

**Advanced Machining Processes**  
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**Lecture 10**

In the last lecture we were discussing about the theory of electrochemical machining I elaborately told about the Faraday's law of electrolysis with the help of which we are able to calculate linear material removal rate, volumetric material removal rate, mass material removal rate.

Then we also discussed about the evaluation of the inter electrode gap with the help of parabolic equation that is applicable for zero feed rate and an implicit equation that is applicable for finite feed rate, both these equations are used in the calculation or theoretical electrochemical machining, but the equation with finite feed rate has certain drawbacks it takes so many iterations before one can arrive at the correct inter electrode gap.

And in case of finite element method, finite difference method, the large number of points at which these equations are to be evaluated so single equation applicable for both the cases and taking very less time had been proposed that was discussed, then we discussed how to evaluate electrochemical equivalent of an alloy, two methods were discussed, percentage weight method and total charge required method.

And after that today let us discuss the maximum permissible feed rate for the given machining conditions and that is very important if the feed rate is very low then productivity will be low, accuracy will be also low, if it is very high although material removal rate will be high but there are all possibilities that short circuit may take place and that short circuit will result in the damage of the tool as well as the work piece.

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**MAXIMUM PERMISSIBLE FEED RATE IN ECM**

THEORETICALLY THERE IS NO UPPER LIMIT FOR FEED RATE IN ECM.

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- FOR THIS → DURING ACTUAL ECM, ELECTROLYTE FLOW RATE SHOULD BE SUCH THAT IT IS ABLE TO CARRY AWAY THE HEAT PRODUCED DURING ECM AND ELECTROLYTE BOILING DOES NOT TAKE PLACE. **WHY?**  
(IT WILL CHANGE  $\eta$  HENCE MRR, AND THE PREDICTED ANODE PROFILE WILL CHANGE.)

**ASSUMPTIONS**

- HEAT PRODUCED ONLY DUE TO OHMIC HEATING ( $= I^2 R$ ) IS CONSIDERED.
- HEAT PRODUCED DUE TO VISCOUS FLOW OF ELECTROLYTE, CHEMICAL REACTION, ETC., ARE NEGLECTED.

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So let us see how to evaluate the maximum permissible feed rate for the given machining conditions. Theoretically there is no upper limit for feed rate in electrochemical machining as we have seen in case of self regulating feature that has the inter electrode gap keep decreasing because of the higher feed rate then material removal rate, linear material removal rate keeps on increasing to maintain the equilibrium condition.

Still there is always a limit for the maximum permissible feed rate that is what we are going to evaluate today for this purpose during actual electrochemical machining, electrolyte flow rate should be such that it is able to carry away the heat produced during ECM and electrolyte boiling does not take place.

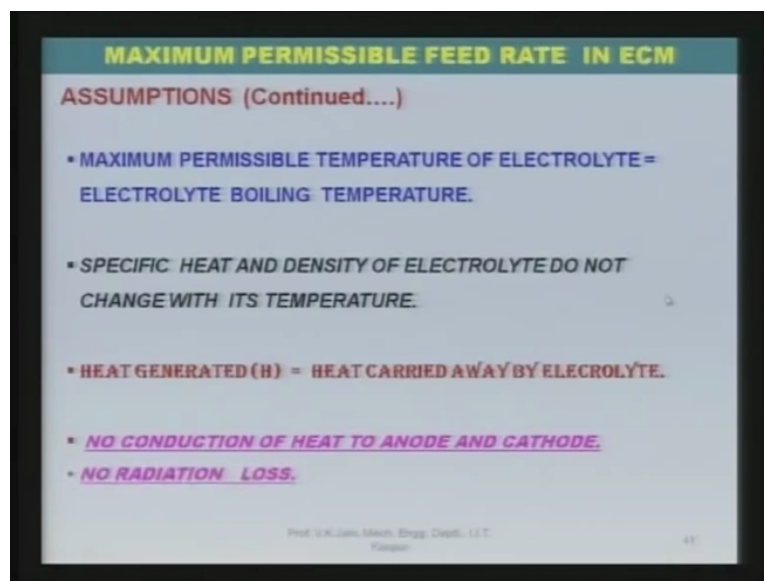
Now here there is a question mark why we want that boiling of the electrolyte should not take place and heat generated in the inter electrode gap should be taken away as quickly as possible because if the heat produced during in the inter electrode gap during ECM is not taken away it is going to change the temperature of the electrolyte as a result of that although conductivity will change, more important is whatever you have designed or expecting the work piece shape or anode shape that will not be obtained because you are not able to control the conductivity of the electrolyte in process.

Secondly boiling should not take place if the electrolyte boiling takes place then vapor generation will take place and once vapor generation takes place those vapors or bubbles will be added to the electrolyte and that is again going to change the conductivity of the electrolyte that simply means again you are not able to get the predicted anode shape or the work piece shape and you are not able to control the conductivity of the electrolyte.

Now, in the derivation of this particular equation for maximum permissible feed rate certain assumptions has been made to simplify the analysis the basic assumption and foremost assumption is heat produced only due to ohmic heating is considered and heat produced by other sources is neglected.

Heat produced due to the viscous flow of electrolyte, also there are certain chemical reactions that take place they also produce the heat they are the exothermic reactions. All these sources of heat production during ECM are neglected.

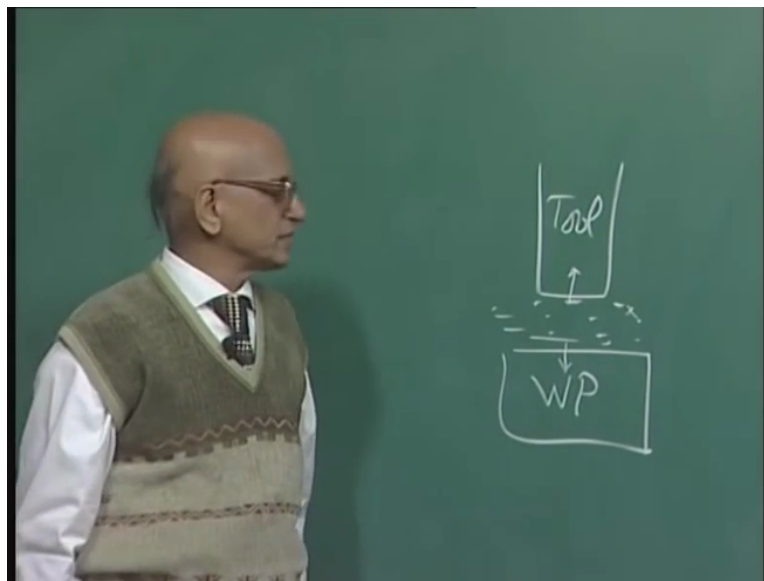
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Maximum permissible temperature of electrolyte is always less than or equal to the electrolyte boiling temperature for the reasons I have just mentioned. Specific heat and density of electrolyte do not change with its temperature. Electrolyte temperature is going to change from the room temperature to say 90 degree centigrade or so and room temperature varies from say 5 to 10 degree in winter to the 45 degree centigrade or more in summer, so there is a large variation.

And above this the temperature can further go during the ohmic heating and it maybe as high as 90 degree centigrade or so and at such a large scale of temperature variation, the specific heat of the electrolyte is going to change and to some small extent the density of the electrolyte may also change but for present modeling or for present development of the equation for maximum permissible feed rate, both these changes are neglected or ignored. So we are going with the assumption that heat generated due to the ohmic heating is equal to heat carried away by the electrolyte.

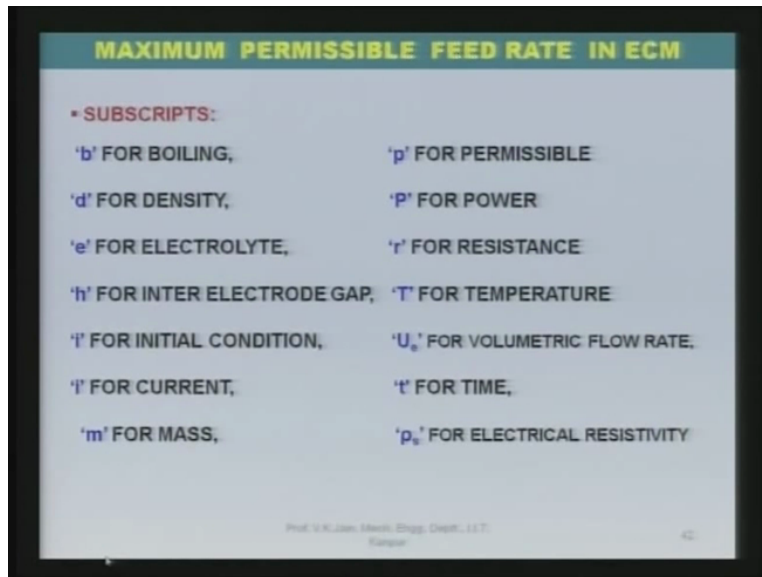
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Now this is another very important assumption that no conduction of heat to the anode and cathode is taking place as you know already that if you recall earlier figures and here you can see tool and work piece both are in contact with the electrolyte where heat is generated, so obviously some heat will be conducted away to the tool and some heat will be conducted away to the work piece but for this simple calculation here we are assuming that heat conducted away to the tool and work piece both is negligible.

The thermal conductivity of the tool is very high however for simplification of the problem we are considering it negligible also we are neglecting here the losses due to the radiation because the temperature developed is not very high so we can easily neglect the heat loss due to the radiation.

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Now they are various subscripts being used in the following equation that you can see here. b is used for boiling, d for density, e for electrolyte, h for inter electrode gap, i for initial condition, also capital I not small i for current, small m for mass, p for permissible and capital P for power, r for resistance, capital T for temperature, U subscript e for volumetric flow rate, small t for time and rho s for electrical resistivity.

(Refer Slide Time: 09:12)

**MAXIMUM PERMISSIBLE FEED RATE IN ECM**

$$H = m_e \cdot c_e \cdot (T_b - T_i)$$

$$= U_e \rho_e \cdot c_e \cdot (T_b - T_i)$$

$m_e$  = Mass of electrolyte flowing,  $c_e$  = specific heat of electrolyte  
 $T_b$  = Boiling temperature,  $T_i$  = Initial temperature,  $U_e$  = Volume of Electrolyte flowing,  $\rho_e$  = Density of electrolyte.

DIVIDE BOTH SIDES OF EQUATION BY 't'

$$\frac{H}{t} = \left(\frac{U_e}{t}\right) \rho_e \cdot c_e \cdot (T_b - T_i)$$

$P$  = Power,  $U_e$  = Volumetric flow rate of electrolyte,  
 $I_p$  = Permissible current,  $R$  = Resistance of the IEG

$$\text{or, } P = 4.186 \dot{U}_e \rho_e \cdot c_e (T_b - T_i)$$

$$= I_p^2 R$$

Simplification of above equation will give,

$$I_p = \sqrt{\frac{4.186 \cdot U_e \cdot \rho_e \cdot C_e \cdot (T_b - T_i)}{\rho_s \cdot \frac{h}{A}}}$$

So we can see here that total heat H is equal to m e c e T b minus T i, so here m is the mass of the electrolyte, c e is the specific heat of the electrolyte T b is the boiling temperature of the electrolyte that is the highest permissible temperature in this case and T i is the initial temperature that the temperature at which electrolyte is entering into the inter electrode gap.

Now this mass m e can be written in terms of the volumetric flow rate or volume of the electrolyte flowing in the given period of time and rho e is the density of the electrolyte, so we have this modified equation. Now divide both sides of this equation by the electrochemical machining time t, then you get H by t is equal to U e over t rho e C e multiplied by T b minus T i.

Now this volume flow U e when divided by time of flowing then you get volumetric flow rate, so you can write and H divided by T, total heat generated divided by time then you get power P. So you can write this equation equal to 4.186 U dot e rho e C e T b minus T i, here U dot e this indicate the volumetric flow rate of the electrolyte and 4.186 is the joules constant and this can be written as equal to I p suffix square R where I p is the permissible current that can flow in the inter electrode gap.

And its value is determined on the basis such that it does not permit electrolyte to boil or it does not permit electrolyte temperature to go above the boiling temperature as we have already made the assumption and R is resistance of the inter electrode gap that is given over here I p square R.

Now simplification of this equation will give us  $I_p$  that is the permissible current in the circuit equal to  $\sqrt{4.186 U_e \rho_e C T b \text{ minus } T_i}$  divided by  $\rho_s h$  over  $A$ . Now this  $R$  has been substituted by  $\rho_s h$  over  $A$ ,  $\rho_s h$  over  $A$  is the resistance and is nothing but the resistance that we already know  $\rho$  that is the  $\rho_s$  is resistance  $h$  is the length and  $A$  is the cross-sectional area through which the current is flowing. Now  $\rho_s$  resistance that can be written as  $1$  over  $k$ , that is the electrical conductivity of the electrolyte.

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**MAXIMUM PERMISSIBLE FEED RATE IN ECM**

WE ALREADY KNOW THAT

$$\dot{m} = \frac{IE}{F}; \text{ SPECIFIC MRR IN TERMS OF } g/A-s \text{ IS GIVEN BY}$$

$$\dot{m}_s = \frac{E}{F} = \left( \frac{A'}{Z} \right) \cdot \frac{1}{F} \quad (A' = \text{Atomic mass, } Z = \text{valency of dissolution})$$

NOW, SPECIFIC VOLUMETRIC MRR ( $MRR_{sv}$ ) IN  $mm^3/(A-s)$

$$MRR_{sv} = \frac{\dot{m}_s}{\rho_w} = \left( \frac{A'}{Z} \cdot \frac{1}{96500} \right) \cdot \frac{1}{\rho_w} \cdot \eta; \quad \rho_w = \text{ANODE DENSITY}$$

$\eta = \text{MACHINING EFFICIENCY}$

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We already know from the, this first lecture of theory of ECM that  $\dot{m}$  that is the mass, mass removal rate is equal to  $IE$  over  $F$ . Now if we want to calculate specific material removal rate in terms of gram per ampere second then you divide this particular equation by the current  $I$  that is amperes then we get  $\dot{m}_s$ ,  $s$  indicates the specific material removal rate is equal to  $E$  over  $F$  because  $I$  in the numerator and denominator cancel is equal to  $A'$  over  $Z$  multiplied by  $1$  over  $A$ , I have already explained in the earlier lecture that  $A'$  is atomic mass,  $Z$  is valency of dissolution,  $A'$  divided by  $Z$  gives you the chemical equivalent  $E$  and  $F$  is the Faraday's constant.

Now specific volumetric material removal rate represented as  $MRR_{sv}$  suffix in cubic millimeter per ampere second this is given by  $\dot{m}_s$  divided by  $\rho_w$  because when mass of the material removed you divide by its density you get the volume of the material removed and here  $s$  indicates specific so you, this can be written as  $A'$  over  $Z$  divided by  $96500$  that is the

Faraday's constant multiplied by 1 over rho w that is the density of the work piece material and zeta is the machining efficiency which you can assume 1 for practical purposes.

(Refer Slide Time: 15:07)

**MAXIMUM PERMISSIBLE FEED RATE IN ECM**

WE ALSO KNOW,

$$f_p = \text{CURRENT DENSITY} \cdot \text{MRR} \left( \frac{\text{Amp}}{\text{mm}^2} \times \frac{\text{mm}^3}{\text{A} \cdot \text{s}} \right) = \frac{\text{mm}}{\text{s}}$$

$$= \frac{I_p}{A} \times \left[ \frac{A'}{Z} \right] \cdot \frac{1}{96500} \times \frac{\eta}{\rho_w}$$

$I_p$  = permissible feed rate

Substitute the value of  $I_p$  and simplify,

$$I_p = \sqrt{\frac{4.186 U_e \rho_e C_e (T_b - T_i)}{\rho_s h / A}}$$

$$f_p = \eta \frac{A'}{Z} \times \frac{1}{96500} \times \sqrt{\frac{4.186 U_e \rho_e C_e (T_b - T_i)}{\rho_s h A}}$$

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We also know that the  $f_p$  that is the permissible feed rate is equal to current density multiplied by specific material removal rate you can see the balance of the units current density is given by ampere per millimeter square and specific material removal rate is given in terms of cubic millimeter per ampere second when you simplify this you get millimeter per second that is the unit of the feed rate.

So the equation shown in the previous slide can be written as  $I_p$  over  $A$  where here  $A$  indicates the cross-sectional area through which the current is flowing multiplied by  $A$  dash over  $Z$  into 1 upon 96500 zeta upon rho w. Now substitute the value of  $I_p$  and simplify this equation because  $I_p$  we have already calculated as given here that we calculated in the earlier slide that 4.186  $U_e$  rho e  $C_e$  multiplied by  $T_b$  minus  $T_i$  divided by rho s h over  $A$  whole under root.

Now substitute this value of  $I_p$  then we get permissible feed rate  $f_p$  is equal to zeta  $A$  dash over  $Z$  multiply by 1 upon 96500 multiplied by under root whatever is the term for  $I_p$  that is 4.186  $U_e$  dot e rho e  $C_e$  multiplied  $T_b$  minus  $T_i$  bracket closed divided by rho s h  $A$ . Now here it should be rho s h divided by  $A$  not multiplied by  $A$  as written over here in the last equation for  $F_p$  so please correct it.



(Refer Slide Time: 17:16)

**MAXIMUM PERMISSIBLE FEED RATE IN ECM**

THIS EQUATION CAN ALSO BE USED TO EVALUATE THE CHANGE IN TEMPERATURE FOR THE GIVEN FEED RATE.

- SUBSTITUTE  $T_b = T_f$  (FINAL TEMPERATURE) IN THE PREVIOUS EQUATION AND SIMPLIFY. THEN,

$$(T_f - T_i) = \Delta T = 2.23 \times 10^3 \left( \frac{\rho_s h A}{U_e \rho_e C_e} \right) \left( \frac{f Z \rho_w}{A n} \right)^2$$

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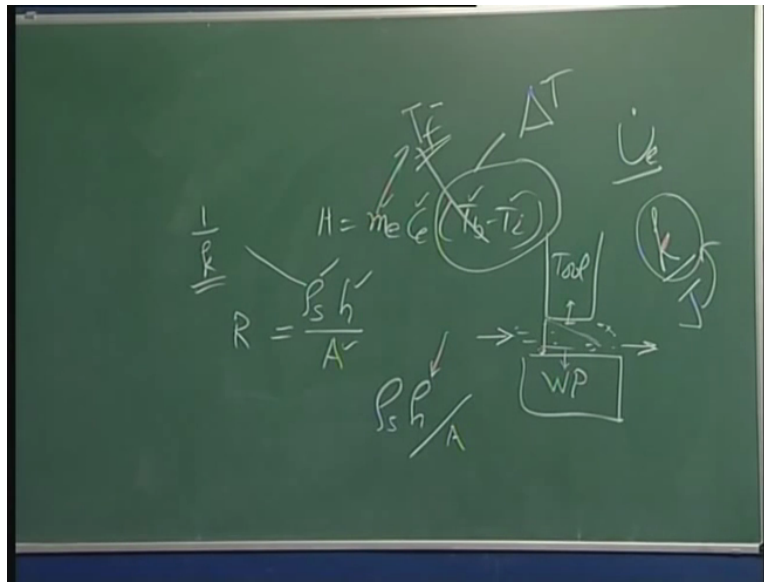
This equation that we have just derived can also be used to evaluate the change in temperature for the given feed rate because feed rate is going to control the temperature of the electrolyte because we know that feed rate will control the inter electrode gap and depending upon the inter electrode gap the length of this or the value of h will change that is the, that is controlling the conductivity of the electrolyte as a result of that the temperature will also will change.

So for the given machining conditions let us calculate what is going to be the change in the temperature, change in temperature is nothing but  $T_b$  minus  $T_i$  that you can represent as  $\Delta T$ , so substitute  $T_b$  is equal to  $T_f$  now here when we are finding out the change in temperature it is not necessary that every time the temperature of the electrolyte reaches to the boiling point so in place of this  $T_b$  you can replace it by  $T_f$  that is the final temperature of the electrolyte in the previous equation.

And then you simplify this and final equation that you get is  $T_f$  that is the final temperature of the electrolyte minus  $T_i$  that is the initial temperature of the electrolyte and this is nothing but  $\Delta T$  that is equal to  $2.23 \times 10^3$  multiply by  $\rho_s h A$  divided by  $U_e \rho_e C_e$  again multiplied by  $f$  is the feed rate  $Z$  over  $A$  dash multiplied by  $\rho_w$  over  $n$  whole square.

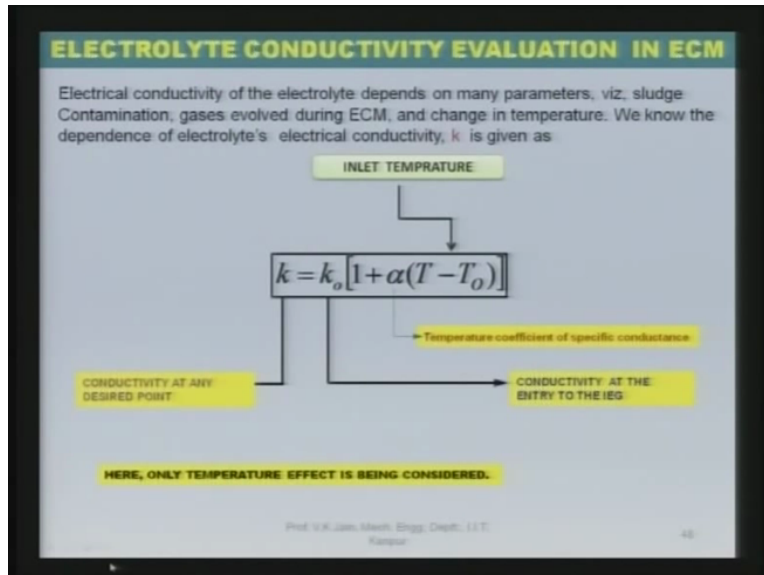
We know that electrical conductivity  $k$  of the electrolyte changes with various parameters namely temperature of the electrolyte, sludge contamination or reaction product of the electrolyte that or reaction product that get mixed up in the electrolyte, vapors or gas bubbles that are formed and get mixed up with the electrolyte they also change the conductivities of the electrolyte.

(Refer Slide Time: 19:51)



Now the question is how to represent all these effects in the form of mathematical equation we will see and another point is the temperature gradient, how the temperature is varying along the electrolyte flow direction suppose the electrolyte is flowing from here towards this direction so what is the temperature over here and how this temperature is varying in this direction, is it increasing, it is decreasing or remaining the constant that also we should know and current density that is the  $J$  because once  $k$  is varying and current density  $J$  is the function of the electrical conductivity of the electrolyte so once  $K$  is varied, how  $J$  is varying that also should be known to us so let us derive these equations for the present purpose.

(Refer Slide Time: 20:54)



Electrical conductivity of the electrolyte depends on many parameters sludge contamination, gases evolved during electrochemical machining and change in temperature that is capital T, we know the dependence of electrolytes, electrical conductivity  $k$  is given as  $k$  is equal to  $k_0$  multiplied by 1 plus alpha T minus  $T_0$  bracket close. Now here  $k_0$  is the electrical conductivity at the inlet point here the electrolyte is entering at this particular point so whatever is the electrical  $k$  value here that is represented as  $k_0$  the temperature at this particular point is represented as  $T_0$ .

And the temperature at the outlet that is here that is represented either as  $T_f$  or in this particular equation it has been written as just T, alpha we already know, let us see, this is the conductivity at any desired point. Conductivity at the entry to the inter electrode gap, alpha is the temperature co-efficient of specific conductance.

(Refer Slide Time: 22:25)

**ELECTROLYTE CONDUCTIVITY EVALUATION IN ECM**

ACCOUNTING FOR VOIDS (GASES EVOLVED) IN THE ELECTROLYTE, THE ABOVE EQUATION CAN BE MODIFIED AS

$$k = k_o (1 + \alpha \Delta T) (1 - \alpha_v)^n$$

exponent

VOID FRACTION < 1.0

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49

Now in the previous equation we have not accounted for the effect of gases evolved during ECM or void fraction so let us see accounting for voids in the electrolyte, the above equation can be modified as  $k$  is equal to  $k_o$ ,  $1 + \alpha \Delta T$  that is the change in temperature multiplied by  $1 - \alpha_v$  whole raise to power  $n$  where  $\alpha_v$  is the void fraction and its value is always less than or equal to 1 and  $n$  is the exponent.

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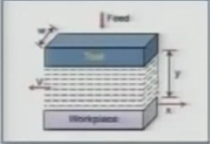
**TEMPERATURE GRADIENT EVALUATION IN ECM**

TEMPERATURE GRADIENT ALONG THE ELECTROLYTE FLOW DIRECTION CAN ALSO BE EVALUATED FROM THE FUNDAMENTAL EQUATION AS FOLLOWS:

FROM THE LAW OF CONSERVATION OF ENERGY, WE CAN WRITE

HEAT GENERATED = HEAT DISSIPATED

OR,

$$\begin{aligned} \text{Heat produced} &= I^2 R \\ &= \frac{(V - \Delta V)^2}{R} \\ &= \frac{(V - \Delta V)^2 k}{\left(\frac{y}{A}\right)} \end{aligned}$$


HERE R OF TOOL AND WORKPIECE ARE NEGLECTED.

$$\text{Heat dissipated} = \rho_e w y v C_e (\Delta T)$$

Temperature gradient along the electrolyte flow direction can also be evaluated from the fundamental equation as follows, from the law of conservation of energy we can write heat generated is equal to heat dissipated that we have already seen, now let us see this particular figure here tool, work piece, inter electrode gap represented by value of Y and w small w represents the width of the tool and also the width of the work piece and V is the electrolyte flow velocity.

So heat generated we have already calculated that is given by I square R now I can also be represented, is equal to V over R so I square becomes V square over sorry yes V square over R square so using this particular equation we can write this as V minus delta V square over R where delta V is the over potential.

Now important point here is that R of tool and work piece are neglected, it is assumed that the conductivity of the tool material and work piece material are very high so that the resistance, electrical resistance of the tool and work piece are negligible so the heat generated due to the resistance of the tool and the work piece are negligible and that are neglected here and we have taken only the heat generated due to the ohmic heating of the electrolyte.

So this can be written like V minus delta V whole square k divided by y over A, now here note it that A is the cross-sectional area through which the current is flowing in the electrolyte. Now

heat dissipated we have calculated earlier also that is  $\rho_e$ ,  $w$  is the width of the path through which the electrolyte is flowing,  $y$  is the gap and  $v$  is the velocity of the electrolyte that is given in terms of centimeter per second or meter per second.

$C$  is the specific heat and  $\Delta T$  is the change in temperature from this we can calculate the heat dissipated. Now note here that we are not considering any heat going to the tool any heat going to the work piece by conduction or any heat loss due to the radiation.

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**TEMPERATURE GRADIENT EVALUATION IN ECM**

Heat produced = heat dissipated

Or, 
$$\frac{(V - \Delta V)^2 \cdot k \cdot w \cdot dx}{y} = \rho_e \cdot w \cdot y \cdot v \cdot C_e \cdot \Delta T$$

$$\Rightarrow \frac{dT}{dx} = \frac{(V - \Delta V)^2 \cdot k}{\rho_e \cdot y^2 \cdot v \cdot C_e} = A \cdot k$$

$(V - \Delta V)$ ,  $\rho_e$ ,  $v$ ,  $C_e$  AND  $y$  CAN BE TAKEN AS **NON-VARYING** FOR A SHORT PERIOD OF TIME  $dt$ . AND FOR A SMALL DISTANCE (OR ELEMENT SIZE)  $dx$ .  $k$  VARIES SUBSTANTIALLY WITH TEMPERATURE. HENCE, IT IS TAKEN AS A VARIABLE.

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So that equation can be written like this  $V$  minus  $\Delta V$  whole square  $k w dx$  divided by  $y$  is equal to  $\rho_e w y v C_e \Delta T$ , now  $dx$  is the small moment in  $X$  direction so this equation can be written as  $dt$  over  $dx$  simplification of the above equation will give you  $dt$  over  $dx$  is equal to  $V$  minus  $\Delta V$  square  $k$  divided by  $\rho_e y$  square  $v C_e$  is equal to  $A k$ .

Now please note here that  $V$  is constant, it is not varying during the process,  $\Delta V$  is also assumed to be constant that is the over potential and that does not vary during the ECM process that is assumption,  $\rho_e$  remains constant that is the electrolyte density, inter electrode gap is assumed to remain constant for a short period of time and at a short length of the flow direction.  $V$  is the velocity of the electrolyte and  $C$  is the specific heat both of them are again considered to be small.

Let us take it like this that this is the tool and this is the work piece and this we are assuming that at a small distance that is dx these parameters are constant specially the inter electrode gap and flow velocity they are considered to be constant that means only k remains as variable and k is electrical conductivity of the electrolyte so we can write it as A dot k where A is constant now please do not confuse this A with the cross sectional area this is not the cross sectional area some constant is there. If you see here w is shown over and y is also shown, both are shown over there you can see this is the w and this is the y.

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**TEMPERATURE GRADIENT EVALUATION IN ECM**

AS IN THE PREVIOUS EQUATION, TEMPERATURE GRADIENT IS A FUNCTION OF K WHOSE EQUATION HAS BEEN DERIVED IN THE PREVIOUS SLIDE (WITHOUT ACCOUNTING FOR VOID FRACTION.)

$$\therefore \frac{dT}{dx} = Ak_0 [1 + \alpha(T - T_0)]$$

$$\int_{T_0}^T dT = Ak_0 \int_0^x [1 + \alpha(T - T_0)] dx$$

Solution of this equation results

$$(T - T_0) = \frac{1}{\alpha} [\exp(\alpha Ak_0 x) - 1]$$

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32

Now as in the previous equation temperature gradient is a function of K that is the conductivity whose equation has been derived in the previous slides without accounting for the void fraction later on we accounted for the void fraction also, so this temperature gradient can be now be represented in place of k you can write down k 0 multiplied by 1 plus alpha T minus T0 parenthesis closed bracket closed.

So this is the equation which can be used to calculate the temperature gradient to find the final solution of this you can simplify this equation like dt is equal to A k 0 multiplied by the value of the k dx, now you can integrate both sides from T0 to T1 or TF or T and dx side you can integrate from 0 to x now integration and then you have to evaluate the constant of integration and if you substitute the values then you find this as T minus T0 is equal to 1 over alpha multiplied by

exponent  $\alpha A k_0 x$ ,  $A$  is the constant that I derived in the last slide, note the cross sectional area  $x$  minus 1.

Now from this equation you can find out what is the temperature variation along the electrolyte flow and from the earlier equation  $dt$  over  $dx$  you can find out what is the temperature gradient along the electrolyte flow path.

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**CURRENT DENSITY EVALUATION IN ECM**

THE CURRENT DENSITY,  $J$ , VARIES ALONG THE GAP AND IT CAN BE EVALUATED AS FOLLOWS

$$J = \frac{(V - \Delta V)k}{y}$$

SUBSTITUTE THE VALUE OF  $k$  IN THIS EQUATION AS DERIVED EARLIER

$$J = \frac{(V - \Delta V) \cdot k_0 [1 + \alpha (T - T_0)]}{y}$$

SUBSTITUTE THE VALUE OF  $(T - T_0)$  IN THIS EQUATION AS DERIVED EARLIER

$$J = \frac{(V - \Delta V) \cdot k_0}{y} [\exp(\alpha \cdot A \cdot k_0 \cdot x)]$$

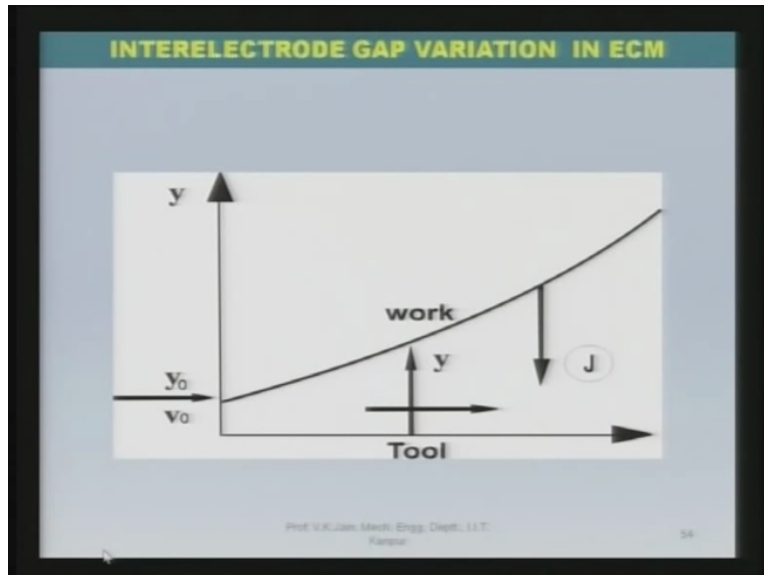
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Now current density can also be evaluated by considering the variation in the electrical conductivity of the electrolyte and we all know this equation we can derive after substituting the value of  $k$  in this equation  $V$  minus  $\Delta V$  divided by  $y$  multiplied by  $k_0$  that is the initial electrical conductivity of the electrolyte multiplied by  $1 + \alpha (T - T_0)$ , substitute the value of  $T - T_0$  in this equation as derived earlier then if you write this if you substitute the equation of  $T - T_0$  you get final equation as like this  $J$  is equal to  $V$  minus  $\Delta V$   $k_0$  divided by  $y$  multiplied by exponent  $\alpha A k_0$  into  $x$ .

From this equation we can finally calculate the current density at a particular point after accounting for the variations in the temperature as well as the void fraction.



(Refer Slide Time: 31:40)



Now this indicates, this figure indicates that how the current density is varying along the electrolyte flow path you can see work piece is shown there, tool is shown and the shape of the work piece is continuously changing from the inlet to the outlet it is not remaining as the straight line and as the electrolyte flow path or you move along the electrolyte flow path, the current density may decrease because of the higher inter electrode gap as shown there the arrow moving showing downward along with the capital J. So this indicates that current density is varying continuously along the electrolyte flow path.

(Refer Slide Time: 32:30)

**Q. 1.** In ECM process, pure iron is taken as a workpiece material, and the desirable  $MRR_v = 5 \text{ cm}^3/\text{min}$ . Determine the current required.

**Solution:** Given data:  $MRR_v = 5 \text{ cm}^3/\text{min}$

From the data book, we get  $\rho_a = 7.85 \text{ g/cm}^3$ ,  $E = 28.0 \text{ g}$   
 $F = 96500 \text{ coulombs}$

$$MRR_g = \rho_a \times MRR_v$$
$$\frac{I E}{F} = \rho_a \times MRR_v$$
$$I = \frac{7.85 \times 5 \times 96500}{28 \times 60}$$
$$I = 2254.53 \text{ A}$$

**Please Note** – All parameters should be taken in the same units (g, kg, mm, m, ....)

Now with this introduction to the basic theory of electrochemical machining let us attempt to solve some of the numerical problems in electrochemical machining cases. Let us take first problem, in ECM process pure iron is taken as a work piece material and the desirable volumetric material removal rate is 5 cubic centimeter per minute determine the current required to achieve this particular material removal rate.

The data given are volumetric material removal rate as 5 cubic centimeter per minute, from the data book or a book from electrochemical machining you can find out the density of the pure iron as 7.85 grams per cubic centimeter and the chemical equivalent of it is taken as 28 gram, we already know that Faraday's constant is 96500 coulombs we also know this equation with the help of which we can calculate the material removal rate in terms of gram per second and this is given by rho a multiplied by  $MRR_v$  where  $MRR_v$  is volumetric material removal rate.

So we can write it like this  $\frac{I E}{F}$  is equal to rho a multiplied by  $MRR_v$  substitute because we already know that  $MRR_v$  required is 5 cubic centimeter per minute so substitute that value rho a is known 7.85 coulombs, Faraday's constant that is 96500 coulombs and we get, we have to convert this per minute into per second that we divide by 60. So finally we get the current as 2254.53 ampere.

Now please note that all parameters should be taken in the same units whether millimeter then for everything you have to account for millimeter or centimeter then everything should be in terms of centimeter or meter.

(Refer Slide Time: 34:58)

**Q. 2.** The composition (% by wt) of Nimonic 75 super alloy is given below:

	Ni	Cr	Fe	Ti	Si	Mn	Cu
% by weight	72.5	19.5	5.0	0.4	1.0	1.0	0.6
Density	8.9	7.19	7.86	4.51	2.33	7.43	8.96
Atomic Weight	58.71	51.99	55.85	47.9	28.09	54.94	63.51

Calculate MRR (cm<sup>3</sup>/min) when a current of 1000 A is used. Use the lowest valency of dissolution for each element.

**Solution:**

Given data:  $I = 1000 \text{ A}$ ,  $E = A/Z$ ,  $F = 96500 \text{ C (=A.s)}$

$$M M R_v = \frac{I E_a}{\rho_a F}$$

Here we do not know  $E_a$  and  $\rho_a$ .

$$E_a = \frac{1000}{\sum \frac{X_i Z_i}{A_i}}$$

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Now let us take next problem, the composition percentage by weight of Nimonic 75 super alloy is given as follows, now here we find that there are various constituents of the alloy like nickel, chromium, iron, titanium, silicon, manganese and copper and it gives the percentage by weight composition of each element in the alloy that is 72.5 percent nickel, 19.5 percent chromium and so on, the density of each element that is nickel 8.9 gram per cubic centimeter, chromium 7.19 gram per cubic centimeter and so on are given over there and the atomic weight of each of them is given like 58.71, 51.99 and so on.

We have to calculate material removal rate in terms of cubic centimeter per minute that is volumetric material removal rate when a current of thousand ampere is used. Use the lowest valency of dissolution for each element. Here you are not given with the valency of different constituents of the alloy that is nickel, chromium, etc. that you have to find out from the handbook or the book on the electrochemical machining or advanced machining processes as I have already told you from there you can find out these values of the valency of various elements.

Other data that are given here is the current, that is 1000 ampere, chemical equivalent  $E$  is equal to  $A$  by  $Z$  that we have already discussed  $A$  is the atomic mass of the element and  $Z$  is the valency at which it is dissolving and you are given that you have to take the lowest valency of dissolution of any element and  $F$  we already know Faraday's constant.

Now volumetric material removal rate we know is given by  $I E_a$  over  $\rho_a F$ , here please note that  $E_a$  is the chemical equivalent of the alloy and  $\rho_a$  is the density of the alloy, we have to find out chemical equivalent of the alloy that is not given and we also have to find out the density of the alloy while we are given the density of the individual element.

We can find out the chemical equivalent of the alloy from one of the methods that we have already discussed that is  $E_a$  is equal to 100 divided by summation  $X_i Z_i$  over  $A_i$ . Now as I have mentioned you have to find out the  $Z_i$  from the handbook or the book on the advance machining processes.

(Refer Slide Time: 37:59)

The slide displays the following calculations:

$$E_a = \frac{100}{\frac{72.5 \times 2}{58.71} + \frac{19.5 \times 2}{51.99} + \frac{5 \times 2}{55.85} + \frac{0.4 \times 3}{47.9} + \frac{1 \times 4}{28.09} + \frac{1 \times 2}{54.94} + \frac{0.6 \times 1}{63.51}}$$

$$E_a = 27.68 \text{ g}$$

$$\rho_a = \frac{100}{\sum \frac{X_i}{\rho_i}}$$

$$\rho_a = \frac{100}{\frac{72.5}{8.9} + \frac{19.5}{7.19} + \frac{5}{7.86} + \frac{0.4}{4.51} + \frac{1}{2.33} + \frac{1}{7.43} + \frac{0.6}{8.96}}$$

Substitute the values of  $\rho$  and  $X$ .

$$\rho_a = 8.187 \text{ g/cm}^3$$

$$MRR_v = \frac{I E_a}{\rho_a F} = \frac{1000 \times 27.68 \times 60}{8.187 \times 96500}$$

$$MRR_v = 2.102 \text{ cm}^3/\text{min} \quad \text{Answer}$$

Now taking those values you can see here  $E_a$  is equal to 100 divided by 72.5 into 2 by 58.71 so here 72.5 is the atomic mass of the element of the alloy that is the nickel, 2 is its valency and so 58.71 is the, that is the atomic mass of the nickel so using these values for different elements we can calculate the value of chemical equivalent of the alloy as 27.68 gram.

Same way we have the equation for calculating the density of the alloy that is 100 divided by summation  $X_i$  over  $\rho_i$ ,  $\rho_i$  is the density of the  $i$ th element  $X_i$  is its percentage contribution to the alloy, substitute the values and take the density from the previous table that is given in the problem you can substitute these values and you get this one and by simplification of this you will get the density of the alloy as 8.187 gram per cubic centimeter.

You need not to take it to 4 decimal place or 5 decimal place, normally you should take it to only 2 decimal places that is good enough so we can now calculate the volumetric material removal rate that is  $I E_a$  over  $\rho_a$  over  $F$  we have already calculated the value of  $E_a$  as 27.68 gram and  $\rho_a$  as 8.187 gram per cubic centimeter substitute these values in this particular equation  $I$  is 1000 ampere and this is a constant and 60 is the conversion of minutes into the seconds.

You get volumetric material removal rate as 2.102 cubic centimeter per minute, now if you want to find out the material removal rate in terms of grams then you can divide this by the density of the alloy you will get the gram per minute as the mass material removal rate.

(Refer Slide Time: 40:32)

**Q. 3.** In an ECM operation, 10 V D.C. power supply is used. The conductivity of the electrolyte used is  $0.2 \Omega^{-1} \text{ cm}^{-1}$  and the feed rate used is 1 mm/min. The work piece is of pure iron. Calculate the equilibrium gap. Take 'over voltage' equal to 1.5 V.

**Solution:** Assume  $A_t = 55.85 \text{ g}$ ,  $Z = 2$ ,  $\rho = 7.86 \text{ g/cm}^3$

**Given data:**  
 $k = 0.2 \Omega^{-1} \text{ cm}^{-1}$ ;  $E = A/Z = 55.85/2$ ;  $\rho = 7.86 \text{ g/cm}^3$   
 $F = 96,500 \text{ A.s}$ ;  $f = 0.1 \text{ mm/min}$ ;  $Y_e =$  Equilibrium interelectrode gap

$$Y_e = \frac{(V - \Delta V) \eta k E}{\rho F f}$$

Substitute the values of different parameters from the given data.

$$= \frac{(10 - 1.5) \times 1 \times 0.2 \times 55.85 \times 60}{2 \times 7.86 \times 96500 \times 0.1}$$

**$Y_e = 0.375 \text{ mm}$**

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Let us take the third problem, in an electrochemical machining operation 10 volt direct current power supply is used, the conductivity of the electrolyte used is 0.2 per ohm per centimeter and the feed rate used is 1 millimeter per minute.

The work piece is of pure iron calculate the equilibrium gap that is  $Y_e$ , take over voltage equal to 1.5 volt. Here let us assume that atomic mass of the iron is 55.85 gram, valency at which the iron is dissolving is 2 and density is 7.86 gram per cubic centimeter. The data that are given in the problem are conductivity is 0.2 per ohm per centimeter, we can calculate chemical equivalent  $A$  by  $Z$  from these two values that comes out that you can calculate and density is already given over here.

We already know Faraday's constant, feed rate is given there as 0.1 millimeter per minute and equilibrium inter electrode gap we have to calculate. Now we know the equation of equilibrium inter electrode gap is equal to  $V - \Delta V - \frac{k E}{\rho F f}$ , now this  $F$  is the feed rate. Substitute the values of this because this is the 10 volt given over there, this is the over potential other values like efficiency, note it here efficiency has been taken as 100 percent that is 1, unless it is given you can always take it as 1.

$k$  is 0.2 and  $E$  is 55.85 and you can take this as the density, Faraday's constant, this is the feed rate, now substituting all these values we get  $Y_e$  is equal to 0.375 millimeter. So this gives you the equilibrium inter electrode gap, now in this way you can solve various real life problems using the theory that we have discussed in these two lectures. So thank you very much.