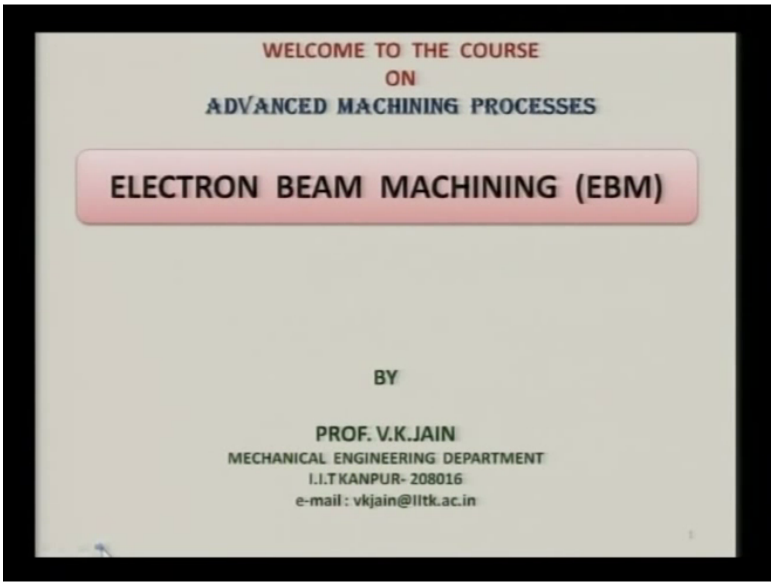
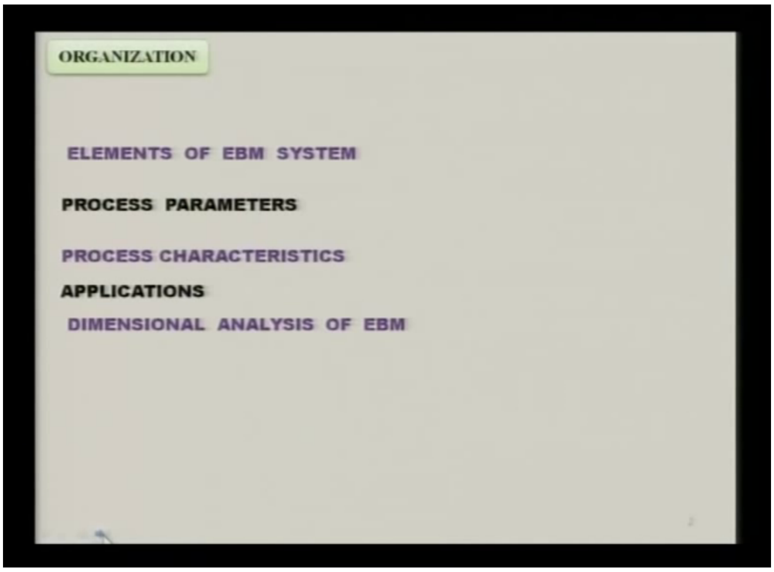
Theory of Computation
Prof. Somenath Biswas
Department of Computer Science and Engineering
Indian Institute of Technology Kanpur
Lecture 28
NFA's with Epsilon transition

(Refer Slide Time: 0:29)

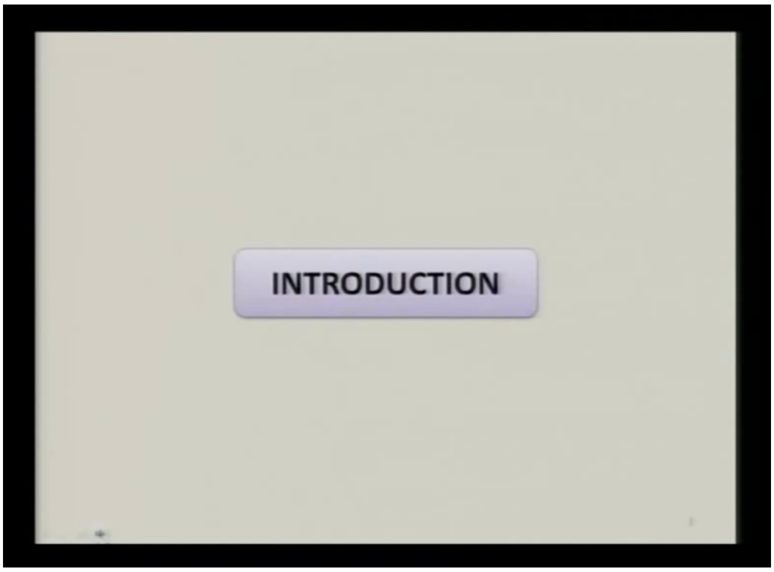


(Refer Slide Time: 1:00)

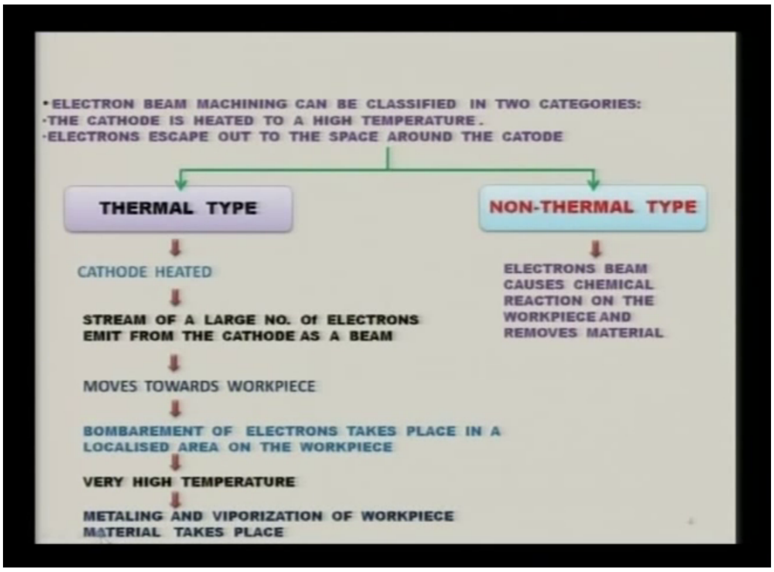


Welcome to the course on advanced machining processes. Today I am going to do this electron being machined EBM process. The organization of this talk is as follows introduction, elements of electron beam machining system, process parameters, process characteristics, applications of electron beam machining process and then I will lastly discuss dimensional analyses of electron beam machining process.

(Refer Slide Time: 1:03)



(Refer Slide Time: 4:37)



Introduction electron beam machining. Electron beam machining can be classified in 2 categories the first on the cathode is heated to a high-temperature and electrons escape out to the space around the cathode because cathode it is very at a very high temperature, so those electrons are emitted. Now let us see how this first one which I have just mentioned is known as thermal type of electron beam machining process.

The second category non-thermal type of electron beam machining process, in this talk I am going to discuss only thermal type of electron beam machining process. We are highly concentrated large amount of heat is generated as a result of that has been will see the work piece material is melted evaporated. As mentioned earlier that cathode is heated to a very high temperature as a result of that stream of a large number of electrons amidst from the cathode as a electron beam.

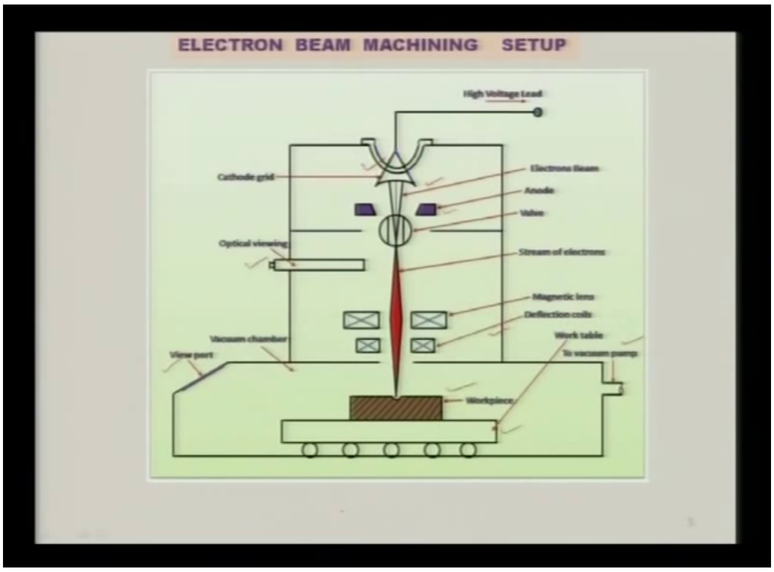
Now this beam moves towards the work piece and once it moves towards the work piece and heats the work piece then bombardment of electrons takes place in a localised area on the work piece. The area on which this bombardment of electrons is taking place is very small in size and this kind energy of the electrons which are heating this work piece converts into the heat and this heat is so highly intends that it can melt and vaporise any material and it leads to a very high temperature.

One thing is to be noted here that electrons are considered as more or less mass less but they still has some mass and the velocity with these electrons are heating the work piece is very close to the velocity of light. So their kinetic energy is substantial to increase the temperature of that small localised area of the work piece to the extent that the work piece material melts and in most of the cases it vaporizes and the material is removed.

In the case of non-thermal type of, okay. So finally melting and vaporization of work piece material is a mechanism of material involved, it is not metaling it is melting. Now in case of non-thermal type of electron beam machining process, electron beam causes chemical reaction on the work piece surface and it removes material once it heats the work piece material.

So here the mechanism of material removal is very different from the thermal type of electron beam machining process. I am going to concentrate as I mentioned earlier only to the first type that is the thermal type of electron beam machining process.

(Refer Slide Time: 4:45)

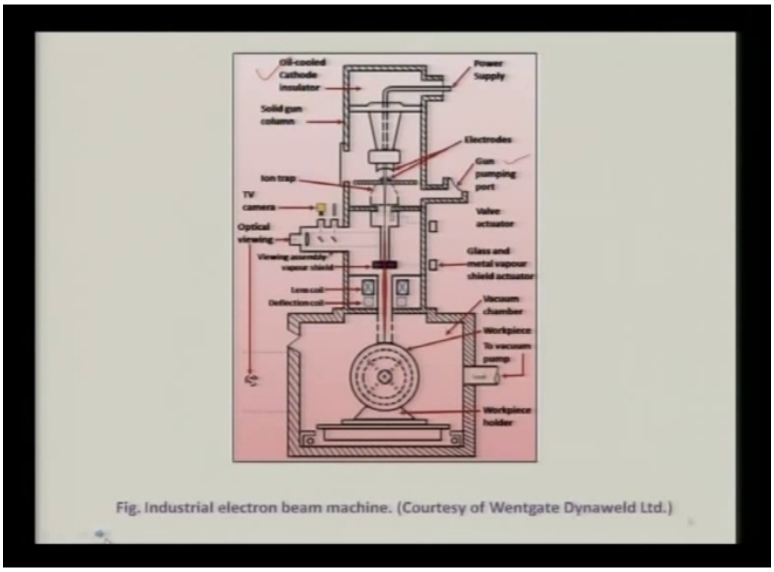


Here are the some salient features of the electron beam machining setup as you can see that here is a cathode grit, as I mentioned that this is heated to a very high temperature and it emits a stream of the electrons which is written here as electron beam then it is attracted towards the anode and that is shown here by this particular shape and then there is a valve through which the beam of the electron passes and here is the optical beam derive through which you can see the beam of the electrons passing through.

And then there is a magnetic length and with the help of magnetic length you can concentrate this particular beam on the work piece and then you want to deflect the beam minor distance left side or right side then there is a deflection coil which helps in deflecting to a very small distance, so that you focus the beam at the desired point and this can be done with the help of deflection coil and there is a work piece table or the work piece is mounted and you can move this work piece table with the help of CNC machine to small distances in X as well as Y direction.

Now whole of this operation of electron beam machining and production of electron beam is done in the vacuum very high vacuum is there and that vacuum can be achieved with the help of the vacuum pump that you can see on the right side and if you want to see the work piece and location of the work piece whether it’s very right with reference to the beam then there is another viewport with the help of which you can see the location of the beam on the work piece and the removal of the material from the work piece. So this is a schematic diagram of electron beam machining setup.

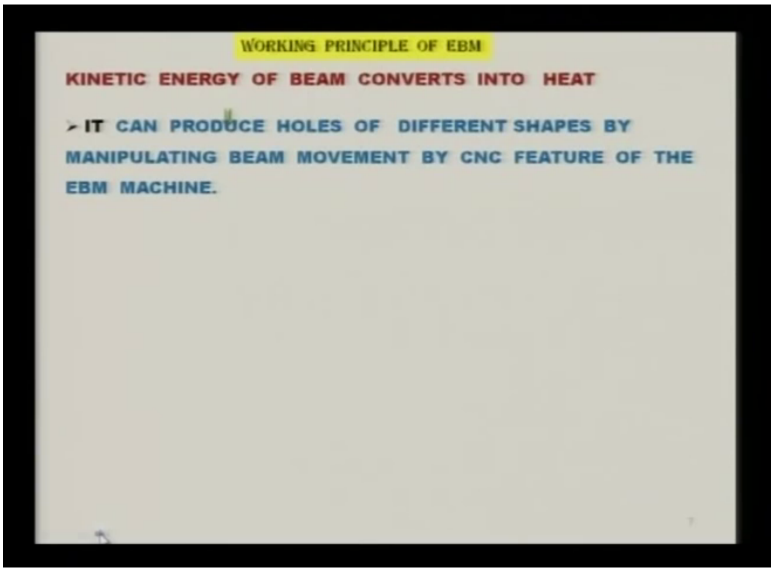
(Refer Slide Time: 6:58)



Some more details you can see in this particular figure, it is just a detailed drawing of the same thing which I have just mentioned to you through the schematic diagram. Now another point which you can see here is why cold cathode insulator? Because temperature is very high, so to safeguard the various devices you have the cooling of the system, over there were cathode grade is there.

So oil cooling can be used for that particular purpose and for creating vacuum again you have one more pump than pumping port where you can create the vacuum then if you see further then rest of the things I have already shown to you through the schematic diagram in the last slide. So this is the industrial electron beam machine which is there.

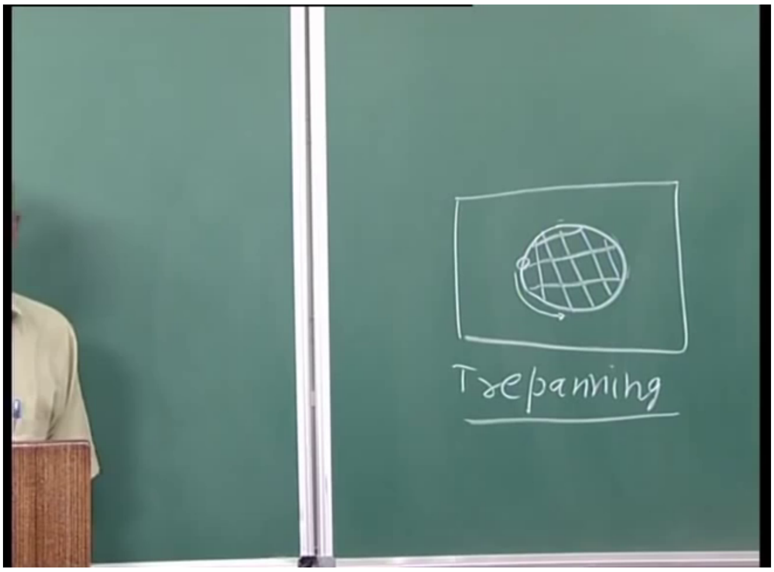
(Refer Slide Time: 8:00)



Now very important is to understand the working of electron beam machining, in brief I have already mentioned to you that the kinetic energy of the electron beam gets converted into the heat in the intensity of the heat is very high which results in the melting and vaporization of the work piece. It can produce holes of different shapes by manipulating the movement by CNC feature of the electron beam machining machine.

Now here you can see you can manipulate the beam to a small distance in XY direction to create different kind of the holes. However you can also move the work piece table, if there is a large hole to be made.

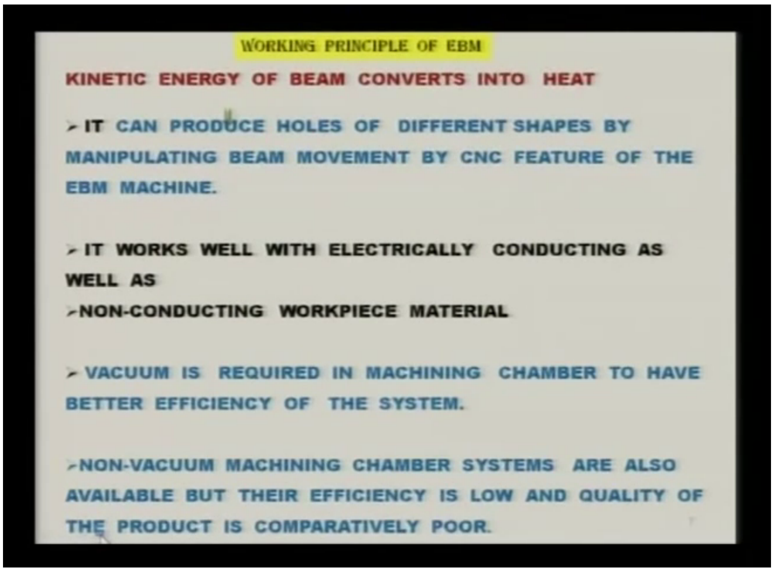
(Refer Slide Time: 8:58)



Just like if you want to do the trepanning operation where large movement of the work piece is important then what you can do as I show you here that suppose this is a work piece and you want to make a large size hole which may be of a few millimetre that then what you can do that you move the table such that the beam is always heating on the periphery from where you want to remove the whole of the material, this is the whole of the material that is to be removed.

So what you can do? Instead of cutting or melting and vaporising whole of this material you can perform the trepanning operation. So when you perform the trepanning operation, this shredded portion will come out as solid piece but normal electron beam machining is not used for a very thick work pieces. So for even thin work pieces you can use this trepanning operation for making the larger sized holes rather than melting and vaporising whole of the work piece material. So this way CNC feature is very useful for creating different kind of the features on the work piece surface.

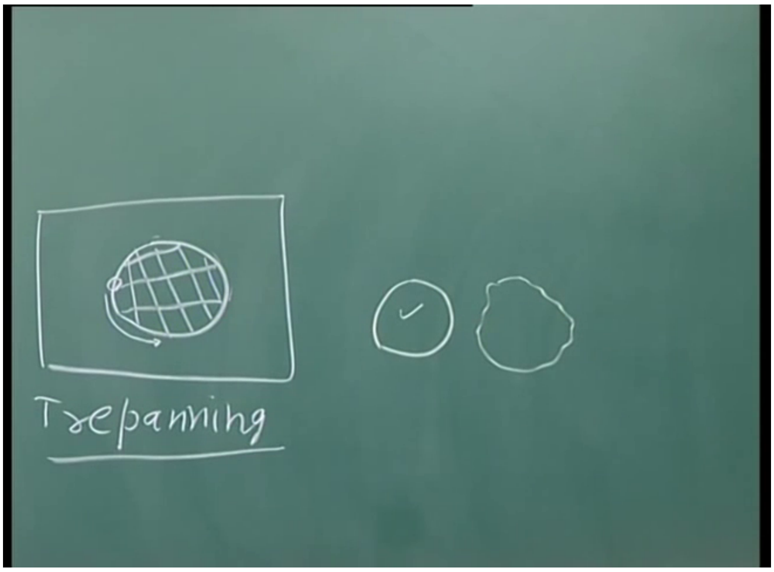
(Refer Slide Time: 10:07)



It works well with electrically conducting as well as electrically conducting material. As we have already seen that electric discharge machining, electrochemical machining these are the processes which are normally used for electrically not normally are always used for electrically conducting materials but this particular process can be used for both electrically conducting as well as electrically non-conducting.

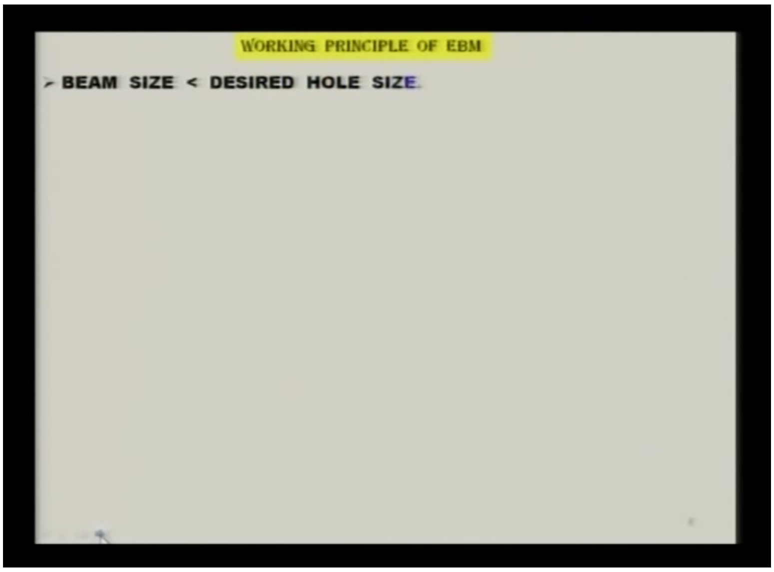
Now vacuum is required in machining chamber to have better efficiency of the system. Most of the EBM systems are vacuum operated means they are operated under the vacuum. However certain systems have been built which can operate on a non-vacuum machining chamber such systems are also available but their efficiency is low, in quality of the product is comparatively poor where what happens in non-vacuum machining system that these electron beam when they hit the air molecules in the chamber they lose their energy to certain extent and you don't get the hole you know of high accuracy just like the one in case of you can get like this.

(Refer Slide Time: 11:35)

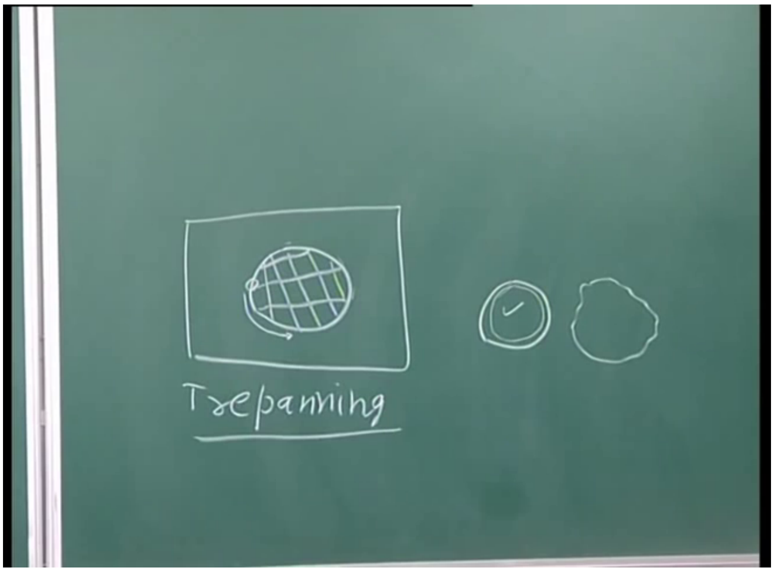


In another case this may not be that accurate as we can obtain in case of vacuum an electron beam machining chamber. So although it is expensive but for better accuracy or higher productivity vacuum machining chamber are more preferred.

(Refer Slide Time: 12:00)



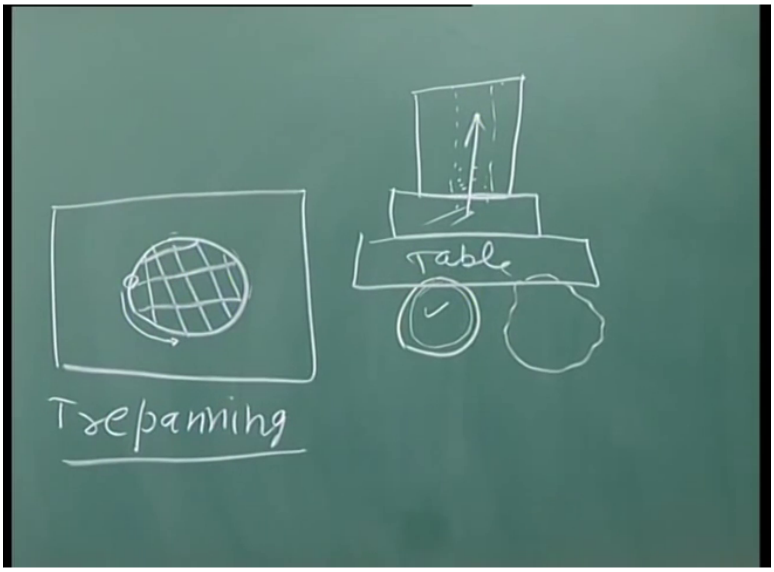
(Refer Slide Time: 12:11)



Now beam size is very important and it should be smaller than the size of the whole, as you can see here suppose this is the size of the hole you want then definitely size of the beam should be slightly smaller than this because the total material that is being removed is slightly larger than the size of the beam.

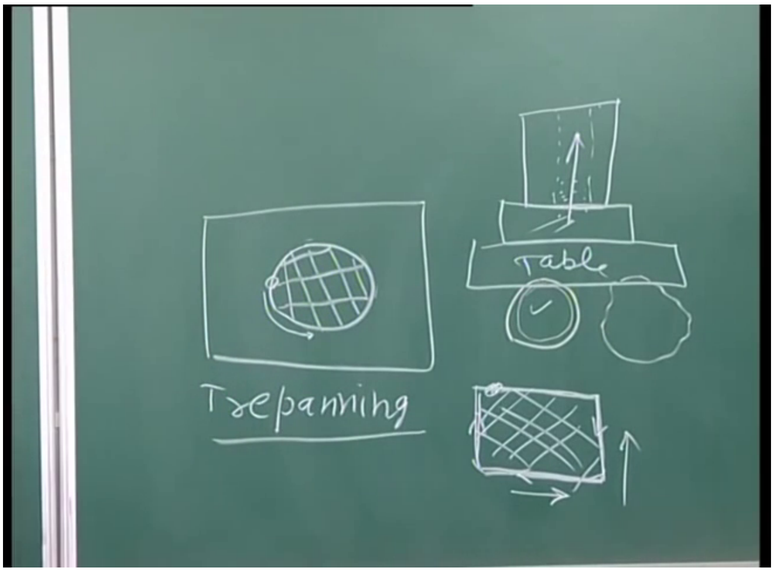
Now injection of molten work piece material is very important and vaporized backing material comes out at high pressure it expels molten material this is important to understand because I have mention that it is not used for high depth and most of the material is removed by melting and vaporization both. So now the question arises when it is not through how this molten material or the vaporized material is going to come out?

(Refer Slide Time: 13:04)



So what happens really is that suppose this is the work piece you're drilling the hole and here is the support. Now when hole has been made then whatever is the molten material or vaporized material that will remain at the bottom, so for this to get out what happens this table not really table but supporting material, this is the table, so what happens this material vaporizes and throws out the molten material and vaporized material out of the hole. For this particular purpose, this as you can see here backing material is very important and it is used so that molten and vaporized material comes out of the hole and you can take out this work piece and you will get through clean hole.

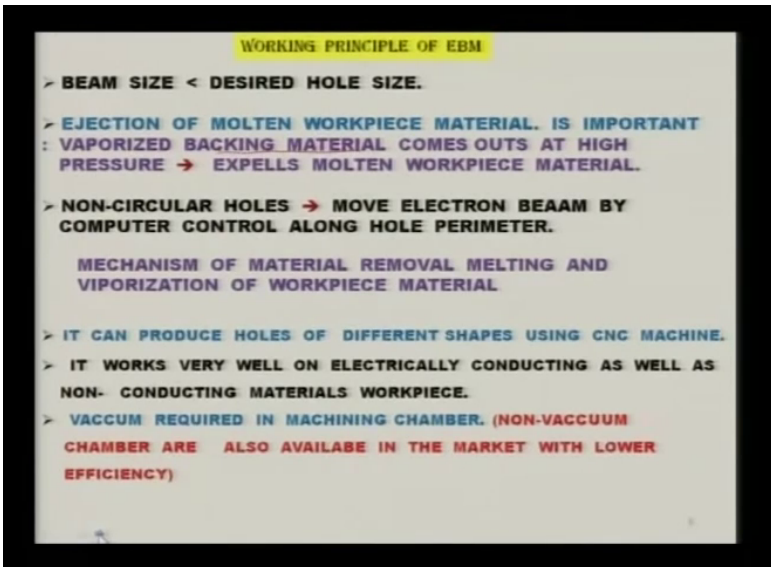
(Refer Slide Time: 14:15)



Noncircular holes can be made with the help of electron beam machining system. What one can do? One can move electron beam by computer-controlled machines along the hole parameter, suppose instead of this circular hole you want rectangular hole. So 2ways are there as I have mentioned earlier that you can move the electron beam along this periphery than the material will be cut and you will get the noncircular hole.

Or you can move the tail in X direction and Y direction in such a way that the beam is always along the periphery that is to be cut. So it can cut this, now the 2 things are there if the size of this is much larger than the size of the beam then definitely you have to do the trepanning operation rather than the melting and vaporization of hole of this material otherwise it is going to be very inefficient and time-consuming unnecessarily. So that point of view the CNC feature helps a lot in getting the desired feature on the work piece material.

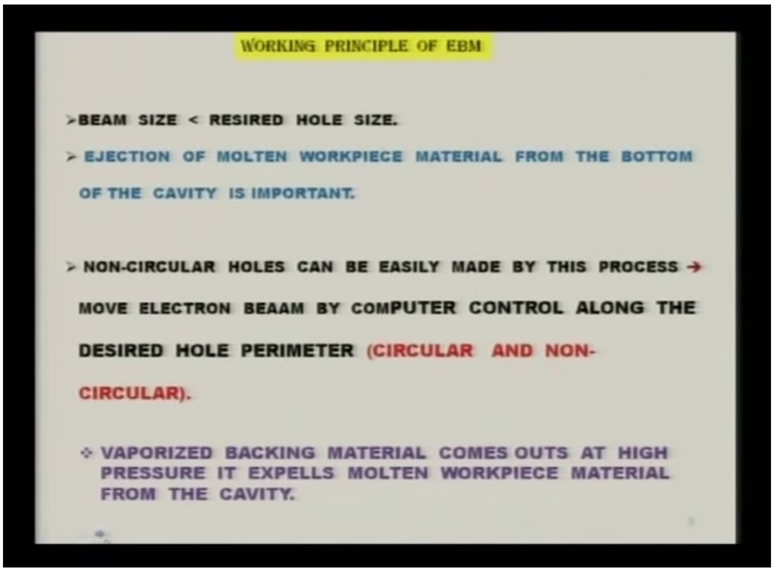
(Refer Slide Time: 15:24)



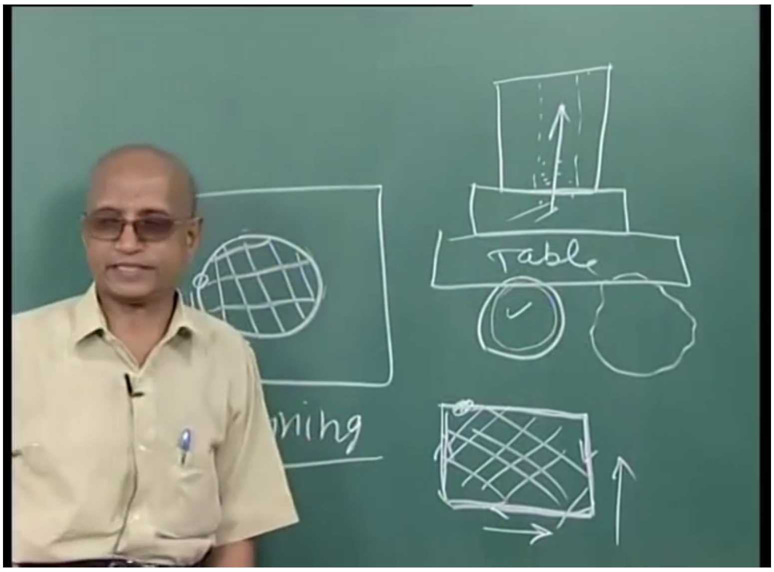
The mechanism material removal definitely melting and vaporization of the work piece material as I have already mentioned. EBM can produce holes of different shapes using CNC machine as I have explained over there. It works very well on electrically conducting as well as electrically non-conducting material which I have mentioned in the earlier slide but there are certain constraint as well as work piece material is concerned which I will show you later on.

Vacuum required in machining chamber. Non-vacuum chambers are also available in the market with lower efficiency. This is to be noted that the efficiency and productivity of non-vacuum machining chambers is lower compared to vacuum machining chambers. However the vacuum machining chambers are more expensive.

(Refer Slide Time: 16:20)

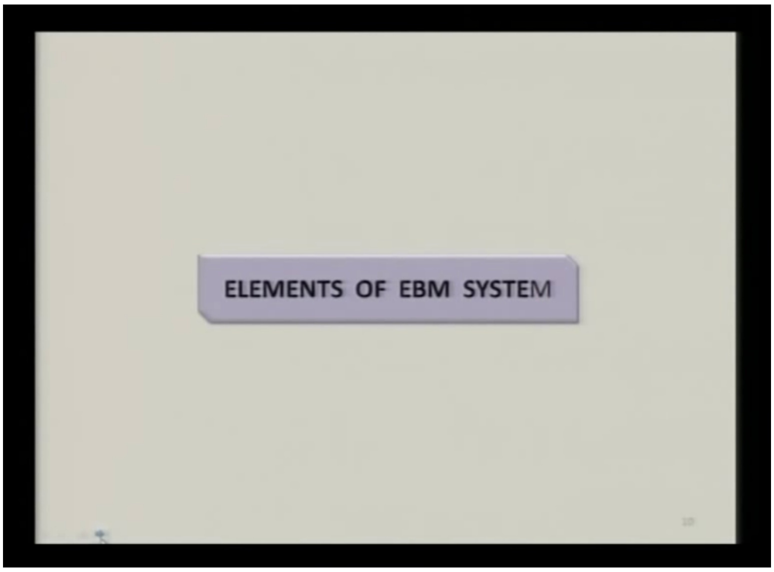


(Refer Slide Time: 16:30)

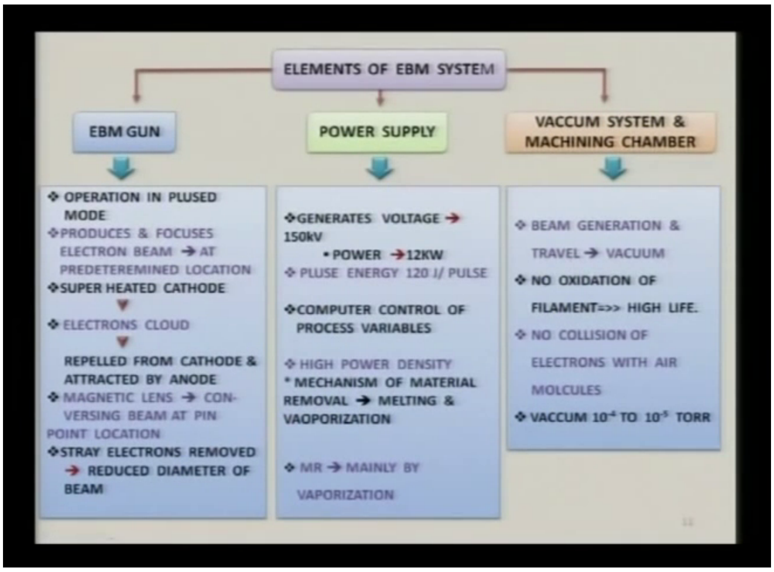


So I have already explained here that ejection of the molten work piece material from the bottom of the cavity is important and this is done with the help of the backing material. Now this is applicable for both circular as well as noncircular holes. Now vaporized backing material comes out at high pressure it expels molten materials from the cavity.

(Refer Slide Time: 16:51)



(Refer Slide Time: 17:12)



Now let us see what are the various elements of electron beam machining? First one is the electron beam machining gun; secondly the power supply to the electron beam machining system and third is a vacuum system and machining chamber. Now electron beam machining gun, now operation is done in pulse mode it reduces and focuses electrons beam at predetermined location, this predetermined location is made with the help of the lenses etc.

Now cathode his superheated as I have mentioned with the help of the diagram also and electrons cloud is created around the cathode and this electron cloud is repelled from the cathode it is attracted towards the anode because they’re having reverse polarity and magnetic lens converges the beam at beam pinpoint location.

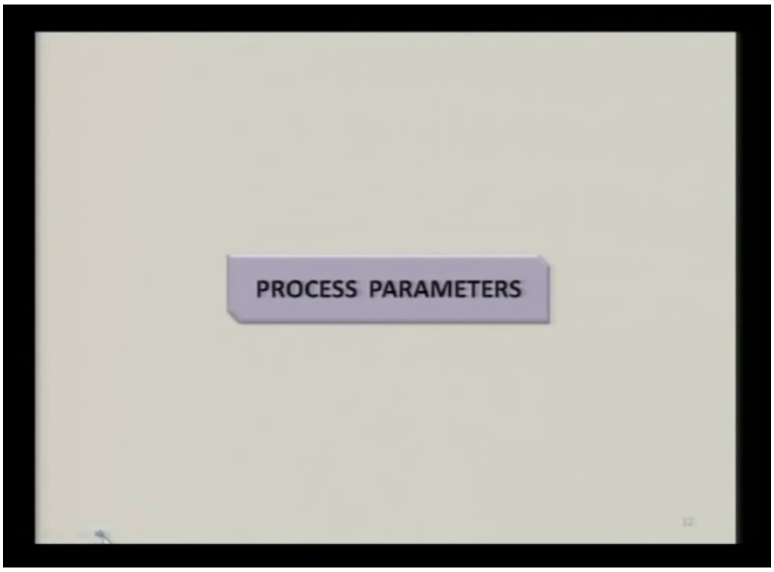
Now stray electrons are removed on the way and reduced diameter of the beam is obtained. As long as power supply is concerned generators give the voltage or you required the voltage approximately 150 kilovolt and the power that is given to the system is around 12 kilowatt and pulse energy is around 120 joules per pulse. Now please note that these values may be very different depending upon the system you buy and the objective for which you want to purchase this particular system.

In 18:43 all systems have computer control of the process variables they have various processes variables which I will explain you in the following slides. Now mechanism of material removal is melting and vaporization. Final material removal is mainly by vaporization process.

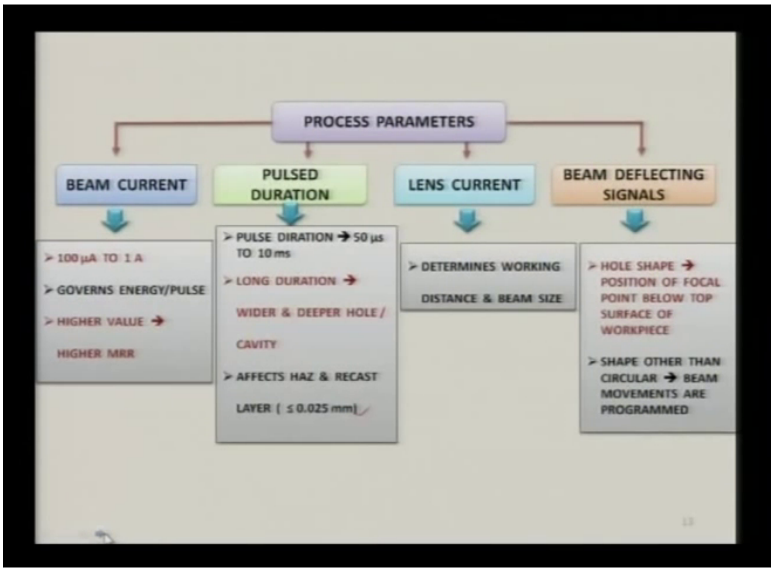
Now beam generation and travel using vacuum. If it is a vacuum system or if it is non-vacuum then definitely in the non-vacuum machining chamber. Now no oxidation of filament takes place because it is a vacuum, so there is no oxygen, no air in the beam generations chamber or machining chamber. Hence there is no oxidation of the filament and filament has high life.

There is no collision of electrons with air molecules because vacuum is there and vacuum value is very high rate maybe 10 raise to power minus 4 to 10 raise to power minus 5 torr. So these are some of the main elements of the EBM system.

(Refer Slide Time: 19:55)



(Refer Slide Time: 20:22)

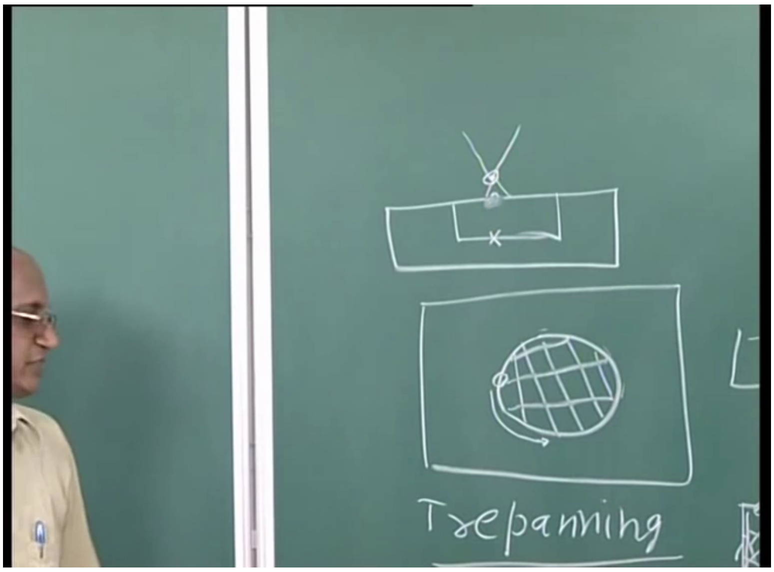


Now let us discuss the process parameters of electron beam machining. Now process parameters you can see they are beam current, pulse duration, plans current and beam deflecting signals. Now beam current normally varies from 100 micro-ampere to 1 ampere and it governs energy per pulse and higher value of the beam current will lead to higher material removal rate but please note it carefully that not necessary that higher material removal rate will retain the same accuracy of the machine components.

So one has to find a trade-off that between the MRR and the accuracy both. Now pulse duration normally varies between 50 microseconds to 10 milliseconds depending upon the requirement, smaller the pulse better is the accuracy but lower is the material removal rate normally. Now long duration gives you wider and deeper hole cavity, deeper hole or deeper cavity it affects the heat affected zone and recast layer. I have already discussed at many places heat affected zone in recast layer, I’m not going to repeat it and the size of the recast layer is normally around 25 microns or plus.

Lens current determines the working distance and the beam size because beam size is going to be governed by the lens current to some extent then beam deflecting signal whole shape as I have mentioned that you can deflect the beam and get the different type of the hole rather than only the circular hole. So you can utilize this feature that position of focal point below top surface of the work piece is there.

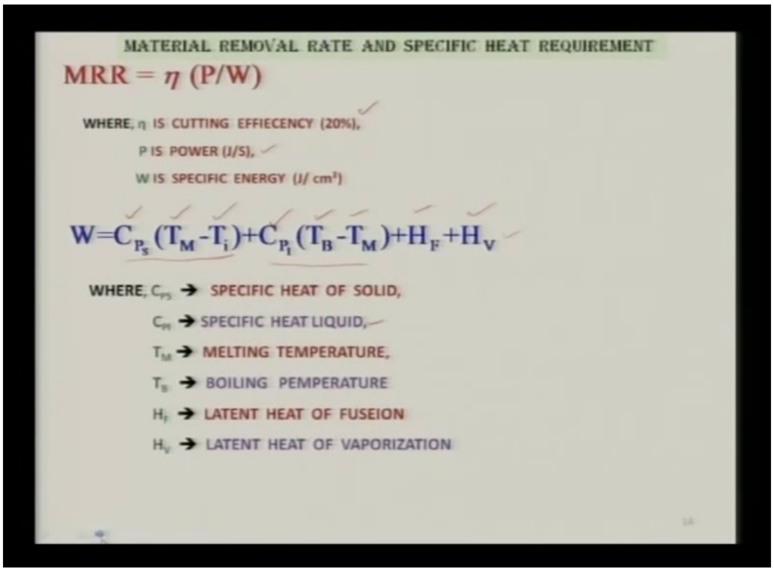
(Refer Slide Time: 22:29)



You know that this is important point that you have to check that the focal point is not far away from the surface which is being machined because the focal point is far away then you will not get the proper machining efficiency just let us take the example. Suppose this is the work piece being removed and you want to remove the material from here and your focal point is beam focal point is here then definitely it is not going to give the total or maximum heat that should be given to the work piece surface.

Part of the heat will be given because your focal point is away from the work piece surface. So this focal point should always be here or suppose you have machined a hole like this and you want to machine it then this focal point should be shifted over here then only you will get the maximum efficiency of the system otherwise not. Shape other then circular can be obtained as I have explained with the help of the movement of the beam or with the help of the move and of the work piece table.

(Refer Slide Time: 23:15)



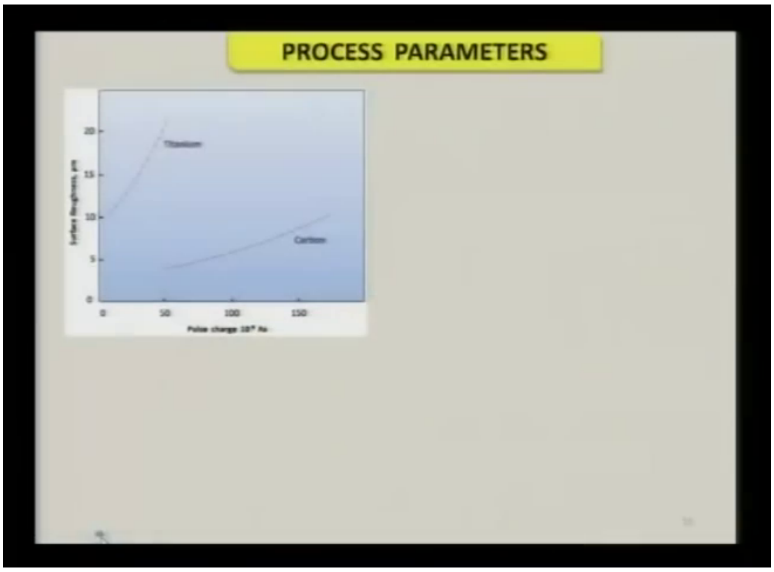
Now for calculation purposes we need to calculate theoretically what is the material removal rate? And what is the specific heat requirement? So that a unit amount of material can be melted and vaporized from the work piece. Material removal rate is normally given by geta P over W where geta is cutting efficiency. Now it's a very high value written over here 20 percent seems to be higher, P is the power in joules per second and W is the specific energy in joules per cubic centimetre.

If you want to calculate the specific heat required to melt and vaporized the work piece material then this is given by W is equal to CPf multiplied by TM minus Ti plus CPl multiplied by TB minus Tm plus HF plus Hv. Now here CPs is the specific heat of solid, so what you're doing really that you are melting the solid to the melting point because TM is the melting point Ti is the initial temperature for the room temperature for the vacuum chamber temperature and CPs is the specific heat of solid.

Same way you have the CPl that is a specific heat of liquid over here and this is the boiling point of the material and TM as we know the melting point of the work piece material, so that will give you the specific heat of the liquid and then you need latent heat of fusion and then latent heat of vaporization. So this way you can calculate the specific heat requirement for melting and vaporization of unit amount of work piece material and this is very essential when we want to theoretically calculate the machining efficiency of system.

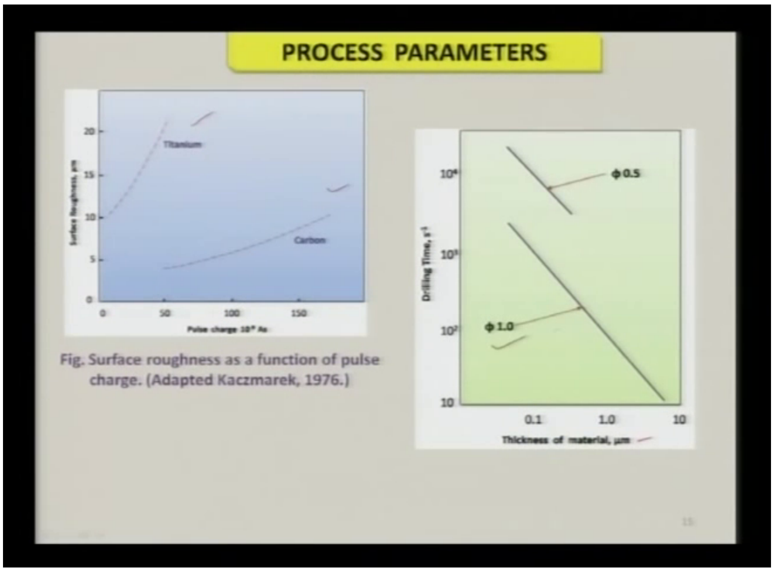
Because whatever amount of material if you know what is amount of the heat required for that and what amount of heat you have given to the system then the ratio of these 2 will give you the machining efficiency of the EBM system and this is normally low as I have mentioned in earlier slide 20 percent or less than 20 percent.

(Refer Slide Time: 25:51)



Now let us see how the surface roughness is being affected by the pulse energy. I have mentioned earlier also that you have to normally make a trade-off between the material removal rate and the accuracy that you’re going to obtain. See similar thing is true for surface roughness. As you can see here that as the pulse energy is increasing the surface roughness value that is the RA value or RZ value is increasing for 2 different types of materials.

(Refer Slide Time: 26:44)

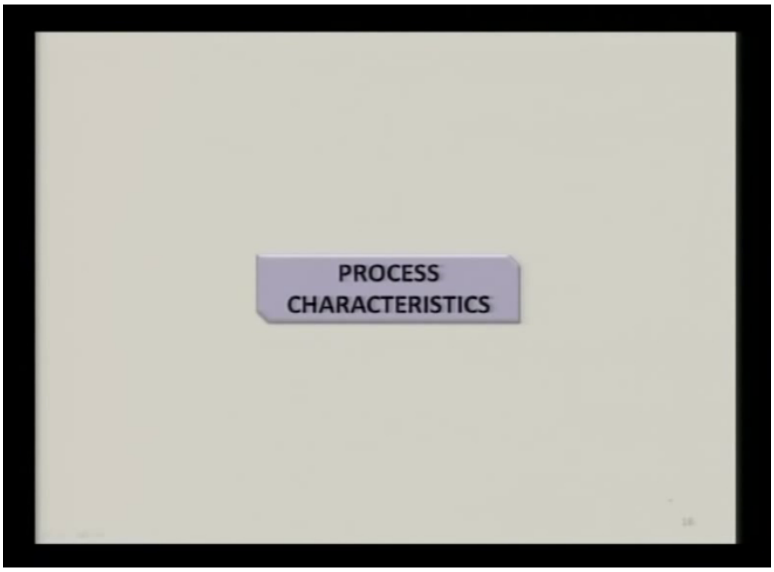


That means smaller the pulse energy better is the surface, smaller is the pulse energy better is the accuracy which is very simple I will explain you that if pulse energy is small then your size of the beam is also going to be small in say in case of low pulse energy and in case of large pulse energy definitely it is going to give smaller or better accuracy as compared to this.

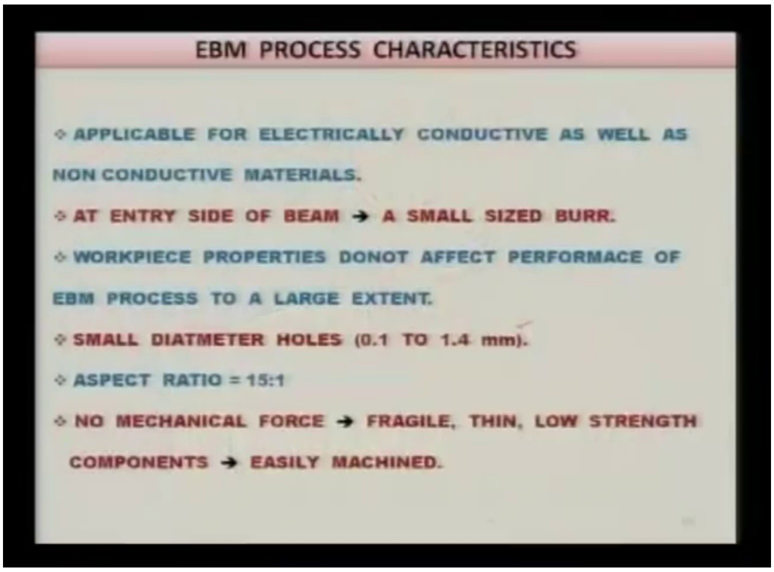
However this can also have a smaller diameter but then because of high energy intensity the melting of the material surrounding this will be larger compared to this and you can have or you will have poor accuracy in case of larger diameter compared to smaller diameter or larger pulse energy compared to smaller pulse energy but material removal rate is going to be low, so it is going to take longer time.

Now here you are given, suppose you are given a work piece material, now if you see that as the thickness of the work piece material is increasing, this is the diameter to be drilled and this is the drilling time.

(Refer Slide Time: 27:55)



(Refer Slide Time: 29:35)

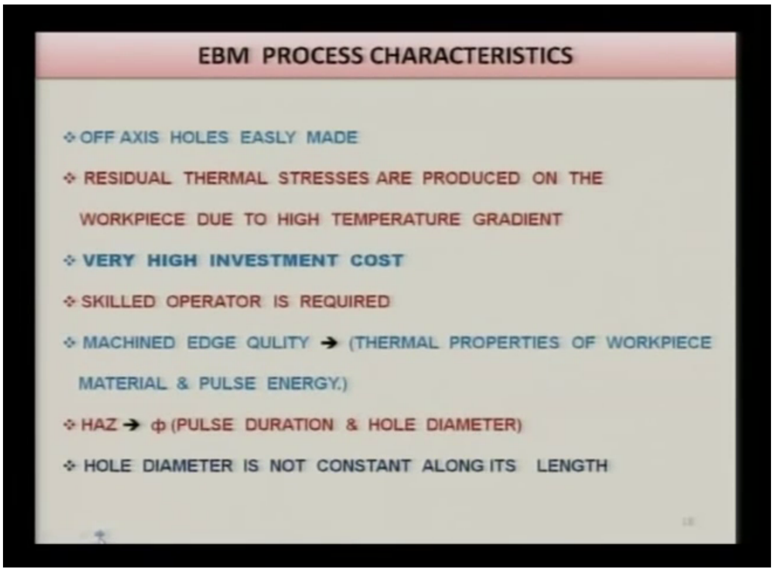


Process characteristics; now let us discuss the characteristics of the electron beam machining process. It is applicable for electrically conductive as well as electrically nonconductive materials. At entry side of the beam, a small sized burr is normally seen. Work piece properties do not affect performance of EBM process to large extent because the amount of the heat that is produced or intensity of the heat that is produced on the work piece surface is very large.

So it really does not matter too much, what are the properties, thermal properties of the work piece material? To some extent, yes it affects but not to a large extent. Small diameter hole can be made as small as 0.1 millimetre and large size 1.4 millimetres this will definitely depend upon various sector like the pulse energy and size of the beam that you are obtaining after concentrating the beam.

Aspect ratio is not greater than 15 ratio 1 in motion of the cases unless special arrangements are made. Now as we have seen in the mechanism of material removal, the process is purely thermal process where material is being removed by melting and vaporization, so no mechanical forces are acting, as a result of that fragile, thin, low strength component can be easily cut or machined or different shapes can be created on these components which is not possible in case of mechanical processes.

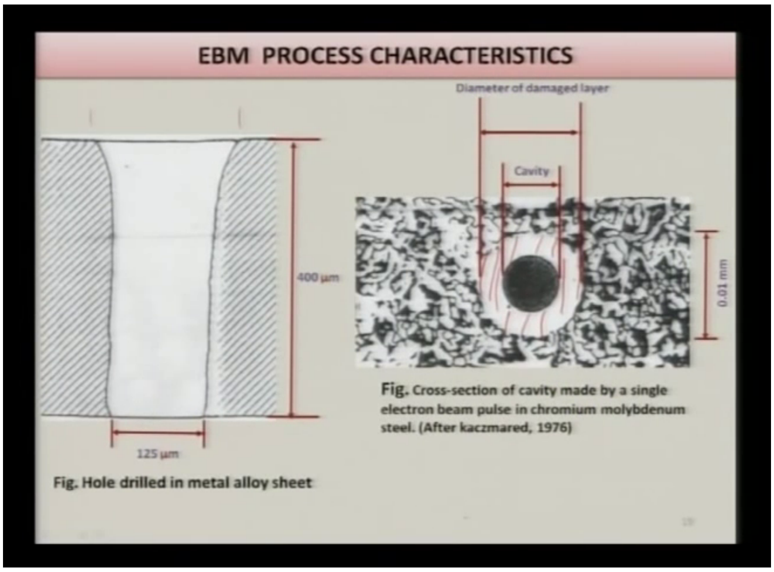
(Refer Slide Time: 29:44)



Off axis holes also can be easily made because there is no mechanical type of the tool, so it is not necessary that always you are making the holes along the axis, off axis holes can also be made by deflecting the beam. Residual thermal stresses are produced on the work piece due to high temperature gradient, this is very important because this is a thermal process material is being removed by melting and vaporization and there is always very high temperature gradient and because of this you are always left the machine component with thermal residual stresses and in many cases you require to anneal such kind of the pieces or remove those thermal residual stresses before putting them into the devices.

Very high investment cost and it requires skilled operators. Machined edge quality is the function of thermal properties of the work piece material and pulse energy and heat affected zone is a function of pulse duration as well as size of the hole being machined and hole diameter is not constant along its length. Although we expect that the diameter of the hole should be constant, the walls of the hole should not be tapered but normally it is not the case.

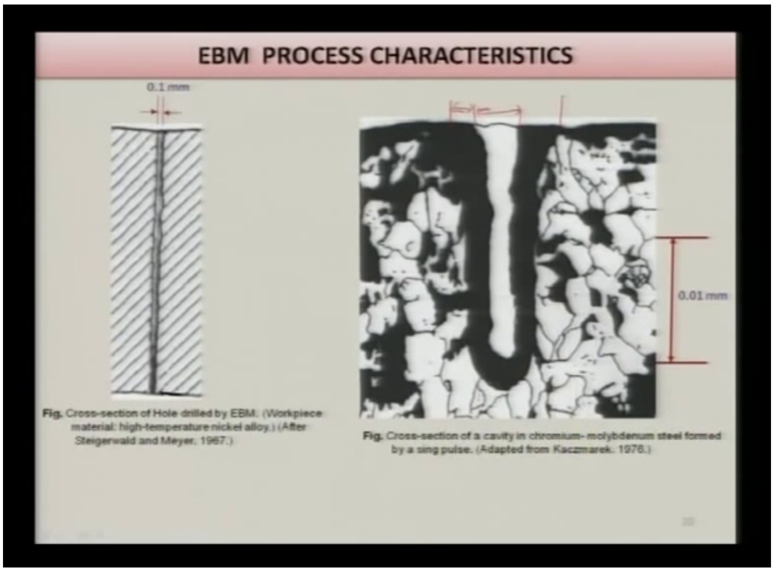
(Refer Slide Time: 31:19)



As you can see here that this is the schematic diagram of a hole that has been machined with the help of the electron beam machining process and the size of the hole at the bottom is just 125 microns while size of the hole at the top is larger than this particular one, although value is not written there and the depth of the hole is 400 micron and you can see the diameter is slightly varying from top to the bottom, so you're not getting the hole of the same size throughout.

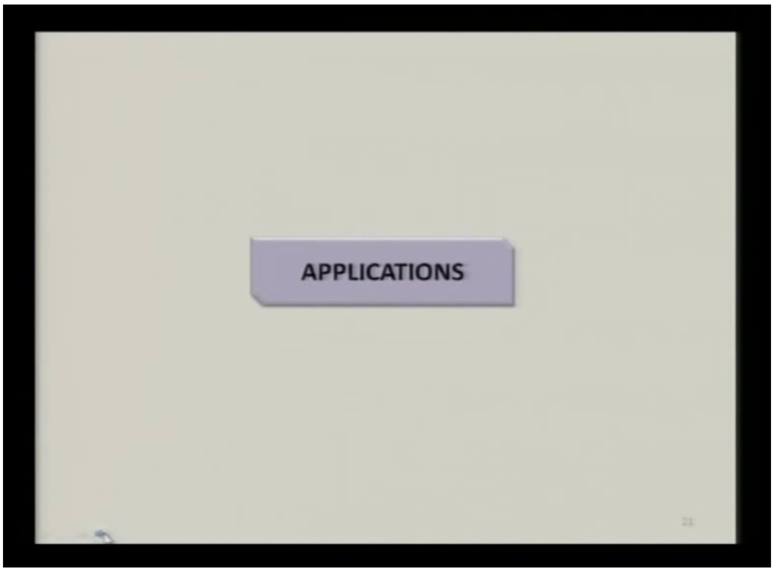
And this you can see here this is the cavity of the whole and this is the diameter of the damaged layer. As I have mentioned earlier here the recast layer is not shown but you can clearly see the kind of the heat affected zone or the zone which has been affected due to the high-temperature gradient. So this is the area which has been affected by the very high temperature gradient. So the cross-section of cavity made by a single electron beam pulse in chromium molybdenum steel.

(Refer Slide Time: 32:38)

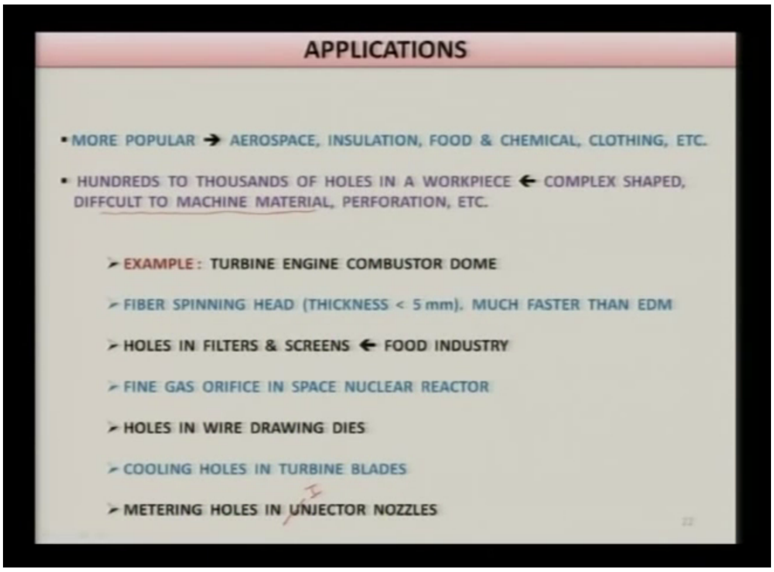


Now this is a deeper hole but a smaller size and again you can see this is not of a constant diameter throughout, the diameter is varying along its length. Now here again this is the heat affected zone and this is the hole diameter and again some heat affected zone and this is the cross-section of the hole that has been drilled and the you can see the scale is given over there. So one can find out what is that the depth of the whole and what is the diameter of the hole? And what is the size of the heat affected zone?

(Refer Slide Time: 33:21)



(Refer Slide Time: 33:30)

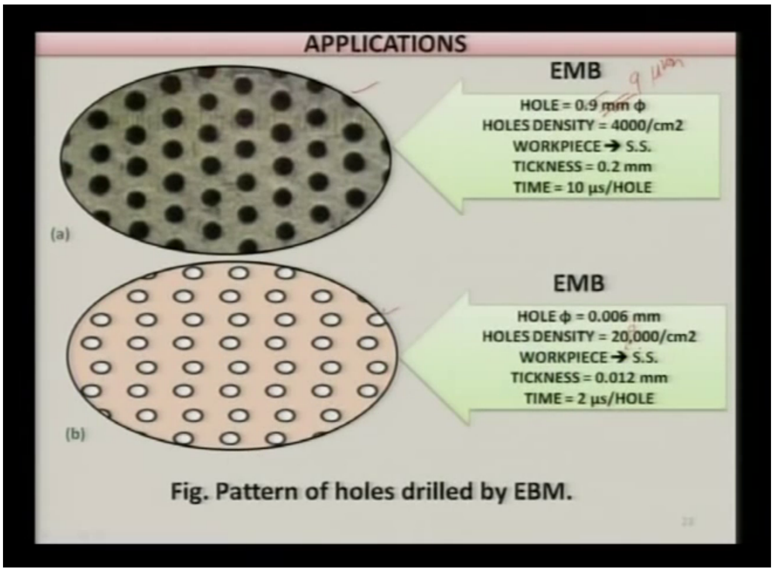


Now applications are very important for this particular process also as for any other Manufacturing process. This particular process is very popular in aerospace industries, installation, food and chemical industries, clothing industries etc. Hundreds to thousands of holes in a work piece may be required many times and these holes not just a circular hole many times you need complex shape, difficult to machine materials, perforation etc in the work piece you may require and to make such small micro-holes in difficult to machine material is really difficult.

Example, turbine engine combustor dome I will show you with a figure of this, fibre spinning head, thickness less than 5 millimetre much faster than EDM process to make hundreds of holes over there, holes in filters and screens especially used in food industries and textile industries. Fine gas orifice in space nuclear reactor. Holes in wire drawing dies because their wire drawing dies which can draw the wire of less than 0.5 millimetre diameter or so.

And in such cases to make forced the initial hole in the dye with the help of electron beam machining then you can draw the wire with the through that particular hole in the dye and cooling holes in turbine blades and metering holes in injector nozzles, this is the injector nozzles not injector notice.

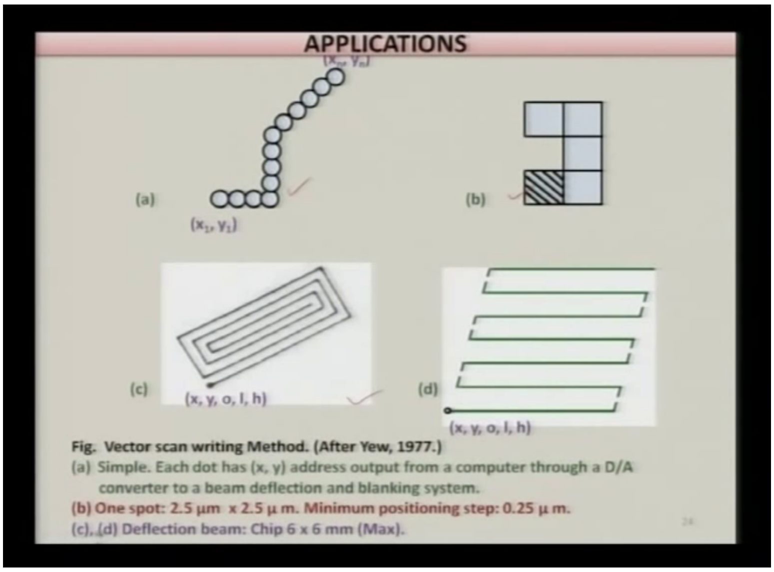
(Refer Slide Time: 35:11)



Now let us go for the whether you asked to the applications of this particular process. Now you see here. Here is a part of the work piece where holes have been drilled and you can see on the right side the hole diameter is this is wrong, this correct this is 0.9 this is really 9 micro-meter and the hole density is here 4000 holes per square centimetre and the work piece is made of stainless steel and the thickness of the sheet in which these holes have been made is made of is made as 0 .2 millimetres that is 200 microns size and the time taken by hole is just 10 microseconds per hole.

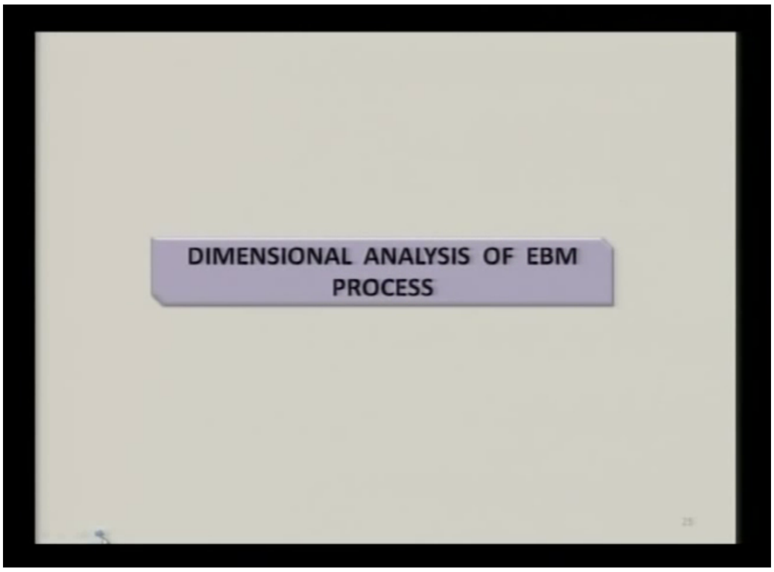
Now people are not satisfied with this quality and this size of the hole especially textile and food industry, they say we want still smaller holes and here you can see in the next slide at the bottom the size of the hole is just 6 microns and here the density of the hole is 200,000 this is again some mistake, 200,000 holes per centimetre square work piece is stainless steel and the thickness of the sheet is 12 microns and the time taken for hole is 2 microseconds. Now you can see such a high-density of holes and such a small-time for drilling a hole you cannot do it by other processes.

(Refer Slide Time: 37:04)

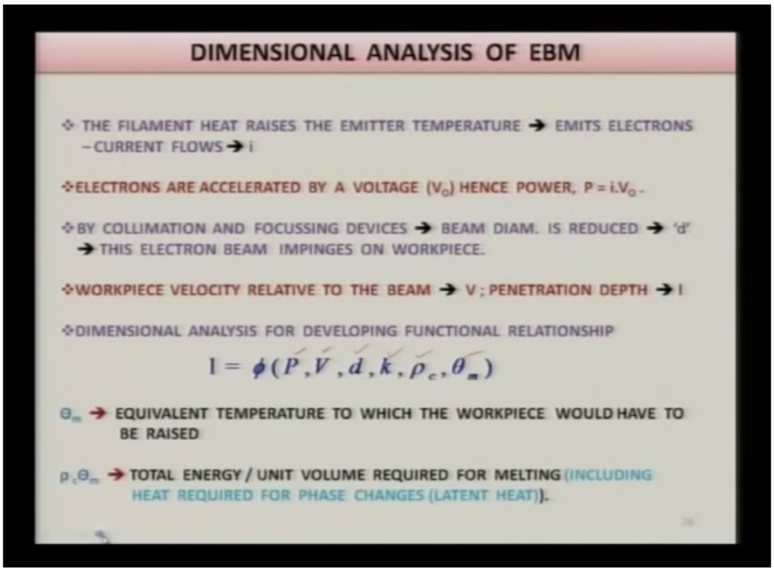


There are some other applications which you can see here that vector scan writing method and each dot has address output from a computer through a DA converter digital to analogue converter to a beam deflection and blanking system kind of thing then one spot 2.5 micron by 2.5 micron as you can see here that has been created with the help of EBM process and in the case of c and d, deflecting beam on the chip they have make certain circuits kind of thing of 6 by 6 millimetre as you can see over here.

(Refer Slide Time: 37:51)



(Refer Slide Time: 38:02)

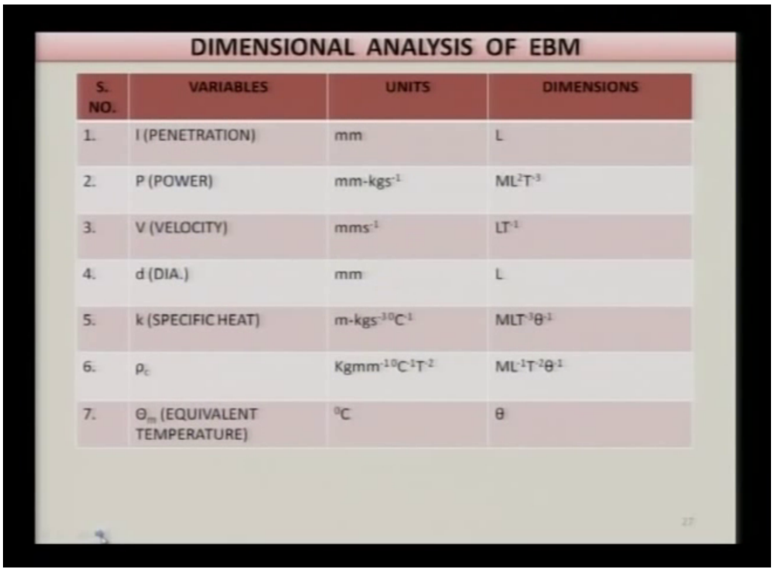


Now I'm going to discuss with this brief introduction of electron beam machining system. Dimensional analysis of electron beam machining process. As we have seen the filament heat raises the emitter temperature to a large value, it emits electrons that flows and electron current is considered as i. Electrons are accelerated by a voltage V0, hence the power of the electron beam becomes i multiplied by V0.

By collimation and focusing devices that we have already seen while discussing the EBM system, the beam diameter is reduced to the d and this electron beam impinges on the work piece wherever you want the feature to be created. Now the work piece velocity is related to the beam is taken as V and penetration depth is taken as i then dimensional analysis for developing functional relationship between the different parameters in penetration depth we proceed as follows.

This is the penetration depth l that becomes the function of power, relative velocity between being and work piece, diameter of the beam, this is thermal properties of the work piece material, density of the work material and temperature of the work piece material wherever the beam is concentrated. Theta M is the equivalent temperature to which the work piece would have to be raise and Rho c theta M is a total energy per unit volume required for melting, it includes heat required for phase changes that is the latent heat for fusion as well as latent heat for vaporization which I have shown to you the equation in the earlier slide.

(Refer Slide Time: 40:10)



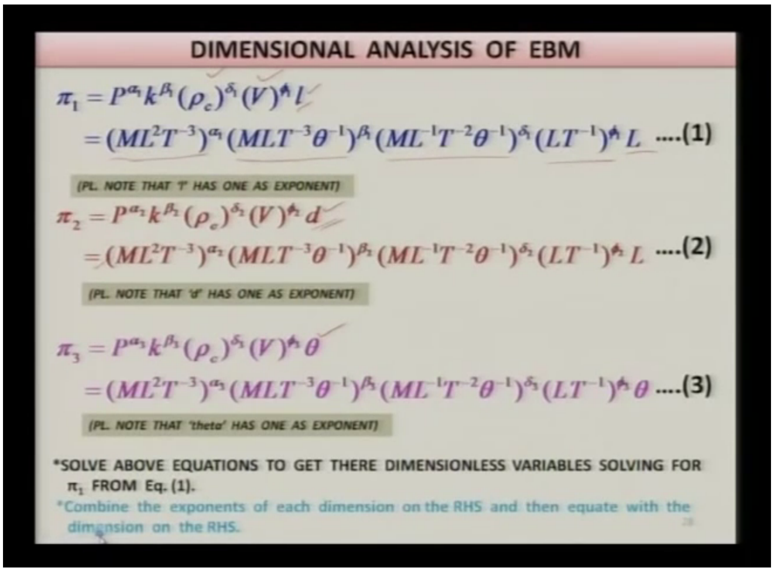
DIMENSIONAL ANALYSIS OF EBM

S. NO.	VARIABLES	UNITS	DIMENSIONS
1.	l (PENETRATION)	mm	L
2.	P (POWER)	mm-kgs^{-1}	ML^2T^{-3}
3.	V (VELOCITY)	mms^{-1}	LT^{-1}
4.	d (DIA.)	mm	L
5.	k (SPECIFIC HEAT)	m-kgs^{-3} $^0C^{-1}$	$MLT^{-3}\theta^{-1}$
6.	ρ_c	Kgmm^{-1} $^0C^{-1}T^{-2}$	$ML^{-1}T^{-2}\theta^{-1}$
7.	Θ_m (EQUIVALENT TEMPERATURE)	^0C	θ

Now this table gives you the name of the variables that we are going to use, units of the variables and dimensions of the variable. Penetration is the millimetre L is that I mentioned. Power is millimetre kg per second and Ml raise to power 2, T raise to power minus 3 is the dimension. Velocity V millimetre per second LT raise to power minus 1. Diameter millimetre L. Specific heat is millimetre kg per second per degree centigrade this is per second per degree centigrade minus 3, second raise to power minus 3 and as you can see here MLT raise to power minus3 Theta raise to power minus 1.

Rho c kg per kilometre per degree centigrade and T raise to power minus 2, T raise to power minus 2 which you can see here in the units also and Theta M is the equivalent temperature that is Theta M degree centigrade. Using this table now we will perform the dimensional analysis. Now one thing you can see here there are 7 variables as listed over here and 4 dimensions, L, M, T and theta. So you can have 3 dimensionless variables and they can be evaluated later on.

(Refer Slide Time: 41:43)



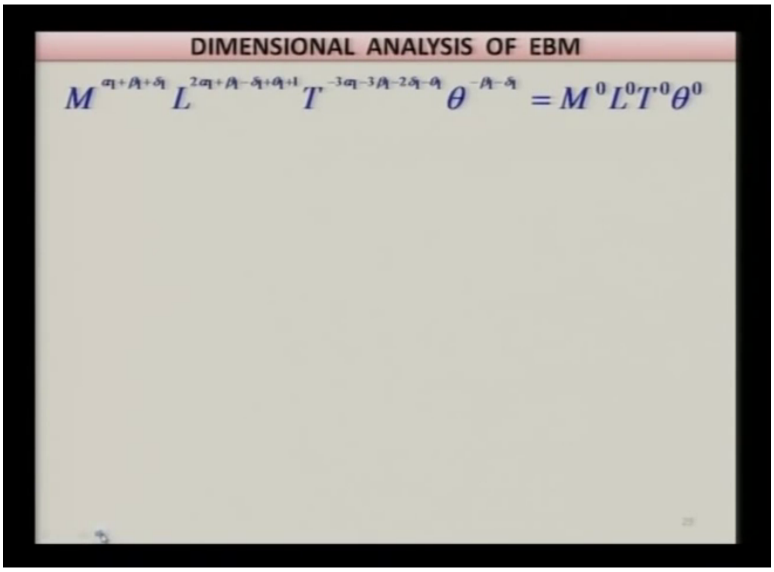
Now using the Pi theorem you can develop these equations which I am going to show you. So let pi1 be the dimensionless parameter and let it be given like p raise to power Alpha 1, k raise to power beta 1, Rho c raise to power Delta 1 and V raise to power phi1 multiplied by l, las by unit power. So now what you can do, you substitute the dimensions like for P, ML square T raise to power minus 3 whole raise to power alpha1, for k MLT raise to power minus 3 Theta raise to power minus 1 beta 1 and then Rho c ML minus 1 T raise to power minus 2 theta raise to power minus 1 delta 1. LT that is the velocity, LT raise to power minus T raise to power minus 1 phi 1 and L that has the unit dimension.

Now similarly you can have second dimension less parameter that is pi2, same procedure is followed here P raise to power Alpha2, k raise to power beta2, Rho c raise to power Delta2, V raise to power phi2 and d. Now you can see the difference here, d is there that is the diameter. Here L is used alone with unit power and then again substitutes the values of the dimensions like ML raise to power 2, T raise to power minus 3 whole raise to power Alpha2 and so on.

Then there is third parameter is the rather variable dimensionless variable, pi3 P raise to power Alpha 3, k raise to power beta 3, Rho c raise to power delta 3, V raise to power phi3 theta. So again substitute the values of the dimensions and you can see again here theta is having single power. Now this way you will obtain 3 different equations. Now these equations solve above equation to get 3 dimensionless variables solving for pi1 from equation 1.

Combine the exponents of each dimension on right hand side and then equation with the dimension on the right-hand side but you see, do you find out the dimensions of each MLT theta on the left side, MLT Theta on the right side and then exponents can be equated on both sides.

(Refer Slide Time: 44:32)



As done here you get these equations M raise to power alpha 1 plus beta1 plus delta 1, L raise to power 2 alpha 1 plus beta 1 minus delta 1, plus theta 1 plus 1, T raise to power minus 3 alpha 1 minus 3 beta 1 minus 2 delta 1 minus theta1 and theta raise to power beta 1 minus beta 1 minus delta 1 is equal to M raise to power 0, L raise to power 0, T raise to power 0 and guitarist to power 0.

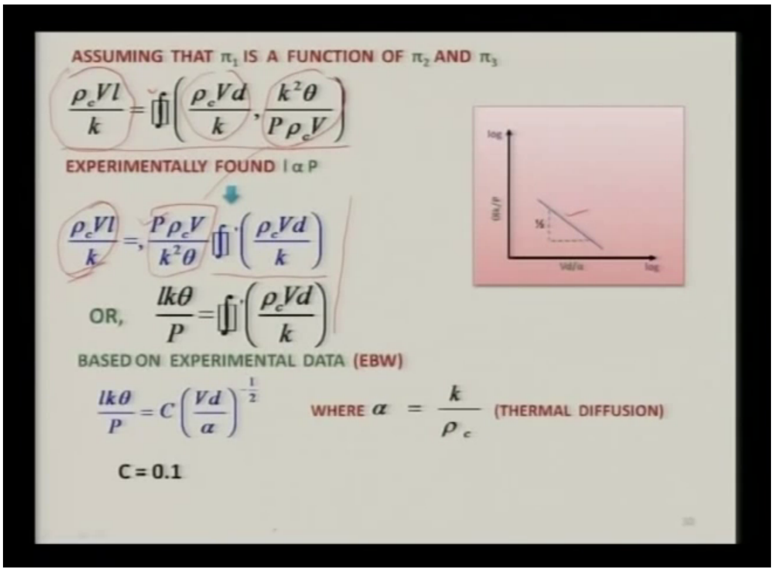
From the above equation one gets alpha1 plus beta 1 plus Delta 1 is equal to 0 because you equate this particular power to this particular power, this particular power to this power and this power to this 0 and again this power to this. So what you will do? You will have 4 different equations and there are 4 different variables, so you can find out the value of each variable.

Then second equation 2 alpha 1 plus beta 1 minus delta 1 plus phi1 plus 1 is equal 0, minus 3 alpha 1 minus 3 beta 1 minus 2 Delta 1 minus phi 1 is equal to 0 and minus beta 1 minus delta 1 is equal to 0, solve these 4 equations for 4variables you get the values as follows.

Alpha1 is equal to 0, beta1 is equal to minus 1, Delta 1 is equal to plus one and phi1 is equal to 1. Now following the same procedure before that now what you get? You get the value of, you substitute these values of alpha1, beta1, Delta 1 and phi1 in the first equation that I showed you in the earlier slide. So you get pi1 is equal to Rho c, Vl divided by k as you can see it is substituting these particular values.

Similarly what you can do? You can follow exactly the same procedure for solving for pi2 again you will have 4 equations, 4 variables, fine evaluate the values of all the 4 variables that is alpha2, beta 2, Delta 2, phi2 substitute these values in the second equation in the earlier slide you will get pi2 is equal to Rho c VD over k. Same way for the third pi3 you will get k square theta divided by P Rho cV.

(Refer Slide Time: 47:07)



Now using these 3 equations what you can, assuming that pi1 is a function of pi2 and pi3 then you will get this particular equation, here is this symbol is the function and this is our pi, this whole of this is our pi1, this is pi2 and this is pi3, so you get this as a function. Now the question arises how we evaluate this particular function? Because we do not theoretically at least what is this function going to be?

So what researchers have done it that they conducted the experiments and then plotted the relationship between these and then found the solution for it? Now we know that this linearly affects this particular pi1 is affected by this particular that is pi3 it is linear because this length or depth of penetration is a direct function of the power P. We can we know this experimentally as (())(48:10) as well as theoretically also that depth of penetration should be direct function of power P.

So what we can do? We can take the single power of this and definitely power is going to directly affect this parameter, so this is reversing that means P Rho c V becomes in the numerator k square Theta becomes in the denominator that means we are still left with what is the relationship between the depth of this particular Rho c Vl over k and Rho c Vd over k that has to be evaluated.

So what they have done? They have plotted the relationship between theta lk over P and Vd over alpha and based on experimental data of electron beam welding they found that lk theta over P is equal to C Vd over alpha raise to power of minus 1 by 2 where Alpha is equal to k over Rho c thermal diffusion C is equal to 0.1. So this way they are able to find out the slope of this particular curve and finally they will be they are able to find out, what is the actual relationship in these pi1, pi2 and pi3.

(Refer Slide Time: 49:51)

And from there you can find with the help of dimensional analysis the value of l that is the depth of penetration in terms of controllable variables, thank you very much.