Bio - Microelectromechanical Systems

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Module No. # 01

Lecture No. # 13

Welcome back to lecture 13 on BioMicroelectromechanical systems. We will just like to begin with the quick review of the last lecture plus we try to derive the zeta potential associated with a surface in contact with the liquid phase. We also tried to interpret how the dual layer comes into existence. Why is it that the charges and the solutions get distributed and forms - as what you know - as the bulk charge or the diffuse layer of a solution and then, we also try to calculate the potential as a function of distance as you move away from this double layer into the bulk layer.

We try to use this concept of double layer charging in order to realize a certain type of micro fluidic flow called electro-osmotic flow. Just to review that if there is a surface which has certain set of dangling bonds like - let us say - salineal bonds and silicon surface Si O H and it comes in contact with a solution of a certain pH, there is a tendency of that surface to acquire a charge that could be a negative charge or positive charge depending on the pH is acidic or basic; because of this charge, if you are able to place a solution close to this charge surface, there is always going to be this diffuse layer which is formulated.

If we talk about micro capillaries carved in such kind of surfaces with charges and try to flow the fluids across it, there is going to be some kind of comparative between the dimension of the channel and the thickness of the diffuse layer. Now, the diffuse layer also tends to drag the fluid in such a micro capillary when you put an external EMF across it.

We try to derive some equations and formulations related to this flow process - the electro osmotic flow process. Basically, we also tried to look into the Couette's flow just for comparison of this electro osmotic flow with flow wherein, there are two plates with one fixed and another moving and trying to drag a fluid layer along with it and then compare it also to a parabolic flow, where there is a pressure gradient and two fixed plates between which the fluid flows.

We saw that in case of electro kinetic flow there is always the tendency of the flow to develop a plug like behavior. So, the velocity profile is like a plug. In case of pressure driven flow, it is something like a parabola. So, if we consider and try to start from that aspect we found out that by solving Navier-Stokes equations. Basically, we try to make a comparison between the different profiles; the velocity profiles within such channels when the flows are pressure driven as opposed to when the flows are electro kinetic in nature. We also tried to as certain that what happens in case of a pressure gradient across such a capillary with two sides fixed or two plates fixed and try to derive an equation of velocity and variation of velocity with respect to the y direction.

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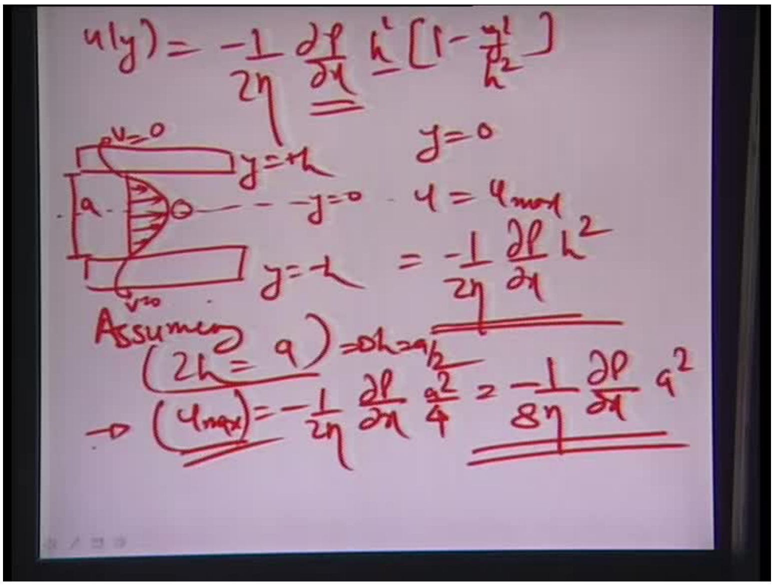

Essentially, the function that we obtained for u in the velocity, in that case was also minus 1 by 2 eta dp by dx h square times of 1 minus y square by h square, where dp by dx essentially is the pressure gradient across the capillary in the x direction. Assume that the two plates are respectively fixed at y equal to plus h and y equal to minus h, the middle of such a plate assembly is really the y equal to 0.

So, if you plot u with respect to the y, you get a parabolic profile; the square on the y. There is a flow velocity variation of this parabolic type and this makes sense also because, the velocity here very close to this channel wall is 0 on either sides and it is maximum somewhere at the center of the channel. If y equal to 0, the u becomes u maximum which is 1 minus 2 eta dp by dx times of h square, this is the maximum velocity that one can have in such a channel flow.

So, if you assume a little bit of different sign convention in the way the plates are laid and assuming that 2h essentially is equal to a or that the distance between these two plates 2h is nothing but a. In that case, h becomes equal to a by 2 and u max in this expression is minus 1 by 2 eta del p by del x a square by 4 also 1 minus 8 eta del p by del x a square, that is what the u max or the u maximum would be in a case like this.

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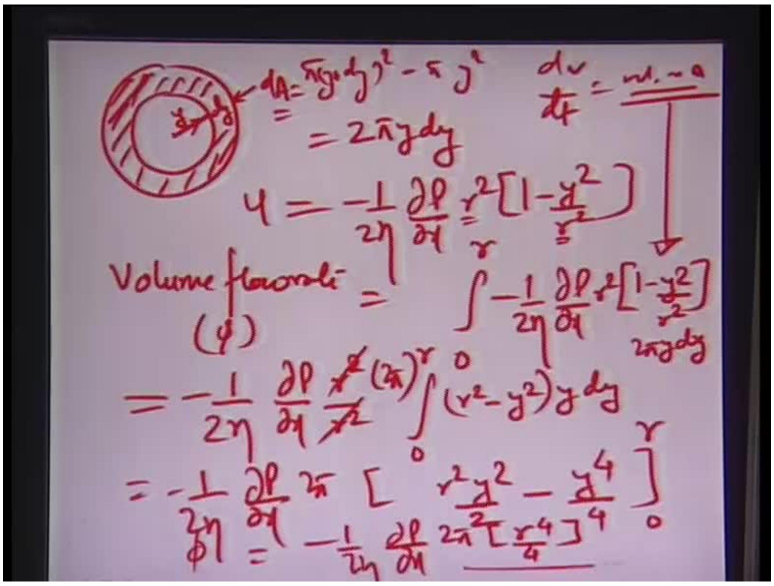

Let us now try to find out the maximum flow rate which would happen because of this function u. So, if you really look at the flow rate, we can assume that in this particular case, there is a circle or there is an annular of area a through which the fluid is emanating out. Add a velocity u which is given by the function shown earlier in terms of the pressure gradient; the square of the y and the square of the distance from the mean plain or y equal to 0, sorry, y equal to plus h and minus h.

If we look at the differential, let us suppose, this value here is y this distance here is dy and we want to find out what the area vector is? So, the area here would essentially be pi y plus dy square minus pi y square, so it will be also equal to twice pi y dy. That is what the da of this particular annular of fluid would be and we already know that u essentially, as a function of y has been represented early has minus 1 by 2 eta dp by dx r square times of 1 minus y square by r square; r is the radius in this case by the by, which is also equal to h. If you want to find out what is the volume flow rate or phi, this becomes equal to 0 to r minus 1 by 2 eta del p by del x r square times of 1 minus y square by r square. This is the u value times of 2 pi y dy.

As you also know, dv by dt the volume flow rate is nothing but the velocity times of area, that is exactly what this is about and the integral, we assume takes place between 0 and r which is the radius of the capillary.

Essentially, if you just solve this equation you would left with this, so we can take this twice eta del p by del x r square and twice pi outside the integral 0 to r, r square minus y square times of y times of dy and there is a divided by r square here these to cancel out (Refer Slide Time: 09:40) and you are left with minus 1 by 2 eta dp by dx 2 pi and this happens to be r square y square by 2 minus y 4 by 4, that is what the integral is in between 0 and r.

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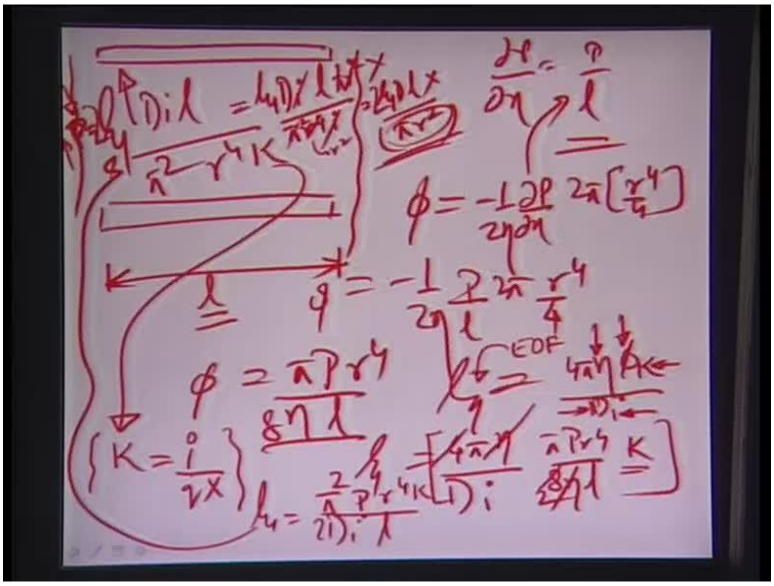

Essentially, this takes the form r square by 4 and we are left with an equation of the type phi equals minus 1 by 2n del p by del x 2 pi times of r 4 by 4. Now, one thing which is of importance here is seen that we consider a channel at the very beginning. If you consider the electro osmotic flow of length l, you consider the channel of length l and if you assume that there is a pressure driven flow in this particular example which is essentially between the length l and the pressure gradient in this case is P then, dp by dx can be represented as p by l.

So, if I put back p by l into this particular equation here, for the pressure driven flow case, phi becomes equal to minus 1 by 2 eta dp by dr times of twice pi; this is dp by dx times of r4 by 4. Essentially, dp by dx is nothing but P by l here, so you get phi is 1 by 2 eta p by l twice pi r4 by 4. If we just read this equation phi comes out to be equal to pi P r 4 divided by 8 times of eta l that is what the volume flow rate is in this particular example.

One more issue here is that if you consider what happened in the electro osmotic flow case is, the zeta potential for the electro osmotic flow - the EOF - this was derived as 4 pi eta phi times of K divided by D i; where K is the conductivity of the medium, phi is a flow rate, eta is the viscosity, i is the current, D is the dielectric constant. If we just substitute these value of phi here, assuming that we equate the electro osmotic flow with the pressure driven flow. We have situation wherein, we have this 4 pi eta D i times of pi Pr 4 divided by 8 eta l times of K. All this essentially can be further reconverted a little bit.

If you remember from before, the value of K is also equated and assumed to be ratio between the current i, the area across section q and the electric field X. That is how we found this relationship earlier from the famous r equal to rho l by a term and then using V equal to IR Ohms law.

So, if we substitute the value of K here in this particular expression, let us find out what it would finally look like. One thing which is very important and very critical to mention here is that, since zeta here is equal to this particular term 4 pi eta Di times of pi Pr4 divided by 8 eta l times of. Here, we can find out tentative relationship between zeta and the pressure. You have the case, where you have pi square times pressure P r to the power of 4 divided by Di l and then, there is a K term here which automatically means that the zeta potential in this particular case would relate to the pressure of the medium P by the relationship, P is equal to zeta times of Di l divided by pi square r 4 K.

What does it really mean? If you consider, the pressure driven flow analogue of the electro osmotic flow. What I am trying to indicate is that if you have an electrostatic osmotic force which is flowing the fluid in the capillary and you consider that to be essentially within the same flow rate everything is contributed by an equivalent flow with a pressure gradient. We can make an analogy between what kind of zeta potential is needed for creating what kind of flow pressures. Here as you see, if the zeta potential of any surfaces more as in this equation, the pressure of the flow would be more and vice versa.

That is very interesting observation which will carry forward later. Here, if we substitute the value of K into this particular equation here (Refer Slide Time: 16:03). Let us look at how this would really behave in this particular case and I need to actually substitute the value of K here, we get P equals eta D i l divided by pi square r 4 and K is i by qx and q pi r square, so i times of pi r square in the numerator times of x.

We can actually do these cancellations here; r4 cancels into r2, pi and pi cancel out, so we are left with the terms zeta D l x divided by and of course, there is going to be a 2 here which we forgot because, essentially there is a 2 here in the denominator, so there is a 2D i l in the denominator here. So, twice zeta D l x divided by pi r square is what this effectively would look like.

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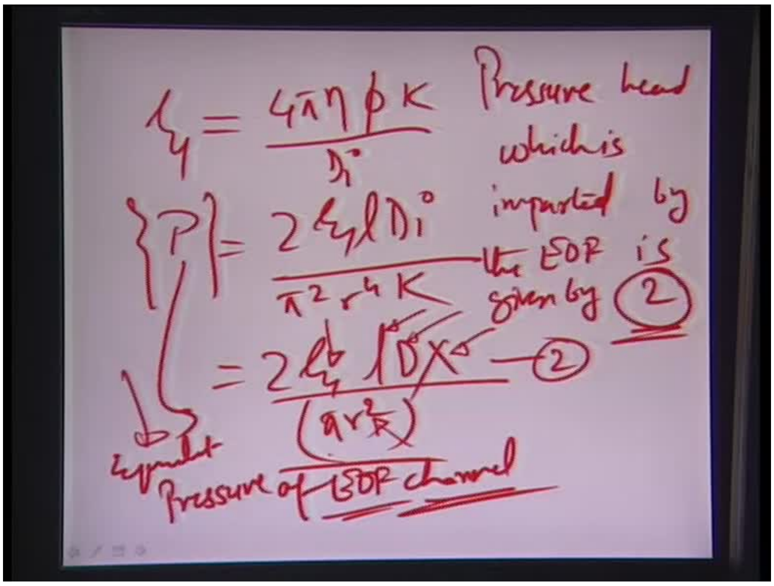

One important thing here is, the pressure that is generated from a flow channel with the zeta potential - let us say - zee is also inversely proportional to the area of cross section of the particular channel. The pressure head p imparted by the electro osmotic flow can be equated using this kind of a formulation which I would just like to write. Therefore in summary, I would like to write down that the zeta potential is equated to 4 pi eta phi K by D i, is also equated to the pressure must P is twice zeta l D i divided by pi square r 4 k is also equated again as twice zeta l D X divided by pi r square the area of cross section.

This is the equivalent pressure of an EOF channel. You can consider the amount of pressure head that is imparted onto the fluid by virtue of the EOF the Electro Osmotic Flow is essentially this P. I can say that the pressure head which is imparted by the EOF is given by equation 2. So there can be design problems, wherein you want to find out what kind of pressure in an EOF flow gives. Essentially, you need the parameters like zeta potential of the channel, the length of the micro channel, the dielectric constant of the medium, the field which is across this channel and then finally, the radius of the micro capillary into question.

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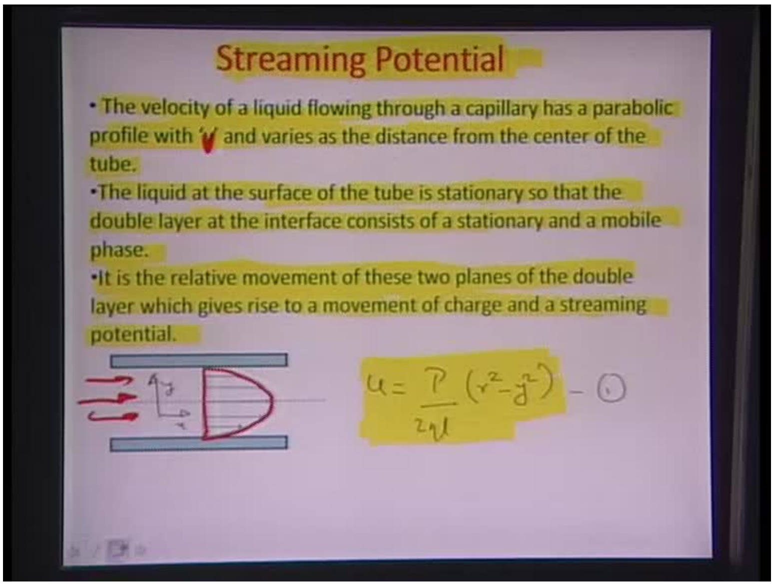

Let us shift our attention to the second electro kinetic phenomena that is the streaming potential. As we talked about before, the velocity of a liquid flowing through a capillary has a parabolic profile with v that varies the distance from the center of the tube to the sides; this is v, the velocity.

So, if you just remember back the various ways and means of predicting the various electro kinetic properties, there can be a case where you produce or give a pressure driven flow and it creates a set of charges on the walls or set of currents. There is a case, when you apply in an EMF from outside and it generates a flow. In one case, they cause the EMF; in another case, it causes the flow. So, flow generates EMF inside the channel; flow generating EMF inside the channel the phenomena is also known as Streaming potential.

The liquid at surface of such a flow being stationary leads to the double layer at the interface consisting of a stationary phase and a mobile phase the solution. The relative motion movement of these two planes of the double layer, one stationary with respect to the wall another moving with the fluid would give rise to the movement of the charge and generate a potential.

Let us derive what this potential level would be in such a micro channel case. Let us say, we have a parabolic flow taking place within this particular architecture here and the velocity flow profile is also indicated by this particular parabola. Maximum velocities at the center and then, these velocities at both sides of the channel are essentially in the no slip zone so there is 0. So, u here the velocity is kind of also defined as P times of r square minus y square by 2 eta l; P is the pressure r square minus y. So, this is again what we found out from the parabolic flows P by eta l r square minus y square.

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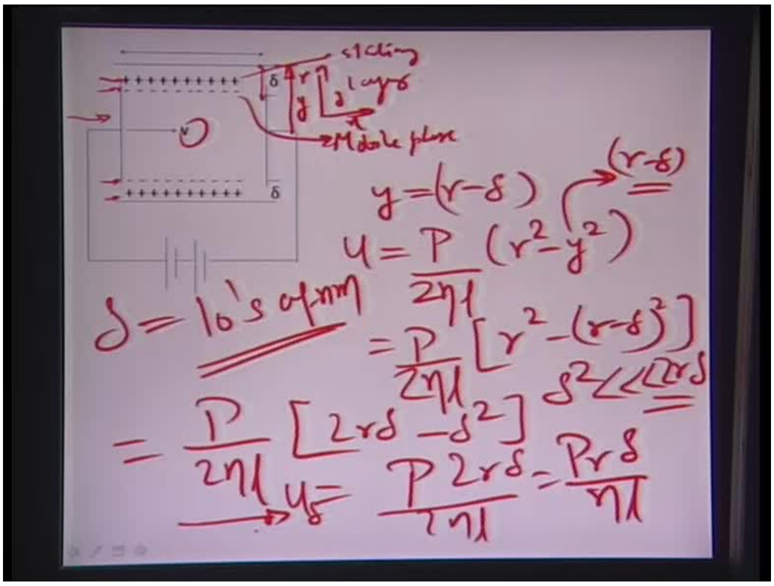

So, if we put this liquid through a pressure difference or push this liquid through a pressure difference across this channel there is going to be a double layer which is generated. In that case, we can model the flow like this figure here (Refer Slide Time: 22:31). So, if you see here there is a double layer of charges which are created on both sides, one is a stationary layer which is the positive layer in this case. So, this is the stationary layer of charges and the other is the mobile phase which is the negative charges on the solution. Whenever there is a fluid flow by means of a pressure this positive layer being stationary and the negative layer being mobile they move relatively with respect to each other.

Let us assume that this delta b, this thickness of the double layer and let us also assume that r be the radius of this particular channel. Essentially, r is the distance from the center of the channel all the way up to the wall of the channel and we decide for a parameter x here, where x is equal to essentially r minus delta.

In this particular case, u the velocity which is actually represented as P divided by 2 eta l times of r square minus y square can be represented in this case, if I substitute the y by the equivalent r minus delta. So let us say, this is y in the notational consistency this direction is the y direction and this is the x direction (Refer Slide Time: 23:57). So, this is actually y and this essentially is also y equals r minus delta for notational consistency. So, r square minus y square can be represented as P by 2 eta l times of r square minus r minus delta square.

If we try to calculate this we are left with 2r delta minus delta square and delta being very very small. We have already mentioned that delta the charge the dual layer thickness is around 10s of nanometers. Therefore this delta square is very small in comparison to twice r delta can be safely neglected and u in this particular case, would be nothing but P times of twice r delta by 2 eta l or Pr delta by eta l. That is what the u delta is going to be in this particular case, the velocity across this double layer.

Now as we need to consider, there are the movements or the front of the liquid forces over these layers of charges and which on the surface essentially produces the current and this current is also given by the product of the total charge around unit length of the tube and the velocity. So, if you have certain surface charge distribution in terms of charge per unit area sigma and in a unit length that means, you have the charge density in terms of per unit surface area, but essentially the total amount of charge we are considering only on a unit length. That particular charge time of velocity that means, how many such unit lengths the relative movement of charge is happening over the surface would comprise of the current.

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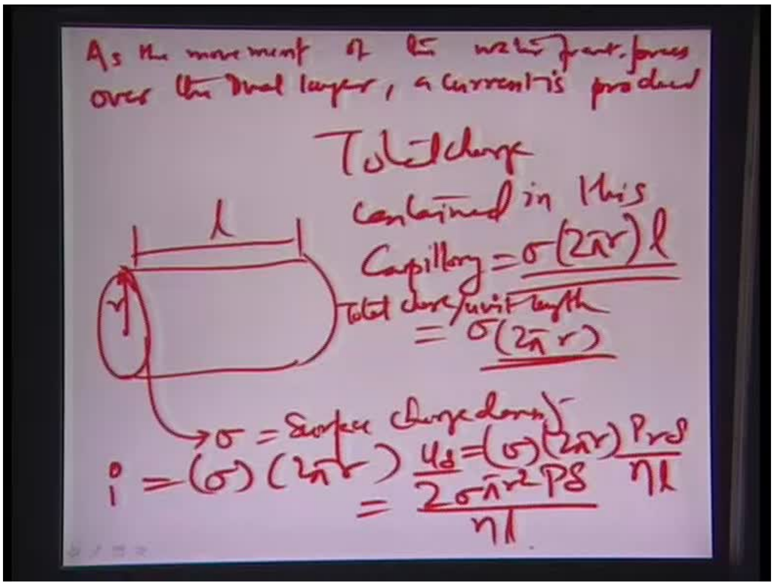

Let me just explain this in a little more detail here that suppose, you have a capillary of length l and radius r; this is the radius r and length is l and we have a surface density of charge inside this capillary sigma - surface charge density. Now the total charge contained in this capillary is given by sigma times of 2 pi r times of l; 2 pi r l being the intervals surface area of the particular capillary. If I consider a per unit length of this total charge, so the total charge per unit length that comes out to be sigma 2 pi r and this essentially if multiplied by the velocity would mean that if suppose, velocity is x meters per second; this per unit length moves so many times in a second, so essentially discharge per second and that is what current is defined as.

Therefore, let me just write this down in totality. As the movement of the waterfront forces over the dual layer, a current is produced and this is also given by the product of the total charge around a unit length of the tube and the velocity of the moving part of the layer. The total current i in this case would be sigma times of 2 pi r charge per unit length times of length per unit time u delta, substituting the value of u delta from the derivation made earlier; we are left with u delta is Pr delta divided by eta l as you found out from the case of the pressure driven flow. Therefore, this current i in this case, is nothing but twice sigma pi r square P delta divided by eta l.

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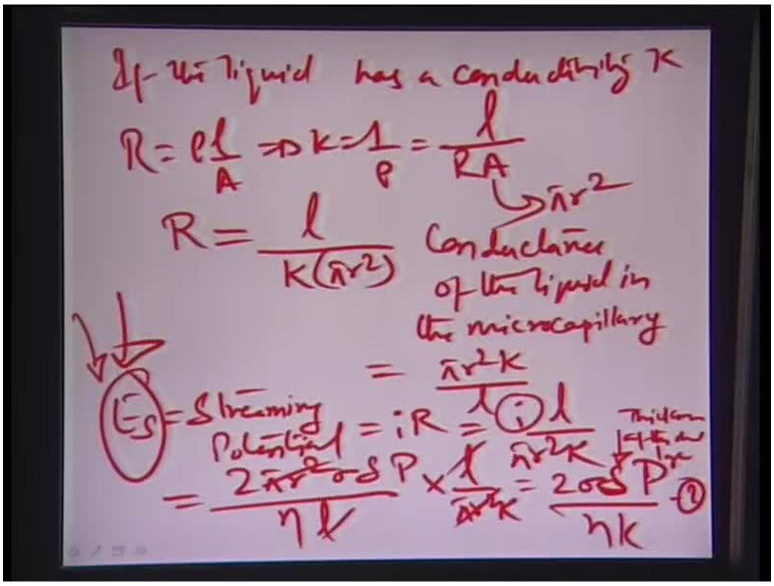

Let us assume that if the liquid in question has a conductivity K liquid and we already know the relationship R equal to rho l by A and essentially, K which is 1 by rho is nothing but l divided R A; A in this case is nothing but the radius square times of pi in the cross sectional area of the circular capillary and the resistance R of the channel is l divided by K times of pi r square in other words, the conductance which is the reciprocal of resistance of the liquid in the micro capillary is just the inverse of that; that is pi r square K by l.

The streaming potential Es developed because of this current and current is formulated by the motion of the charge across the dual layer is given by i times R, i times l by pi r square K. If we further try to put the value of current here from the previous derivation which we obtained is 2 sigma pi r square p delta by eta this term here was the i (Refer Slide Time: 30:55). Here the final expression would come out to be 2 pi r square sigma delta P by eta l times of l by pi r square K, these go off, the l's go off and we are left with the term twice sigma delta P by eta k.

So, that is what the streaming potential would really be. One important point here to be mentioned is that you can easily find out from this potential - the length delta the thickness of the dual layer which can be of immense utility in almost all electrochemistry so that is what the streaming potential is. In case of just flown through a pressure gradient over dual layer of the surface, we have already derived the streaming potential here Es in terms of 2 sigma delta P by nk.

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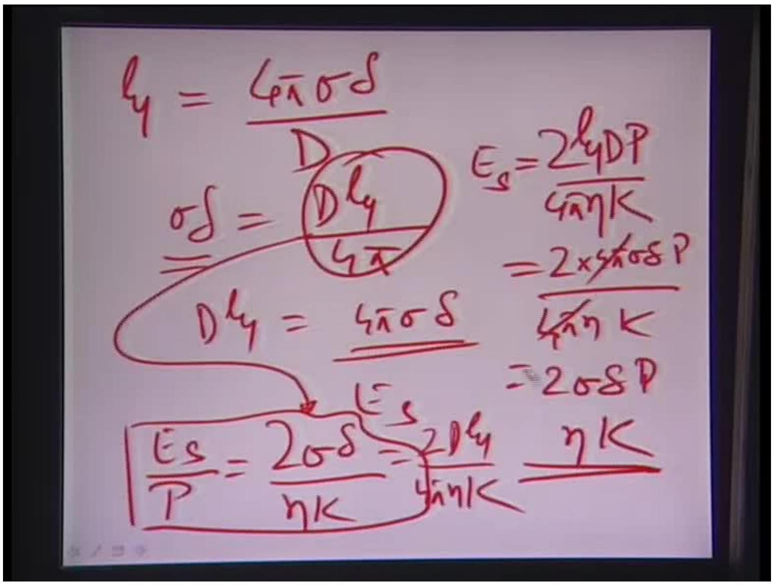

Let us modify a little bit in terms of the zeta potential; try to involve what the zeta potential of a surface would be. As we know from earlier equations that the zeta potential of such a system of micro channel in contact with the liquid phase is also given by 4 pi sigma delta by D; sigma as the surface charge density, delta is one of the double layer charge - double layer thickness -, D is dielectric constant of the medium of interest.

If you just do a little bit of a mathematical manipulation here sigma, S sigma delta comes out to be equal to D times of zeta by 4 pi and essentially, if you had a look earlier at the Es the streaming potential it came out to be twice zeta DP - P the pressure - divided by 4 pi eta K. So, if we just try to substitute the value of this delta into sigma into this particular equation here, we are left with that - let us say - D zeta is also equal to 4 pi delta sigma.

Essentially from the Es value, if you substitute this 4 pi sigma delta P divided by 4 pi eta K, you are left with twice sigma delta P by eta K. Essentially that is what the Es or streaming potential would result in and we can also write this down as Es the streaming potential per unit pressure which is given as a difference between both ends of capillary, which is causing this fluid to move is also twice sigma delta divided by nK, that is what the final form of the ratio between the streaming potential and the pressure.

Another interesting thing here is to find out that if you really want to write these whole term in terms of quantities which are measurable, let us actually try to understand this more in terms of measurable quantities like flow rate phi then, things like measurable quantities like current across the micro capillary so on so forth.

There would be a little modification though which will need here. Let us re-substitute back this value of sigma delta back into this equation (Refer Slide Time: 34:47). You are left with twice D zeta divided by 4 pi eta K. We can easily find out a correlation between the flow rate phi and the current i by looking at the equation that we had derived earlier, which talked about the relationship between the zeta potential and these other parameters 4 pi eta phi K divided by Di.

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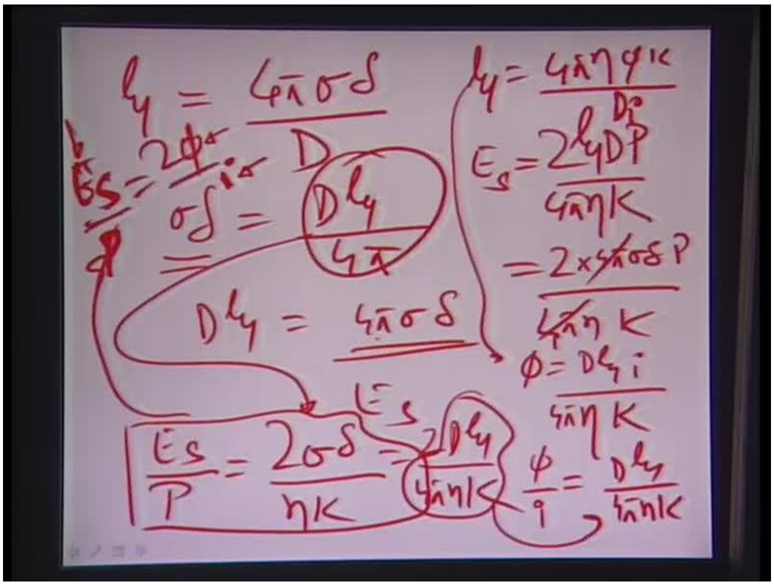

So, if you try to readjust these parameters here, you are left with the value of phi - the flow rate - as D zeta i divided by 4 pi eta K and this can also be represented as phi by i - is D zeta by 4 pi nK. This quantity looks similar to what this is, so essentially Es by p here is nothing but twice this quantity here phi by i. So, if you can measure the flow rate in such a streaming flow and also measure the amount of current that is produced this can easily give you this ratio Es Es by p and Es is the streaming potential and p is the pressure. Let us say, the pressure gradient that were driving or the pressure difference that were driving the flow is the P and phi was the flow rate which was created and it generated a current i. You could easily back calculate, what is a streaming potential in that particular application. So that is what is used sometimes as a sensing mechanism for investigating combinations of surfaces and their behavior with respect to flowing solutions.

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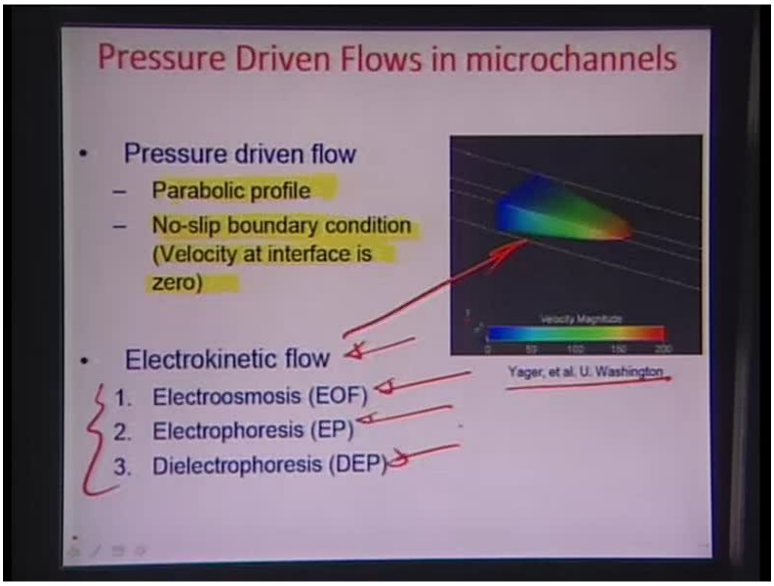

Let us try to summarize that in pressure driven flows essentially, the flow profile is parabolic in nature and there is no slip boundary condition at all, the walls or the edges of such a channel the flow profile pretty much looks something like this here (Refer Slide Time: 37:09).

You have like a parabola with the velocity of maximum at the center and which is actually 0 at all these different walls. Electro kinetic flows on the other hand electrophoresis; electroosmosis and the dielectrophoresis and these are some of the mechanisms for doing electro kinetic flows, this right here is simulation by Yager's group up at the University of Washington.

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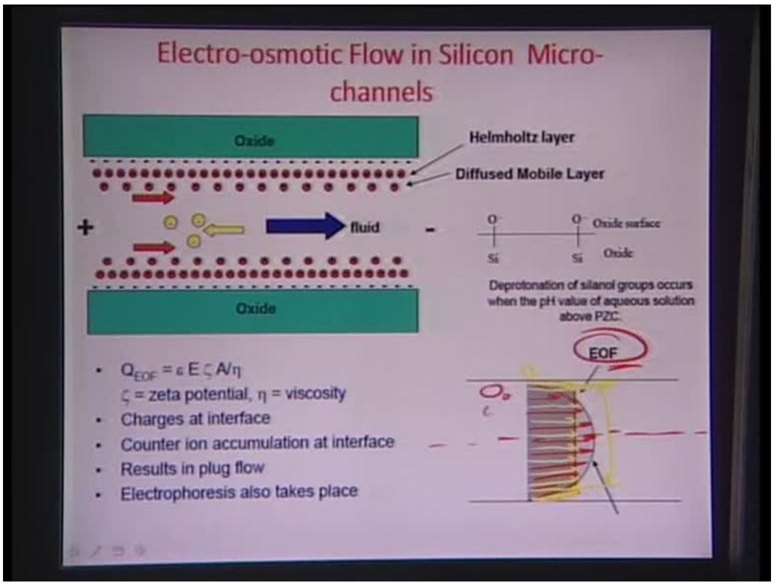

Essentially, if you just compare the other flow profiles which are generated from the EMF and a parabolic flow. If you may recall, in a pressure driven flow the flow profile is something like a parabola of the sort.

The velocity vectors keep on maximizing as they go from walls towards the center of this particular capillary. On the other hand, for the EOF flow or the Electro Osmotic Flow as we know, the flow really takes place as a plug flow from the start of 1 double layer which is essentially this site here, all the way up to the other double layer which is on the other side of the channel. This area is having a constant velocity (Refer Slide Time: 38:15).

Let me just use a different color here to represent the flow rates related or flow velocity is related to the electro osmotic flows. So if you see here, as represented by these yellow color arrows; these are the flow vectors between the double layers in an electro osmotic flow. So, it is like a plug flow all the velocity is between these two double layers starting from here all the way up to here are uniquely similar to each other (Refer Slide Time: 38:50). I opposed to the parabolic whether there is a slow increase of velocity as it goes from sides all the way to the center.

Other interesting point here is the behavior of the flow rate around that the surface in question till the double layer starts, so this is essentially the Helmholtz plane. You are been talking about this a lot on the Helmholtz plane. This particular layer here as you see is though having a kind of parabolic profile, which means that this is the layer which is a sheers to give way to this plug like flow.

The velocity really close to this surface here is 0 and the velocity here is some maximum value v and the flow profile between the two were really parabolic on both sides. So that is how we interpret both these electro osmotic flows and the parabolic flows. Now, there are several important issues which emanate from this is that the electro kinetic flow is being a plug like flow, there is almost a continuity or uniformity of velocity vectors across such a channel. For characterization sake, particularly when you do particle image velocimetry, it may be of immense utility even though flow in cross section of such a channel is having all uniform velocity.

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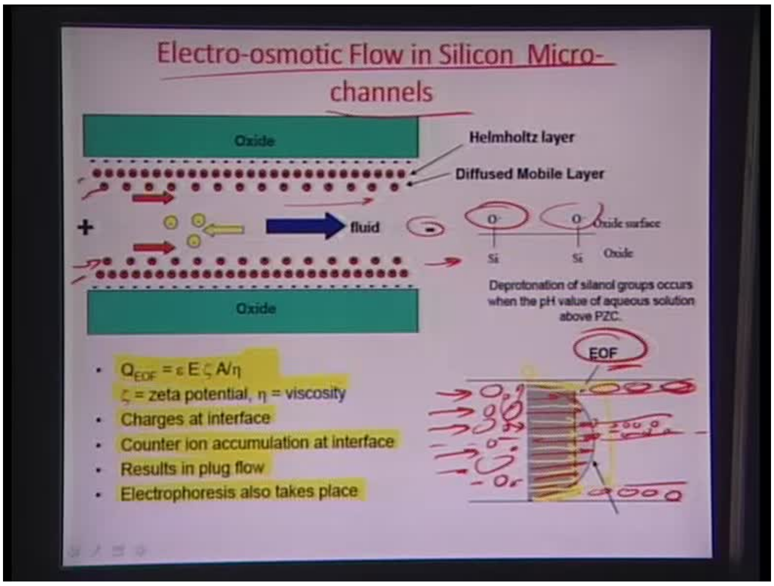

This also would mean that these kinds of flows do not lead to any particulate separation particularly when the particles concerned are all different sizes. So, if there are parabolic flows as we have seen in cases of channels which are pressure driven essentially. One aspect that comes into picture is that if suppose, there are different cells of all different sizes moving across such channels with the continuity and with this velocity variation in the parabolic profile, the heavier masses tend to move towards the side and the lighter masses tend to move more towards the center by the principle of conservation of momentum.

Ultimately, the retented here at the center is all consisting of smaller entities and the retended at the sides here all containing the bigger entities is known as leukocyte; this is also known as the Margination. In the human body, the micro capillaries essentially do this job, where leukocytes which are heavier in comparison to the RBCs would migrate slowly to the walls of the capillary and would be rich towards the vasculatures is walls essentially.

So that is sometimes an advantage but then in some cases it may be desirable to move these kinds of a different sizes and masses throughout the micro channel uniformly without the separation coming into picture. Electro kinetic flows are the best in those kinds of cases because, they essentially do not have any velocity variation, it is a plug like flow like behavior. This is the kind of talks about what happens on a silicon dioxide surface I have been repeating it off and on.

You can see here silicon dioxide surface, first of all the surface gets hydrolyzed and then later on forms Si O minus and there is a net negative charge on the surface due to which there is a positive charge of counter ions, which is developed in the bulk charges or diffuse layer charges and then, when you apply a potential across that all these bulk charges try move towards the negative electrode as dragging the fluid around it. Therefore, electro osmotic flow and silicon micro channels are a great area of study.

Some important observations here as summary: the flow rates of such flows in proportional to the dielectric constant xylene; the electric field external to the channel, the zeta potential of a surface and also proportional to the area and inversely proportional to the viscosity of the particular medium which is flowing through these surfaces. Essentially depends on the charges at the interface and there is a counter ion accumulation and which actually drags along the fluid along with it and it results in a plug like flow.

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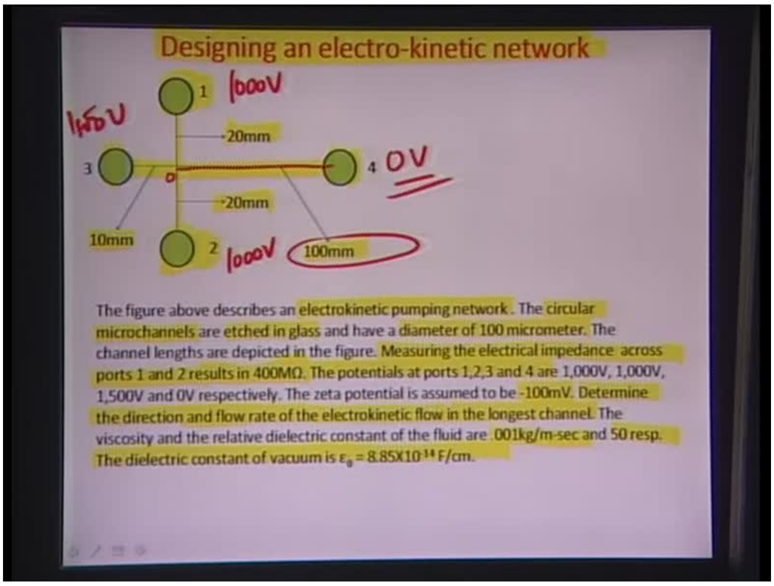

Electrophoresis on the other hand is the motion of charged particles directly in a fluid medium and will be dealing with this a little bit later in more details, but let us look at one illustration here, where we can use this electro kinetic techniques for designing micro pumps.

In this particular problem here, we want to design somehow a electro kinetic pumping network. So, using this principle of double layer and formulation of charge etcetera on the surface and the bulk of the fluid, we would like to design a pump network here.

The dimensions that are given is basically that the micro channels are circular in nature; they are etched in glass, so there will be Si O minus layer and they have a diameter of about 100 micro meters. So, this here are the two channels and the channel lengths are actually depicted in the figure, so the length here is 10mm between, let say 3 and there is a point of intersection, let us say 0; so between 3 and 0 the length of the channel is 10mm between 4 and 0 this is 100mm between 1 and 0, and 2 and 0 both are same 20mm each.

So that is how this pumping network is being laid out. These essentially are the reserve wires 1, 2, 3 and 4 and we have the following measurements. So, if you measure the electrical impedance across ports 1 and 2 that means, this port here and this port here; it results in 400 mega ohms of resistance. The potentials at ports 1, 2, 3, 4 are 1000 Volts, 1500 volts and 0 volts respectively, so let me just write that down here.

The potential at port 1 is about 1000 volts of applied potential, same goes to for port 2 another 1000 volts and then, port 3 it is about 1500 volts and port 4 is about 0 volts, so there is essentially no potential applied on port 4.

So this zeta potential here is assumed to be about minus 100 millivolts as can be defined by the surface in connection to another liquid phase. You have to determine the direction and the flow rate of the electro kinetic flow formulated in this case of the longest channel that is the one, which measures about 100 millimeters or this channel between 0 and port 4. The viscosity and relative dielectric constant of the fluids are given to be 0.001kg meter second and 50 respectively and the dielectric constant of vacuum is found to be 8.85 10 to the power of minus 14 farad per centimeter.