**Acoustics**
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**Lecture 3**
**Module 2**
**Waveguides, Transmission Line Equations**
**And Standing Waves**

Hello again, so in the last class we had derived relation for one-dimensional wave equation and we were just starting to develop solutions for this equation and that is what we will continue today. So as we had developed it earlier, so I will just briefly rewrite that equation once again.

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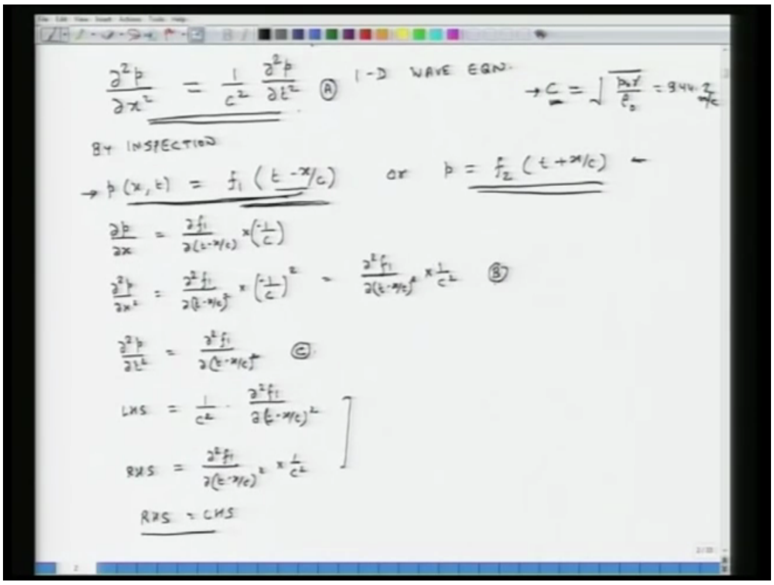


So far one-dimensional equation, the equation for pressure is second derivative partial derivative partial derivative of pressure with respect to x that is the space dimension and that equals 1over C square del 2 P over del t square, so that is once again one-dimensional wave equation, okay. And what we had said in the last class was that if I compute the value of C, so C is equal to square root of P not gamma over rho not and that commuted value of C it comes to about 344.8 meters per second.

And if I measure the speed of, so excuse me, so this is 344.2 meters per second and if I measure the speed of sound in air at sea level at room temperature conditions then the value of C, so the measured value of speed of sound is 344.8 meters per second. So what we see here is that this value of C and the measured value of speed of sound they are fully close.

And what we will see is the reason why C comes to be extremely close the actual speed of sound and the reason why this constant which is nothing but square root of P not gamma over rho not is nothing but actually (()) (2:52) speed of sound. So from the last class we had just started on a journey and we said that a solution by inspection I can write the solution for this partial differential equation, second order partial differential equation in such a way that either P which is dependent on x and time is a function of t minus x over c or P is a function of t plus x over c.

So this is another form of solution which we can say that is a valid solution by inspection and now what we are going to do is, we are going to prove that there is relation indeed is an actual solution of this partial differential equation. So I know that del P over del x if I use this equation is del f1 and then I am differentiating this f1 with respect t minus x over c times the derivative of this entire term t minus x over c with respect to x, so that is 1 over c negative.

Similarly if I take the second derivative I get del2P over del x square is second derivative of f1 with respect to t minus x over c times 1 over c whole square, so what I get is del 2 f1 over del t minus x over c square times 1 over c square. Similarly I can write that the second derivative of pressure with respect to time is nothing but del 2 f1 over del t minus x over c square.

So let us call this relation A, we will call this relation B, the third relation is C and now what I will do is I will plug B and C back into A and what I get is that the left hand side of the equation is 1over C square times del 2 f1 over del t minus x over c square and the right-hand side also comes out to be the same thing. So right-hand side is 1 over c square times del 2 P over del t square which is this thing into1 over c square.

So what we find is that once I B and C into this equation A which is the one-dimensional wave equation for pressure then this particular function for pressure is satisfied. So RHS equals LHS, so essentially what that is, that this particular form of a function is a valid form S in the context of being valid solution for this equation. Likewise we can also prove sufficiently easily that this particular form where this function f2 which depends on t plus x over c also is a valid solution for one-dimensional wave equation. So this is a general solution for wave equation.

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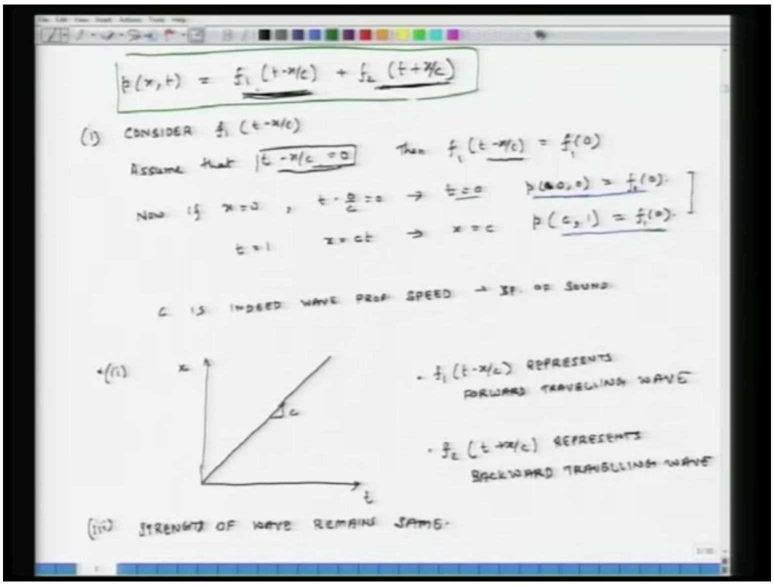


So in general what we can write that a valid solution for one-dimensional wave equation is P of x,t is nothing but f1 t minus x over c plus f2 t plus x over c because it is a linear system, so all individual solutions if I add them up they will also be a valid solution for a linear partial differential equation. So in the next 5 to 10 minutes what we will try to explore is the meaning of the solution.

What does this solution mean? What does this solution mean? The physical interpretation of these functions, so that is what we are going to do. So we will start with ft minus x over c. So we will consider f1 t minus x over c and we know we have proved just now that this is a valid solution for one-dimensional wave equation. So now in this case we take special situation.

Let us consider that we assume that t minus x over c equals 0 then f t minus x over c is nothing but f of 0. Now what we are going to do is we are going to plot for this condition t and x but before we do that let us consider one more case. So if this is the condition that t minus x over c is 0 then we can say that now if x equals 0 then t minus 0 over c equals 0 implies t equals 0 and the pressure is P is 0, 0 is equal to f of 0.

Now let us assume that t equals 1 then x equals ct implying, so then x equals c and thus we get P and the value of x is c, value of time is 1 and that is equal to f of 0. So consider these 2 relations, what these 2 relations are showing is that an instant time when time was 0 and x was 0, so the value of pressure was f1 of 0.

Now after 1second time increases from 0 to 1 and x increases from 0 to c and f1 the value of f1 remains the same. So essentially what I am seeing is that in 1 second this disturbance pressure which was initially P00 it has moved by a distance of c, in 1 second this disturbance has moved by a distance c. So what that tells me is, that if there is a function f1 t minus x over c because it satisfies the 1-D wave equation and thus it really presents a pressure disturbance and speed of propagation of this pressure disturbance is c meters in 1 second that is the velocity the speed of propagation is c. The speed of propagation is c.

So what we see here is that c is indeed wave propagation speed and this is also called speed of sound. So this is the first implication of the fact that a general function which can be expressed as in this form f1 t minus x over c because it satisfies the one-dimensional wave equation, the speed of sound is nothing but indeed c and we can make a similar conclusion if we assume that the solution is f2 of t plus x over c you will get exactly the same conclusion that the wave propagation speed for pressure comes out to be c which is same as speed of sound.

So the second inference from this is that if for this specific case t minus x over c is 0 then I plot, let us say I plot t and x, so the second inference we can draw is this function which is of the form f1 t minus x over c represents a wave which is travelling forward which is a former travelling wave.

So what does that mean? So let us plot again t minus x over c is 0 in this case and let us say for this we plot this equation. So, on the horizontal axis I am plotting t and on the vertical axis I am plotting x. So essentially I get a straight line and the slope of this is c meters per second and what this line tells me is that as time is growing, so is x that is in physical terms as time is increasing the disturbance is travelling in the positive x direction.

So f1 t minus x over c represents forward travelling wave. Likewise I can argue with validity that f2 t plus x over c represents backward travelling wave because if I plot t plus x over c and let us say I assume t plus x over c equals constant let us assume that constant to be 0 then if I plot that then the slope of the line would be negative and what that means is that as time is growing x is moving in the negative direction.

So what that tells me is that as time is growing the wave is travelling in the negative x direction, so it is a backward travelling wave. A good example of a backward travelling wave could be a reflected wave. So you have a wave moving forward it hits a rigid surface it gets reflected and the reflected wave is essentially f2 t plus x over c. The third conclusion I can make is that as f1 or f2 is moving forward or backward respectively the strength of the wave essentially remains constant.

So that is what we saw earlier, that at t equals 0 and x equals 0, P00 was f1 of 0 then after 1 second once the wave has travelled forward by c meters, the strength of the wave still remains f1 0, so what that tells me is that the strength of the wave remains same.

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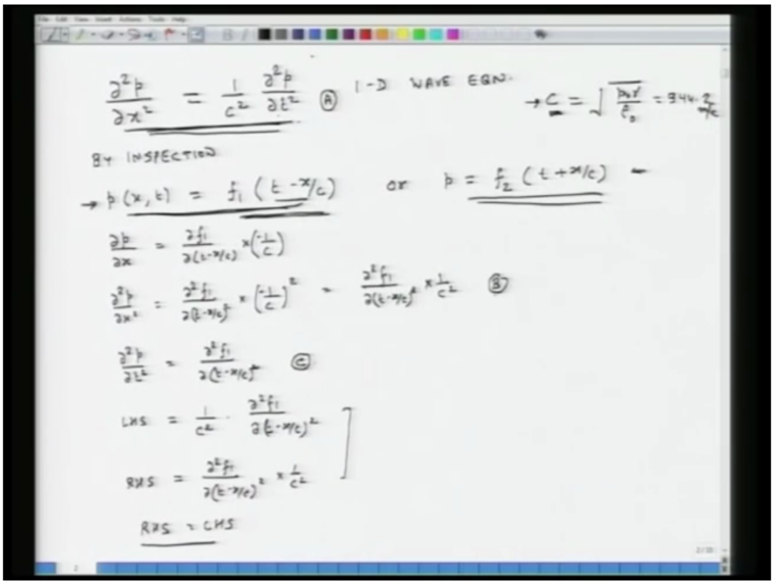


So the wave which is represented by one-dimensional wave equation which is this relation is essentially a way which does not change its strength over a period of time and also over x and that is essentially because in our entire formulation we did not assume that there was damping present in the system. If we had modelled damping also in the system then we would have seen that the strength of the wave starts decreasing as we march ahead on the time or x axis on the time axis. So that is pretty much the overall interpretation of this general solution.

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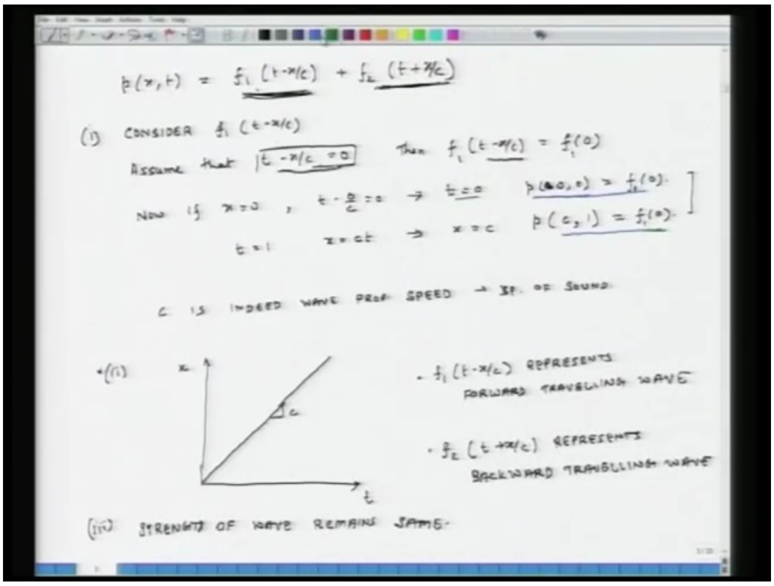


So this is the general solution and the interpretation of f1 t minus x over c is that the speed of sound is nothing but same as c which is a constant is calculated through the relation P not times gamma divided by rho not. Second thing is that f1 t minus x over c represents so forward travelling wave and f2 t plus x over c represents a backward travelling wave and the third thing is that the strength of the wave remains the same over a period of time.

So then the next question, a logical question to ask is that where do we in reality encounter such waves because when I am speaking my sound is heard in all the directions, it is not only travelling in just x direction but it is moving in X, Y and Z directions. So a lot of sound propagation phenomena is such that the propagation happens in all the directions. Now the one-dimensional wave equation assumes that the variation in x excuse me variation in y and variation in z is exactly 0. So essentially it is a one-dimensional wave equation, so again the question is that where do we encounter such waves.

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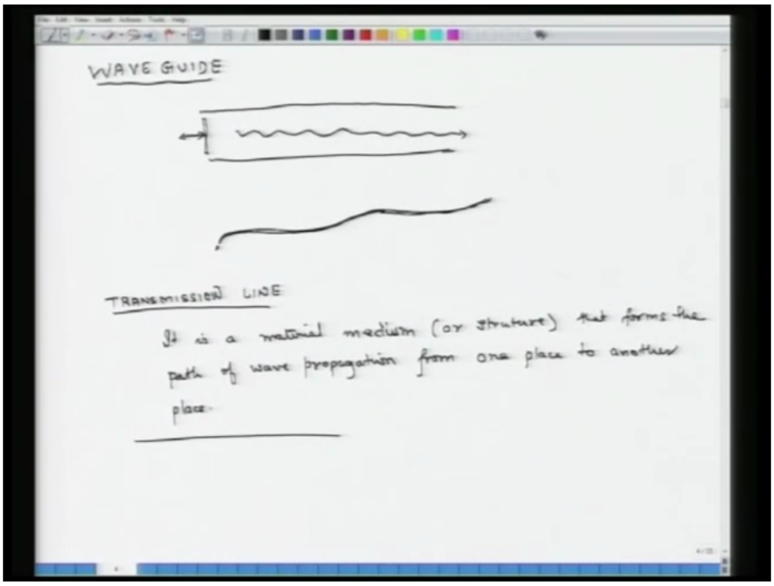


So in this context we introduce 2 terms the first term is a waveguide. So what is the meaning of the word waveguide? It is essentially a structure or a device which guides are wave. So for instance and the this waveguide is a term which is not only used in the area of acoustics but it is also used in area of optics, in area of electrical waves and so on and so forth. So one example of a wave guide could be a tube.

It could be a tube and I am generating some pressure fluctuation through some piston mechanism, so this piston is moving back and forth is generating some pressure wave and this pressure wave is travelling along the waveguide in just one single dimension there is no wave travelling in the y direction or in the z direction. Another example of a waveguide could be a fibre-optic cable.

So you have a light source here and because of the way this fibre-optic cable is designed the light travels along the length of this very long fibre optic cable, so this is also another example of a waveguide. One more example of a waveguide could be essentially just very long electrical wires.

So across the whole length of the wire electricity travels and in some cases it travels thousands of kilometres essentially from the generating stations or generating power plants to the home where electricity is being consumed and all that transmission happens through waveguide like devices.

The second term I would like to introduce is transmission line. So a transmission line is a term which is which has kind of similar implications as of a waveguide but it is used in a more general sense. So the definition of a transmission line could be that it is a material medium or it could be a structure that forms the path of wave propagation from one place to another place.

So a transmission line is a material medium or it could be a structure that forms the path of wave propagation from point A to point B and examples of it could be electrical wires, coaxial cables and waveguides for sounds, tubes and hollow ducts for sounds, electrical power lines, dielectric slabs and fibre-optic cables and so on and so forth.

So what we are going to do now is in the context of acoustics still itself we are going to develop equations which help us understand propagation of sound in tubes and ducts and these equations are called transmission line equations and please remember that these equations are specific they are specific to propagation of sound.

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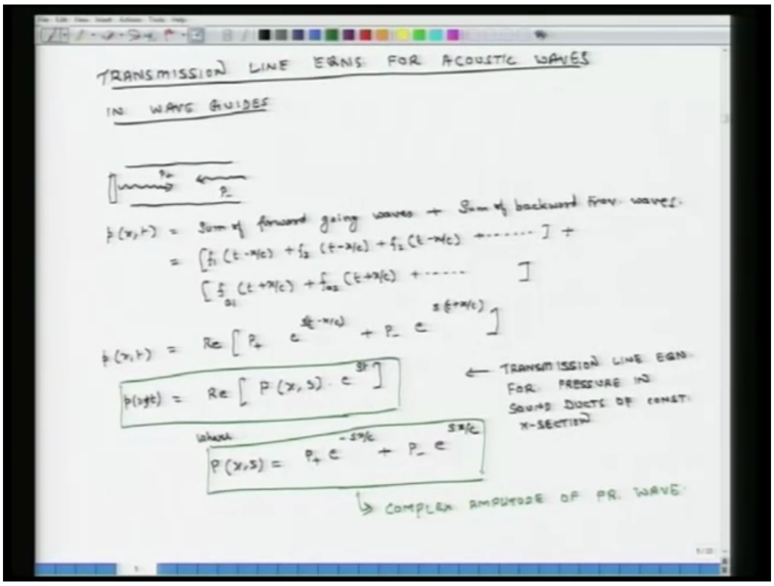


So what we are going to develop is transmission line equations for acoustic waves in waveguides. So you have a waveguide and it could be a shortwave guide or a long waveguide and the aim is to develop equations which help us understand how is sound propagating along this waveguide? So this sound could be moving forward and let us say it is complex amplitude is P plus and part of the sound could also be getting reflected and the complex amplitude of the reflective wave could be that let us say P negative.

So we can write that P of x, t which is the pressure it is a function of space that is x and time and that is essentially sum of forward going waves plus sum of backward travelling waves. So the sum of forward travelling waves could f1 t minus x over c plus f2 t minus x over c plus f3 t minus x over c and so on and so forth.

So all these are forward travelling waves, so I bracket them and then the sum of backward travelling waves, so I said I designate that as fa1 t plus x over c plus fa2 t plus x over c, so all that is the reflected waves. Now if we have a situation that the forward travelling wave is harmonic in nature let us say I have piston and it is generating sinusoidal waves.

At this point if I have a piston then the forward travelling wave and the backward travelling wave they will be harmonic in nature they will be sinusoidal or sinusoidal in nature. So in that case I can rewrite this equation as P of x, t is nothing but real of, so here now I start using complex variables P plus which is a function of s e st minus x over c and please remember that s is a complex frequency here.

So this is the forward travelling wave and P minus e st minus t minus x over c and this is again a function of complex frequency, so excuse me this should be positive and this I can rewrite as real part of t x, s times e st where P x, s is nothing but P plus e to the power of minus x, x over c plus P negative e to the power of sx over c. So this is my equation for pressure and this equation is called transmission line equation, this is transmission line equation for pressure in sound ducts of constant cross-section, okay.

So this is the transmission line equation for sound ducts, sound travelling in sound ducts and these sound ducts have constant cross-section as I move in x and this term P x of s is also called as, so actually this is not right, so P x of x is also called complex amplitude of pressure wave. So here P depends on x and P depends on s both.

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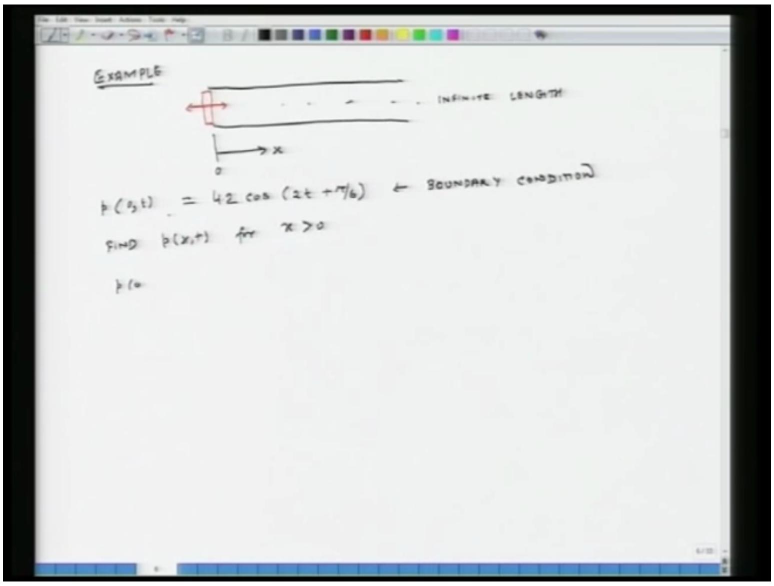


So let us do an example where we will try to calculate the complex amplitude and from that values of P plus and P minus. So this is an example where we have a straight tube and I have a member let us say a piston and this piston is vibrating back and forth, so it is generating some sound waves. Let us say my coordinate system starts from here, so I am counting x from this point.

So x is 0 at this point, this tube is of infinite length, so it starts from 0 but it goes on till infinity it has in finite length and once again it is one-dimensional, so its cross-section is not changing over distance and I know as a boundary condition that the pressure generated by this piston at x equals to 0 and for varying and for at x equals 0 it changes with time and that can be expressed as 42 cosine 2t plus pi over 6, so this is my boundary condition.

So then the question is that find P of x, t for x greater than 0. So if I know the boundary condition that near the piston the pressure is 42 cosine 2t plus pi over 6, how is pressure changing in time and as I also move along x that is a question.

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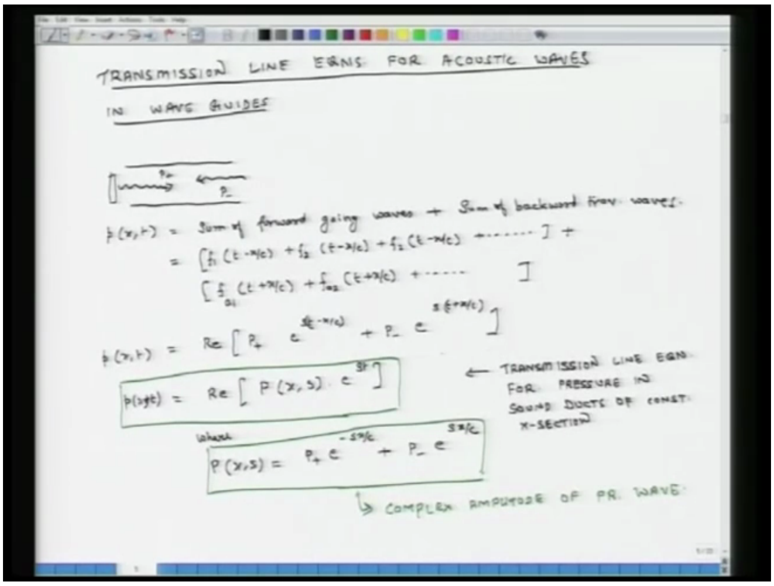


So as the first step what we do is that we plug-in this boundary condition in this long relation and there we put x equals 0, so that is what we are going to do.

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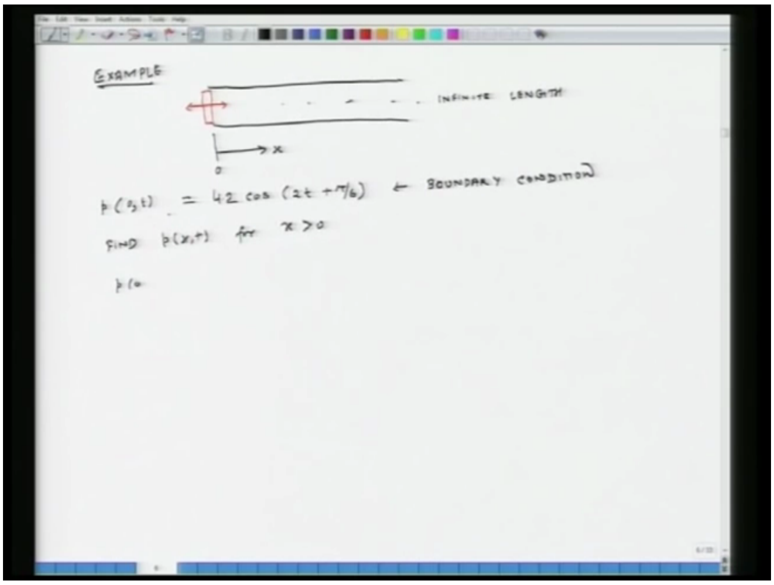


So P 0, t equals 42 cosine 2t plus pi over 6 and that is equal to am going to use this relation where P plus P is this entire thing and I am going to put x to be 0 here. So what I get is, so let refer back, so I get P x, s is essentially P plus plus P minus because x is 0. Now the next thing I am going to do is I am going to represent this term in exponential firm. So that once I do that I get real 42 exponent of 2t plus pi over 6 times j equals real of P plus plus P minus e st and now I resolve this into 2 specific components, so I get real of 42 e 2jt times t pi over 6 j equals real of P plus plus P minus e st.

So now from inspection I can say I can compare this term and this term and I conclude that S equals 2j and also I conclude if I compare this term and this term, so the ones in blue they are all constants they are not changing with time or space, so then I say 42 e pi over 6 times j equals P plus plus P minus but we know that this is an infinitely long wave. So one starts from one end it just keeps on propagating and it never gets a chance to get reflected.

So what that tells me is P minus equals 0, so again what that tells me is that P plus equals 42 e to the power of pi j over 6. So now we have calculated through the boundary conditions that P minus is 0 because this is for wave travelling in a tube which is infinitely long, so all the waves which are getting generated here they keep on travelling forever and they never hit a surface or change an impedance which causes reflection.

So P minus is 0 and as a consequence we figured out that P plus is 42 e pi j over 6 and finally the complex frequency was which is s is same as 2 times j. So now what we will do is, we will rewrite the original wave propagation equation for a transmission line with constant cross-section and in that equation we will plug in these values.

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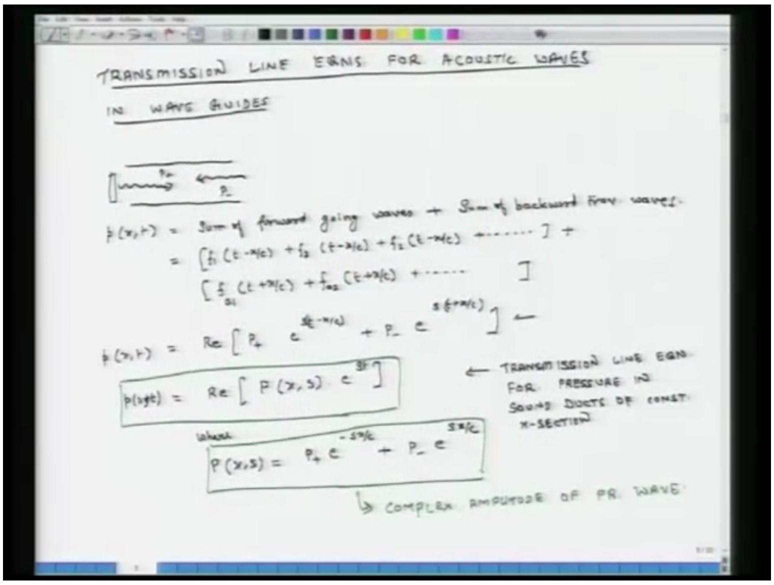


So revisiting this equation which is this one we will rewrite it.

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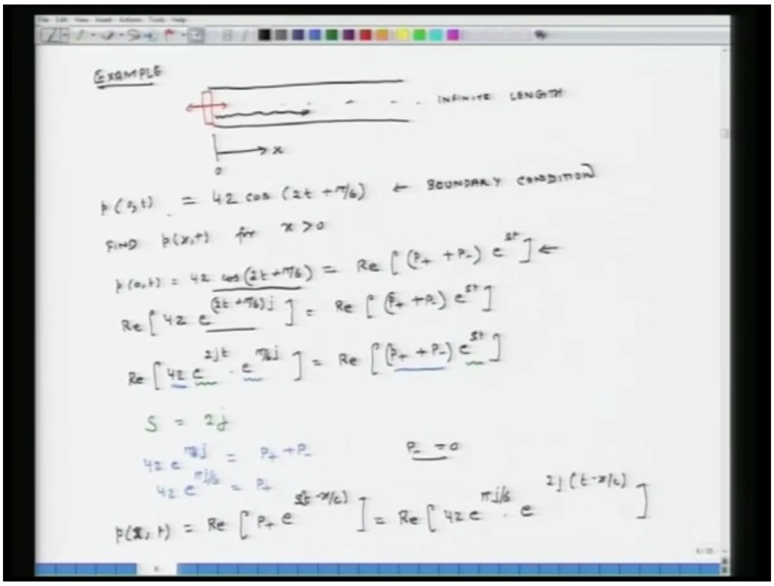

So P x, t is real P plus e to the power of st minus x over c and we know that P minus is 0, so I am going to (()) (37:09) that term. So I have just rewritten this particular equation and I have dropped out the term associated to P negative because there is no reflection or backward travelling wave in this vertical example.

And now I start plugging in the values of P plus and S and what I get is real of 42 e pi j over 6, so that is P plus times e to the power of st minus t minus x over c. So s is 2j t minus x over c, so that is my wave propagation equation for this particular example. So now what I will do is I will go one further Step and simplify it and take its real component.

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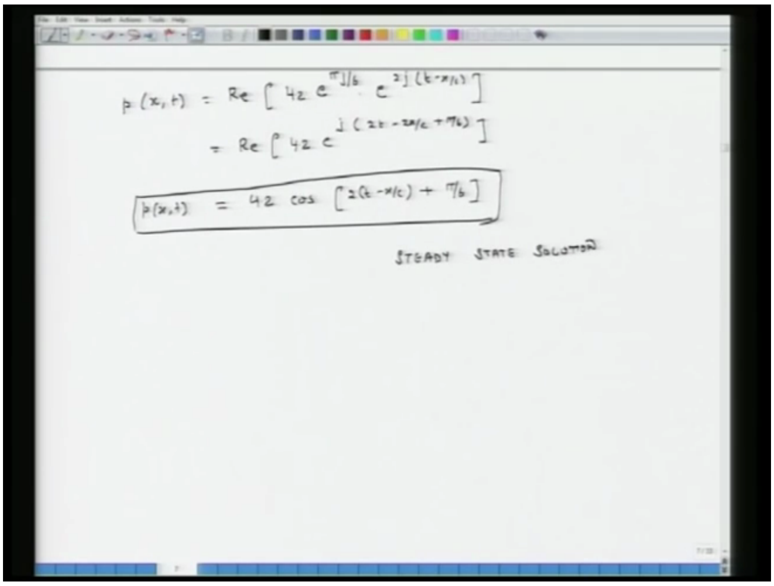


So moving on to the next page, I will just rewrite the original equation x, t is real of 42 e pi j over 6 times e to the power of 2j, so I think I have to t minus x over c this equals real of 42 e to the power of I take j out 2t minus 2x over c and then I have to add this pi over 6, so this is nothing but 42 cosine of 2t minus x over c plus pi over 6. So I will just make it cleaner, so this is my relation for P of x and t.

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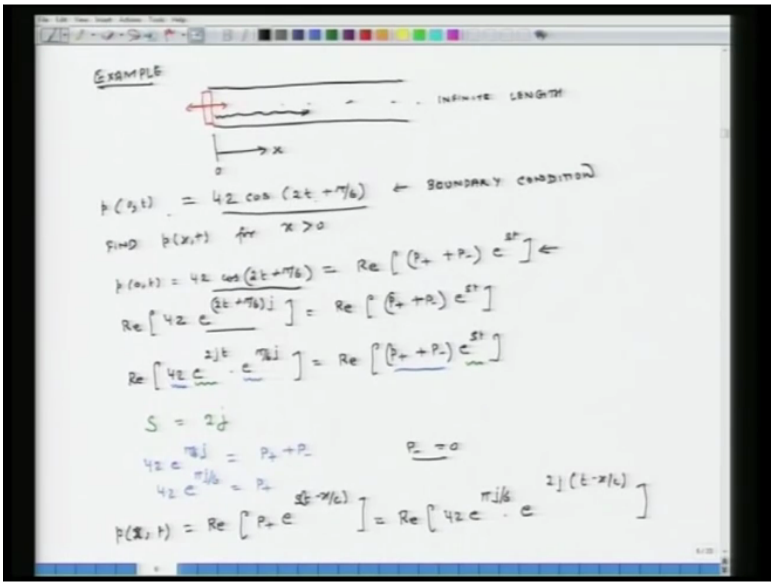

So this is the steady State solution for the example of a wave travelling in a tube which is infinitely long and at x equal to 0 there is a piston which is generating a forward travelling wave whose form is of this type 42 cosine 2t plus pi over 6.