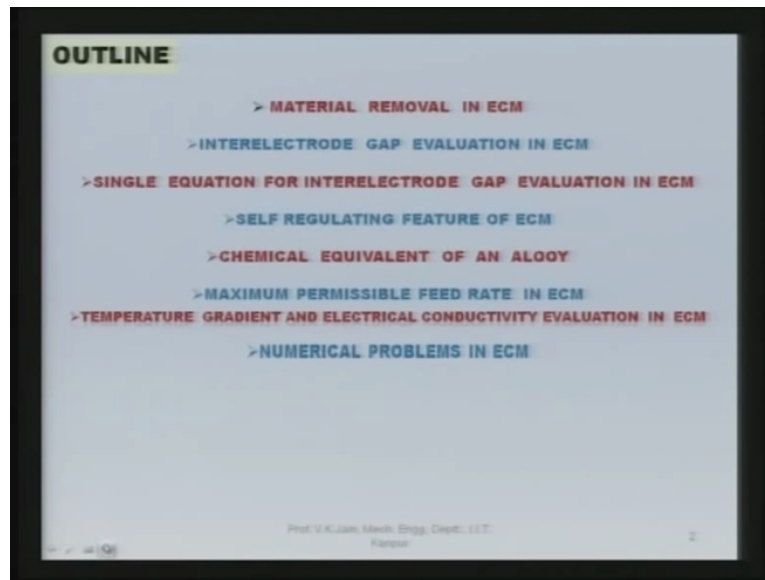


Advanced Machining Processes
Professor Vijay K. Jain
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
Lecture 09
Theory of Electrochemical Machining ECM-2

(Refer Slide Time: 00:16)



Welcome to the course on Advanced Machining Processes, we are going today to the theory of electrochemical machining part 2. The outline of the today's lecture is as follows material removal in electrochemical machining, inter electrode gap evaluation in electrochemical machining, single equation for inter electrode gap evaluation in ECM, self regulating feature of ECM process, chemical equivalent of an alloy, maximum permissible feed rate in ECM, temperature gradient and electrical conductivity evaluation in ECM, then some numerical problems will be solved related to electrochemical machining process.

(Refer Slide Time: 01:26)

MATERIAL REMOVAL IN ECM

MATERIAL REMOVAL (m) IN ECM FOLLOWS FARADAY'S LAWS OF ELECTROLYSIS:

$$m = \frac{ItE}{F} \dots\dots(1)$$

MATERIAL REMOVAL RATE (MRR) CAN BE OBTAINED AS:

$$\frac{m}{t} = \dot{m} = \frac{IE}{F} \dots\dots(1a)$$

Where, 'm' is amount of material removed in grams, 'I' is current flowing through the IEG in Amperes, 't' is time of current flow (or ECM), 'E' is gram chemical equivalent of anode material, 'F' is Faraday's constant (Coulombs or A.s) and \dot{m} is material removal rate in g/s.

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

Let us see how to evaluate or calculate material removal and material removal rate in electrochemical machining process. Material removal is abbreviated as small m in electrochemical machining as I mentioned in the earlier lecture that is in part one, electrochemical machining follows faraday's laws of electrolysis which clearly tells that mass of material removed is equal to ItE divided by F where I is the current flowing in the circuit or in this particular case through the inter electrode gap, t is the time of flow of the electric current or electrochemical machining, E is the chemical equivalent of the anode material and F is the Faraday's constant which has 96500 coulombs as its value which is constant.

Material removal rate in ECM can be calculated when we divide m by time t of machining that becomes m by t as you can see here which is indicated as m dot, which becomes equal to IE over F, now here m is the amount of material removed in grams I is current flowing through the inter electrode gap in amperes t is time of current flow or electrochemical machining, E is gram equivalent or gram chemical equivalent of anode material that is the work piece material, F is Faraday's constant given in coulombs or ampere seconds and m dot is material removal rate in grams per second.

Note it that here I have mentioned material removal rate in grams per second, it can also be represented as MRRg where the suffix g indicates that it is in grams per second, there are other ways also of representing the material removal rate which we will see.

(Refer Slide Time: 03:35)

MATERIAL REMOVAL IN ECM

MRR CAN BE OBTAINED AS:

$$\frac{\rho_a \check{V}_a}{t} = \frac{\rho_a A_a (y_a)}{(t)} = \frac{IE}{F}$$

WHERE, ρ_a = DENSITY OF ANODE, \check{V}_a = VOLUME OF MATERIAL REMOVED FROM THE ANODE IN TIME 't', A_a = X-SECTIONAL AREA ON THE ANODE FROM WHICH MATERIAL IS BEING REMOVED IN TIME 't', y_a IS THE THICKNESS OF MATERIAL REMOVED IN TIME 't'. ΔV IS OVER POTENTIAL, k = ELECTROLYTE'S ELECTRICAL CONDUCTIVITY

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

Material removal rate can be obtained as $\rho_a \check{V}_a / t$ and volume of anode removed \check{V}_a , let us see it is given by A_a cross-sectional area the inter electrode gap, y_a that is the gap divided by t is the time and ρ_a is the density of the anode material.

Now as I have written here ρ_a is density of anode, \check{V}_a is the volume of material removed this \check{V}_a and this \check{V}_a they are the same and A_a is the cross-sectional area on the anode from which material is being removed or in other better way we can say through which the current is flowing in time t , y_a is the thickness of the material removed in time t and ΔV is over potential which will be shown in the following equations and k is electrolytes electrical conductivity that will be used in the following equation.

(Refer Slide Time: 05:01)

MATERIAL REMOVAL IN ECM

FROM ABOVE EQUATION, WE CAN WRITE

$$\therefore MRR_t = \frac{(y_a)}{(t)} = \frac{IE}{F \rho_a A_a} \checkmark$$

$$MRR_t = \frac{JE}{F \rho_a} \dots\dots(2)$$

(J = CURRENT DENSITY)

ABOVE EQUATION CAN BE WRITTEN AS

$$MRR_t = \left(\frac{V - \Delta V}{A_a} \right) \left\{ \frac{kA_a}{y} \right\} \frac{E}{F \rho_a}$$

$\frac{I}{A_a} = J$
 $I = \frac{\text{Voltage}}{A_a}$

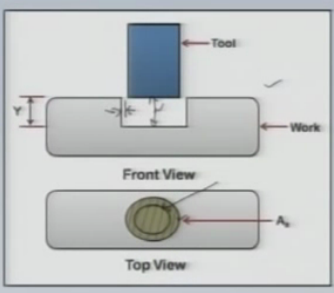
Prof. V.K. Jain, Mech. Enng. Deptt., I.I.T. Kanpur

From above equation we can write MRR_t where MRR suffix t indicates linear material removal rate whose units will be millimeter per second and that is y_a over t, y_a is the thickness of the material that has been removed and it is given by y_a over t is given by IE over F rho_a A_a. So we write this MRR_t that is the linear material removal rate is equal to JE over F rho_a because I over A_a can be written as J as has been done in this particular equation rest of the terms remains the same.

Current density J, this above equation can be written as MRR_t is equal to V minus delta V over A_a multiplied by K A_a divided by A into E over F rho_a. Now here you can see J has been replaced we know I is equal to voltage divided by cross-sectional area and here delta V as I mentioned in the earlier slide is over potential, so V minus delta V becomes the effective voltage which is working across the inter electrode gap, so this particular equation can be used for evaluating linear material removal rate from the work piece material I have already shown in the last lecture what this MRR_t means.

(Refer Slide Time: 06:54)

MATERIAL REMOVAL IN ECM

$$MRR_1 = (V - \Delta V) \frac{k E}{y_a F \rho_a} \dots\dots(2a)$$


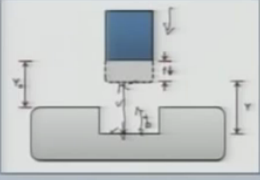
Prof. V.K. Jain, Mech. Engg. Deptt., I.I.T. Kanpur

So MRR1 again written in the same form, now let us understand it like this that y is the amount of material that has been removed in time t from the work piece now this is the front view of the work piece and tool combination showing the inter electrode gap that is the or the gap that is shown over here, now here MRR1 is in front gap only, side gap are not considered here we are considering only this front gap that is this one the gap that is in the sides is not considered in this particular equation however you can evaluate MRR1 from the side gap also by developing an equation on the same lines.

Now if you see this assembly from the top you will find that the cross-sectional area through which current is flowing is shown over here and this is the one which is the diameter of the tool, tool diameter and this is the one which is the diameter of the cavity that has been formed on the work piece.

(Refer Slide Time: 08:17)

MATERIAL REMOVAL IN ECM



BUT, $\frac{dy}{dt}$ IS RATE OF CHANGE OF IEG

$$\frac{dy}{dt} = \frac{y}{t} - f$$

"y" and "f" both are in the same direction.

$$= \frac{E(V - \Delta V)k}{F \rho_a y} - f$$

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

Now $\frac{dy}{dt}$ is the rate of change of inter electrode gap, what is this rate of change of inter electrode gap, here let us see we are giving the feed to the tool that is moving downwards and due to the removal of the material the surface of the work piece is also moving downward so both are moving in the same direction.

Now if we see here if the movement of the surface, top surface of the work piece is say y here then at which rate the gap between the bottom surface of the tool and top surface of the work piece is changing with time, we have to find it out here this becomes Y in place, this is the surface that has been removed from here, this becomes Y then rate of change of the gap between the bottom surface of the tool and the top surface of the work piece that is given by $\frac{dy}{dt}$ and since this is the inter electrode gap, so it say the rate of change of inter electrode gap, that is this particular gap.

So we have to find out this and this is given by $\frac{dy}{dt}$ is equal to $\frac{y}{t}$ the rate, the inter electrode gap remove or the material removed from there in time t , now this f because both are moving in the same direction so effective inter electrode gap becomes the difference between the amount linear material removal rate and feed rate that is f .

And this you can find out like this we have already found $\frac{dy}{dt}$ that is $\frac{E(V - \Delta V)k}{F \rho_a y}$ and this is the minus f that is there so we add it here so this will give you the rate at which the inter electrode gap is changing with time, now the question is how to evaluate this inter electrode gap in ECM.

Now the two cases are there, one is the case of zero feed rate that is when the value of f is 0 that means no feed or no movement of the tool only the surface of the work piece is recessing downwards, another case is that where the feed rate is not zero that means tool is moving and work piece is also recessing downwards work piece surface is recessing downwards.

(Refer Slide Time: 10:52)

INTERELECTRODE GAP EVALUATION IN ECM

From previous equation on page 8,

$$\frac{dy}{dt} = \frac{c}{y} - f \quad \dots(3)$$

WHERE, $C = \frac{E(V - \Delta V)k}{F\rho_a}$

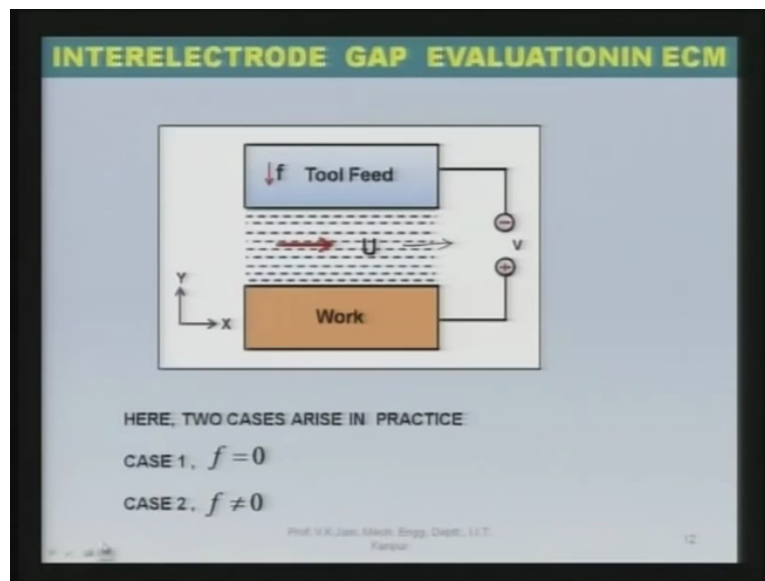
or, $C = \frac{EV_e k}{F\rho_a}$;

where, $V_e = (V - \Delta V)$

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

So let us see for the first case, from the previous equation we can write down dy over dt is equal to c over y minus f , where C is given by E into V minus ΔV k over F into ρ_a , now we are writing here V minus ΔV as V effective V_e now one thing is to be seen over here that whole of this term has been represented as C and it is varying with y that is the inter electrode gap, so this is nothing but this represented by $V_e k$ is there and $F\rho_a$ is there and this is given by V_e is equal to V minus ΔV .

(Refer Slide Time: 11:41)



Now in this particular figure we can clearly see that tool is feeding, is being fed downward, there is the flow of the electrolyte in this particular direction with a certain velocity and work piece and tool both are connected to the power supply tool being cathode and work piece being anode. Now here as I have mentioned earlier two cases arise one is the case where feed rate f is equal to 0 and another is the case where feed rate is not equal to 0.

(Refer Slide Time: 12:13)

INTERELECTRODE GAP EVALUATION IN ECM: ZERO FEED RATE

WHEN FEED RATE OF THE TOOL TOWARDS WORK PIECE IS ZERO.

CASE - I ($f = 0$)

SUBSTITUTE IN $f=0$ IN EARLIER EQUATION AND SOLVE FOR Y. WE GET

$\frac{dY}{dt} = C$ (3)

INTEGRATE BOTH SIDES AND SOLVE. IT GIVES,

$Y = \sqrt{(Y_0^2 + 2Ct)}$ (3b)

For $Y=Y_0$ at $t=0$. This equation is well represented by the Figure in the next slide.

ASSUMPTIONS

- 1) $k = \text{CONST}$ IN IEG
- 2) ΔV is a SMALL FRACTION OF V
- 3) \sqrt{k} of work piece and tool $\gg k$ of electrolyte

- PLANE PARALLEL ELECTRODES Machining is NORMAL TO FEED DIRECTION.
- ELECTROLYTE PROPERTIES ARE UNCHANGED IN Z - DIRECTION.

Let us evaluate the inter electrode gap in ECM for zero feed rate, when feed rate of the tool towards the work piece is 0 that means tool is stationary, now substitute in earlier equation f is equal to 0 then we will get that dY as function of Y here is equal to C into dt . Integrate both

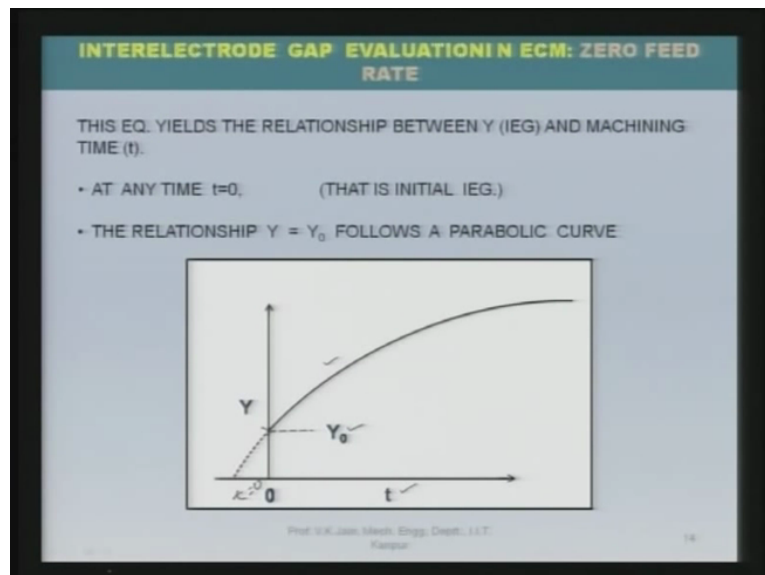
sides of this particular equation it will give Y is equal to Y_0 square plus $2 C t$ under the root whole.

Now this equation has been derived taking the initial condition to evaluate the constant of integration for Y is equal to Y_0 at time t is equal to 0 , that is the initial inter electrode gap if at t time whatever inter electrode gap we maintain in the beginning that is indicated by Y over here and that is the initial condition this equation is well represented by the figure in the next slide I will show you, here there are certain assumptions in deriving this particular equation.

Here K is taken constant that is the electrical conductivity of the electrolyte in the inter electrode gap. ΔV is a small fraction of V , ΔV is the over potential which is a very small fraction of the voltage being applied and K of work piece and tool are very very large compared to the electrolyte, k is the electrical conductivity of the work piece and the tool.

Let us take the case of plain parallel electrodes machining that is normal to the feed direction as you can see over here, here is the plain parallel machining. Electrolyte properties are change in Z direction that means we are considering 2D problems, third direction that is the Z direction we are assuming that the electrolyte properties are the same along the Z direction there is no variation.

(Refer Slide Time: 14:34)



Now this equation yields the relationship between inter electrode gap and machining time, any time t . Now you can see here that as the time t is increasing inter electrode gap is increasing according to the equation which we derive in the earlier slide now here you can

see the time t is equal to 0. There is a certain inter electrode gap that is represented by Y_0 and that is what we have used in the earlier equation for as the initial condition.

So the relationship Y is equal to Y_0 follow a parabolic, this relationship follow a parabolic curve that is shown over here. Now most of the time in electrochemical machining, feed rate is not equal to zero rather there is a finite feed rate so that means F is not equal to 0 and let us see how to evaluate the inter electrode gap when feed rate is not equal to zero that is the finite feed rate.

(Refer Slide Time: 15:49)

INTERELECTRODE GAP EVALUATION IN ECM: FINITE FEED RATE

CASE II ($f \neq 0$)

WHEN CONSTANT FEED IS GIVEN TO THE TOOL ($f \neq 0$) FOR BETTER PERFORMANCE OF THE PROCESS, IT IS DESIRABLE TO MACHINE UNDER EQUILIBRIUM CONDITIONS THAT IS $\frac{dy}{dt} = 0$ (CONSTANT IEG IS MAINTAINED)

FROM EQ(3), ONE CAN GET

$$\Rightarrow y = \check{y}e = \frac{\check{c}}{f} \left(\text{for } \frac{dy}{dt} = 0 \right)$$

Ye IS IEG UNDER THE EQUILIBRIUM CONDITIONS.

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

Case two, now when constant feed is given to the tool that is f not equal to 0 for better performance of the process it is desirable to machine under equilibrium condition that is dy over dt is equal to zero constant inter electrode gap is maintained. Now this is very important to understand that here in this particular case when dy over dt is equal to zero that simply means that this gap is maintained constant and this gap is nothing but equal to equilibrium gap and this can be maintained only when the condition that linear material removal rate is equal to the feed rate then only the gap between the bottom face of the tool and top face of the surface being machined will remain constant.

So y that is the inter electrode gap is equal to y_e that is the inter electrode gap under equilibrium condition and this is given by c by f as we have seen earlier for the case dy over dt is equal to 0. Here y_e is inter electrode gap under the equilibrium condition.

(Refer Slide Time: 17:03)

INTERELECTRODE GAP EVALUATION IN ECM: FINITE FEED RATE

FOR THE GENERALISED ANALYSIS, IT IS ALWAYS BETTER TO GO FOR NON - DIMENSIONALIZATION OF THE PARAMETERS. HENCE,

$$y' = \frac{y}{y_e} = \frac{y f'}{C}; \quad t' = \frac{t f^2}{C}$$

$$\rightarrow \frac{dy'}{dt'} = \frac{1}{y'} - 1$$

$$\int t' = y'_e - y' + \ln \left\{ \frac{(y'_0 - 1)}{(y' - 1)} \right\} \dots\dots(4)$$

THIS EQ. IS DERIVED AS FOLLOWS:

THIS EQ. IS VALID FOR $Y' > 0$

IF $Y' < 0$ IT INDICATES SHORT CIRCUIT

For the generalized analysis it is always better to go for non dimensionalization of the parameter hence here the parameters have been non dimensionalized as you can see y dash is equal to y over y e where y is the inter electrode gap and y e is the inter electrode gap under equilibrium condition and this can be shown is equal to y f over C because we have shown in the earlier slides that y e is equal to C over f.

Now t dash is equal to the t multiplied by f square over C and this becomes non-dimensional, so we can write the equation, earlier equation of dy dt can be written as dy dash over dt dash is equal to 1 over y dash minus 1. Now we can evaluate this equation or we can simplify this equation by taking integration on both sides and we will find finally this equation as t dash is equal to y 0 dash minus y dash plus ln that is natural log of y 0 dash minus 1 divided by y dash minus 1.

Now how to get this particular equation we will see in the following slides, this equation is valid for the case when y dash is greater than zero this is very important condition one should note down if y dash is less than zero then it will indicate that short circuit has taken place between the tool and the work piece and this is highly undesirable condition, no short circuit should take place between the tool and the work piece otherwise both will get damaged.

(Refer Slide Time: 18:57)

INTERELECTRODE GAP EVALUATION IN ECM: FINITE FEED RATE

THIS EQUATION CAN BE WRITTEN AS

$$\frac{dt'}{dy'} = \frac{y'}{1-y'} = \frac{(-1+y'+1)}{1-y'} = \left[\frac{-(1-y')+1}{1-y'} \right]$$
$$= \left\{ -1 + \frac{1}{1-y'} \right\}$$

Prof. V.K. Jain, Mech. Engrg. Dept., I.I.T. Kanpur

18

Now this equation can be written as $\frac{dt'}{dy'}$ the earlier equation which I wrote there is equal to $\frac{y'}{1-y'}$ this becomes equal to, if we add minus 1 and plus 1 then it becomes 0 so we get the same equation so we can write it as minus 1 plus $\frac{y'+1}{1-y'}$ divided by 1 minus y' and this can be written like this that is minus 1 plus $\frac{y'+1}{1-y'}$ we can simplify it and write it as minus 1 plus $\frac{1}{1-y'}$ because this term we separate out this term and this term and 1 minus y' and 1 minus y' of this term will get cancel so you will get minus 1 over there plus 1 over 1 minus y' .

(Refer Slide Time: 19:58)

INTERELECTRODE GAP EVALUATION IN ECM: FINITE FEED RATE

OR,

$$\int_0^{t'} dt' = \left\{ -1 + \frac{1}{1-y'} \right\} dy'$$

INTEGRATE BOTH SIDES,

$$t' = -y' - \ln(y' - 1) + K$$

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

Now if you simplify the earlier equation or rewrite the earlier equation you will get dt dash is equal to minus 1 plus 1 over 1 minus y dash, dy dash. Integrate both sides of this particular equation that is you can integrate it, you will get the after integration and simplifying it you will get this becomes 0 to t dash and you can integrate this particular part also and you get finally t dash is equal to minus y dash minus ln y dash minus 1 plus K.

(Refer Slide Time: 20:49)

INTERELECTRODE GAP EVALUATION IN ECM: FINITE FEED RATE

TO EVALUATE THE CONSTANT OF INTEGRATION, K.

SUBSTITUTE THE INITIAL CONDITION AS $y' = y'_0$ at $t' = 0$,
IT GIVES THE FOLLOWING:

$$0 = -y'_0 - \ln(y'_0 - 1) + K$$

OR

$$K = y'_0 + \ln(y'_0 - 1)$$

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

Now we have to evaluate the constant of integration K, now for this particular purpose substitute the initial condition as y dash is equal to y 0 dash where we have already seen that initial inter electrode gap is given by y 0 and if initial inter electrode gap is y 0 then you non dimensionalize this by dividing by y then you get y 0 dash and this true at t dash is equal to 0.

It gives the following relationship as 0 is equal to $\ln y$ dash 0 minus $\ln y$ dash 0 minus 1 plus K , from this you get if you substitute this value and simplify this particular equation then you get K is equal to y dash 0 plus $\ln y$ dash 0 minus 1 .

(Refer Slide Time: 21:49)

INTERELECTRODE GAP EVALUATION IN ECM: FINITE FEED RATE

SUBSTITUTE THE VALUE OF 'K' IN ABOVE EQUATION

$$t' = -y' \ln(y' - 1) + y'_0 + \ln(y'_0 - 1)$$

$$= y'_0 - y' + \ln \left\{ \frac{(y'_0 - 1)}{y' - 1} \right\} \quad \dots\dots(4)$$

IN THIS EQUATION, ONLY +ve VALUES OF Y' ARE POSSIBLE.

Y' = 0 IMPLIES A SHORT CIRCUIT BETWEEN THE TOOL AND WORK PIECE.

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

21

Substitute the value of K in above equation you get t dash is equal to $\ln y$ dash 0 minus $\ln y$ dash 0 minus 1 plus y dash 0 plus $\ln y$ dash 0 minus 1 . Now you can simplify this equation then you will get this as y dash 0 minus y dash plus $\ln y$ dash 0 minus 1 over divided by y dash 0 minus 1 . You can take this and this together they will give this equation divide this by this because this has the minus term you will get this particular term.

In this equation only positive values of y dash are possible, y dash is equal to 0 implies a short circuit between the tool and work piece that I have already mentioned and for negative value of, negative value it will not work, this particular equation will not work.

(Refer Slide Time: 22:44)

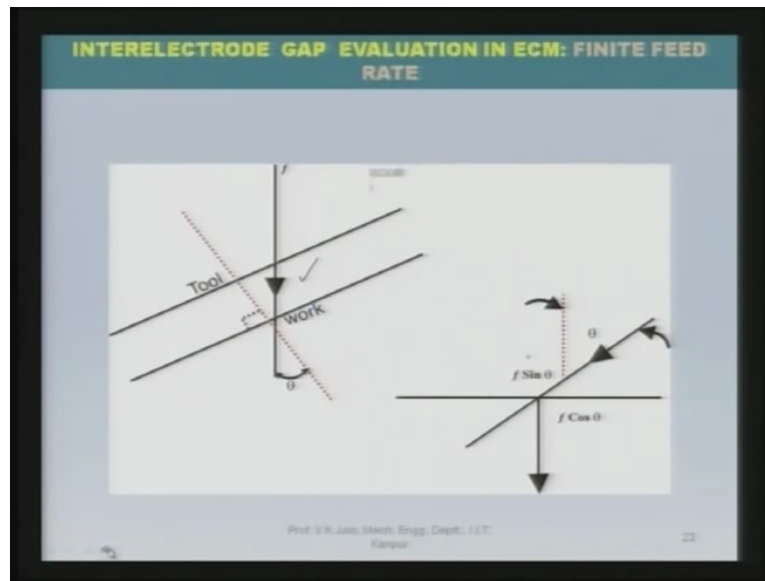
The slide features a title bar at the top: "INTERELECTRODE GAP EVALUATION IN ECM: FINITE FEED RATE". Below the title, it states: "WHEN THE PARAMETERS ARE NOT NON- DIMENSIONALISED, THE SOLUTION OBTAINED CAN BE WRITTEN AS". The central equation is:
$$*t = \frac{1}{f} \left[y_0 - y_t + y_e \ln \left(\frac{y_0 - y_e}{y_t - y_e} \right) \right] \dots\dots(4a)$$
 There is a handwritten symbol y_e to the right of the equation. At the bottom, a yellow bar contains the text: "REF - ELECTROCHEMICAL MACHINING BY J.A. McGEOUGH, CHAPMAN AND HALL, LONDON". Below that, in smaller text, it says "Prof. V.K. Jain, Mech. Engrg. Dept., I.I.T. Kanpur".

When the parameters are not non-dimensionalised the solution obtained can be written as t is equal to 1 over f within bracket y 0 minus y t plus y e ln y 0 minus y e divided by y t minus y e, this bracket. Now this equation is an implicit equation when you want to evaluate the inter electrode gap y t right. So this equation is given in the book on electrochemical machining by J. A. McGeough published by Chapman and Hall, London.

Solving this particular equation takes lot of time because this is an implicit equation and with the help of computer if you want to solve it for one particular value of inter electrode gap it may take 30 to 40 iterations which takes lot of computer time because solving the ECM problem with the help of electrochemical machine, with the help of finite element method or boundary element method or any other finite difference method then there are thousands of node and at each node you have to iterate it for 30 to 40 time.

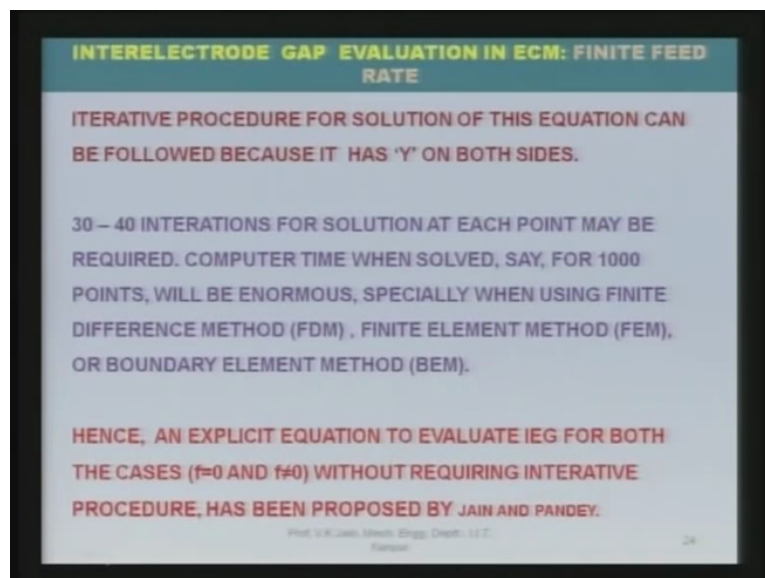
Total time taken by computer will be too large so it is quite cumbersome and difficult to solve by the help of even, difficult in the sense a lot of time it will be taking on the computer.

(Refer Slide Time: 24:20)



Now most of the time you find that the feed rate is inclined to the surface of the work piece as I can see here in that particular case you have to take the component of the feed rate which is normal to the work piece surface and that becomes $f \cos \theta$, so in place of f you have to use $f \cos \theta$ at appropriate places wherever feed rate term f appears.

(Refer Slide Time: 24:52)



Now iterative procedure for solution of this equation can be followed because it has Y on both sides when you are evolving Y or inter electrode gap with the help of this particular equation, so it takes 30 to 40 iterations for solution at each point. Computer time when solve say for 1000 points will be enormous specially when using finite difference method, finite element method or boundary element method.

Hence an explicit equation to evaluate inter electrode gap for both the cases that is feed rate is equal to 0 and finite feed rate without requiring iterative procedure has been proposed.

(Refer Slide Time: 25:39)

SINGLE EQUATION FOR BOTH CONDITIONS (f=0 and f≠0)

EQUATION FOR EVALUATING IEG FOR NUMERICAL ANALYSIS OF ECM PROCESS, WE KNOW,

$$\frac{dy}{dt} = \frac{E(v-\Delta v)k}{F\rho_a y} - f$$

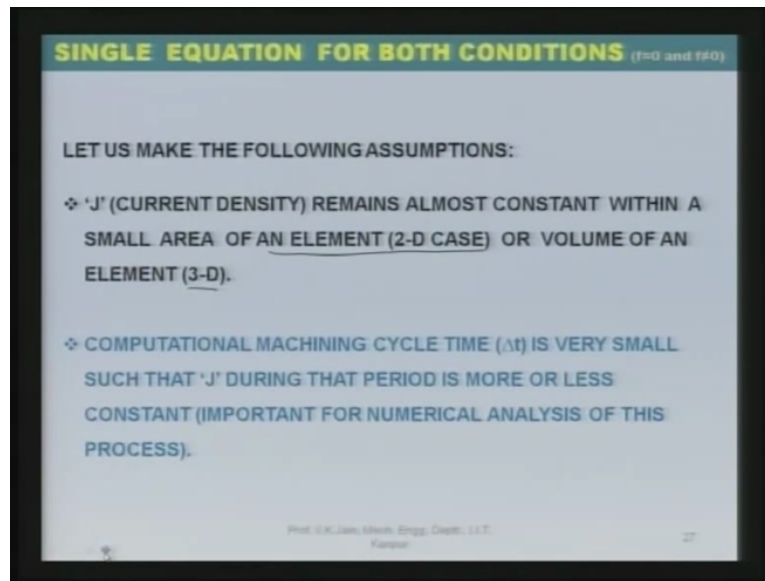
$$= \frac{EJ}{F\rho A} - f$$

$$dy = \left(\frac{EJ}{F\rho A} - f \right) dt$$

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

Now let us see how to evaluate this single equation for inter electrode gap evaluation in case of electrochemical machining, equation for evaluating inter electrode gap for numerical analysis of electro chemical machining process. We have already derived this particular equation that is dy over dt is equal to E v minus delta v k over F rho a y minus f where v is the applied voltage and delta v is the over potential and F is the feed rate This can be written as EJ over F rho a minus f, this can be rewritten as dy is equal to EJ over F rho A minus f into dt.

(Refer Slide Time: 26:50)



Let us make the following assumptions, J that is the current density remains almost constant within a small area of an element when I say element that means I am talking with reference to the finite difference method or finite element method of 2D problem or volume of an element that means I am talking of 3D problem.

Computational machining cycle time Δt is very small, it is normally taken as fraction of a second such that J that is the current density during that period of time is more or less constant, important for numerical analysis of this particular process.

(Refer Slide Time: 27:39)

SINGLE EQUATION FOR BOTH CONDITIONS (f=0 and f≠0)

INTEGRATION OF ABOVE EQN WOULD GIVE

$$y = (c' - f)\Delta t + k'$$

TO EVALUATE THE VALUE OF INTEGRATION CONSTANT, K'

WHERE $C' = \frac{EJ}{F\rho_a}$

AT TIME: $t = 0, y = y_0 \Rightarrow k' = y_0$

SUBSTITUTE THIS VALUE IN THE ABOVE EQUATION.

$$y = y_0 + (c' - f)\Delta t$$

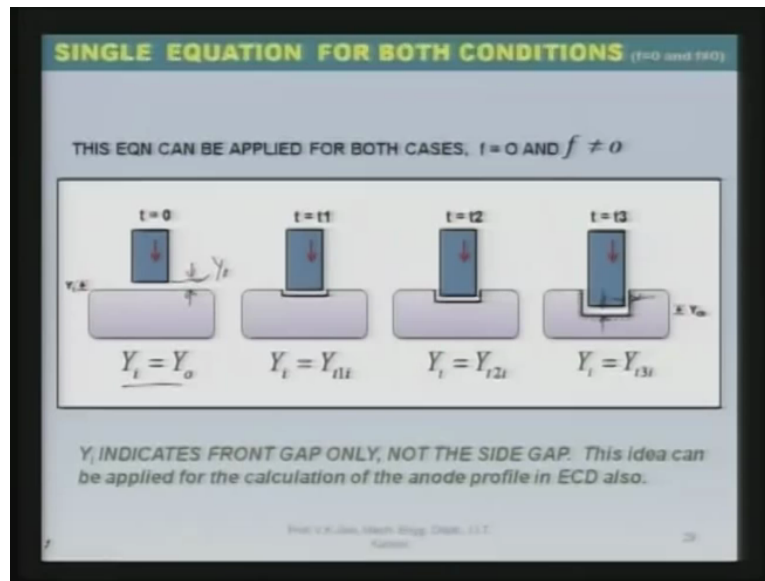
since 't' is very small hence, $t = \Delta t$

Prof. V.K. Jain, Mech. Engg. Deptt., I.I.T. Kanpur

Integration of above equation would give y is equal to c dash minus f into Δt plus k dash, to evaluate the value of integration constant k dash where C dash is equal to EJ over $F\rho_a$ at time t is equal to 0 , y is equal to y_0 I have already mentioned if you substitute this value T is equal to 0 and y is equal to y_0 in this particular equation you will get k dash is equal to y_0 , substitute this value of k in this particular equation then you will get y is equal to y_0 plus c dash minus f into Δt , here I have written Δt since t is very small hence t is equal to Δt which I have already written over here.

This equation can be applied for both cases, f is equal to 0 and f not equal to 0 as if you see again previous equation here you can, because feed rate is here if you are taking the case of feed rate is equal to 0 substitute the value F is equal to 0 if it is not 0 then you substitute here the value of f whatever is the feed rate in appropriate units you will be able to evaluate the inter electrode gap in both the cases whether zero feed rate or finite feed rate from the same equation.

(Refer Slide Time: 29:22)



Now what is this Y_0 , you know when you are beginning the machining then whatever is the initial inter electrode gap that is Y_0 . Now when we deal with the finite element method or other procedure then we in place of Y we can use Y_i that is the I S node and Y_i is equal to Y_0 and if it is at time T is equal to 0, if time T_1 is there then it becomes Y_i is equal to Y_{T1} and so on, as I have shown over here.

Y_i indicates front gap only not the side gap as I have mentioned earlier also we are not considering this side gap over here, we are considering only the front gap as shown over here, this idea can be applied for the calculation of the anode profile in electrochemical drilling also so this equation which we have just derived as single equation for finite feed rate as well as for zero feed rate is very very important especially when you are solving large problems having thousands of nodes and in a very small machining time Δt you are taking.

Then total computational time will be substantially reduced if this single equation used in place of parabolic equation and implicit equation which requires solution for 30 or 40 iteration for each point solution. Now apart from this evaluation of inter electrode gap either by parabolic equation or implicit equation or by single equation applicable for both, there are certain interesting features with this electrochemical machining process has got and this self regulating is one of the very important feature of electrochemical machining process.

(Refer Slide Time: 31:25)

SELF REGULATING FEATURE OF ECM

- ECM PROCESS ALWAYS ATTEMPTS TO ATTAIN AN EQUILIBRIUM GAP CONDITION SUCH THAT $y = y_e$ OR, $f = MRR_{le}$.
- IN CASE THE EQUILIBRIUM CONDITION IS DISTURBED, TWO POSSIBILITIES EXIST, AS DISCUSSED BELOW.

CASE-1 PLEASE NOTE THAT LOWER VALUE OF IEG (y) WILL RESULT IN HIGHER VALUE OF J THAT IS HIGHER VALUE OF MRR.

IF $y > y_e$, THEN MRR_l WILL BE LOWER AS COMPARED TO MRR_{le} OR, $f > MRR_l$ BECAUSE $f = MRR_{le}$.

AS A RESULT OF THIS CONDITION ($f > MRR_l$) THE IEG WILL TEND TO DECREASE. HENCE, y TENDS TO DECREASE OR ATTEMPTS TO BE EQUAL TO y_e .

71

ECM process always attempts to attain an equilibrium gap condition such that y is equal to y_0 , it always attempts to attain this condition or in other words you can say f is equal to MRR_{le} , here suffix l indicates the linear material removal rate and e indicates the linear material removal rate under equilibrium condition.

Now one point is to be noted that if f is equal to MRR_{le} then inter electrode gap will always remain constant and once inter electrode gap remains constant we call it as equilibrium condition, in case the equilibrium condition is disturbed two possibilities exist as discussed below, disturbed means either inter electrode gap becomes more than equilibrium gap or it becomes less than equilibrium gap.

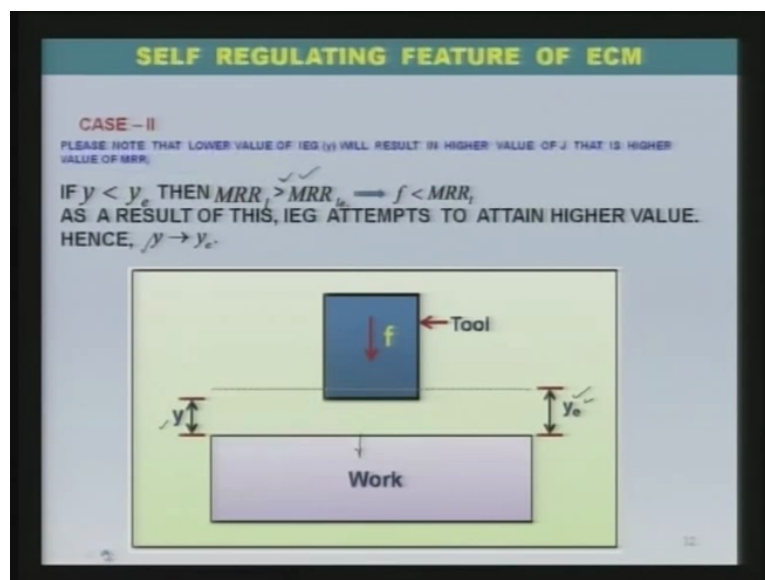
Now let us see what happens when initial inter electrode gap is more than equilibrium gap or less than equilibrium gap. Case one, if inter electrode gap y is greater than equilibrium inter electrode gap y_e then MRR_l , linear material removal rate will be lower as compared to MRR_{le} because y is greater than y_e as you can see here that means the current density will become lower as the inter electrode gap increases current density becomes lower and this is greater than equilibrium gap so current density compared to the case when equilibrium gap is y_e it will become lower or f becomes greater than MRR_l because we can use this particular equation for this particular purpose that when y is greater than y_e then feed rate becomes greater than linear material removal rate.

And you can see here this is the equilibrium gap y_e and this is the initial gap or the gap inter electrode gap y so in this particular case this one feed rate becomes greater than MRR_l so

what will happen, also please note that lower value of inter electrode gap will result in higher value of J that is current density that is higher value of MRR because once the value of J becomes higher than the or the current density becomes higher than the material removal rate increases.

As a result of this condition that is the feed rate greater than MRR the inter electrode gap will to decrease hence y tends to decrease or attempts to be equal to y e because when this y is trying to reduce then definitely this y will be tending to move towards the equilibrium gap y e. So we are discussing self regulating feature of electrochemical machining, case one is the case where inter electrode gap is greater than equilibrium inter electrode gap.

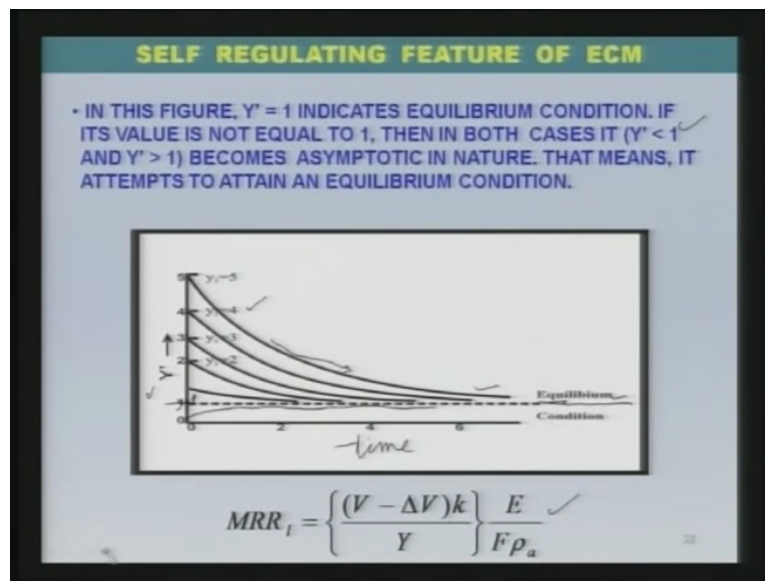
(Refer Slide Time: 34:43)



Now case two is the one where inter electrode gap or initial inter electrode gap is less than equilibrium gap, then MRR that is linear material removal rate will be greater than the linear material removal rate under the equilibrium conditions and f is smaller than MRR compared to the equilibrium condition.

As a result of this inter electrode gap attempts to attain higher value because linear material removal rate is greater than feed rate so gap will continuously keep increasing so y that is the gap will always try to attain the equilibrium gap as you can see here this is the y e and this is the y. y is smaller than y e now since material removal, linear material removal rate in this particular case is greater so this will try to move faster this surface will try to move faster than the feed rate and the gap will try to increase so that it attains the value of equilibrium gap that is y e.

(Refer Slide Time: 36:04)



So in this figure which I am going to show, we have seen earlier that whether the inter electrode gap was lower than the equilibrium gap or larger than the equilibrium gap in both the cases inter electrode gap automatically was attempting to attain the equilibrium gap, the same thing which we discussed in the or we saw in the earlier slides is represented here in the form of the graph as you can see here, here is the Y dash and here is the time.

Now you can see here if it is 1 then equilibrium gap and inter electrode gap both are the same that means the machining will continue under equilibrium condition, if gap becomes more than that in this zone then you can see all the curves they are attempting to attempt asymptotic condition where the inter electrode gap becomes equal to the equilibrium gap.

If it is lesser than that then also it will be, it is trying to attempt, to get the asymptotic condition and to attain the equilibrium condition even if it is less than and so it is very clearly visible here that in both the cases inter electrode gap attempts to attain equilibrium inter electrode gap the same thing is written here that in this figure y dash is equal to 1 indicates equilibrium condition that is this one, dash dash condition.

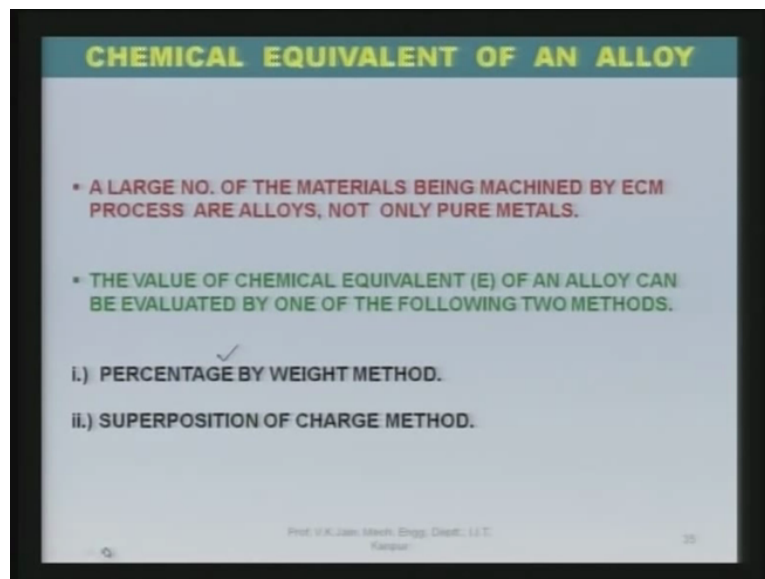
If its value is not equal to 1, then in both cases it, that is y dash less than 1 and y dash greater than 1 becomes asymptotic in nature that you can see here that means it attempts to attain an equilibrium condition and that condition is represented by 1 over here and we all know already that MRR is given by this particular equation.

Now we know that electrochemical machining process is applicable only for electrically conducting material now electrically conducting materials they can be, they are the metals

only they can be, mostly they are the metals, they can be pure metal that means iron, there are no other constituent.

Or second case is where other constituents are there that means alloy, more than one metals are there and they form the alloy now the question arises especially when alloy is there what is going to be the value of the chemical equivalent that is E , because E is different for different metals when more than one metals are there in the alloy how to evaluate this? So there are two methods to evaluate chemical equivalent of an alloy.

(Refer Slide Time: 39:11)



Let us see, a large number of the materials being machined by ECM process are alloys not only pure metals, the value of chemical equivalent E of an alloy can be evaluated by one of the following two methods, first one is known as percentage by weight method and here I will like to emphasize what this percentage means.

Say suppose an alloy is there and five constituents are there then each constituent will be in different weight percent, someone may be 80 weight percent another maybe 5 percent, third may be 2 percent something like that. So that percentage by weight method is the one and second is known as super position of charge method we will discuss both of them.

(Refer Slide Time: 40:15)

CHEMICAL EQUIVALENT OF AN ALLOY

(i) PERCENTAGE BY WEIGHT METHOD

THE SUM OF THE CHEMICAL EQUIVALENTS $\left(\frac{A_i}{Z_i}\right)$ OF EACH ELEMENT $\cdot X_i$ IN THE ALLOY IS MULTIPLIED BY ITS RESPECTIVE PROPORTION BY WEIGHT (X_i) IN THE ALLOY. IT GIVES A VALUE FOR THE CHEMICAL EQUIVALENT OF THE ALLOY $(A/Z)_a$

$$\left(\frac{A}{Z}\right)_a = \frac{1}{100} \sum_{i=1}^n \left(\frac{A_i}{Z_i}\right) X_i$$

(n IS THE NO. OF THE CONSTITUENT ELEMENTS)

USE THIS VALUE OF $E (=A/Z)_a$ IN THE EQUATION OF MRR

Handwritten notes:
 $\frac{A_1 + A_2 + \dots}{Z_1 Z_2}$
MRRl
MRRg
MRRv
MRRa

Percentage by weight method, now in this particular case the sum of the chemical equivalent A_i divided by Z_i , i indicates the i th element maybe written as A_1 over Z_1 plus A_2 over Z_2 and so on for different elements of each element i in the alloy is multiplied by its percentage, respective proportion by weight that is X_i in the alloy. It gives a value for the chemical equivalent of the alloy that is abbreviated as A over Z with suffix small a .

It can be written the equation is written like A over Z , A over Z is nothing but E that is the chemical equivalent where A is the atomic mass of the element and Z is the valency at which it is dissolving, please note here I have mentioned earlier lecture also that some elements have more than 1 valencies say 2 valencies, 3 valency, then you should clearly know at which valency it is dissolving then only you will get the correct value of E otherwise it will be a misleading value of E then calculated MRR as well as predicted anode shape or design tool shape all will be erroneous.

So A by Z a is equal to $\frac{1}{100} \sum_{i=1}^n \left(\frac{A_i}{Z_i}\right) X_i$, where X_i as mentioned over here is the weight percent of a particular element i , n is the number of constituent elements. Use this value of E that is equal to A over Z a in the equation of MRRl this is useful for the equation whether it is for MRRl or MRRg or MRRv or even MRRa if it is being used.

(Refer Slide Time: 42:42)

CHEMICAL EQUIVALENT OF AN ALLOY

(ii) SUPERPOSITION OF CHARGE METHOD

FOR THE DISSOLUTION OF 1g OF ALLOY, THE TOTAL AMOUNT OF ELECTRIC CHARGE REQUIRED FOR DISSOLUTION BY INDIVIDUAL ELEMENT IS EQUATED TO $\left(\frac{Z}{A}\right)_a F$ COULOMBS.

$$\left(\frac{Z}{A}\right)_a F = F \sum_{i=1}^n \frac{X_i}{100} \left(\frac{Z_i}{A_i}\right)$$

or, $\left(\frac{A}{Z}\right)_a = \frac{100}{\sum_{i=1}^n X_i \left(\frac{Z_i}{A_i}\right)}$

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

Now super position of charge method, the basic philosophy of super position of charge method is for the dissolution of 1 gram of alloy the total amount of electric charge required for dissolution by individual element is equated to Z over A suffix small a multiplied by F that is the Faraday's constant.

And it is written like this Z over A F is equal to F summation i is equal to 1 to N , X_i over 100, Z_i over A_i , where Z_i is the valency of highest element, A_i is the atomic mass of the highest element and X_i represents the percentage by weight of that particular highest element in the alloy.

This can be simplified like A by, because in the earlier equation we try to find out A by Z that is the E so we can write this equation, this particular equation in this particular form so that we directly get the value of E and A by Z a is equal to 100 divided by summation i is equal to 1 to n X_i Z_i over A_i so whatever value of E you get from here this value can substitute in the equation of MRR.

(Refer Slide Time: 44:12)

CHEMICAL EQUIVALENT OF AN ALLOY

COMPARISON OF TWO METHODS FOR EVALUATION OF "E"

THEN, MASS REMOVAL RATE (\dot{m}) CAN BE FOUND

$$\dot{m} = \left(\frac{A}{Z}\right) \frac{I}{F}$$

THESE TWO METHODS GIVE DIFFERENT VALUES OF $\left(\frac{A}{Z}\right)$

CASE	I METHOD	II METHOD
I	26.8 ✓	25.1 ✓
II	39.9 ✓	34.7 ✓

THESE TWO METHODS RESULT IN DIFFERENT MRR.

Prof. V.K. Jain, Mech. Engrg. Deptt., I.I.T. Kanpur

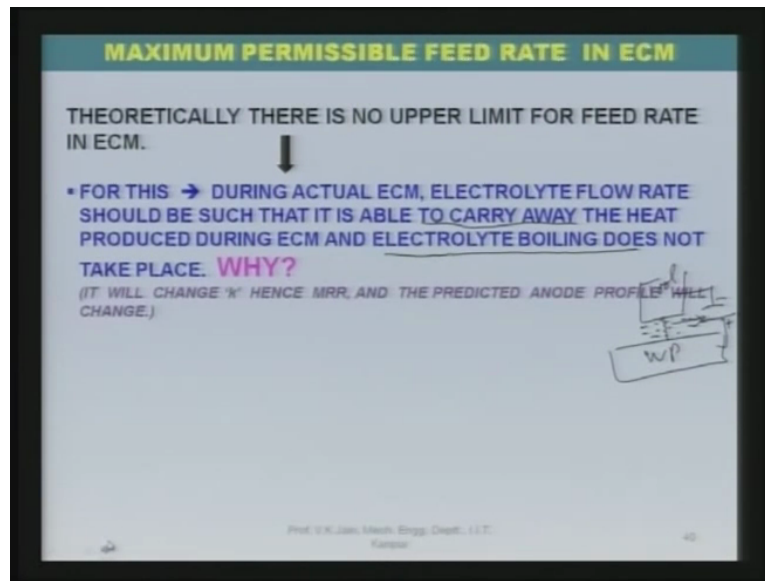
Comparison of two methods for evaluation of E, mass removal rate can be obtained from this particular equation \dot{m} is equal to $\frac{A}{Z} \cdot \frac{I}{F}$. $\frac{A}{Z}$ represents for alloy and we are able to calculate from one of the two methods the value of $\frac{A}{Z}$ for the alloy.

Current we already know Faraday's constant we already know, we can find out the mass material removal rate that is also represent it as MRRg. These two methods give different values of $\frac{A}{Z}$ that is the chemical equivalent of the alloy, now some values that have been calculated for a particular case and you can see from case one, for case one or for one particular alloy method one gives the 26.8 value while method two gives you the 25.1

While second case method one gives you 39.9 and method two gives 34.7 so you can see clearly there is a difference in the value of E, now if the value of E is different as you can see over here then definitely material removal rate either in gram or linear or volume these also going to be different so you have to be careful which method gives better results as compared to the experimental results you should use that particular method for the case that applies in your industry or research work.

Now as I have mentioned already that when material is being removed the tool is being fed continuously towards the work piece so that you can maintain the constant inter electrode gap during machining and if inter electrode gap is constant then other parameters are constant then material removal rate also becomes constant but what is that maximum value which you can use during ECM because larger the value of feed rate, larger becomes the material removal rate.

(Refer Slide Time: 46:39)



Theoretically there is no upper limit for feed rate in ECM, however there are certain practical constraints because of which we have to limit the feed rate because if feed rate is very very high then what will happen, the tool will move very fast towards the work piece and it will touch the work piece and short circuit will take place and in any case we do not want short circuit to take place, for this during actual ECM electrolyte flow rate should be such that it is able to carry away the heat produced during ECM and electrolyte boiling does not take place.

This is very important, first thing is it should be able to take away the heat, carry away the heat produced during the ECM process because we all know that this is the tool and this is the work piece and here is the electrolyte, it is flowing, these two are connected to the positive and negative terminal so this is working as the electrical resistance and in this resistance or inter electrode gap heat is being generated and this heat should be taken away out of the machining zone.

If it remains there then electrolyte temperature will keep increasing and once electrolyte temperature is increasing as we have seen earlier, electrolyte conductivity will keep changing and once electrolyte conductivity changes then material removal rate will keep changing continuously this is one which we do not want.

Second thing is the electrolyte boiling we do not want that electrolyte should boil in the inter electrode gap first thing is once it starts boiling it will create or generate the water vapour which will change the electrical conductivity of the electrolyte. Second thing the rise in the temperature will be quite high this will also affect the conductivity of the electrolyte which

will affect the material removal rate as well as the shape of the work piece that you are going to get on the work piece.