Advanced Machining Processes
Professor Vijay K. Jain
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
Lecture 04
Ultrasonic Machining (USM)

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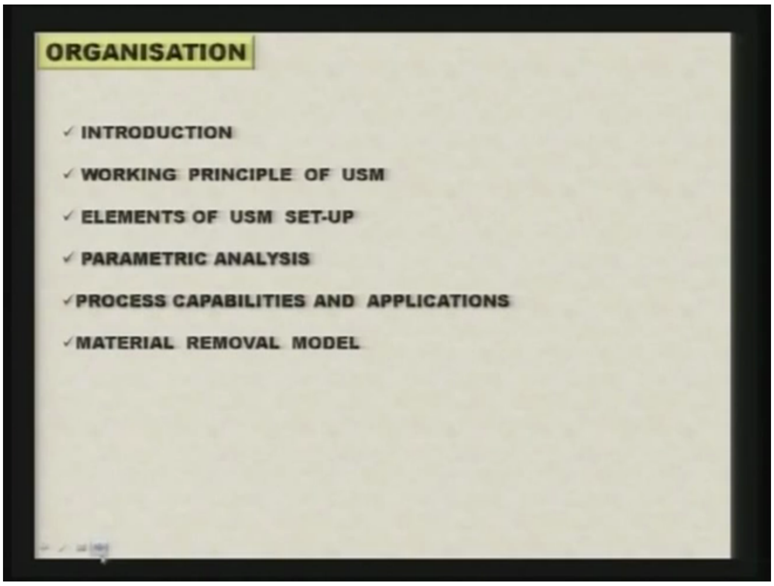

Welcome to the course, Advanced Machining Processes, today I am going to talk about Ultrasonic Machining. Organization of the talk is as follows, introduction working principle of ultrasonic machining process, various elements of ultrasonic machining setup, parametric analysis of the process then we will see about the process capabilities and applications. Then there are various material removal models proposed by various researchers one of them proposed by M. C. Shaw I will discuss that material removal model.

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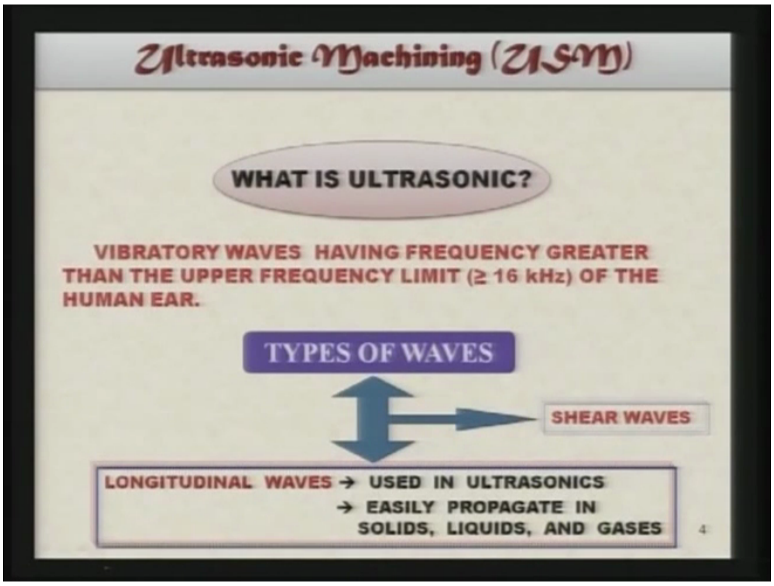

Introduction, first question is what is ultrasonic? Ultrasonic is vibratory waves having frequency greater than the upper frequency limit of the human ear and that is greater than or equal to 16 kilohertz or 16000 hetrz per second. Types of waves, there are basically two types of the waves one of them is known as longitudinal waves another is known as shear waves. In case of ultrasonic machining we normally use longitudinal waves they easily propagate in the solids, liquids as well as in the gases.

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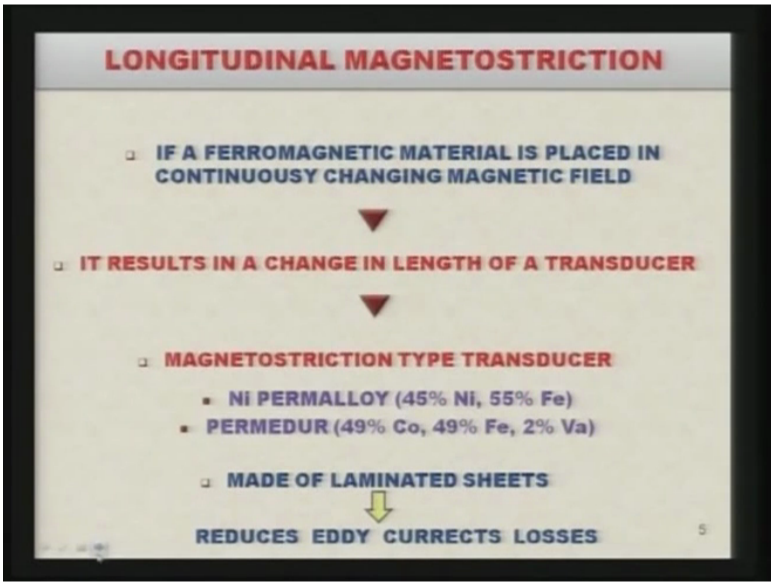

Longitudinal magnetostriction, if a ferromagnetic material is placed in continuously changing magnetic field then what happens, it results in a change in length of a transducer that is placed in the continuously changing magnetic field. So magnetostriction type transducer is there and they are normally made up of nickel permalloy which consist of 45 percent nickel 55 percent of iron then there is permedur which consists 49 percent cobalt, 49 percent iron and 2 percent vanadium. These transducers are made of laminated sheets because there are certain reasons for it, so they make the make of laminated sheets, it reduces the eddy current losses that is why they are made of laminated sheets.

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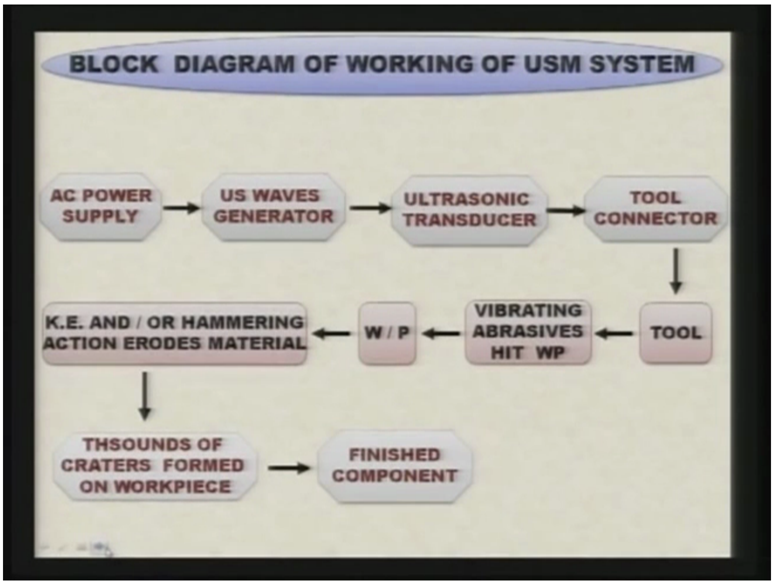

Now ultrasonic transducer converts appropriate form of energy into ultrasonic waves, normally in ultrasonic machining electrical energy is converted into mechanical energy with high-frequency of vibration. Let us see, what is the working principle of ultrasonic machining process. Block diagram of ultrasonic machining system, now first we have AC power supply which is connected to the ultrasonic waves generator and this ultrasonic wave generator is connected to the ultrasonic transducer and this ultrasonic transducer is connected to the tool connector and this tool connector holds the tool and because ultrasonic vibrations are generated so this tool vibrates at ultrasonic frequency.

And these vibrating tool between the tool and the work piece as I will show you in the figure there is a vibrating tool there is a work piece and in between two there is a slurry and this slurry consist of liquid as well as abrasive particles so when this tool is vibrating at ultrasonic frequency it is hitting thousands of abrasive particles which are present in the slurry under the tool.

And these particles when they are hit by the tool they move towards the work piece and because of their high velocity and large number of abrasive particles they hit the work piece and their kinetics energy or due to hammering action they erode the material from the work piece and thousands of very very fine craters are formed on the work piece and this results in finished component.

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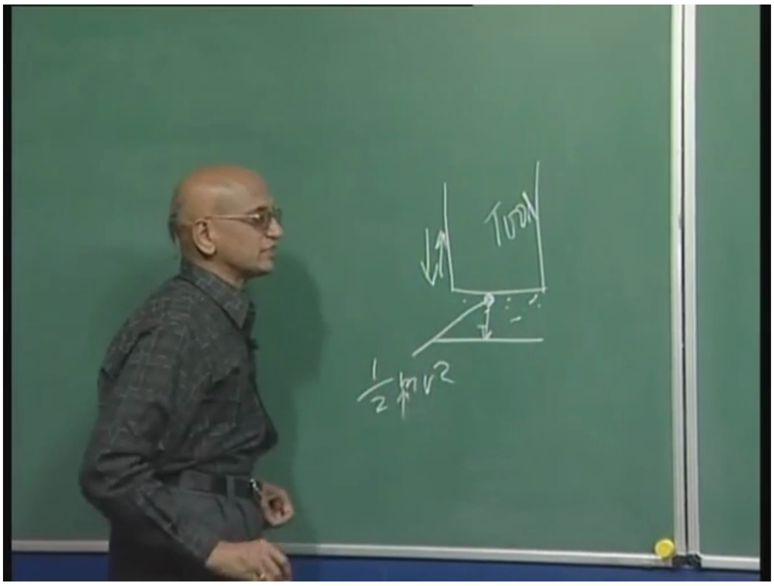

Actually what happens in the work piece that if you see there is a tool and thousands of the abrasive particles are there in the tool and this under the tool and when this tool is vibrating it is hitting an abrasive particle and this abrasive particle goes and hits the work piece with the kinetic energy and that kinetic energy is nothing but half MV square where M is the mass of the abrasive particle and V is the velocity which is hitting the work piece.

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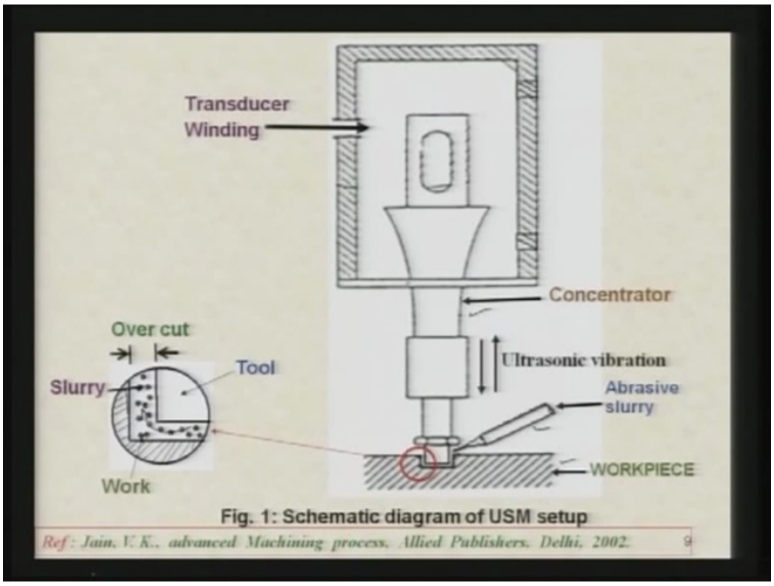

As you can see the schematic diagram of this, here is the transducer winding and there is a concentrator over here and it is moving at or vibrating at ultrasonic frequency and here is the nozzle which is supplying the abrasive slurry and here is the work piece. If you see the enlarged view of this particular portion you will find that tool is there, work piece is there and slurry is there.

Now this slurry and this over cut is created because the slurry has to go out from here so additional material is removed because of the action between the abrasive particle and the side surface of the work piece as well as some material is removed from the tool also, so due to this you will find that the material is removed continuously from the work piece and thousands and thousands of verifying craters are created on the work piece.

So they produce a replica, approximately replica of the tool on the work piece and this figure has been taken from the book on advanced machining processes published by Allied publishers.

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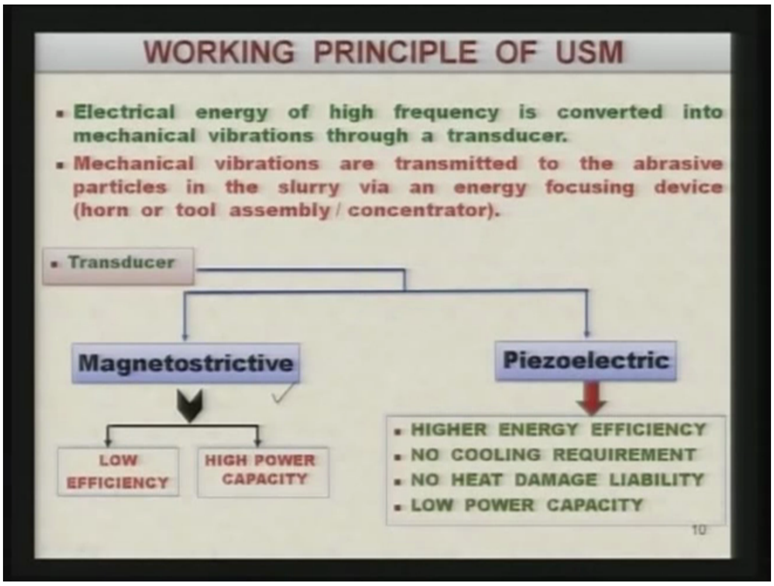

Now as we have seen electrical energy of high frequency is converted into mechanical vibrations through a transducer and mechanical vibrations are transmitted to the abrasive particles in the slurry via an energy focusing device that is known as horn or tool assembly or concentrator.

Now this transducer are basically of two types, one is known as Magnetostrictive type of transducer which has certain characteristics that it has low efficiency but they can produce high power, on the other hand you have the piezoelectric type of transducer they have higher energy efficiency they do not require any cooling, there is no heat damage, liability and the problem is their power capacity is low. In this particular case, in case of magnetostrictive type of transducer, most of the time you require cooling so that again adds to the cost of the whole system.

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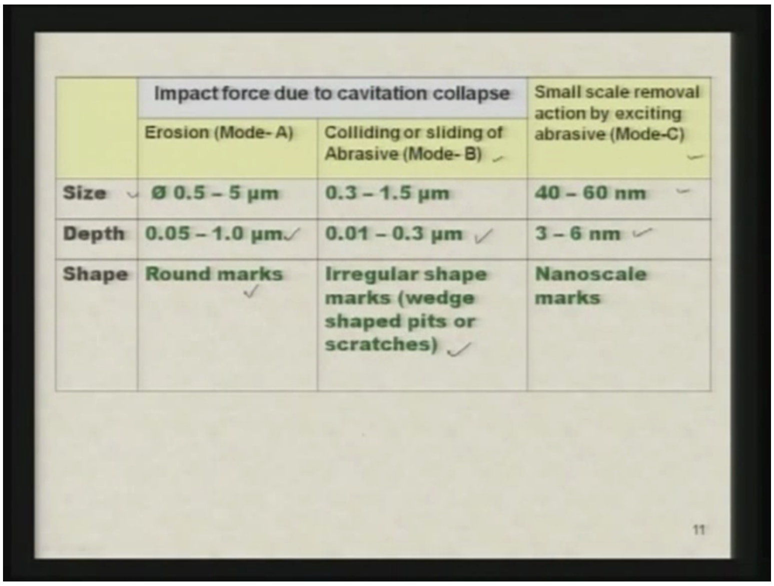

	Impact force due to cavitation collapse		Small scale removal action by exciting abrasive (Mode-C)
	Erosion (Mode- A)	Colliding or sliding of Abrasive (Mode- B)	
Size	Ø 0.5 – 5 μm	0.3 – 1.5 μm	40 – 60 nm
Depth	0.05 – 1.0 μm	0.01 – 0.3 μm	3 – 6 nm
Shape	Round marks	Irregular shape marks (wedge shaped pits or scratches)	Nanoscale marks

11

Now when abrasive particles are hitting the work piece, some of the researchers have proposed that erosion may take place into three different modes, one is the mode A that is the erosion, second is mode B that they have called as colliding or sliding of abrasive and mode C that is the small scale removal action by exciting abrasive particles, the size of the crater formed in which three different modes are different in the first A, it is somewhere 0.5 to 5 microns micrometer size in the second B type it is 0.3 to 1.5 micrometer size and in the C type it is 40 to 60 nanometer.

The depth to which abrasive particle is penetrating in mode A it is point 0.5 to 1 micrometer in mode B 0.1 to 0.3 micrometer and in mode C it is just 3 to 6 nanometer which is very very small. Shape you normally get round marks in case of mode B irregular shape marks are obtained and in case of C nano scale marks are obtained.

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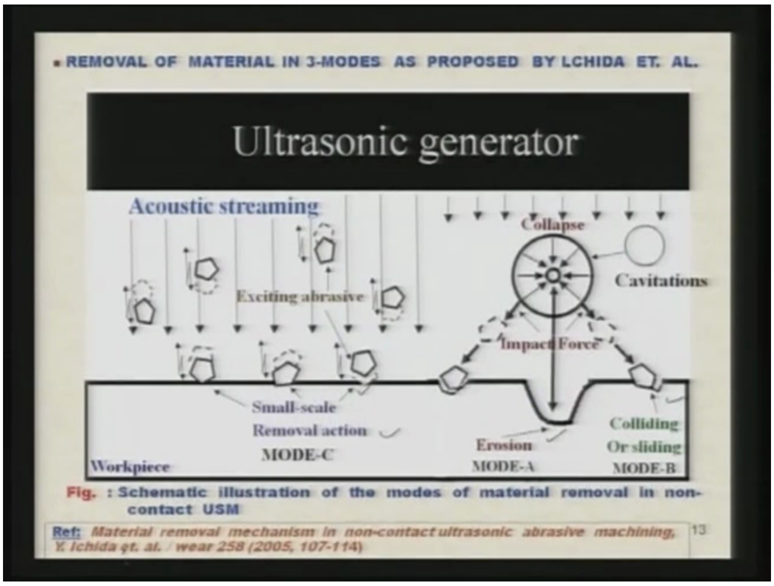

Now this gives you a picture of the small scale removal process, now as you can see here this is the tool which is vibrating at ultrasonic frequency and here you have the slurry in the gap between the work piece and the base or the bottom of the tool and these are the all abrasive particles which are being hit by the tool and once they are hit by the tool with certain kinetic energy they come and hit the work piece surface over there.

Now you can see abrasive grain here, here and here that are removing the material from the work piece of very small size. So this is known as small scale removal process. Now you can see here the work piece hitting over there then going back and you can see the mark obtained over here is very very small.

In this one all the three modes of the removal of the material are shown you can see here, this is the ultrasonic generator, this is the work piece and in between there is the slurry and then you can see the (())(10:38) excited abrasive particle here in case of mode C you can see the kind of the crater formed over there.

In case of mode A here is the crater formed over there and in case of mode B here is the crate shown over there now you can clearly see the cavitation process also contribute to the removal of the material to some extent apart from the hitting of the work piece by direct impact from the tool.

Now this particular diagram has been taken from the paper by Ichida that was titled as Material Removal Mechanism in non-contact ultrasonic abrasive machining, it was published in the international journal of wear.

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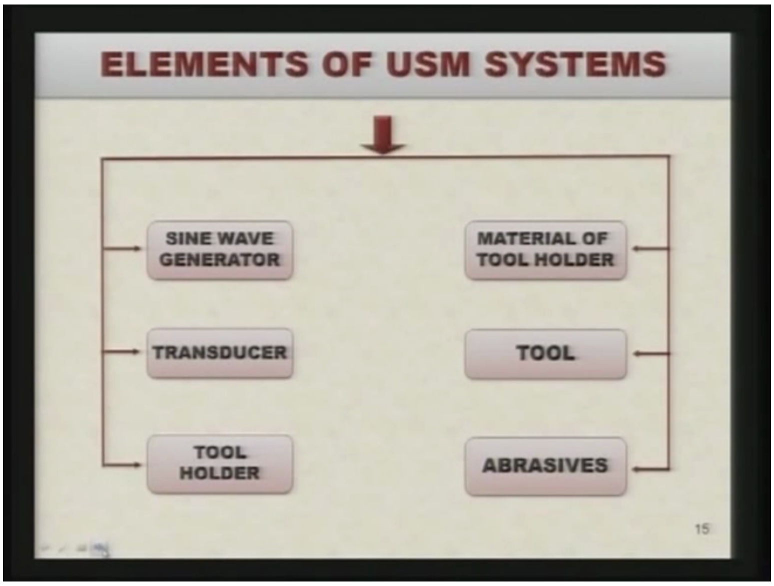

Now let us see in brief the various elements of ultrasonic machining, it includes sine wave generator, transducer, tool holder, material of tool holder, tool and the abrasive particles. Now in brief we will discuss all these elements.

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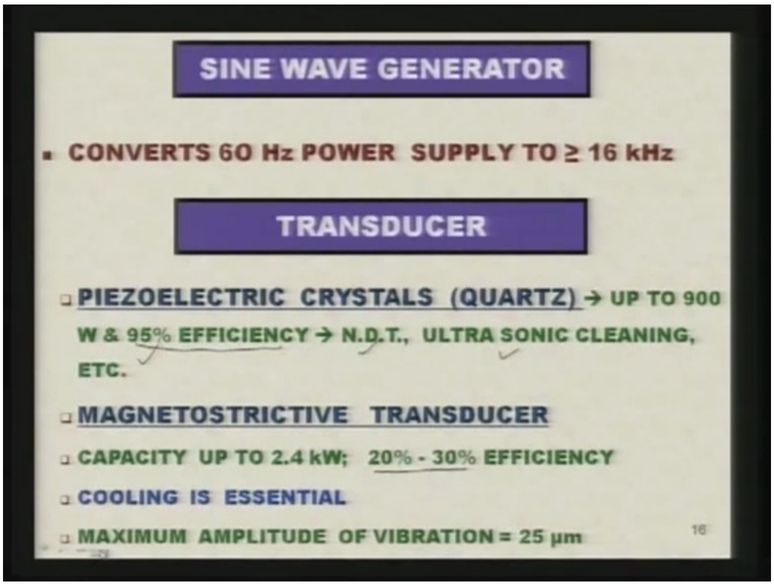

Sine wave generator normally converts 60 hertz power supply which we normally have in the labs as well as the houses into the frequency having more than or equal to 16 kilo hertz. This transducer there are piezoelectric crystal which can have the power upto 900 watts and the efficiency in this particular case is very high that is the 95 percent efficiency, piezoelectric crystals they are commonly used in non-destructive testing that is the NDT, they are also used for ultrasonic cleaning and some other application even in very low power ultrasonic machining process you can use the piezoelectric crystal for that purpose.

Magnetostrictive type transducer they have high capacity as compared to piezoelectric type of crystal, they can have as high as 2.4 kilo watt but their machining efficiency as you can see over here is just 20 to 30 percent while in case of piezoelectric type it is 95 percent. Now in case of magnetostrictive type transducer you always require cooling so it becomes essential and the maximum amplitude of vibration that you obtained in case of magnetostrictive type transducer is 25 micrometer.

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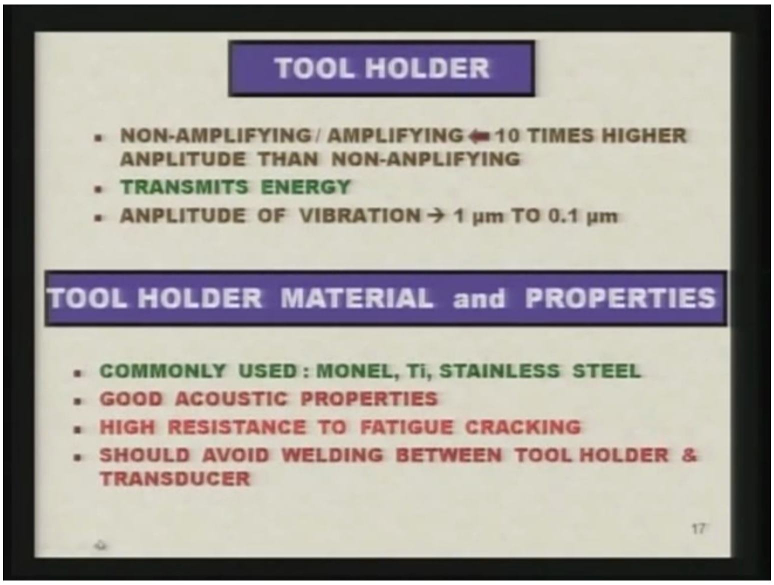

Tool holder there are two types of the tool holder one is known as non-amplifying tool holder another is amplifying tool holder. In case of amplifying tool holder you can get 10 times higher amplitude of vibration as compared to non-amplifying type of tool holder. It transmits energy, amplitude of vibrations can be as high as 1 micron meter to 0.1 micrometer. Tool holder material and its properties commonly used tool holder materials are monel, titanium, stainless steel. It should have good acoustics properties, it should have resistance to fatigue cracking and it should avoid welding between tool holder and transducer during function or during working.

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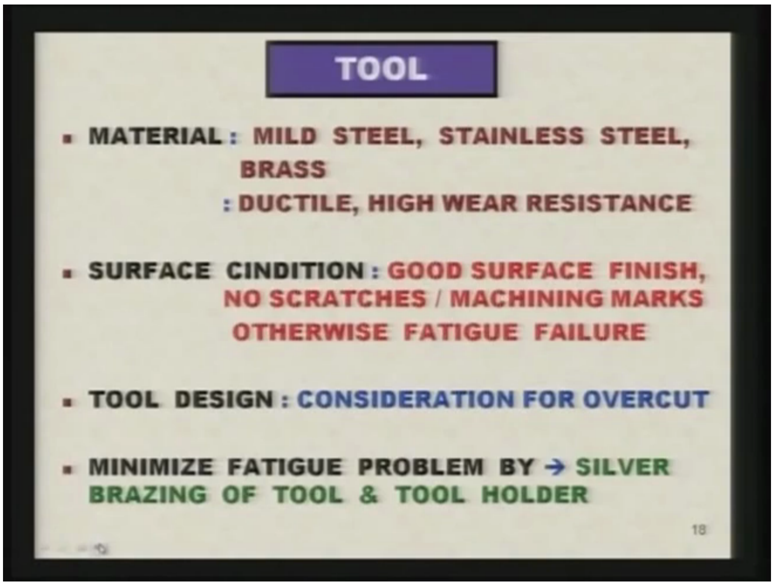

Tool materials normally used are mild steel, stainless steel, brass. The main properties of the tool material are ductile and it should have high wear resistance, surface conditions are concerned it should have good surface finish, there should be no scratches or machining marks on the tool otherwise they will be reflected on the work piece as well as fatigue failure of the tool will take place because tool is subjected to the very high frequency load changing so fatigue failure of the tool may take place if it has certain scratches or marks left over there or machining marks left over there.

Tool design consideration for over cut should be taken into consideration at the time of tool design so that you get the desired shape and size of the machine work piece. Minimum or minimized fatigue problem by silver brazing of tool and tool holder you can minimize the fatigue failure, fatigue problems if tool and tool holder are silver brazed.

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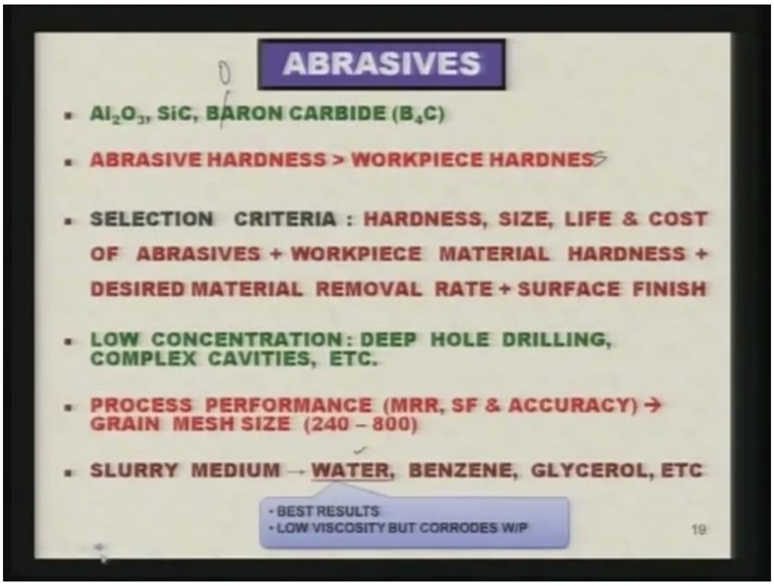

Abrasives, there are various kind of abrasives that are used in case of ultrasonic machining some of them are alumina AL2 O3, silicon carbide, boron carbide B4C, this is O not A, abrasive hardness should be definitely greater than the work piece hardness otherwise abrasive itself will fracture or break rather than removing the material from the work piece.

Selection criteria while selecting the abrasives following points should be taken care of hardness, size of the abrasive, life of the abrasive and cost of the abrasive, plus what is the work piece material hardness this is very important while selecting the abrasive work piece material hardness should be known and it should be taken into consideration also desired material removal rate and surface finish should be considered while selecting the abrasive particles.

Low concentration of the abrasive is recommended for deep hole drilling so that you know the abrasives and the liquid carrying the abrasives can penetrate inside specifically in case of complex cavities. Process performance is evaluated in terms of material removal rate, surface finish and accuracy obtained on the machine component and it is a function of grain size of the or grain mesh size of the abrasive particles, normally mesh size selected for USM ranges between 240 to 800 mesh size.

Slurry medium or the abrasive carrying medium normally selected is either water, benzene, glycerol, etc. especially you know low viscosity material or liquid like water is recommended so that it can reach to the fine details or complicated shapes easily if it is having a high viscosity then it may not be able to reach to those fine cavity or fine details on the machine component. Water gives best result because of low viscosity but the problem with water as carrier medium is that it corrodes the work piece so one has to take this also into consideration.

Let us briefly see the parametric analyses of ultrasonic machining process, there are various parameters which affect the performance of the ultrasonic machining process, I have just mentioned to you that there are various parameters with the help of which you can evaluate the performance of the ultrasonic machining, like material removal rate, surface finish and accuracy.

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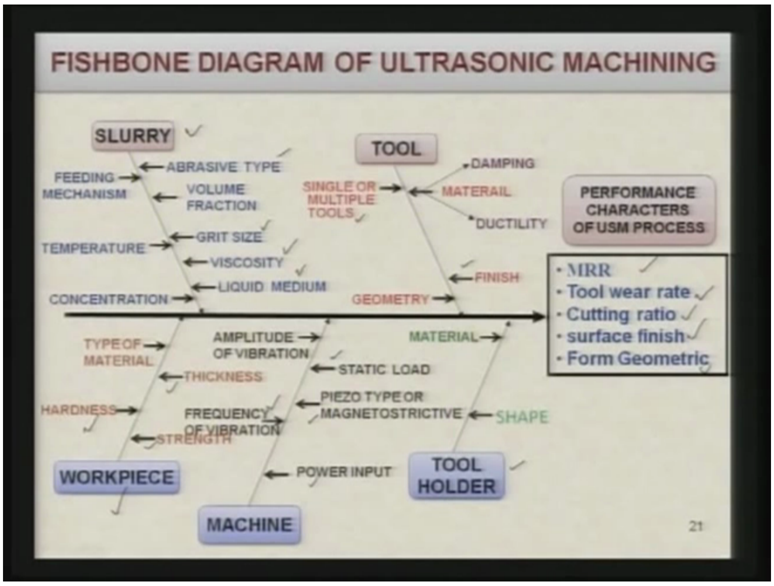

Now it gives you the fishbone diagram for ultrasonic machining which will show you various parameters which affects the ultrasonic machining process. Now in case of slurry the parameters which are affecting the performance of ultrasonic machining include feeding mechanism temperature of the slurry concentration of slurry or percent concentration of abrasive particles in the slurry, what are the abrasive type that you are using, volume fraction of the abrasive in the slurry, size of the grit or the abrasive particle as I have showed to you, it normally varies between 240 to 800 mesh size.

Viscosity of the slurry and this is very important and then what liquid medium you are using then another important parameter is tool whether you are using single tool for ultrasonic machining or multiple tools. Multiple tools means more than one tools can be used for machining at the same time I will show you one of the figures later on to you.

Now material of the tool what is the damping characteristics of the material of the tool, what is the ductility of the material of the tool this will also affect the performance of the ultrasonic machining and another very important is finishing of the tool because finishing of the tool as I have just mentioned sometimes that if scratches are there or micro cracks are left it will quickly fail because of ultrasonic vibration and load acting over it there and also if finish is not good this will be reflected to some extent on the finish of the machine component and also definitely the geometry of the tool will affect the geometry of the machine cavity that you are going to get on the work piece.

Work piece properties like type of material, what is the type of material of the work piece will also affect the selection of other parameters, what is the hardness of the work piece because according to the hardness of the work piece you have to select the type of the abrasive particles what is the thickness of the work piece and what is the strength of the work piece material these are the parameters which affect the performance of the ultrasonic machining processes.

Amplitude of vibration, let us see the machine definitely amplitude of vibration is going to affect the performance of the process, static load how much static load is acting that will decide with what velocity the particle is going to hit the work piece surface. Piezo type or magnetostrictive type of transducer what is the power input to the transducer or the force with which you are hitting the abrasive particles which will carry with it to the work piece and then the frequency of vibration.

Finally there is another parameter which affects the performance of the process that is a tool holder so shape of the tool holder and the material also will affect to some extent the performance of the process. Now the question arises in what way you are going to evaluate the performance of the ultrasonic machining process as I have mentioned earlier also you can evaluate the performance of the process in terms of material removal rate, tool wear rate, cutting ratio which is nothing but the ratio of the material removal rate and tool wear rate we will see later on. Surface finish and form geometry that you are obtaining.

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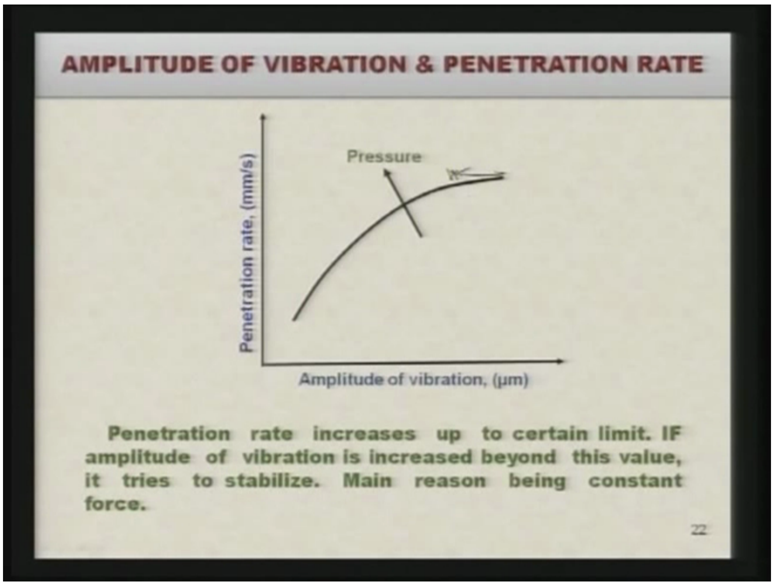

Amplitude of vibration and penetration rate, this shows the relationship between the amplitude vibration of the tool and the penetration rate, penetration of the abrasive particle into the work piece as you can see here that as amplitude of vibration is increasing the penetration rate is also increasing but beyond a certain value this is trying to become more or less stable so penetration rate increases upto certain limit if amplitude of vibration is increased beyond this value it tries to stabilize as you can see beyond this one it is trying to stabilize like this. Main reason being constant force, force remains the same so it tries to stabilize beyond a certain value of amplitude of vibration.

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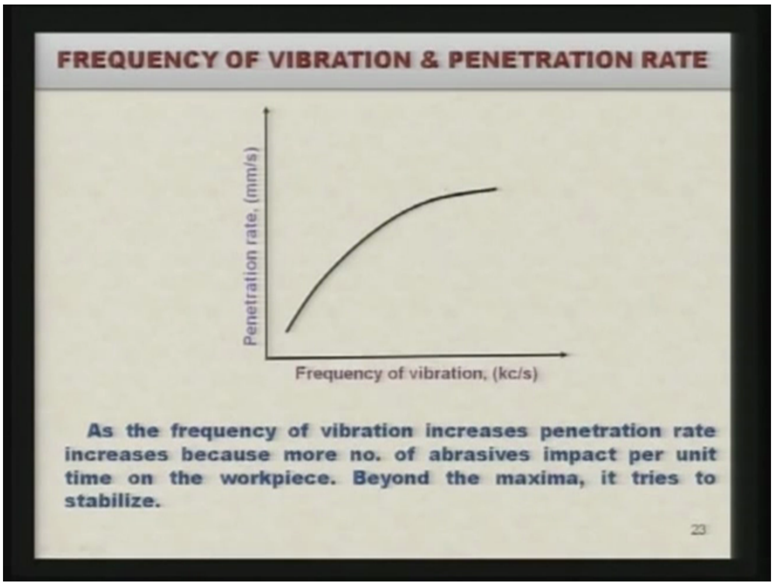

Frequency of vibration and penetration rate relationship is shown over here as we can see over here as the frequency of vibration is increasing, penetration rate is also increasing but it also stabilizes so as the frequency of vibration increases, penetration rate increases.

Because more number of abrasive impact per unit time on the work piece and because more number of abrasives are impacting per unit time on the work piece definitely penetration rate will increase and beyond the maxima it tries to stabilize as we can see over here it is trying to stabilize over there.

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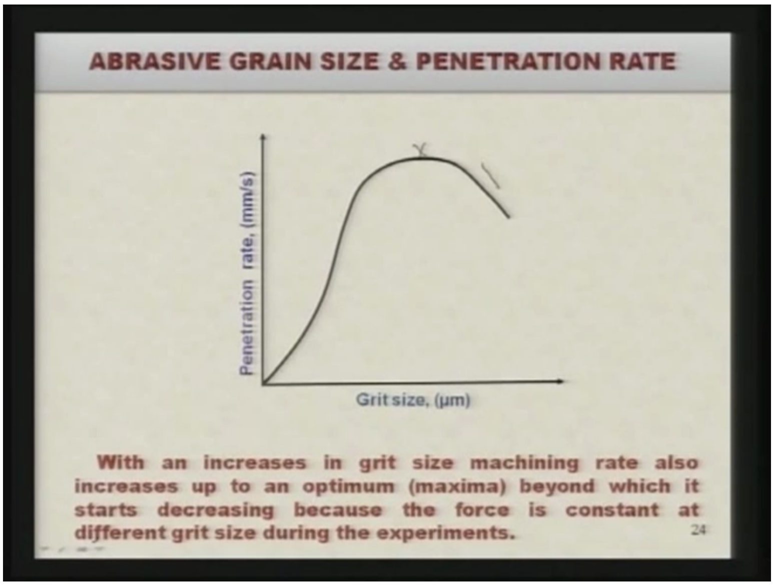

Now grit size, this is another important parameter which affects the penetration rate. As grit size is increasing there is optimum value of grit size where you get the maximum penetration rate in terms of millimeter per second and beyond that, with an increase in grit size machining rate also increases here machining rate means penetration rate also increases upto an optimum value where you are getting the maxima beyond which it starts decreasing because the force is constant at different grit size during the experiment so as the grit size increases the penetration rate start decreasing beyond the optimum value.

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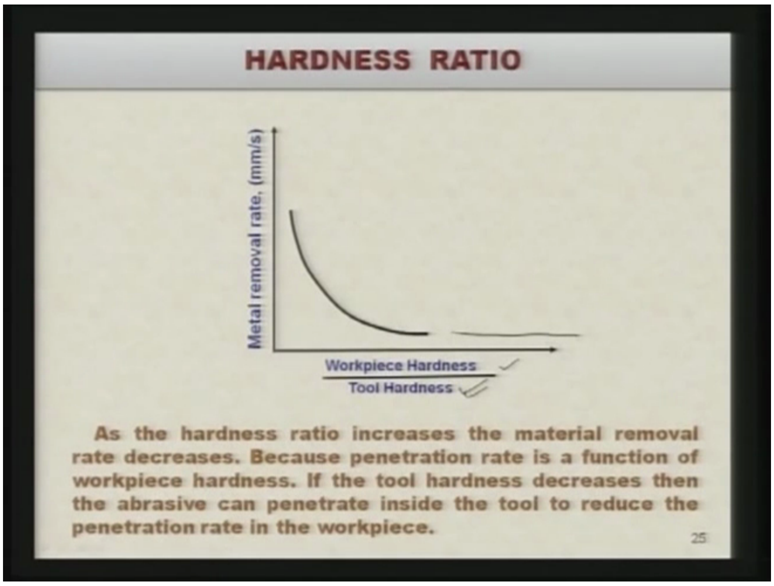

Hardness ratio, the ratio of the work piece hardness and the tool hardness also plays an important role as we already know that as the hardness of the work piece increases the material removal rate is going to go down so you can see here as the ratio of work piece hardness and tool hardness is increasing the material or metal removal rate in terms of millimeter per second is decreasing and it becomes more or less asymptotic beyond a certain value but if the hardness of the work piece is very very high then the material metal removal rate may decrease substantially or tending towards the zero.

So as the hardness ratio increases the material removal rate decreases because penetration rate is a function of work piece hardness if the tool hardness decreases then the abrasive can penetrate inside the tool to reduce the penetration rate in the work piece as you can see if this tool hardness decreases then this ratio becomes higher but the abrasive particles will penetrate inside the tool and this rate will decrease.

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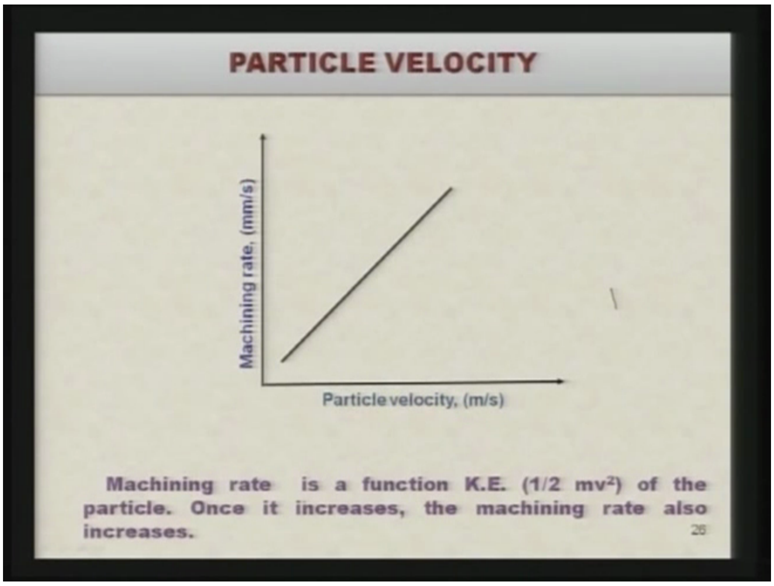

Particle velocity, as the particle velocity is increasing machining rate in terms of millimeter per second is also increasing and we already know that the penetration (())(26:36 to 26:44 - inaudible) so the machining rate is also going to increase. So machining rate is a function of kinetic energy of the particle once it increases the machining rate also increases.

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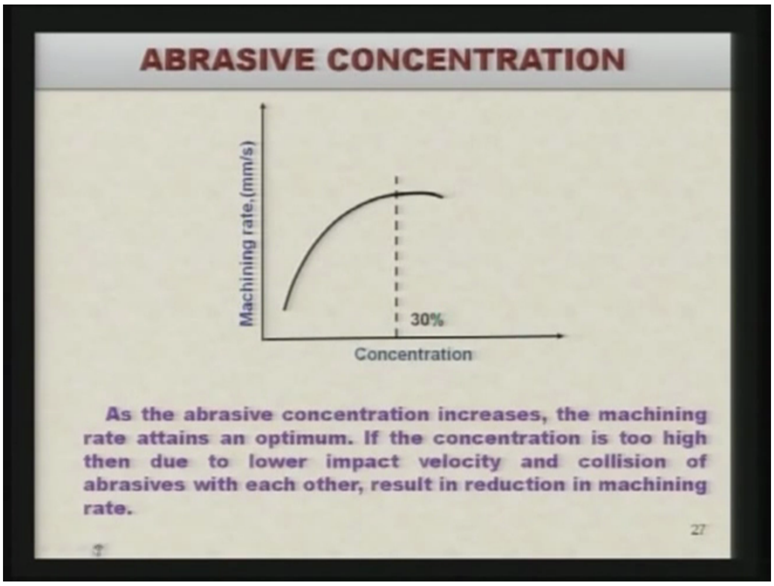

Abrasive concentration, concentration of abrasives as increases it keeps increasing the machining rate in terms of millimeter per second but beyond a certain concentration it starts decreasing, as the abrasive concentration increases the machining rate attains an optimum value if the concentration is too high then due to lower impact velocity and collision of abrasives with each other result in of reduction of machining rate.

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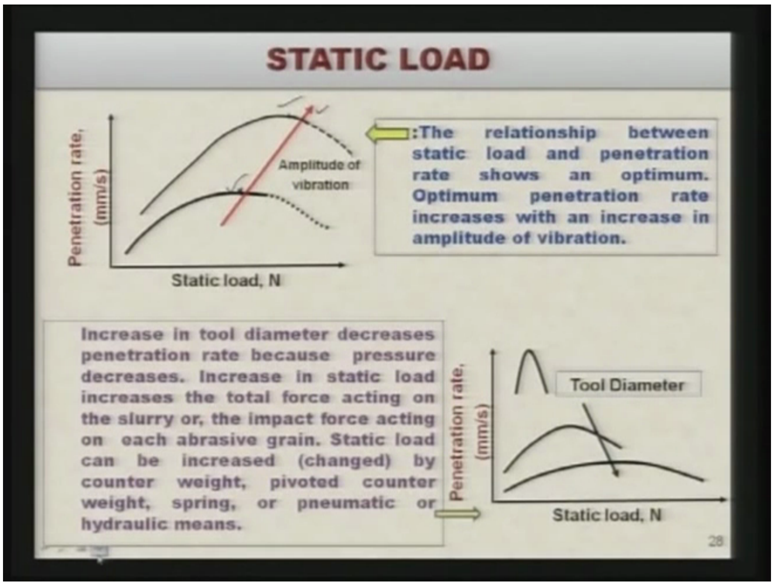

Now static load plays an important role as we can see over here, here as static load is increasing the penetration rate is increasing in terms of millimeter per second but beyond a certain value as you can see in both the cases it starts decreasing and as the amplitude of vibration is increasing the penetration rate is increasing.

The relationship between static load and penetration rate shows an optimum value, optimum penetration rate increases with an increase in amplitude of vibration this also can be seen that this optimum value is higher for higher amplitude of vibration.

Now if we vary the static load as well as the tool diameter this is the kind of the relationship we obtain that as the static load is increasing, penetration rate is increasing but beyond a certain value as we have seen in the earlier figure it start decreasing but as the tool diameter is increasing the penetration rate is decreasing, increase in tool diameter decreases penetration rate because pressure decreases.

Increase in static load increases the total force acting on the slurry or the impact force acting on each abrasive grain. Static load can be increased by counter weight pivoted counter weight, spring or pneumatic or hydraulic means as I will show you in the following slide.

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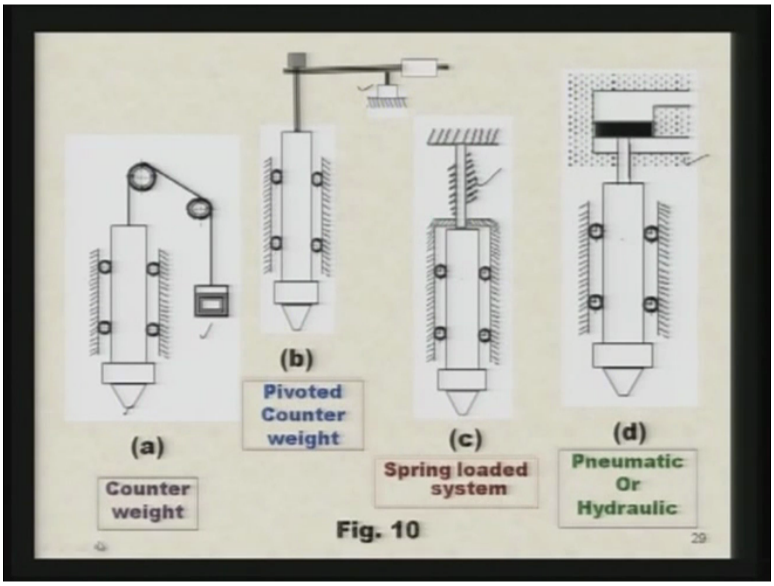

These are the four different ways with which you can increase the static load here you can see as counter weight is changed the load on the tool here will change you can increase the or change the counter weight with the help of pivot over here or you can change the static load with the help of the spring as can be seen over here or it can also be changed by pneumatic or hydraulic means over there and once the static load is changing it changes the performance of the process as we have seen in the earlier slide.

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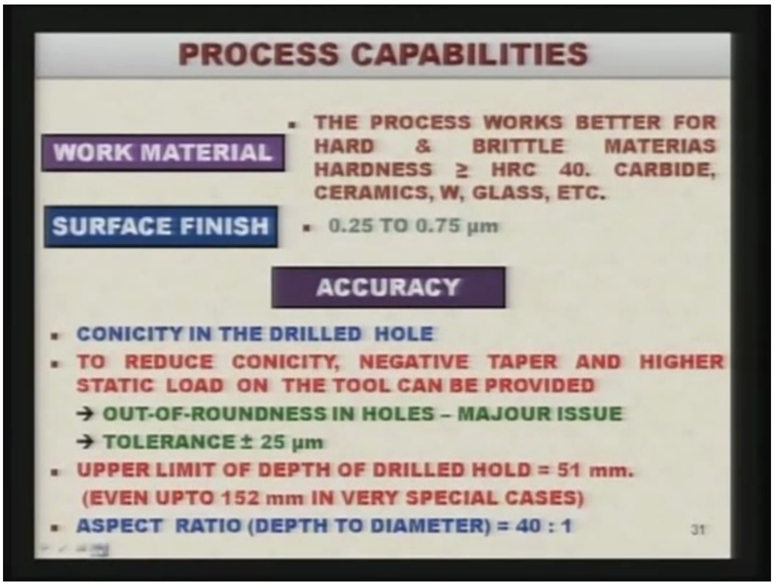

Now after studying the parametric effect of the different parameters on the performance of the process let us see what are the capabilities of this particular process, the process works better for hard and brittle materials, the hardness normally recommended of the work piece should be greater than or equal to Rockwell hardness on scale C as 40. The materials which can be comfortably machined include carbide, ceramics, tungsten, glass, etc. Surface finish that can be obtained by this particular process is in the range of 0.25 to 0.75 micrometer, however all this will depend upon the selection of the parameter of the ultrasonic machining process.

Accuracy, normally the holes that are obtained from ultrasonic machining process, they have the conicity or the shape is no or the size of the hole drilled is not straight rather they are tapered to reduce a conicity negative taper and higher static load on the tool can be provided. Out of roundness in holes is a major issue in case of ultrasonic machining and the tolerance is that is and can be achieved in this particular process are plus minus 25 micron.

Upper limit of depth of drilled hole is normally 51 millimeter but in very special cases people have gone upto 120 millimeter as the depth of penetration but you require their very special kind of the slurry pumping devices so that they can penetrate deeply inside. Aspect ratio that has been obtained is normally 40 ratio 1.

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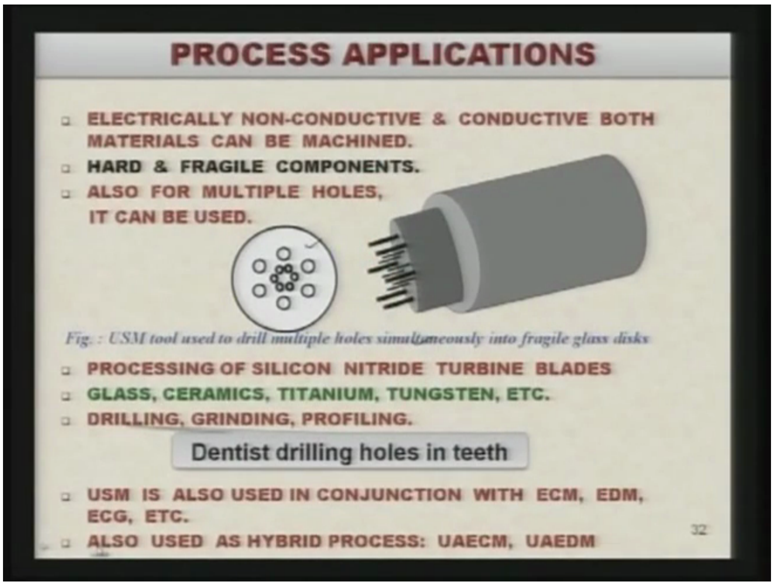

Process applications, it can be applied for electrically non-conductive material as well as conductive material because we are not making either tool or work piece as cathode or anode. It is very good for the hard and fragile component or the component made of hard and fragile material.

Now this is another very important capability of this processes, multiple holes or multiple cavities can be machined at the same time. It can be used as you can see here this is the tool which is having more than one tools and this is the shape that has been produced on the work piece and you can see there are 5, 6 bigger size hole and 6 smaller size holes and all these 12 holes haves been produced at the same time using this particular tool.

So it is ultrasonic machining tool use to drill multiple holes simultaneously into fragile glass disc. Processing of silicon nitrite turbine blade is also possible by this particular process. Glass, ceramics, titanium, tungsten, etc. are the material which can be machined by this process. Drilling, grinding, profiling and other kind of operations can be performed.

Now this is very common application that dentist while drilling the hole in the teeth they use this particular process and this is very interesting. USM is also used in conjunction with electrochemical machining, electric discharge machining, electrochemical grinding, etc.

So this ultrasonic vibration is also used with electrochemical machining that time we call it as ultrasonic assisted electrochemical machining, it is used with EDM we call it ultrasonic assisted electric discharge machining with ECG, ultrasonic assisted electrochemical grinding. What really we are doing in case of ultrasonic assisting that the tool is vibrated by ultrasonic frequency or at ultrasonic frequency once the tool is vibrating say in case of ECM then the removal of the electrolyte in case of electrochemical machining becomes much more effective than without ultrasonic frequency, ultrasonic vibration. So it is found that in case of ECN, EDM the ultrasonic has improved the performance of the process to large extent also it is used as hybrid process like ultrasonic assisted ECM, ultrasonic assisted EDM.

Material removal model, various researchers have evolved the theoretical or empirical models for the removal of the material or to (())(34:58 to 35:03 - inaudible) ultrasonic machining, the most, the first one that was proposed by M. C. Shaw Milton C. Shaw, I will discuss that particular model here.

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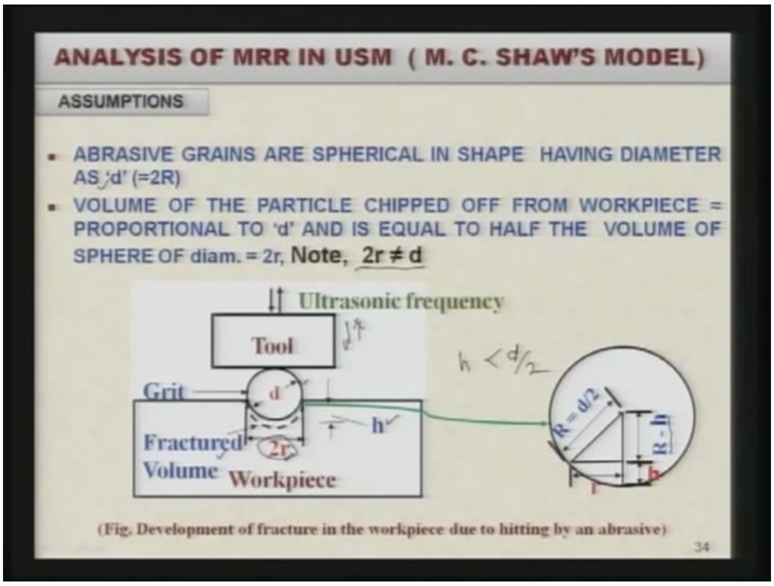

Now the model proposed by M. C. Shaw there were certain assumptions which were made in this particular model let us see here it is assumed that all abrasive particles are spherical in shape and they have the diameter as small D which is equal to the 2 R, where R is a capital R, volume of the particle chipped off from the work piece is approximately proportional to the, is equal to half the volume of sphere of diameter 2 R where note it that 2 R is not equal to D, this will be very clear from this particular diagram.

This is the tool which is vibrating at ultrasonic frequency, now here is the abrasive particle which is hitting the work piece and removing the material now note it that this has penetrated into the work piece but that penetration depth is less than the half of the diameter and this is designated as H, small H so small H is smaller than D by 2.

Now whatever is this width that is taken as spherical or 2 R and it is assumed that if we draw a sphere of this diameter equal to 2 R then that will give you half of that the volume of half of that will give you the amount of material removed and that is shown over here as the fractured volume.

If you see the enlarged view of this it is very clear over here R is capital R is the radius of the spherical particle and H is the penetration and small R is the radius of the semi sphere created by the abrasive particle into the work piece surface. So if you see this one it becomes R minus H and this is the H and this is the center of the radius of the sphere of the abrasive particle.

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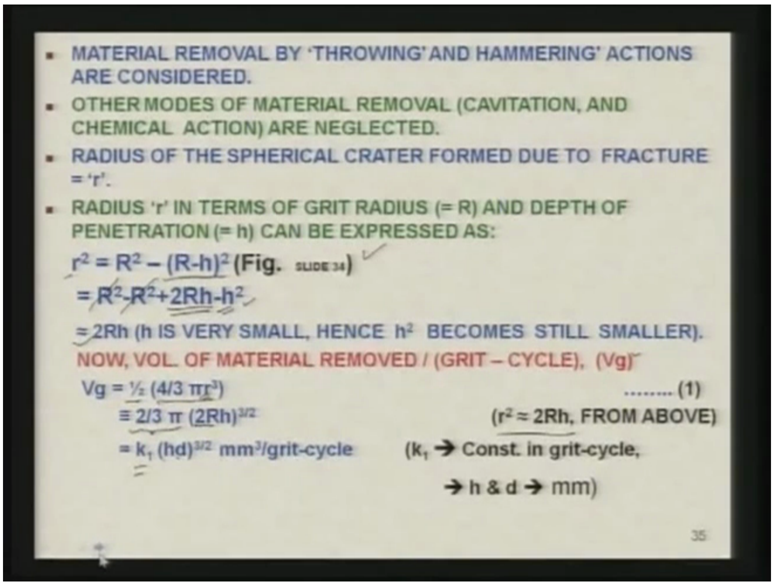

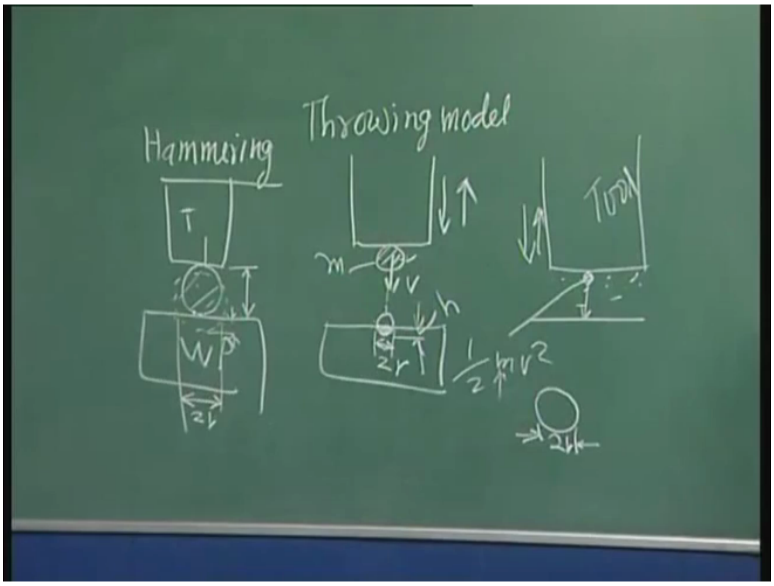

So material removal by throwing mechanism and hammering action, M. C. Shaw proposed two models for the evaluation of the material removal in the ultrasonic machining one of the way is he assumed that the tool, I will show you here he assumed two ways of the removal of the material and analyzed and evaluated it in the first model he assumed that when the tool is vibrated it is hitting the abrasive particle and this abrasive particle is moving towards the work piece at a velocity V and it has got the mass M this is model he called as throwing model.

Now in this as I just mentioned throwing model, the tool is hitting the abrasive particle it is moving towards the work piece and hitting it there and creating a cavity in the work piece. In the second model what he is assuming that here is work piece here is the tool and the abrasive particle is large size enough and the gap between the tool and the work piece at a particular point is smaller than the size of the abrasive particle.

So once this gap is smaller than the size of the abrasive particle, it will hit the abrasive particle and the abrasive particle will move down and it will penetrate inside the work piece, so here it is nothing but hammering the abrasive particle by the tool that is why he called it as a hammering. So using these two models he calculated the material removal rate by both the models. Other models of material removal, other methods or modes of material removal like cavitation and chemical action are neglected he did not consider in this particular case.

Radius of spherical crater formed due to fracture is equal to R which I showed you in the earlier transparency also as we can see here in throwing model if this is the one which is being crater being formed then this is equal to 2 R or when it is crating over here in throwing one this is taken as 2 R and this 2 R in this case and this case both are different one as we will see there in the slides.

So R square is equal to capital R squared minus R minus H whole square as we have in the previous slide so if you simplify this particular equation then you get R square, you open this term then you get R square minus R square plus 2 RH minus H square.

Now H that is the depth of penetration here, this is the depth of penetration which is very small already, this H is very small and if you square it then it will becomes further small so that is why this can be neglected so it becomes this this cancel so you are left with 2 RH so R square becomes approximately equal to 2 RH, H is very small hence H square becomes still smaller.

Now volume of material remove per grit per cycle that is indicated or represented by VG, G is the suffix over there so VG becomes half 4 by 3 pi R cube this 4 by 3 pi R cube gives the volume of the sphere, now this has been taken up as a sphere and this sphere consists the diameter equal to 2 R so half of that is taken as the volume of that particular sphere so if you simplify it becomes 2 by 3 pi.

Now here small R square value can be taken from here and we substitute that value R square is equal to 2 capital R H, then you get VG equal to 2 by 3 pi multiplied by 2 RH whole raise to power 3 by 2, now we know already R square is equal to 2 capital R H.

So this term including this two you can make it as a constant K1 and then this becomes H 2 R becomes D small D and H so H into D raise to power 3 by 2 so this becomes K1 H D raise to power 3 by 2 cubic millimeter per grit per cycle and K1 is a constant. H and D are to be given in terms of millimeter where H is the depth of penetration as I have shown over there.

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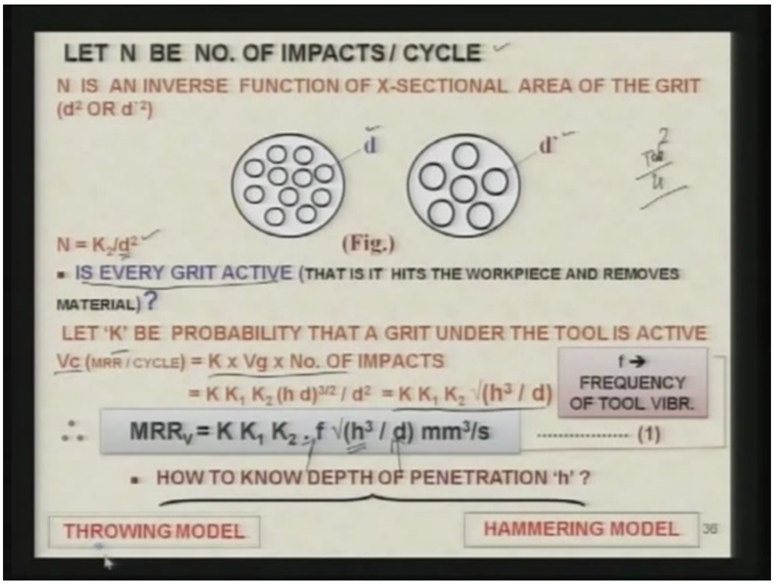

Now let N be number of impacts per cycle now when we are considering this tool there are thousands of abrasive particles which are under the tool and when this tool is vibrating in each cycle or in one vibration it is considered that N number of abrasive particles are hitting the work piece surface.

N, capital N is an inverse function of cross sectional area of the grit, now the question arises how many abrasive particles are going to hit because there are thousands of abrasive particles in the slurry so how many are going to hit this will depend upon what is the size of the abrasive particles if the size of abrasive particle is small then larger number of abrasive particles will be under the tool and they will be hitting the work piece if size of the abrasive particle is large then lesser number of the abrasive particles will be under the tool and then lesser number of the abrasive particles will be hitting the work piece.

So really it will depend upon the diameter of the abrasive particle that will decide how many number of abrasive particles are impacting the work piece surface. So N is the number of particles hitting the work piece surface is a inverse function of D square because D, pi D square by 4 gives you the cross-sectional area of the spherical ball that is why it becomes N is equal to K2 divided by D square. Is every grit active, now this is a very important question that whichever particle is under the tool whether it is active or not, let me explain it clearly.

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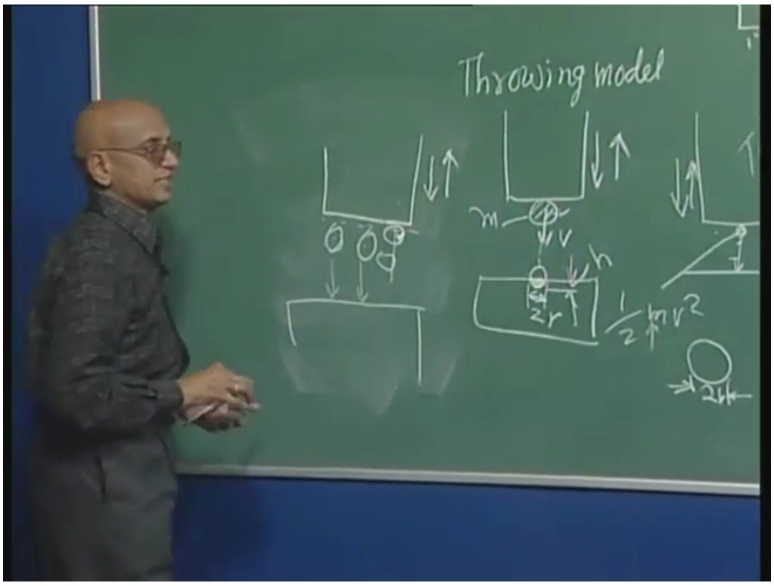

This is the tool, there are so many abrasive particles over here now when it is vibrating some of the tools say this is hit by the tool this particular abrasive particle quite possible that other particle like this or this may not get the impact from the tool and they may not be able to hit the work piece surface so the question is, is every grit or every abrasive particle active? That is it hits the work piece and removes the material, that is question mark.

So what it is assumed that all abrasive particles which are under the tool are not active, that simply means that each abrasive particle is not hitting the work piece surface so what they do they assume that the probability of an abrasive particle of hitting the work piece surface let it be K.

Then VC is the volume of material removed per cycle, suffix C indicates the cycle becomes equal to, K is the constant and as we have seen above. VG is the volume of material removed per cycle and number of impacts, so it becomes K, K1, K2 H D raise to power 3 by 2 divided by D square so K, K1, K2 under square or under root H cube by D this is what you get over here. Now in this particular formula we see very clearly K, K1, K2 they can be evaluated from the experimental results.

F is the frequency that is known to us well in advanced, diameter D of the abrasive particle or grit size we know already from the mesh size, now H is not known and this H will be different for throwing two types of model that I mentioned throwing model and hammering model.

So what we have to do now is that we have to find the value of H for throwing model and hammering model separately and F is the frequency of the tool vibration we already know so question is how to know depth of penetration H. So he proposed two models hammering models and throwing models we will try to find out the value of H from throwing model and hammering model.

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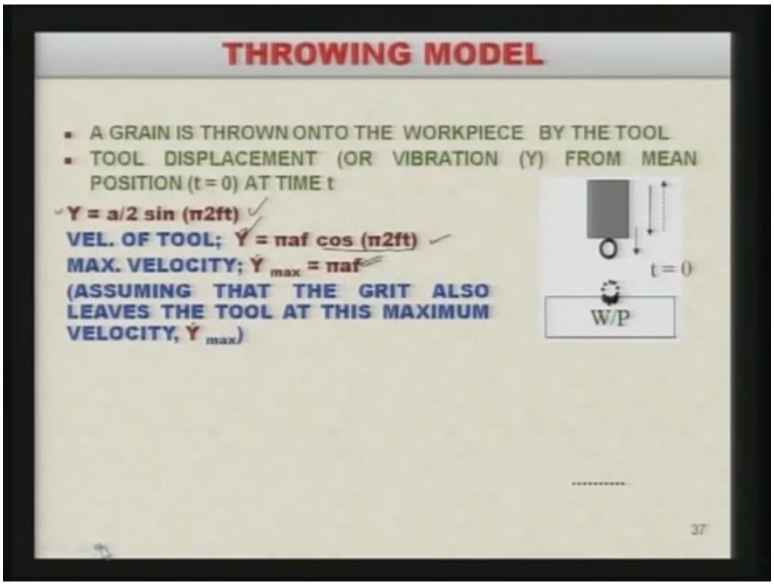

Throwing model, a grain is thrown onto the work piece surface by the tool, tool displacement or vibration Y from mean position at time T can be given by Y is equal to A by 2 sin 2 pi FT now where F is the frequency, T is the time, A by 2 is the amplitude of vibration. Now this velocity of the tool which is vibrating and its vibration is controlled by this equation we can find out by differentiating this equation with respect to time T and that is given over here by dot that is the velocity of the tool becomes equal to pi A F cos 2 pi T.

Now we want to find out what is going to be the maximum velocity of the tool during its vibration so the maximum velocity will be obtained when the value of the cos 2 pi FT is maximum and the maximum value of cos function can be 1, so you get Y dot maximum as pi AF assuming that the grit also leaves the tool at this maximum velocity.

Now this is very important assumption over here, it is assumed that with whatever velocity the tool is moving downwards with the same velocity abrasive particle is also moving towards the work piece however it is not very true it is not realistic because when abrasive particle moves throw the slurry it will hit other abrasive particles it will pass through the carrying medium that is water in this particular case so frictional losses will be there so actual velocity of the abrasive particles with which it is hitting the work piece will be different than by this maximum velocity.

However for simplicity of analysis we are assuming that maximum velocity of the abrasive particle hitting the work piece surface is pi AF itself.