Acoustics

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Lecture 4
Module 1
Review: transfer function and bode plots

Good afternoon welcome to this course on acoustics in the last lecture we had touched upon the area of bode plots, pole zero plots and phase and magnitude plots for transfer functions in the context of providing the students are people who are interested in learning about acoustics, some grounding in some of the basic concepts which will be quite often used later in the course of acoustics.

So with that intention in the last class I had covered in somewhat detail on the concept of bode plots and specifically I had developed plots for magnitude and bode plots for phase for complex 0 functions and for transfer functions which includes simple zeros or for transfer functions which include just simple poles. So today we will use all that information which we talked about in the last class and we will go a step further we will start with a transfer function which will have which will be essentially a combination of poles and zeros and for this transfer function we will construct a bode magnitude plot and a bode phase plot.

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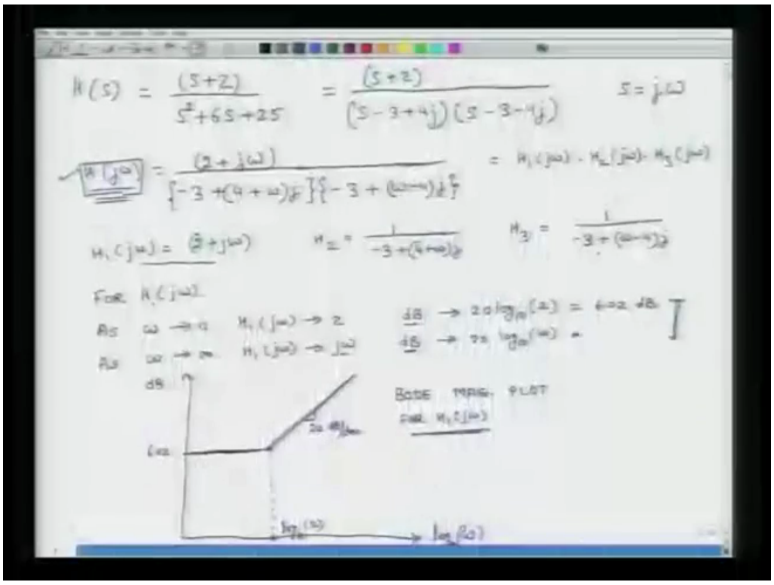

So in this context let us say that the transfer function for which we are interested in developing bode plot for magnitude and phase it looks like this S Plus2 over S square plus 6S plus 25, okay. So if I find the roots of the denominator essentially what I get is in the numerator I still have S Plus2 and in the denominator I have S minus 3 plus 4j times S minus 3 minus 4j.

Now remember that these bode plots are 4 the case that S equals j omega. So in this transfer function I replace every S by a j omega. So what I get is HS, excuse me, H of j omega is 2 plus j omega in the numerator and in the denominator I have minus 3 plus 4 plus omega times j this is one linear factor of the denominator and the other linear factor is minus 3 plus omega minus 4j, so this transfer function is essentially a product of 3 individual functions transfer functions which is H1 j omega times H2 j omega times, H3 of j omega where H1 j omega is 2 plus j omega H2 of j omega is 1 over minus 3 plus 4 plus omega times j and H3 j omega is minus 3 plus omega minus 4 times j. So H1 is 1 over minus 3 plus 4 plus omega times j and this is the expression for H3.

Now as we are trying to develop a bode plot for magnitude and a bode plot for this transfer function what we will do is first, let us start we are just thinking about the magnitude part, so we will construct a bode plot for magnitude for H1, we will construct the same thing for H2 and the same thing for H3 and then because these bode plots are essentially especially the magnitude is a logarithmic function because on the vertical axis we are plotting decibels, albeit have to do is add up these 3 bode plots and we will get the final bode plot for magnitude for the original function which is H, so that is what we are going to do now.

So now what we are going to do is, we are going to construct for bode magnitude plot for H1. So once again that is my original function as omega is approaching 0, when omega becomes extremely small H1 j omega approaches to H1 j omega, so in decibels this approaches 20 log in base 10 of the number 2 which is essentially 6.02 decibels similarly as omega is approaching infinity H1 of j omega approaches j omega, so if am going to calculate the strength of H1 in terms of decibels that approaches 20 logarithm in base 10 of omega because the magnitude of j omega is omega.

So these are the 2 asymptotes, once again this is the low frequency, the first one is low frequency asymptote and the second one is the high frequency asymptote. So far H1 the bode plot, bode magnitude plot is going to look something like this bode magnitude plot for H1. So, on the horizontal axis I am plotting logarithm of omega and again it is logarithm on base 10 and on vertical axis I am plotting decibels.

I know that my low frequency asymptote is a horizontal line. So horizontal line which cuts the Y axis at 6.02 decibels also I know the crossover point is such that when omega becomes to then the high frequency then after that the high frequency asymptote becomes more important. So my crossover point, this is my crossover point and that is essentially log in base 10 of 2 and my high frequency asymptote is a positively sloped line, the slope being 20 decibels per decade. 20 decibels per decade, so this is my high frequency asymptote and this is my low frequency asymptote. So this is my bode magnitude plot for H1 of j omega.

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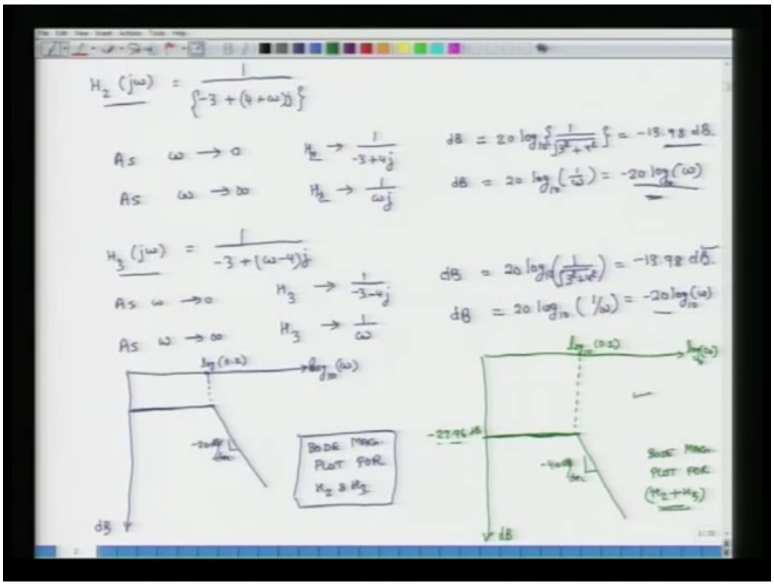

Likewise am going to construct a bode magnitude plot for H2 and S3 functions. We will rewrite the relation for H2 and that is 1 over minus 3 plus 4 plus omega times j that is H2, as omega approaches 0, so now again we are trying to construct a low frequency asymptote and also high frequency asymptote. So as omega approaches 0 H1 approaches 1 over minus 3 plus 4j or in decibels this is essentially 20 log to base 10 1 over 3 square plus 4 square the whole thing under square root.

So once again the low frequency asymptote comes out is a constant and when I calculate this value essentially what I get is minus 13.98 decibels and as omega approaches a very large number or infinity H1 approaches one 1 over omega j. So in decibels essentially I get 20 log of 1 over omega which is essentially minus 20 log of omega. So once again we see that the low frequency asymptote is essentially a we represented by constant horizontal line which is 13.98 decibels away negative 13.98 decibels away from the horizontal axis and the high frequency asymptote will be a straight line but it will have a negative slope of minus 20 decibels per decade.

let us do the same thing for H3 j omega, so the relationship is minus 3 plus omega minus 4 times j. So as omega approaches 0, so earlier I had written H1, this should be H2. So as omega approaches 0 H 3 approaches 1 over minus 3 minus 4j or in decibels again it is 20 log 10, 1 over 3 square plus 4 square whole thing under square root and that is again minus 13.98 decibels and as omega approaches infinity H 3 approaches 1 over omega.

So in terms of decibels I get 20 log to the base 10, 1 over omega which is minus 20 logarithm of omega. So essentially what we are seeing here is that the magnitude part for H2 and H3 they are same the low frequency asymptote for the magnitude of H2 is minus 13.98 decibels constant line and same thing for H3 and the high frequency asymptote for H2 is essentially a negative sloped line with the slope of minus 20 decibels per decade and it is the same thing for H3.

So if I have to construct a bode magnitude plot for H2 on the horizontal axis I am going to plot log of omega on the vertical axis I am going to plotting decibels, so my low frequency asymptote, so let us say I am plotting for, I am doing a bode magnitude plot for H2 and H3 because they are the same. So my low frequency asymptote is once again a horizontal straight line.

My high frequency asymptote is negatively sloped line and the slope of this is minus 20 decibels per decade and the crossover point is corresponds to this point and this number is such that when omega equals, the crossover point is 0.2. So when omega equals 0.2 then low frequency asymptote under high frequency asymptote after the crossover point the high frequency asymptote becomes more important. So this is log of 0.2, so this plot is for H2 as well as H3.

Now what I am going to do, is I am going to develop a bode plot magnitude plot H2, so it is not H2 and H3 but rather I will develop a plot for H2 plus H3. So essentially I will add these 2 lines, so my low frequency asymptote will be once again a constant line it will be a constant line such that the Y intercept will be negative 13.98 times 2 decibels. So this is my low frequency asymptote and that is minus27.96 decibels on the horizontal axis I am plotting log of omega.

On the vertical axis I am plotting decibels low frequency asymptote once again is minus 13.98 times 2 which is negative 27.96 the high frequency asymptote is steeply sloped line and the slope is negative 20 plus negative 20 decibels per decade. So the overall slope is minus 40 decibels per decade and once again the crossover point remains as is and it does not change when I am just adding up the bode lots for H2 and H3.

So when I am summing up the bode plots for H2 and H3, as I mentioned just now the crossover point still remains omega being equal to 0.2 which means on the horizontal axis it gets plotted as log of 10 of 0.2. So this plot i is a bode magnitude plot for a combination of 2 functions or a product of 2 functions which are H2 and H3.

So now we will construct the final bode magnitude plot which will be essentially a sum of 3 individual magnitude plots for H 1, H2 and H3, once I sum all these 3 up I get the bode magnitude lot for function H1 times H2 times H3 which is same as the original H in the transfer function which we have developed.

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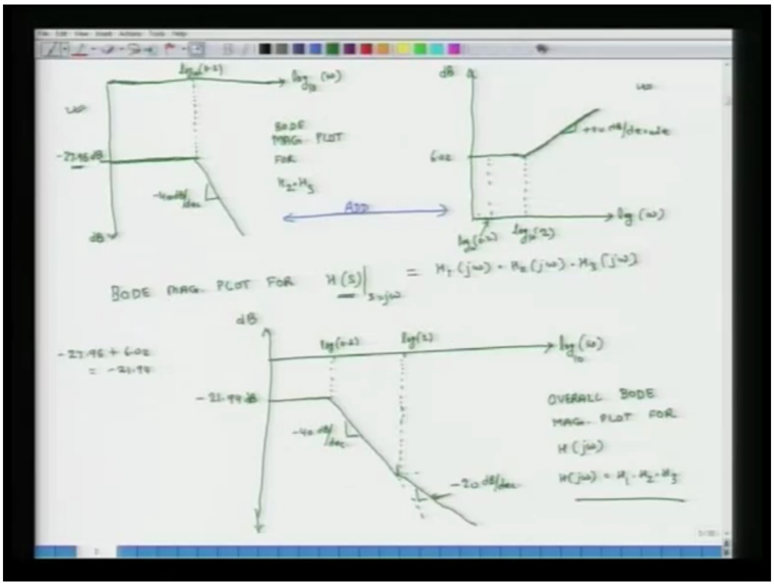

So for purposes of clarity I will just redraw the magnitude plot H2 times H3. So the low frequency asymptote is a horizontal line with Y intercept at minus 27.96dB and this low frequency asymptote dominates the response of the system represented by H2 times H3 up to specific crossover point which is which corresponds to this point when omega is larger than 0.2 then the high frequency asymptote starts dominating the response. So that is my crossover point and after that I have my high-frequency asymptote and here the slope is minus 40 decibels per decade. This is MAG plot actually I will be more specific bode magnitude plot for H2 plus H3.

And then we had constructed a bode magnitude plot for H1 and the low frequency asymptote was again a horizontal line with Y intercept of 6.02 decibels the crossover point was omega equals 2 and after the crossover point high-frequency response starts dominating and that is represented by a straight positively sloped line, the slope being plus 20 decibels per decade.

So now I am going to construct a bode magnitude plot, so just to be mathematically consistent this is a bode magnitude plot for a function which is equal to H2 times H3 not H2 plus H3 but rather H2 times H3. So now we will construct a bode magnitude lot for HS which is our original transfer function when S equals j omega and that is nothing but product of 3 individual transfer functions H1 j omega times H2 j omega times H3 j omega.

And the way we are going to do it is we are going to add these 2 plots, we add these 2 plots and we get our final answer. So that is how I am going to construct it and this is how the final plot looks like. Once again my horizontal axis is a logarithm axis where I am plotting omega, on the vertical axis I am plotting decibels, so what we see here is that up to omega equals 0.2 the response (()) (22:50) are essentially straight lines.

So let us say this is log of 0.2 this is log of 2 and up to this point I just add 2 horizontal lines 1 line is plus 6.02 decibels away from the X axis, the other line is minus 27.96 decibels away from the X-axis. So my first segment of the line will be something like this and this is minus 21.94, now how did I get this 21.94? Essentially it is minus 27.96 which is this number plus 6.02 equals minus 21.94 decibels.

So the response curve for the system which is represented by this transfer function H is essentially a constant horizontal line which is minus 21.94 decibels away from the horizontal axis up to a frequency of 0.2, angular frequency of 0.2. Now after that I have another segment of frequency, another band of the frequency which is from 0.2 to 2 and for this reason we see that this particular transfer function it is having a slope of negative 40 decibels per decade while this transfer function is still in the constant range.

Because this is let say log of 10 of 0.2, so the overall transfer function will be negatively sloped line it will be a negatively sloped line and the slope will be minus 40 decibels per decade up to angular frequency of 2. Once I have exceeded the angular frequency of 2 then this part of the curve for H1 also starts to kick in. So my overall slope becomes minus 40 plus 20 decibels per decade.

So my slope comes lesser and here the slope is minus 20 decibels per decade. So this is my overall bode magnitude plot for a transfer function H of j omega where H of j omega is essentially a product of 3 individual transfer functions H1 times H2 times H3. So we will use a very similar approach for also developing a bode phase plot for this complex transfer function H and that is what we are going to do in next several minutes.

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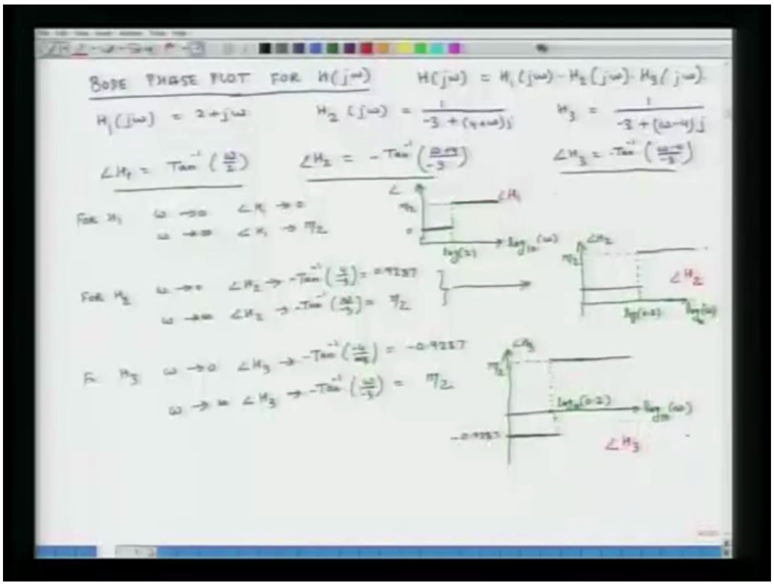

So now our objective is to develop a bode phase plot. A bode phase plot for H of j omega, so once again my H of j omega is a product of H1 j omega times H2 j omega times H 3 of j omega and once again for purposes of clarity I am going to rewrite the functions for H1, it is 2 plus j omega for H2 it is 1 over minus 3 plus 4 plus omega times j and for H 3 it is 1 over minus 3 plus omega minus 4 j.

So the phase of all the specific individual functions will be something like this for H1 phase will be Tan inverse of omega over 2, for H2 the phase will be negative because it is in the denominator of Tan inverse of omega plus 4 over 3, actually in the denominator I will have negative 3 and for H3 it will be negative of Tan inverse omega minus 4 over minus 3. So these are the 3 different relations for phase for each particular sub transfer function.

Now we will start construct thing bode phase plots for each of these transfer functions and then finally we will just add them up and we will get our final bode phase plot. So for H1 when omega goes to 0 phase of H1 goes to 0 when omega goes to infinity, phase of H 1 goes to pi over 2, so my bode plot is going to look like for low frequency it is a constant line at 0. So once again let me just label the axis.

Horizontal axis is log of omega, vertical axis is phase and this is in radiance. So this is my low frequency asymptote and my high frequency asymptote is going to look like this and the crossover point where the intercept is going to be pi over 2, this is 0 and my crossover point is log of 2. So now I am going to do same thing for H1, H2 as omega is approaching 0 phase of H2 is approaching negative of Tan inverse of 4 over minus 3 and that is 0.93 here little more precise 9287 radiance.

And as omega is approaching infinity phase or phase 2 is approaching negative of Tan inverse, omega over minus 3 and when omega becomes extremely large then essentially comes plus minus pi over 2, so when omega has a positive value and it is extremely large then it becomes pi over 2. I will again, so the bode plot for this H2 the phase plot for this will look something like this.

So again I am having phase of H2 been plotted here on Y axis, on the horizontal axis I am plotting log of omega and base 10 and this is my low frequency asymptote, the crossover point is log of 0.2 and my high frequency asymptote is once again a constant line and the value Y intercept is pi over 2. For H3 as omega approaches 0 phase of H3 approaches minus Tan inverse minus 4 over minus 3 and that is essentially minus 0.9287.

And as omega approaches infinity from the positive side then H3 phase approaches negative of Tan inverse omega over negative 3 and that is pi over 2. So just for purposes of clarity let me just label this, this is phase plot for H1, this is phase plot for H2 and now I am going to construct a phase plot for H3. So my low frequency asymptote is again a horizontal line with a Y intercept of minus 0.9287, once again I am plotting here phase and here I am plotting log of 10 of omega.

So my low frequency asymptote for phase is a horizontal line with a Y intercept of minus 0.9287 then after the crossover point which corresponds to omega being equal to 2. I have a high frequency asymptote and the high frequency asymptote also has a Y intercept of Pi over 2 and once again this crossover point is log 10 of 0.2, so this is H3. I am plotting phase plots. So I have plotted phase plots for H1, H2 and H3 on log scale. So my next logical step is just to add these 3 up because we know that if there is a function which is a product of 3 individual functions then the phase for this complex function is nothing but a sum of phases of each individual sub function.

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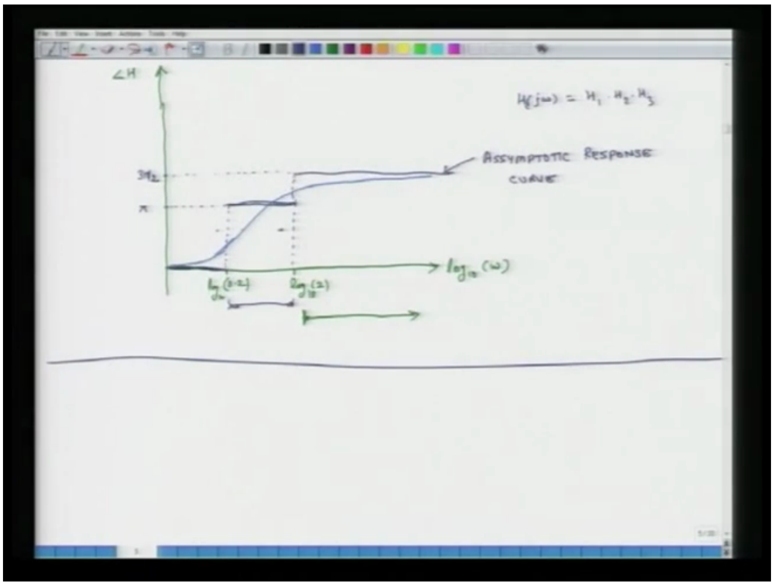

So once I do this exercise, my final curve it looks like this, on the horizontal axis I am plotting log of omega, on the vertical axis I am plotting phase of the original transfer function and I know that for the range 0 to 0.2 the contribution of H1 and H2 cancel each other and the contribution from H1 is identically 0. So for the range 0 to 0.2 my curve for the phase is a horizontal line which is exactly cutting the vertical axis at 0.

Then for the range 0.2 to 2 the contribution from H1 is pi over 2, no excuse me, so for the range of frequencies or angular frequencies between 0 and 0.2 we have seen that the phase plot is essentially coincident with the horizontal axis and Y intercept is 0 radiance. So this is what it corresponds to this dark blue curve for the angular frequency range from 0.2 to 2 the contribution from H2 and H3 is pi over 2 radiance.

We see it from here, contribution from H2 and H 3 is pi over 2 radiance while contribution from H1 still remains 0 because the low frequency asymptote for H1 is 0 up to 2 radiance per second. So what that means is that for this range my face curve will once again be a constant line but it will be a constant line however the Y intercept will be at pi radiance, right?

Then for the range where angular frequency exceeds to which is for this range let us look at the contribution from H1. From H1 the contribution is pi over 2, for H2 the contribution is pi over 3 again a constant then from H3 the contribution is once again pi over 2. So pi over 2 times pi over 2, no pi over 2 plus Pi over 2 plus Pi over 2 is 3 pi over 2.

So for this range the phase response is 3 pi over 2. So once again I have a horizontal line and it extends to infinity and that is why phase response curve my phase curve bode phase curve for a transfer function which is again a product of 3 individual transfer functions H1, H2 and H3. Now they are in mind that this is the asymptotic response curve. However in real systems the transitions from this range to this range to this range they are more or less fairly smooth.

So the actual response curve will be bounded by this asymptotic response curve but however this asymptotic response curve it gives us some qualitative understanding how the system is going to behave. The actual curve in this case may very well look like something like this and that is what I am going to plot in light blue. So at extremely low frequencies it is bounded by this dark blue line curve which is essentially horizontal line cutting the vertical axis at 0 at extremely high frequencies that is bound by another horizontal line which has Y intercept of 3 pi over 2. So this is how we can construct bode magnitude plots and bode phase plots for transfer functions which can be broken up into individual poles and zeros.

And once we have an individual curve for every single pole and every single zero then all we have to do is we have to add these up and we get a bode magnitude plot and we also get a bode phase plot for the original transfer function which could be a combination of several individual pole and zero related transfer functions.

So that is pretty much what I wanted to cover in terms of bode plots and I think this introduction will help you develop bode plots for magnitude and phase for fairly complex transfer functions and that will come in really handy once we start talking about acoustics in detail. So we have covered a few of topics complex algebra, complex numbers bode and bode magnitude plot, bode phase plots, pole zero plots and so on and so forth and all these concepts will come in fairly handy once we start talking about acoustics in detail.

So having said that now will move to the wave equation which is the first concept we are going to talk about in context of this course on acoustics. So before I start talking about the wave equation per se just wanted to give you a brief understanding of what this equation means and what is it that we are trying to do through the wave equation. So as we have talked about earlier in acoustics we can break up this whole range of issues and problems which we try to solve in the area of.

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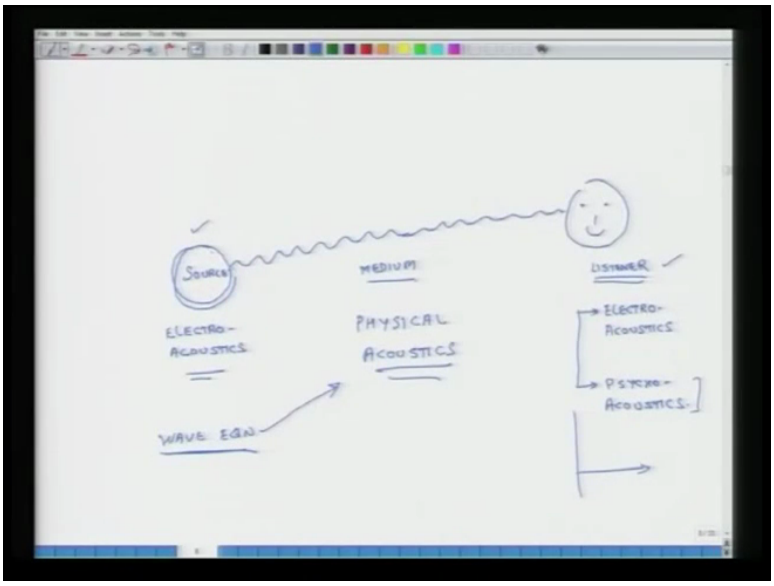

The first group relates to situation when a noise or sound is being produced. So you have a source and this source produces some and then I have a listener and this listener could be a human being, an animal, a microphone or whatever. So I have a source, I have a listener and I have a medium and there are different branches of acoustics which deal with these 3 things there is a branch which helps us understand how sound is generated and a lot of that information comes from ideas in electro acoustics.

So electro acoustics along with some related area of acoustics they help us understand how sound gets generated then in terms of listening or recording sound or analyzing sound or sensing sound here we deal with 2 broad areas, one is electro acoustics, electro acoustics helps us design for instance microphone which help us sense sound and then that can be recorded.

And then another very big area is psycho acoustics and what psycho acoustics help us do is, it helps us understand sound from the standpoint of the human individual. So it is one thing to record data and see it on a computer screen but how does brain understand sound how does brain interpret sound signals that is what psycho acoustics help us understand.

So this is about sound generation, this whole area is about sound, sensing of sound and interpreting a sound and as sound travels from the source to the listener through a medium the branch of acoustics which deals with this propagation of sound is called physical acoustics, so that is physical acoustics. So sound gets generated and there we seek the help of electro acoustics to understand that phenomena. It gets propagated from point A to point B and there we seek the help of physical acoustics to understand that propagation phenomena and then finally there is a listener who is listening to sound and there the understanding is derived through principles of once again electro acoustics or psychoacoustics.

So wave equation it helps us understand how sound gets transmitted through a medium. So that comes in the ream of physical acoustics. So that is a very broad context in which I wanted to place wave equation , where does wave equation fit in the overall scheme of things? So what we will be doing in this course is that first now starting from today onwards we will do several lectures on physical acoustics and then we will try to understand how sound gets propagated through a medium.

Once we have done that then we will move on to electro acoustics and there we will try to understand how we can generate sound. So we will address this part and then finally we will go to the listener and there will try to understand how sound gets recorded, interpreted and so on and so forth. So that is the overall course landscape and that is the context where we can place wave equation.

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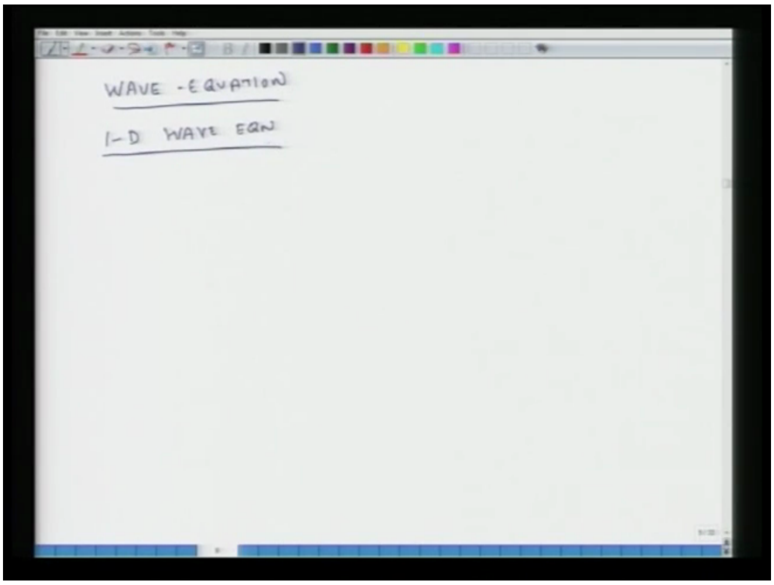

So what we are going to talk about is wave equation and once again what this equation helps us do is, helps us understand how does sound get propagated as it travels through a medium. Now this medium could be a piece of solid as in a piece of steel or it could be volume of gas, so most of the times when we speak we are sitting in air and sound gets propagated from our mouth to an individual sphere, so that is again where wave equation comes in handy and there are different versions of wave equation for different types of medium.

Or it could be the case of a fluid to be more specific the case of a liquid where you have sound getting propagated in water through miles and miles or kilometres and kilometres of distances and how does get sound propagated there in that I of a medium? So in each of these different media types it is the wave equation which helps us understand how sound gets propagated.

What we are going to talk about today is specifically propagation of sound in elastic media and more specifically in air at atmospheric conditions? And even more specifically we will talk about the 1-D wave equation. So a typical example of 1-D wave equation could be I have a long queue and I am speaking in that tube and the tube may be bent but sound gets it travels to that tube through the air inside the tube and it can be heard at the other end of the tube. So that is typical example of 1-D wave equation.

Another example could be, again a rectangular pipe or a channel let us say you have each (()) (51:43) duct at one end of the duct you have a blower which is blowing air into the duct and in that process it is also generating some noise and that noise moves through the duct and a sound gets it comes out from the duct into an air-conditioned room that sound get heard.

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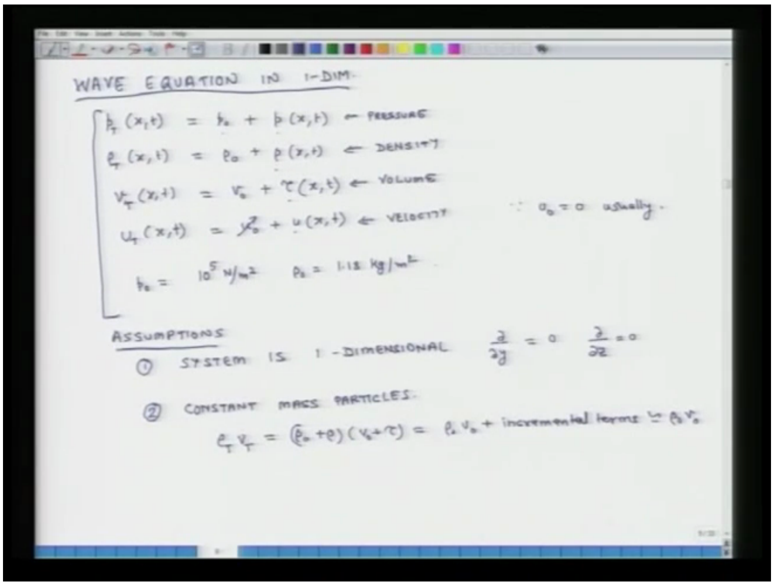

So what is the phenomena and how is that propagation happening through the duct? That is something 1-D wave equation will help us understand. So what we will be talking about is wave equation in one-dimensional. Now initially we will start this discussion by having propagation of the wave in one-dimension where my coordinate system is a rectangular

Cartesian coordinate system but later I will use mathematics to also explain the wave equation in a radial or a spherical coordinate system.

So let us consider a 1-D piece of air, one-dimensional piece of air where sound is getting propagated and let us say the initial pressure, initial density, initial volume and initial velocity of this air are P not, Rho not, V not and U not and then because of the disturbance due to sound propagation these values change, so the initial values have a subscript of 0 and the final values have a subscript of PT.

So my PT which is the final pressure and it can depend on X which is the position, so again it is not X, Y and Z but it is just X because it is only in one-dimension, so my final pressure which is PT is dependent on X and it is also dependent on time. So that is equal to P0 which is my initial pressure which was again dependent on x and t it was P0 plus some disturbance which is dependent on x and t.

So once again here I am saying essentially that P0 does not change with time and x because it corresponds to standard atmospheric pressure which is approximately 10 to the power of 5 newtons per square meter. So that is the relation for pressure. Likewise the density of the air under consideration the final density could be rho subscript t and that is equal to my initial density which is rho not plus incremental change in the density which is rho which is again a function of position and time. So that is my relation for density.

Then my volume of air let us say my final volume of air is V subscript t that is equal to initial volume V0 plus increment some change in volume which is Tau, again a function of position in time. So that is the relation for volume and finally I have a relation for velocity, velocity being U, so my final velocity is UT which is equal to an initial velocity U not plus some change term which is represented by U which is again a function of position and time.

Now usually U not is essentially 0 because we assume that in the initial state of affairs there is no motion in air. So U not is 0, so I can drop this term because U not is 0 usually. Also it is important to provide some values at this point of time, so my P not equals approximately 10 to the power of 5 newtons per square meter, my rho not which is the density of air at MSL is 1.18 KGs per square meter and initial volume could be variable because I can take as much I can consider as much volume as I want.

So once again P not, rho not, V not and U not which is 0 in this case, there my initial pressure, values of pressure, density and volume and velocity while P, rho, Tau and U are my incremental values for pressure, density, volume and velocity and why when I add these 2 up the initial value and the incremental value I get final values which are designated by a subscript t. So this is how I am going to frame my problem.

So what I am interested in knowing is how are P, rho, Tau, U all interrelated and as I am trying to develop these relationships I will make 2 very important assumptions. So I will make 2 important assumptions the first assumption is that this is a 1-dimensional system and so my system is 1 dimensional and what that means is that Del over del y that is the partial derivative of any variable with respect to Y is equal to 0 and del over del Z is also equal to 0.

Second assumption I am going to make is that I have constant mass particles. So I hope piece of air which has a initial mass, its mass is not changing as it moves through the system. So essentially what that means is rho t, Vt is same as rho not plus rho times V not plus Tau and that is same as rho not V not plus incremental terms and that is equal to rho not V not. So these are the 2 key assumptions and then we will also talk a little bit more about some other assumption in the next class.

But this is how I am going to frame my problem and in the next class what we will do is, we will try to develop an equation of how pressure and velocity and all these variables are interrelated through 3 important equations, one is the newtons equation which is an equation for Force equilibrium, so if I have a piece of fluid it has to have equilibrium from using newtons laws of motion.

The second equation I am going to use relationship I am going to use is that for conservation of cars and that is what I call continuity equation and finally I will use a relationship which links pressure and volume through a material constitutive equation and through these 3 equations, equation of momentum that is newtons law, equation of mass conservation and material constitute equation help develop wave equation in one-dimension, thank you very much.