-sensing or diagnostic.

However, if you go to a certain different domain in the Nano scale, there the properties of the fluid itself vary with time because as, I am going to illustrate this concept in next slides. What happens if you scale down to smaller scale but, then essentially in most engineering applications related to sensing and diagnostics we still continue to believe the fluid as 1 big continuum, one indivisible substance be it that the molecules are moving in a certain motion with respect to one another. Still they are bound by a certain domain a bigger domain and that domain keeps on changing shapes and sizes, but it keeps on doing so continuously and the fluid continuity is maintained.

The continuum breaks down, whenever the mean free path of the molecules become of the same order as any significant characteristic dimension of the problem. So, if you are considering the volume element to be too small in comparison to the mean free path of individual molecule, the probability of the continuum to break down is more in that respect. So, as a result of the continuum each fluid property is assumed to have a definite value because you have one continuum the fluid is behaving as one medium and spatially or at every point in space these fluid properties would show a fixed value average value. If it is of a certain finite size where this problem of the mean free path to be equalized to one characteristic dimension that is no longer there. The mean free path is much smaller in comparison to one of the characteristic dimensions. So, properties such as density, temperature, viscosity etcetera are considered to be continuous function of position and time. In such instances where the continuum still exists, where the continuum breaks down the change with time, they become totally independent of the space as well as time. They keep on varying space to space, time to time etcetera.

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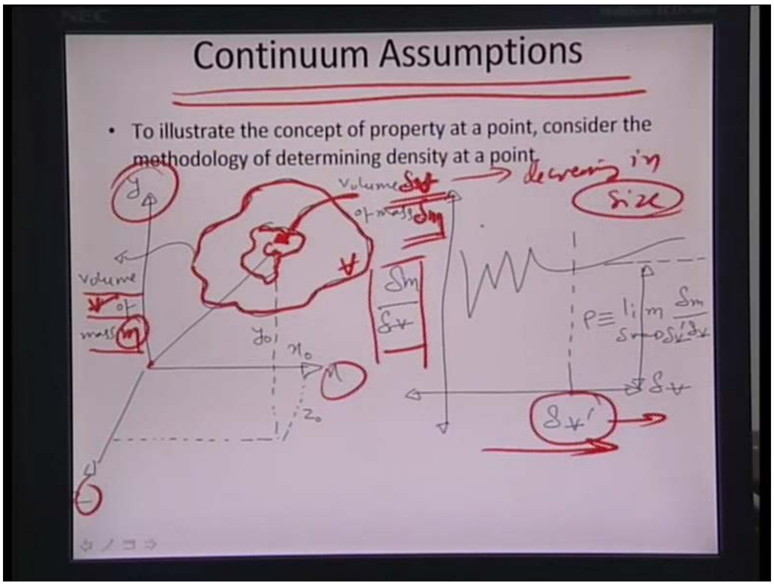

Continuum Assumptions

- To illustrate the concept of property at a point, consider the methodology of determining density at a point.

Let us illustrate this concept a little more in details suppose, we are talking about plotting the volume v of a mass of fluid m, as can be illustrated here with in this x y z Cartesian coordinate system as you can see. Let us suppose, there is a point C in space, which is that radius vector r from the origin and you assume 1 big volume around v tilda; this whole thing is v cross. We assume the mass of such an element m. Now, around the point C, we also take a much smaller volume delta v cross and take the mass of this particular element delta m.

So, this volume here is much smaller around C in comparison to the overall volume, which is v cross with a mass m. This smaller volume has much smaller volume, much smaller volume delta v cross and a much smaller mass delta m. So, what I would like to do is, to go ahead and plot on the ratio mass per unit volume with respect to the volume change. So, this is essentially the variable volume, we are assuming that this delta v is going on in decreasing in size to a certain extent. So, it is kind of closing on the point C and becoming more towards the point C that means it is becoming a point slowly from a finite volume and so it approaches continuously a smaller value.

So, what would happen to properties like density that means mass per unit volume; in case it achieves this, let us have a look at what that would be.

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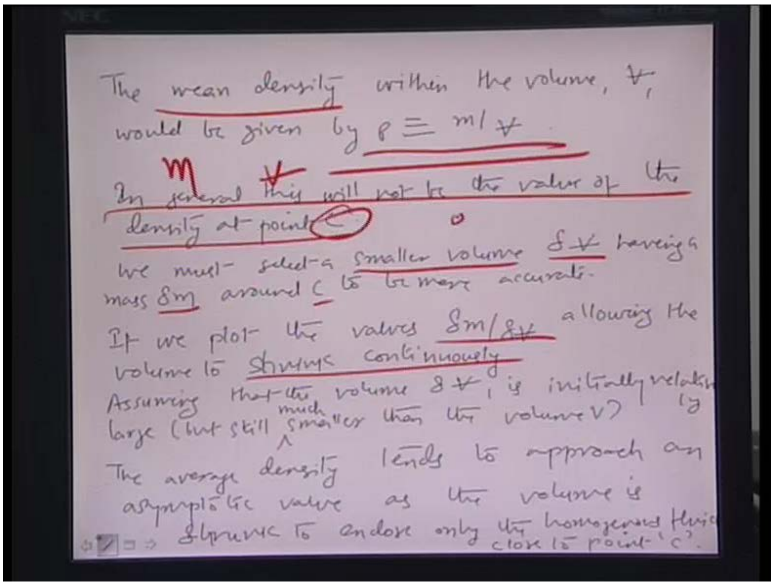

The mean density within the volume, $\forall$, would be given by $\rho \equiv m/\forall$.

m $\forall$

In general this will not be the value of the density at point C

We must select a smaller volume $\delta\forall$ having a mass δm around C to be more accurate.

If we plot the values $\delta m/\delta\forall$ allowing the volume to shrink continuously

Assuming that the volume $\delta\forall$, is initially relatively large (but still much smaller than the volume V)

The average density tends to approach an asymptotic value as the volume is shrunk to enclose only the homogeneous fluid close to point 'C'.

So, the mean density of this bigger volume v, where the mass is given by mass per unit volume. In general, this will not be the value of the density at point C, though very close to C. If you are just having a point dimension of C, we have to select a smaller volume delta v tilde. Let us assume, we have a mass delta m around c. Now, you are plotting this, allowing the volume to shrink continuously.

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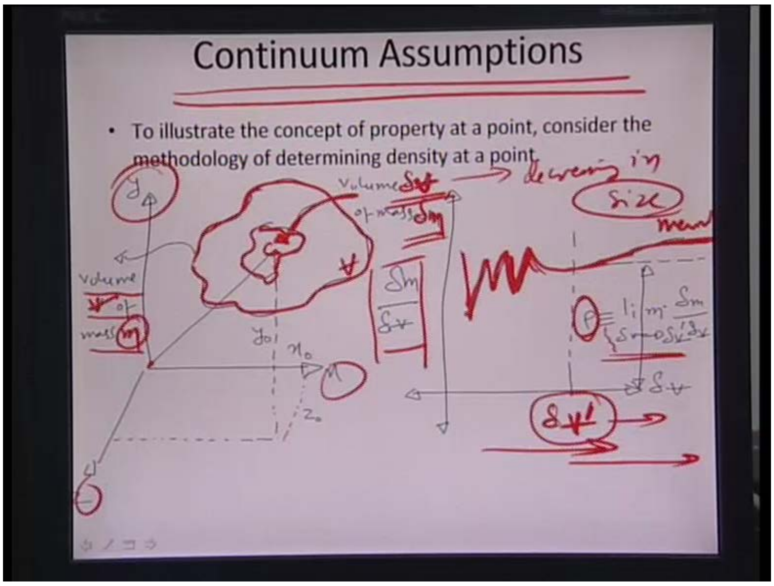
Continuum Assumptions

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So, there is a certain volume here as you can see, let us say delta v cross dash beyond which, the property density rapidly increases and asymptotes or becomes asymptotic to a fixed mean value, but below this you can assume that the dimension is so small that one of the characteristic dimensions is more like the dimension of individual mean free path of the molecules. So, the volume is small and the mean free path is greater; you never know there is an uncertainty or whether the molecule is within the volume or outside the volume. Thousands of these molecules, which are criss-crossing the surfaces of this small dimension and therefore, you are not 100 percent certain as to how much amount of a mass or how much amount of volume is there? Within the infinitesimally small point volume, that you are considering. Therefore, the density should be a function of time.

So, delta m by delta v dash below a certain v cross dash volume delta v cross dash volume becomes very erratic, because of this motion of the molecules or continuous seeping of the molecules through its boundaries.

Let us say, you behave the density behaves like this; it is very erratic with time. So, really the continuum assumption is really valid beyond this delta v cross dash value. Beyond which you can say that as the delta v is approaching a limit delta v dash cross, rho becomes delta m by delta v cross, as a continuum property. But, less than v cross dash, the rho is really a function of rho t. So, function of time it keeps on varying.

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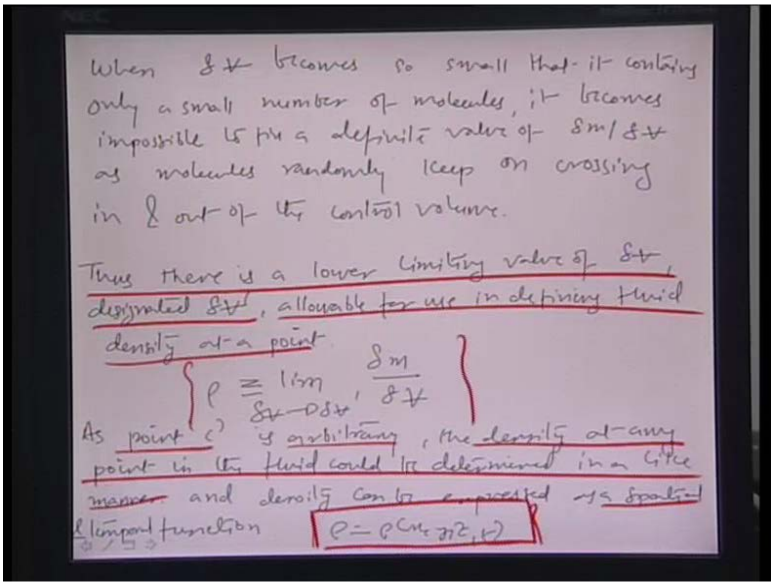

When δV becomes so small that it contains only a small number of molecules, it becomes impossible to fix a definite value of $\delta m / \delta V$ as molecules randomly keep on crossing in & out of the control volume.

Thus there is a lower limiting value of δV, designated $\delta V'$, allowable for use in defining fluid density at a point

$$\left\{ \rho = \lim_{\delta V \to \delta V'} \frac{\delta m}{\delta V} \right\}$$

As point 'c' is arbitrary, the density at any point in the fluid could be determined in a like manner and density can be expressed as spatial & temporal function $\boxed{\rho = \rho(x, y, z, t)}$

So, that is what continuum essentially means in the real physical sense. In summary, there is a lower limiting value of delta v cross designated at delta v cross dash allowable for use defining fluid density at a point. So, at point c being arbitrary the density at any point in the fluid could not be or could be determined in a like manner. The density can be expressed as a spatial temporal function or spatio-temporal function as a function of x y z and t as you can see here in this particular equation (Refer Slide Time: 27:20). So, rho becomes a space time function.

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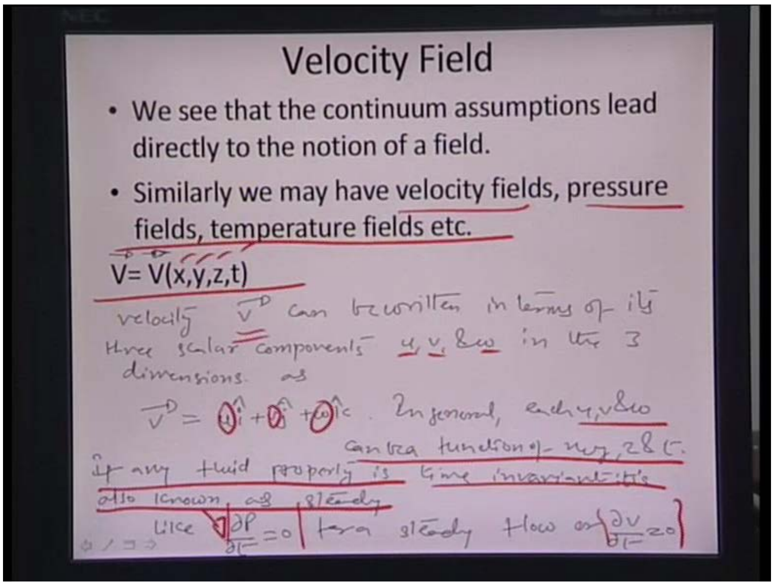

Velocity Field

- We see that the continuum assumptions lead directly to the notion of a field.
- Similarly we may have velocity fields, pressure fields, temperature fields etc.

$\vec{V} = \vec{V}(x, y, z, t)$

velocity $\vec{V}$ can be written in terms of its three scalar components u, v & w in the 3 dimensions as

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$. In general, each u, v & w can be a function of x, y, z & t.

If any fluid property is time invariant it's also known as steady like $\left\{\frac{\partial \rho}{\partial t} = 0\right\}$ for a steady flow or $\left\{\frac{\partial V}{\partial t} = 0\right\}$

So, important concept to assume or to find out is what a velocity field would be? By the continuum assumption, we see directly there is some kind of a notion of a velocity field developing; that means if velocity really is a function of x y z and time, but it is continuum property, it is an average velocity property. We should have a kind of trajectories along which this velocities or meet a certain condition and they follow a certain space-time relationship. So, these trajectories can then act to define the flow paths of the various molecules, which are transporting through this fluidic medium. They could be timelines, they could be streak lines, they could be streamlines, and they could be path lines and all different kind of lines. So, we will like to have some illustration as to what this velocity field is and then go into this various aspects of various methods of defining the flow paths within the fluid.

So, we may have similar equations as velocity fields, pressure fields, and temperature fields etcetera within this volume. Let us say, vector V is a function of x y z and time, that is typically the case because, V can be written as a summation of a 3 components u v w in the x y z direction as u i cap plus v j cap plus w z cap and in general, each these u v and w can be a function of x y z and time. We can assume that a thing, if the fluid properties time invariant, it is also known as Steady-state flow. So, the d rho by d t or as a matter of fact any other property like d v by d t or d P by d t, P being the pressure; they are all equal to 0, there is no time variation of the particular flow. However, in flows which are heated up this may not be the case as because the density would keep on changing from the cold surface towards more towards the hot surface in touch with the fluid in question and. Therefore as we will see later on especially in micro scale fluidics we hardly need this component of heating and we can do away with the energy equation of the Navier stokes just because of that reason. So, in microfluidic situations, mostly we use time invariant flows d v by d t or d P by d t or d rho by d t or any other property with respect to time, does not really vary at all within such domains.

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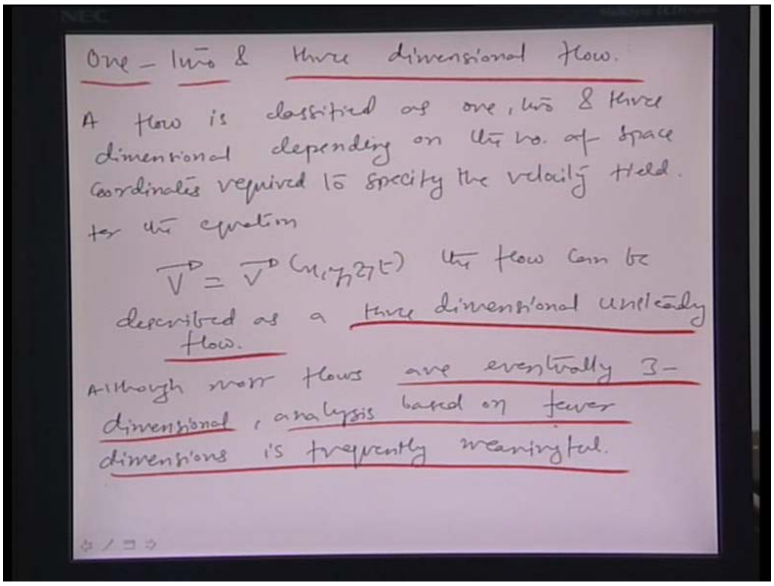

One - two & three dimensional flow.

A flow is classified as one, two & three dimensional depending on the no. of space coordinates required to specify the velocity field.

for the equation

$\vec{V} = \vec{V}(x, y, z, t)$ the flow can be described as a three dimensional unsteady flow.

Although most flows are eventually 3-dimensional, analysis based on fewer dimensions is frequently meaningful.

So, flows can further be described as 1 2 and 3-dimensional. The name suggests, it really depends on the number of space-time coordinates that is needed to specify the velocity field for certain equation. So, if it is just dependent on x and not on y and z or time, then that is called a One-dimensional velocity field and which is steady-state and if it varying on time it can be also known as Unsteady one -dimensional. Therefore, it varies with all the 3 coordinates you can call a 3-dimensional unsteady flow system. Although, the flows are eventually 3-dimensional analysis almost has always helpful, if you can convert it somehow into a fewer dimensions 2 or 1 in most of the cases. Depending on the symmetry of the situation of the geometry through which the fluids are flowing, you could easily convert it into more 1-dimensional than 2 or 3. Although, the actual real-life it is more towards 3 dimensional. So, this is just for calculation simplicity sake, that you are eventually assuming all 3-dimensional flows or predicting all 3-dimensional flows on basis of analysis on one or more dimensions but fewer a than 3.

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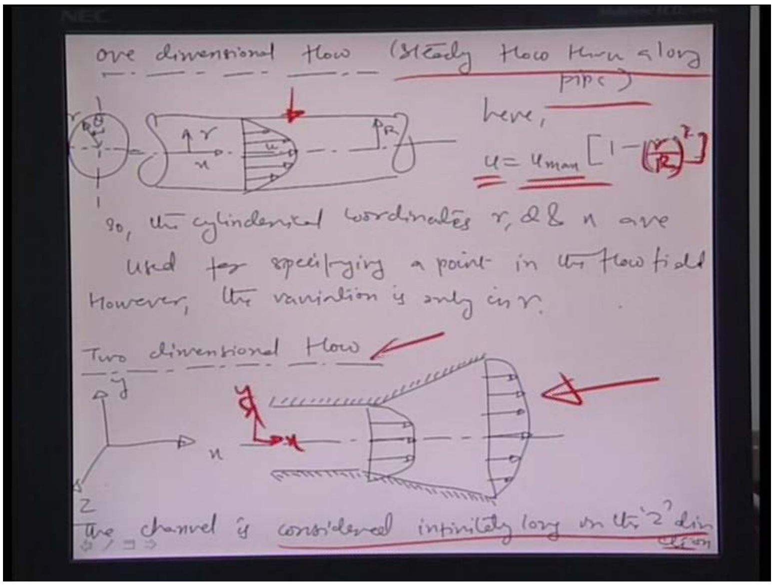

One dimensional flow (steady flow thru along pipe)

here,

$u = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$

So, the cylinderical coordinates r, θ & x are used for specifying a point in the flow field. However, the variation is only in r.

Two dimensional flow

The channel is considered infinitely long in the 'z' dir

Let us discuss some examples of 1-dimensional flows, especially steady flows through long pipes. You can see here, this really is a variation of velocity profile with respect to the radius r. So, u here is actually equal to u max, which is somewhere in the centre of the pipe, times of 1 minus small r by big radius R square. So, it is varying only on the radius vector. It is symmetric along the angle phi to describe the cylinder it is only varying along the radial direction.

So typically, this is the case in cylindrical coordinates or same is true for the z direction, z is too long in comparison to the width of the pipe, the radius of the pipe and so if you talking about micro-flows, where the dimensions of the pipe itself is very small in comparison to the length in which it is laid out. You typically can consider, it to be 1-dimensional flow problem, which is radially symmetric. If you assume that, the continuum assumptions or approximations hold true. Two-dimensional flows are represented here (Refer Slide Time: 32:54) in this particular figure. As you can see, the channel is considered infinitely long in the z direction that means in the plane of this particular illustration or in the board. Therefore, the variation though is mostly in the x and y if x and y are these dimensions here the flow velocity would be vary in only 2 dimensions in this particular case illustration.

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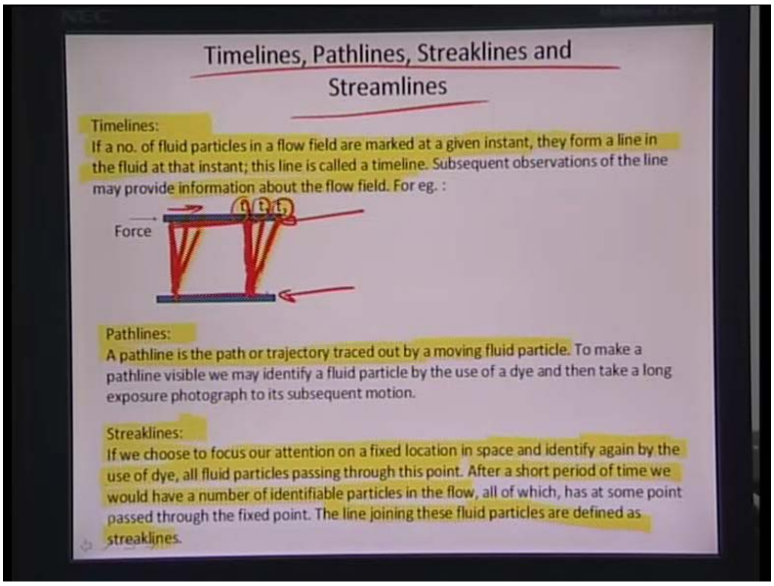

Timelines, Pathlines, Streaklines and Streamlines

Timelines:
If a no. of fluid particles in a flow field are marked at a given instant, they form a line in the fluid at that instant; this line is called a timeline. Subsequent observations of the line may provide information about the flow field. For eg. :

Pathlines:
A pathline is the path or trajectory traced out by a moving fluid particle. To make a pathline visible we may identify a fluid particle by the use of a dye and then take a long exposure photograph to its subsequent motion.

Streaklines:
If we choose to focus our attention on a fixed location in space and identify again by the use of dye, all fluid particles passing through this point. After a short period of time we would have a number of identifiable particles in the flow, all of which, has at some point passed through the fixed point. The line joining these fluid particles are defined as streaklines.

Let us talk, a little bit about these concepts of timeline, pathline, streaklines and streamlines. These are important for understanding of later on topics of microfluidics or micro flows. So, if you talk about timelines here- the timelines are these various instances t 0 t 1 and t 2, what they are typically is that, if a number of fluid particles in a flow field are marked at a given instant of time at a certain point in space. Then, they form a certain line; it can be a straight line. This line is known as the Timeline and the variation of this line would provide information about the flow field like for example, in this particular case, as you can see the fluids at the very outset were held in a very symmetrical manner between the mobile plates here on the top and the fixed plate. The mobile plates starts moving and it being fluid and the property dictates it to be or take the shape of the relative displacement between the upper plate and the lower plate, which kind of varies with time and kind of increases with time. As you see here, from time t 0 to t 1 to t 2, you have all these different lines which are getting formulated. So, this was a time t 0 all the particles which were here in this particular edge of the fluid over defined by this timeline at t 0. Now, it moves to t 1 the timeline also changes to t 1 and t 2. So, the flow field here would be defined by tracing these timelines that is what timeline means.

What are pathlines? A pathline is a path or trajectory traced out by a moving fluid particle. We can use identification techniques to make these timelines or these pathlines visible things like, we can introduce a small dye or an ink in water and we can see how this ink diffuses through or across. So, the ink particle moves in a path or trajectory traced out by moving the fluid around it or the ink itself with time. So, that is a pathline.

So, we can take a long exposure photograph of such a flow situation and see where this dye is slowly going into or diffusing into. So that is what the pathline of the dye would be in let us say, water.

The other important classification is streaklines and this is in little bit indirect way of describing flow fields. So, if you choose to focus our attention on a fixed location in space and identify again by the use of dye, all fluid particles passing through this point. After the short period of time, we would have a number of identifiable particles, which flow through that point in space in the flow field. All of these particles flow through that same fixed point as indicated before. Now, the lines that would typically join these fluid particles, once they are out of that small point and they are somewhere along the fluid after that points. The lines which would join all those particles, which came from this 1 spatial location is known as a streakline. So, we have already seen what a timeline pathline and streaklines would be and they have all different connotations. You could use these to explain fluid flow behavior or motion of molecules in a continuum fluid mechanics very easily.

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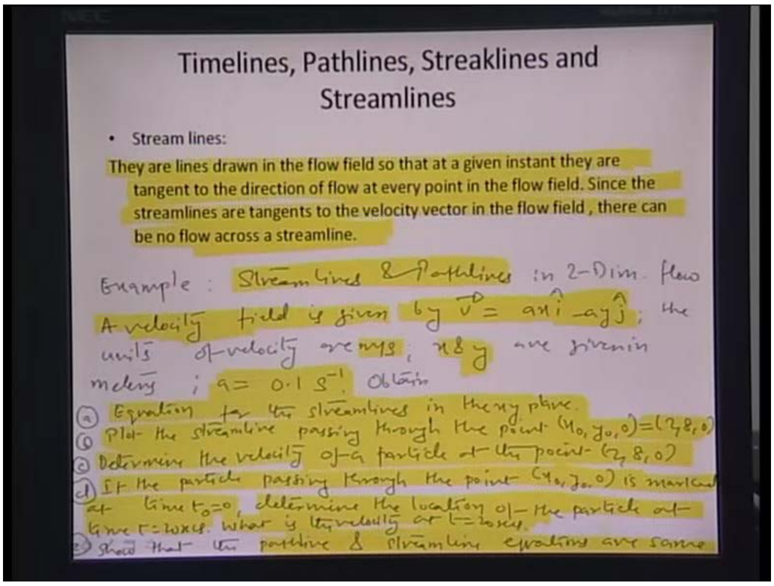

Timelines, Pathlines, Streaklines and Streamlines

- Stream lines:

They are lines drawn in the flow field so that at a given instant they are tangent to the direction of flow at every point in the flow field. Since the streamlines are tangents to the velocity vector in the flow field , there can be no flow across a streamline.

Example : Streamlines & Pathlines in 2-Dim. flow

A velocity field is given by $\vec{v} = ax\hat{i} - ay\hat{j}$; the units of velocity are m/s; x & y are given in meters; $a = 0.1\ s^{-1}$. Obtain

ⓐ Equation for the streamlines in the xy plane.

ⓑ Plot the streamline passing through the point $(x_0, y_0, 0) = (2, 8, 0)$

ⓒ Determine the velocity of a particle at the point $(2, 8, 0)$

ⓓ If the particle passing through the point $(x_0, y_0, 0)$ is marked at time $t_0 = 0$, determine the location of the particle at time $t = 20$ sec. What is the velocity at $t = 20$ sec.

ⓔ Show that the pathline & streamline equations are same

The other very interesting factor is a streamline. So what are streamlines? definitionally They are lines drawn in the flow field, so that at a given instance they are tangent or they are tangential to the direction of flow at every point of the flow field. By drawing, let us say, you have a particle which moves along a certain trajectory and this trajectory’s like curvilinear like this and there is a directional variation in the velocity. This variation in the velocity by virtue of the direction and that is a reason why the particle describes this trajectory. To combining all these tangents together would give you what you call the streamline or flow. Very interestingly, the streamlines do at a microscale, define at all possible scales, whether the flows are laminar or turbulence if they are very streamlined and they are very laminar in nature that indicates the less mixability between 2-3 different flows, which would have similar lines parallel to each other as they are going along in a small channel side by side. Since, the streamlines are tangents to the velocity vector in flow field, there can be no flow across a streamline.

So, what I would go ahead is to give you an example problem, to calculate how we can really estimate an equation on a 2-dimensional flow between the x and y coordinate to estimate, what we call a streamline.

This particular example of velocity field is given by the expression v is equal to a x i cap minus a y j cap, x and y are the position coordinates and a is some constant and i cap j cap are unit vectors in the x and y direction, x and minus y direction and the units of velocity here are meters per second, a is given by the expression 0.1 second inverse. Just to be consistent on this equation and the dimensional aspect. So, we want to obtain the equation for streamlines in this particular case in the x y plane. Basically, as in this example the velocity field is given by this equation v vector is equal to a x i minus a y j, i cap and j cap are the unit vectors in the x and y direction and a basically is having the units of second inverse. So, a is given as 0.1 second inverse, x and y are the x y coordinates and basically the velocity v is given meters per second in this particular expression.

So, the first problem here is to find equation for streamlines in the x y plane, we also want to plot the streamlines passing through the point x 0 y 0 0, where x 0 and y 0 are 2 and 8 respectively. Essentially, these two things are something like a plot, that we would be able to a define in terms of a geometrical relationship between x and y. We would also like to determine the velocity of the particle at a point 2 8 0 and also that if the particle passing through the point x 0 y 0 0 is marked at time t 0 is equal to 0,we have to determine the location of the particle at time t equal to 20 seconds. So, typically it is almost everything related to streamlines and pathlines and how to plot them and also we need to ascertain the velocity at t equal to 20 seconds. So, one thing is determined the location another at that particular new location we have to also find out what the velocity of the particle at t equal to 20 second. Actually that is what we had approved also that the path and streamline equations are one and the same in this particular case. So, let us actually start this problem.

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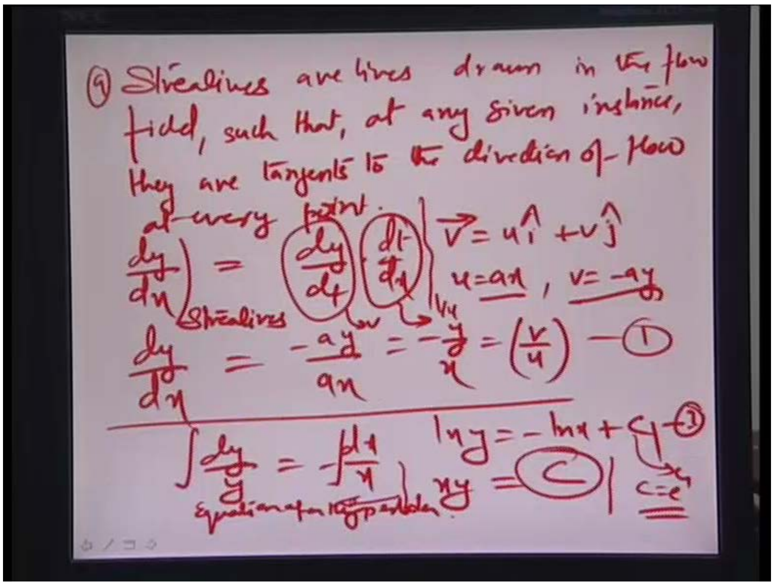

⑨ Streamlines are lines drawn in the flow field, such that, at any given instance, they are tangents to the direction of flow at every point.

$$\left.\frac{dy}{dx}\right)_{\text{Streamlines}} = \underbrace{\frac{dy}{dt}}_{v}\,\underbrace{\frac{dt}{dx}}_{1/u} \qquad \vec{V} = u\hat{i} + v\hat{j}, \quad u = ax, \quad v = -ay$$

$$\frac{dy}{dx} = -\frac{ay}{ax} = -\frac{y}{x} = \left(\frac{v}{u}\right) \quad \text{— ①}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x} \qquad \ln y = -\ln x + c_1 \quad \text{— ②}$$

$$xy = \boxed{C} \qquad C = e^{c_1}$$

Equation of a [illegible]

What are really streamlines? If we just go back to the definition of streamlines; streamlines are lines drawn in the flow field and such that at a given instant, they are tangent to the direction at every point of the flow field. So, in a flow field such that, at any given instance they are tangents to the direction of flow at every point. Here, we have to find out d y by d x, which is the tangent for the streamlines and this can also be represented as d y by d t times of d t by d x by using the Chain rule. As can be seen here in this equation, d t by d x; what we really know the velocity relationship here has been given as u i plus v j, where u is given as a x and v is given as minus a y. That is how the velocity is defined really in the problem statement.

Therefore, the d y by d x for the streamline really is nothing but minus a y by a x. This is a x or minus y by x. Typically, it is v by u that is exactly what this equation here says d y by d t is v the rate of variation of y dimension in terms of time and d t by d x its 1 by d x by d t that is 1 by u, rate of variation of x with respect to time. Therefore, if we just solve this equation number 1 here, we are left with d y by y equals essentially d x by x with a minus sign and we can integrate this to obtain l n y on 1 side is equal to minus l n x plus c. In the words, C 1 maybe and so in other words we have from this particular equation number 2 we can find out x y is equal to a constant maybe C and C is essentially e to the power of C 1 C 1 being a constant e to the power C 1 is also a constant.

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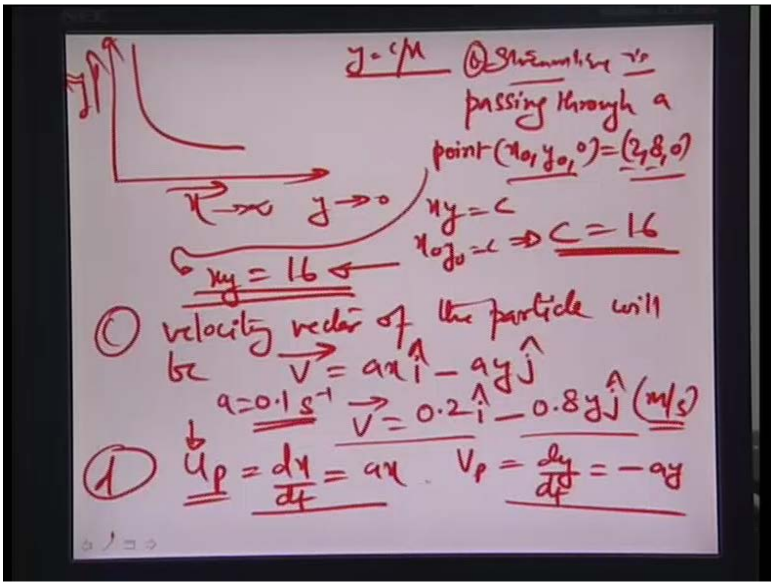

$y = C/x$ ⓑ Streamline passing through a point $(x_0, y_0, 0) = (2, 8, 0)$

$x \to \infty$ $y \to 0$

$xy = C$

$x_0 y_0 = C \Rightarrow C = 16$

$xy = 16$

Ⓒ velocity vector of the particle will be $\vec{V} = ax\hat{i} - ay\hat{j}$

$a = 0.1\ s^{-1}$ $\vec{V} = 0.2\hat{i} - 0.8y\hat{j}$ (m/s)

Ⓓ $u_p = \frac{dx}{dt} = ax$ $v_p = \frac{dy}{dt} = -ay$

This is in fact represents the equation of a hyperbola. It is very convenient to plot all the x y points in a manner, whereas x tends to 0, the y value tends to infinity and vice versa, as y tends to 0 then again the same the x value tends to infinity. So, you can actually plot this equation normally as a set of streamlines. Let us say, this is y, this is x and so these equations would be typically something like this or of a parabola, where if x goes to infinity; let us say y would assume 0 value, y being equal to C by x and vice versa. If y goes to infinity x would assume a 0 value. They are asymptotic to both the x and y axis as can be seen here.

So, for a streamline, this is essentially the equation of the streamline. The part B of this problem statement says that, for a streamline passing through the point x 0 y 0 0, where x 0 y 0 are 2 and 8 respectively. What would be the equation of the streamline? It is an interesting question. Let us say, in this particular case the streamline is passing through a point x 0 y 0 0 equal to 2 8 0 respectively. Assuming that the streamline follows an equation x y equal to C. Therefore, x 0 y 0 should be also equal to C and hence, C becomes 16 in this particular case. Therefore, the equation of such a particular point here, would be typically x y equal to C equal to 16. So, this is the equation of the streamline passing through the point x 0 y 0 0 2 8 0.

The third part of the question says or asks or determines, it tries to investigate the velocity of a particle, which passes through this point. So, you have to determine what the velocity vector of the particle will be, when it is on the particular streamline x y equal to 16. Therefore, in this particular case, a velocity V was given by a x i cap minus a y j cap, where a is 0.1 second inverse and velocity was given in meters per second.

Therefore, velocity here(Refer Slide Time: 48:50) would be represented as 0.2 i cap minus 0.8 y j cap in meters per second assuming and this 2 and 8 has the units of distance in meters. So, that is what essentially third parts of the question can be addressed as.

Now, the next part asks that, if the particle is passing through the point x 0 y 0 0 and it is marked at time t 0 equal to 0. Then, we had determine the location of the particle at time t equal to 20 seconds and also we had to further determine what is the velocity at time t equal to 20 seconds.

Let us suppose that you are actually trying to move from the point x y equal to 16 to a new trajectory. The velocity here from the earlier equation is given by this point 2i minus 0.8y and we have to see that at the new position, we have to first calculate what the new position really would be; if we assume this velocity. As we know, u p which is actually equal to the d x by d t equals to a x. So, this is u at a particular position or velocity or the x component of the velocity at a particular point P. Similarly, v of the particular particle will be d y by d t equal to minus a y from these 2 equations, what we really need to ascertain is, what the relationship between x y and time t because, we need to find out what will happen at time t equal to 20 seconds. Assuming that the initial curve here x y equal to 16 satisfies a streamline which is formulated at time t equal to 0 seconds.

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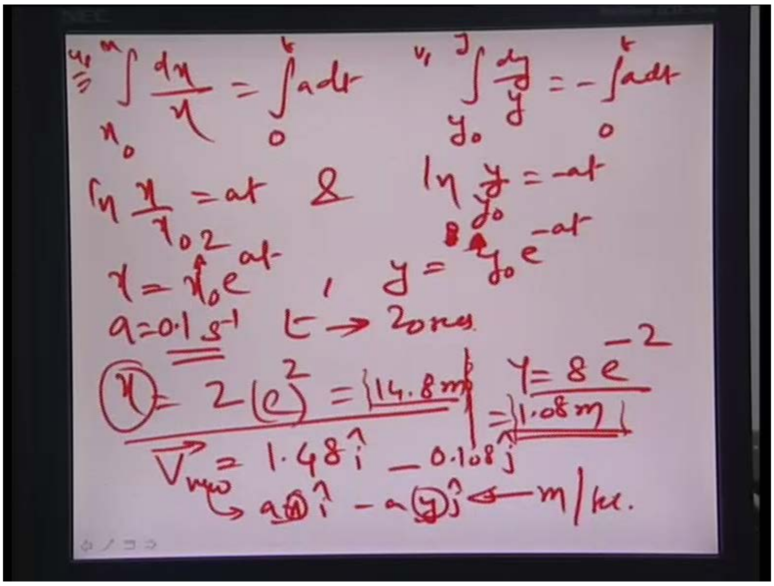

$$\int_{x_0}^{x} \frac{dx}{x} = \int_0^t a\,dt \qquad \int_{y_0}^{y} \frac{dy}{y} = -\int_0^t a\,dt$$

$$\ln \frac{x}{x_0} = at \quad \& \quad \ln \frac{y}{y_0} = -at$$

$$x = x_0 e^{at}, \quad y = y_0 e^{-at}$$

$$a = 0.1\ s^{-1} \quad t \to 20\ s$$

$$x = 2(e^{2}) = 14.8\ m \qquad y = 8e^{-2} = 1.08\ m$$

$$\vec{V}_{new} = 1.48\hat{i} - 0.108\hat{j}$$

$$a(x)\hat{i} - a(y)\hat{j} \leftarrow m/s.$$

Therefore, if we integrate this particular equation, d x by x on the left side and a d t on the right side from 0 to time t, assuming the initial position to be x 0 and the final position to be x here. Similarly, this is for the velocity of the particle in the x direction and for the y direction, we do a similar thing as d y by y from y 0 to y equal to minus a d t from time t equal to 0 to time t equal to t. Therefore, l n x by x naught here is equal to a t and l n y upon y naught here equal to minus a t and hence it is an exponential relationship. So, x becomes equal to x 0 e to the power of a t and y new coordinates become equal to y 0 e to the power of minus a t. We already know, the value of a to be 0.1 second inverse and the time here really is from 0 seconds to 20 seconds, which means the new x coordinate here assuming x 0 to be 2 and y 0 to be 8 as the two different points through which this earlier streamline would have passed from can be found. So, the new position x and y could be found out as 2 times of e to the power of 2 e square, which is about 14.8 meters and y similarly would be 8 times of e to the power of minus 2, which is about 1.08 meters; that is what the new x y coordinates of this particular point is really. So, if the particle is moving at the velocity a x i cap minus a y j cap and it is supposed to be at time equal to 0 along the streamline or at a point x 0 y 0 0 at a particular point in the streamline x y equal to 16. Then, from the relationship between x y and t, we find out that the new positions x and y of the particles would be represented by 14.8 meters and 1.08 meters.

Therefore, the new streamline equation first of all, the new velocity that the particle will possess in this instance V new of the particle is equal to 1.48 i cap minus 0.108 j cap. Velocity again relationship a x i cap minus a y j cap holds true, the only difference here is that the x and y are the new values of x and y, which are 14.8 meters and 1.08 meters respectively. So, these again is in meters per second, also we would like to determine the pathline equation for this particular example and for that, we also need to find out first the parametric equations, that means the relationship between x and t and y and t as we had done in the earlier example and then try to solve for time and from there we can find out what the pathlines are really.

So, definitionally again what are pathlines? Pathlines are typically lines, which are a kind of paths or trajectories traced out by a moving fluid particle. So, this is something that a particle at a particular point of time, when introduced into the fluid would follow temporarily as it goes ahead inside its domain.

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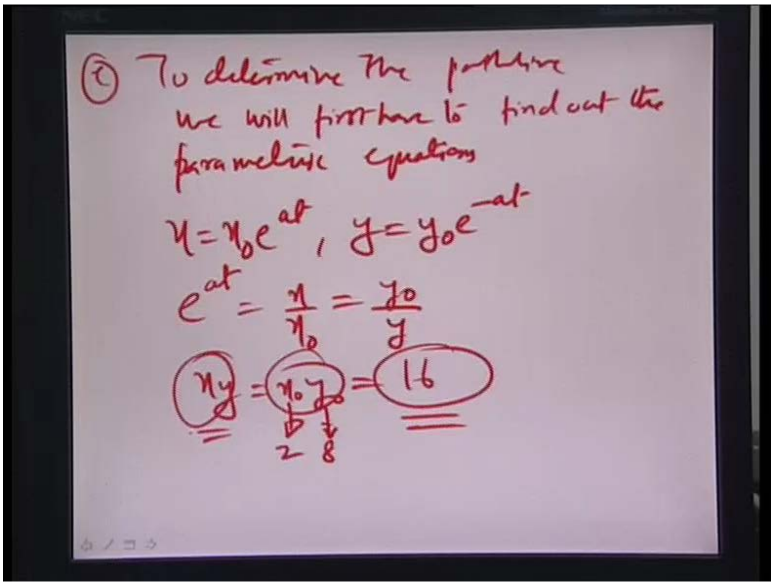

② To determine the pathline we will first have to find out the parametric equations

$$x = x_0 e^{at}, \quad y = y_0 e^{-at}$$

$$e^{at} = \frac{x}{x_0} = \frac{y_0}{y}$$

$$\underline{\underline{xy}} = x_0 \underset{\substack{\downarrow \\ 2}}{} \, y_0 \underset{\substack{\downarrow \\ 8}}{} = \underline{\underline{16}}$$

Let say, in order to determine this pathline, we will first have to find out the parametric equations x equal to x 0 e to the power a t and y equal to y 0 e to the power of minus a t. So, from this expression what we get is a relationship between x x 0 y y 0; let us say e to the power a t in this case is x by x 0 and in this other case is y 0 by y. Therefore, x y really equal to x 0 y 0, which is again equal to 16 x 0 being equal to 2 and y 0 being equal to 8 from the earlier example.

Therefore, what we find out here is that if suppose by definition what a pathline means really is, the location of the particular point at a certain time instance. You have to make, if you want to join all such points after all such particles, which have moved past a certain point. You have to make them independent of time and in order to make them independent of time, you find out the equation which has come out is same as the equation of the streamline as before. Therefore, the pathline equation derived here is same as the streamline equation in this example. This kind of led us to get a feel about all these different concepts of how to trace trajectory or lines of fluid particles within fluid flow systems. These will be immensely helpful later, for understanding of microflows because they are highly streamlined and laminar in nature.

So, with this I would like to close today’s lecture and then start with derivation of the stress field and then correlation of all these parameters together, to derive something, which is very fundamental and unique to the fluid mechanics or the Navier stokes equations. So, we will cover this in the next lecture. Thank you.