Acoustics
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Module – 07
Sound in Public Spaces and Noise Management
Lecture – 05
Design of a muffler

So what we will talk about today is about mufflers, so we have talked a lot of different applications of sound, we have talked about speakers, microphones. We have also seen how shakers were accelerometers which use principles which assimilate to what is used in acoustics.

Student: Seismometer

Yes seismometers we have also seen how sound moves in architecture spaces. So mufflers in another thing I wanted to talk about and to understand mufflers we have to understand how sound propagates as it moves through three different media. So we will, what we will do is we will initially see how sound moves through three different media and then we will use those principles in context of mufflers and see how mufflers work.

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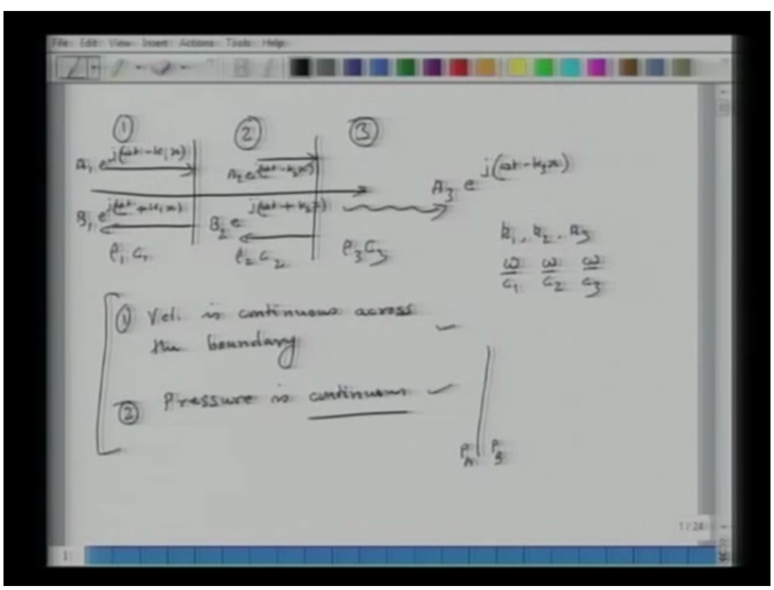

What you can have is you have sound and this is medium 1, medium 2, medium 3. These could be physically separated boundary or you could have three different you know units where properties of the materials are significantly different so let us say Z not is rho 1 C1 first medium rho 2 C2 in the second and rho3 C3 in the third.

Now we know that as sound moves from one medium to the other medium there is some reflection right, so let us say the incident wave is A1 e j omega t minus k1 x and then when it hits the first boundary, it gets reflected and the reflection is B1 e j omega t plus k1 x, similarly here now when it moves to second medium and it hits the second boundary let us say that the incident wave is A2 e j omega t minus k2 x and it gets reflected part of it.

So what you get is B2 e j omega t plus k2 x, okay and finally the final wave which emanates into the third medium is A3 e j omega t minus k3 x, k1 k2, what are k1, k2, k3?

Student: Omega by C1

Yeah, so this is omega over C1, omega over C2, omega over C3. Now think about it as this wave hits the first boundary, what is the boundary condition at that point?

Student: Sir, velocity is at point by the way considered boundary 1 or boundary 2 or same?

Velocity just inside the boundary and just outside the boundary is the same, so velocity is continuous across the boundary, what else?

Student: Displacement, amplitude.

That will be consequence of velocity itself but pressure will also be continuous across the boundary and why will pressure be constant because if pressure is jumpy, so this is my boundary and just inside you have PA and just here you have PB or P1 and P2 and if that film is extremely thin which of size thickness is its vanishingly thin.

So the mass of it is 0 but you have across a film of zero thickness, a non zero pressure difference and what that means is that it will have infinite acceleration but in reality it does not happen. So pressure is continuous. So we use these two boundary conditions and get several equations and from those equations we compute A1, A2. A1 we already know whatever is the incident wave at the end of the day what we are trying to find is what is the value of A3, right.

We know A1 we have, what we do not know is B1, B2, A2 and A3, four unknowns so from these two boundary conditions if we can generate four equations then we should be able to compute all these unknowns B1, A2, B2 and A3.

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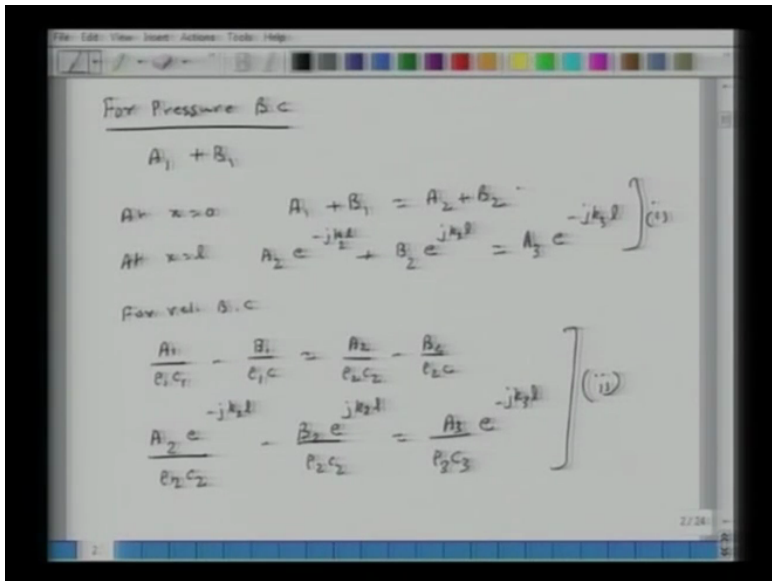

For the pressure boundary condition, what you have is A1 plus B1 at x equals 0, okay so here x equals 0 and here I think x equals L. So at x equals 0 my boundary condition is A1 plus B1 equals A2 plus B2 and at x equals L, I get A2 I just put x equals L in these expressions and what I get is A2 e minus j k2 L plus B2 e and here I will have a positive j k2 L equals A3 exponent minus j k3 L, right. So this is my pressure boundary condition.

And for velocity what I get is A1 over rho 1 C1 minus B1 over rho 1 C1 equals A2 over rho 2 C2 minus B2 over rho 2 C2 at x equals 0 and then the other boundary conditions at x equals L is A2 exponent minus j k2 L over rho 2 C2 minus B2 e j k2 L over rho 2 C2 equals the velocity on the other side of the layer rho 3 C3 e minus j k3 L.

We have four equations, 1, 2, 3 and 4, and using these four equations I can compute all the unknowns in terms of A1. So what we are interested in understanding is that how much power is transmitted and how much power is lost as sound moves through this media.

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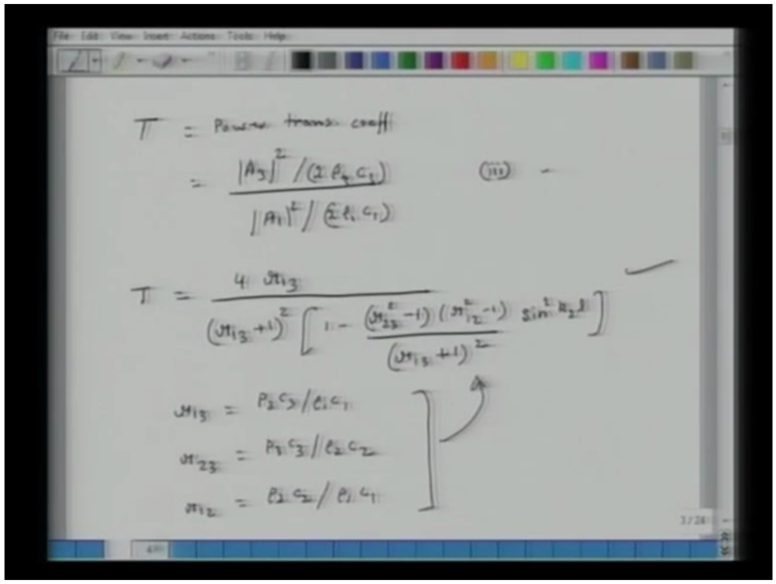

So we will define a ratio T and T is power transmission co-efficient and what is power transmission co-efficient? It is the ratio of A3 square divided by 2 rho 3 C3 over whatever is coming in to rho 1 C1, right. So if I use all these equations and I find A3 in terms of A1 at the end of day what I get is transmission co-efficient of power is 4 a number r13 divided by r13 plus 1 entire thing squared r23 minus 1 square minus r13 this entire thing divided by r13 plus 1 whole square times sine square of k2 L.

So how did I get this equation? By solving A3 in terms of A1 and plugging those values in this equation 3, okay and these r's are ratios essentially r13 is rho 3 C3 over rho 1 C1, r23 is rho 3 C3 over rho 2 C2 and r12 equals rho 2 C2 over rho 1 C1. So I know the properties of the medium so I can find these ratios, I can plug them in this relation and I know how much power is getting transmitted through the medium and how much is getting lost.

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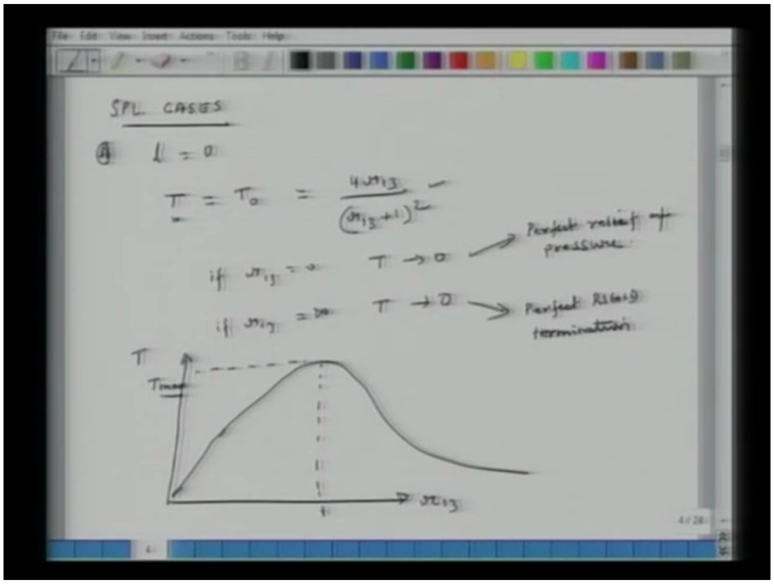

So now what we will do is we will do some two special cases and then we will the third special case will be of a muffler. Special cases, first special case is that what happens when this L vanishes and becomes 0, L equals 0 once that happens then T equals what we call T0 and that is essentially 4 r13 over r13 plus the entire thing square.

This value of T for, now if r13 equals 0 then T is 0 and also if r13 equals infinite then T is again 0, this is the case when you have perfect relief of pressure so as sound is moving from first medium to third medium because second medium is just not there then the pressure just gets perfectly relief then r13 is 0, this is the case when you have perfect rigid termination.

So you have a wall and it is infinitely rigid it just does not move then nothing gets (())(11:49) that is it and if I plot this, I plotted against r13, this is my T and my curve looks something like this and this is Tmax and this is 1, for r13 equals 1 I get maximum value (())(12:22)

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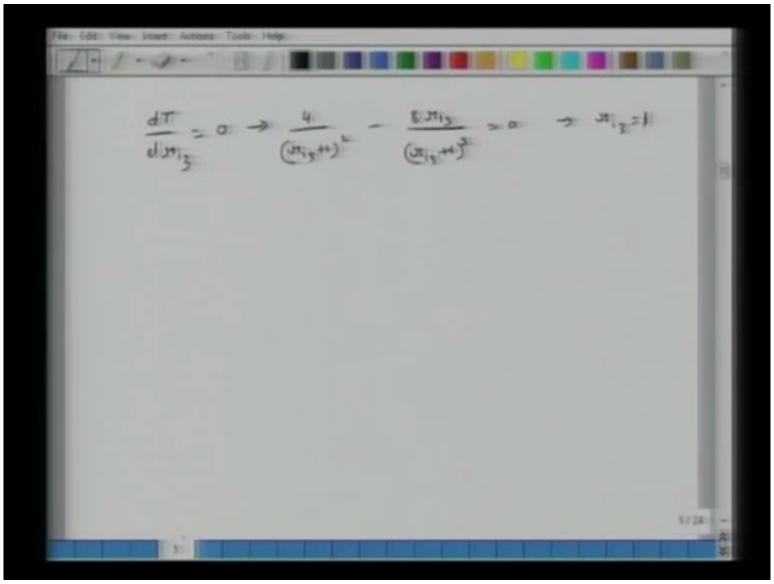

And I can find that also analytically by doing this dr1 is equal to 0 which gives me if I differentiate T with respect to r13 what I get is 4 over r13 plus 1 whole square minus 8 r13 over r13 plus 1 cube here equals 0. So if I solve it what I get is for extremer condition r13 is equals 1. Bear in mind this is the definition of r13 it is rho 3 over C3 divided by rho 1 over C1.

Suppose you have an acoustic transducer and let us say it is underwater and it is generating sound and that sound has to be transmitted lets in water but also because of the fact that it is emerged in water you want to protect it from leakage of moisture into the system, electrical circuit has to be protected, salt is there so it has to be protected in a mechanical since so there is no corrosion and all that.

If you apply some sort of a layer on top of it there will be loss of transmission if you want to maximize the loss of transfer, you want to minimize that transmission lost. The property of the material has to be such chosen that r13 equals 1, so there are special rubbers for which they have actually tuned the material properties of that particular rubber to seawater and they are called rho C rubbers.

So what those specific materials if they used it to encapsulate the whole transducer, the impedance is perfectly matched and there is no transmission loss so you get TL because your r13 is 1, so that was the first case.

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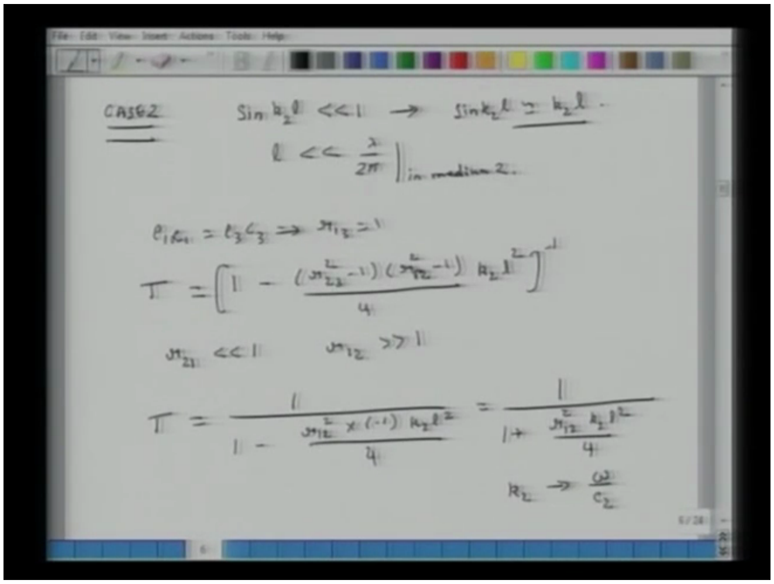

Case 2, case 2 could be where the middle medium, medium number 2 is of a thickness which is very small it is not very thick. So in that case sine k2 L is extremely less than 1 which means I can approximate sine k2 L as approximately equal to k2 L and this condition is valid only when L is very small compared to lambda over 2 pi.

And lambda is the wavelength of the sound where, in the second medium so in medium 2. So if that is the case now we take a special case where rho 1 C1 equals rho 3 C3 equals so such that r13 equals 1 so this is a special case of case 2 so then my T becomes 1 minus r23 minus 1, r12 minus 1 over 4 k2 L square and this entire thing is inverted.

Now we know that r21 is very small compared 1 and for the same reason r12 is very large compared to 1 so my transmission factor is 1 over 1 minus r12 square times minus 1 times k2 L square over 4 and this becomes 1 over 1 plus r12 square k2 L square over 4, so now if I put k2 as omega over c2.

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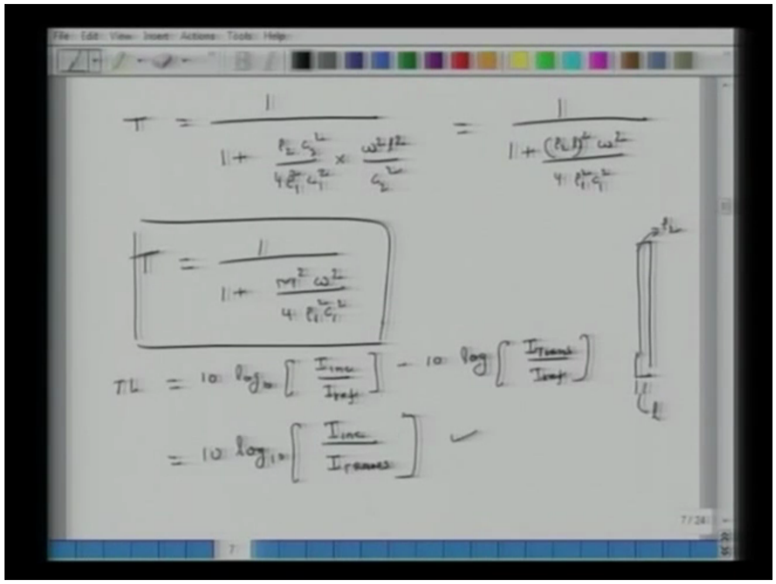

Then what I get is T equals 1 over 1 plus rho 2 C2 square over 4 rho 1 C1 square times omega square L square over C2 square and that perform simplification and I get is 1 plus rho 2 L square omega square over 4 rho 1 C1, everything is squared.

Now what is physically rho 2 L, I have a medium which is very thin it has the density rho 2, it has a length L, so physically what does it mean, mass per unit area right. So rho 2 L is mass per unit area and I call that upper case M so this is basically 1 plus M square omega square over 4 rho 1 C1 square.

Now on this one if I do a transmission loss this is the power transmitted so if you want to compute transmission loss which I call TL and if want to do that in decibel scale then that will be 10 logarithm on base 10, I incident over I reference minus 10 log I transmitted over I ref and then this essentially becomes 10 log of 10 log in base 10 I incident over I (())(19:05)

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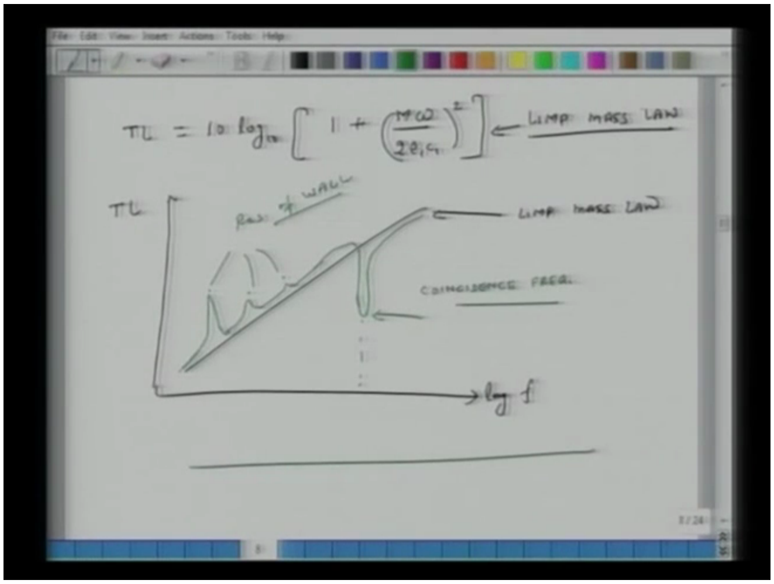

So this is basically inverse of T so my transmission loss is basically 10 log 10 over 1 plus M omega over 2 rho 1 C1 then (())(19:25) this relation a lot of people call it Limp Mass Relation or Law. So limp mass law or limp mass relation and why it is called limp based because you are not assuming any stiffness. Suppose you have sound and there is a thin wall in between you have room 1 big room then you have a thin wall and then you have a another big room, what you are not factoring in is the stiffness of the wall, so that is why it is called limp mass law.

And typically if I plot this law against logarithm of frequency of omega and here I am plotting in decibels transmission loss then I get some sort of a straight line okay, so this is limp mass law, now in reality the structure does something like this which I am going to plot it in green. Something like this, these peaks are related to the resonance modes of the wall itself.

So whenever you have a resonance you lose a little bit more energy so it just happens that the shape of the wall is out of phase with the pressure wave so you lose a little bit more of it and this is a place which is called at this point, for this frequency is called co-incidence frequency.

And what is this frequency, physically what it means is that when the frequency in the air matches the frequency of the bending waves in the wall then you have this thing called co-incidence frequency so rather than going to map at this you have a depth in transmission loss but otherwise this limp mass law is the fairly good estimate of how much energy gets transmitted or lost to the medium. So with this background now we will move to another case of mufflers.

Student: Sir what is the definition of the co-incidence frequency.

That co-incidence frequency is when the bending flexural waves of the wall co-inside with the incident frequency that is the frequency of this.

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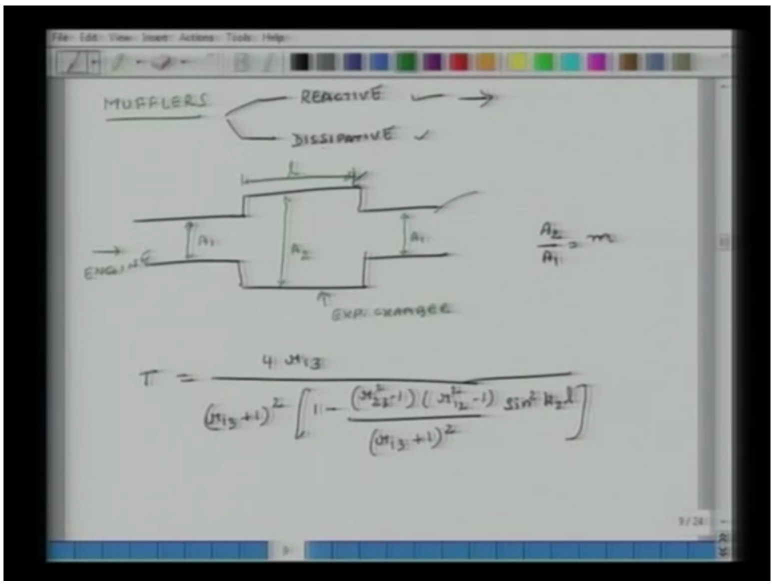

So now we will talk about mufflers, so in a very broad sense you have two types of mufflers first type are reactive and the other type dissipated. In dissipative types of mufflers the reliance in terms of reducing the noise coming out of engine or whatever is the system is, reliance is on the dissipative characteristics of the materials inside the muffler so that is why it is called dissipative and here it is firmly how you are tuning and the bigger reliance is how to cancel pressure waves inside the muffler so that the total noise emitted out is minimized.

In reality mufflers are hybrid, they rely on both reactive plus dissipation rates so they are mix of these two, so within reactive what we will today talk about is a particular type of muffler which is called expansion type muffler and in general the construction is something like this, so you have air coming in from engine of course goes to some catalytic converter where all the poisonous gases are catalyzed and emissions are made safe.

So it comes from a pipe then you have an expansion chamber which is this and then it gets emitted out so another tube, shock tube, let us say the cross sectional area of the inlet tube is A1 and we also assume outlet tube is still A1 and the middle tube is A2 such that A2 over A1 equals m, the ratio m.

So this is the three medium problem, you have the first medium where the cross sectional area is A1 then you have A2 and then you have another one K1 except we cannot idealize this directly as a lumped volume because in this in case of a lumped volume what is the property of sound, velocities are 0 in that, here velocities are not 0.

We cannot also not idealize it as lumped mass because in lumped mass it moves in a rigid way, air moves in a rigid way, it is not happening here so pressure gradient in the lumped mass is negligible, it is not the case here. So we use the three medium approach which we talked about just now and understand this problem.

From the earlier discussion we know that transmission co-efficient is 4 r13 over r13 plus 1 whole square 1 minus r23 plus 1 minus 1 r12 minus 1 square over r13 plus 1 whole square sine square k2 L and I mention that this distance is L, length of the expansion chamber, now what we will do is we will compute what are the different values of r13, r23, r12.

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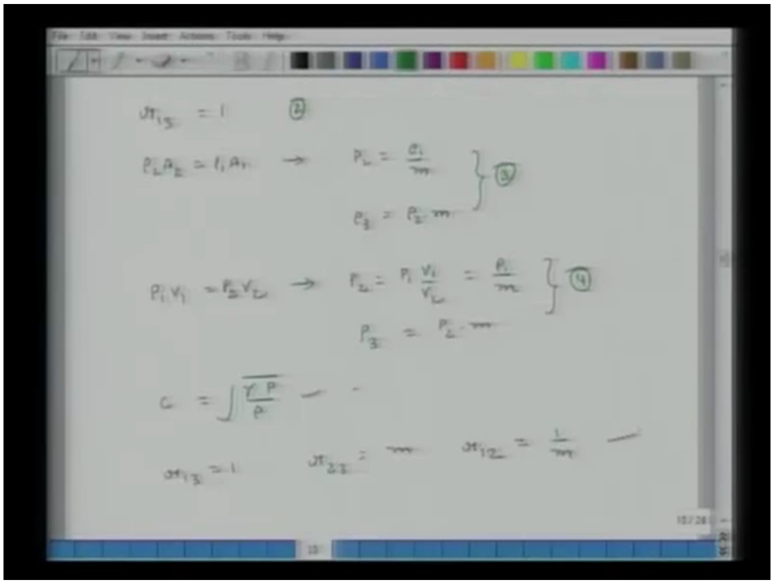

So r13 equals 1, also we know that rho 2 A2 equals rho 1 A1 because of conservation of mass that gives me rho 2 equals rho 1 over m, similarly rho 3 equals rho 2 times m. I also want to find how are C's behaving, C velocity of sound behaving in each of these chambers. So we know that P1 V1 equals P2 V2 which gives me P2 equals P1 V1 over V2 equals P1 over m and similarly P3 equals P2 times m. Now velocity of sound is got gamma, gamma does not change P over rho.

Student: Sir why have you considered endothermic process?

We have to understand this, the flow of the air we have a different nature compared to flow of the sound air may be going through a different process but as sound is propagating through it, it is going through (())(27:35) air is essentially going through this. So we have valid and making one set of, using one set of rules for air and another set of rules for sound itself.

Student: No but sir sound is travelling in air only.

Sound travels in air but just because air moving like that sound does not travel with air, in the first lecture what we have seen is that the motion of particle is not same as like you drop a stone in a wave the wave travels but in that case the lake is sitting at one place, right. So with using a similar though process we are using similar thought process.

Student: Sir why are you considering the travel of air?

Because we have to find pressure, these are not, these are remember these are not in, these P1, P2 and P3 they are basically average pressures, bulk pressures in each of these chambers, first chamber, second chamber. Around these mean values of pressure there are fluctuations in the pressure because of sound but even as sound is not travelling through this whole expansion chamber still P1, P2, P3 will hold. C is gamma P over rho so if I use this relation in conjunction with 3 and 4 what I get is r13 equals 1 r23 equals m, r12 equals 1 over m.

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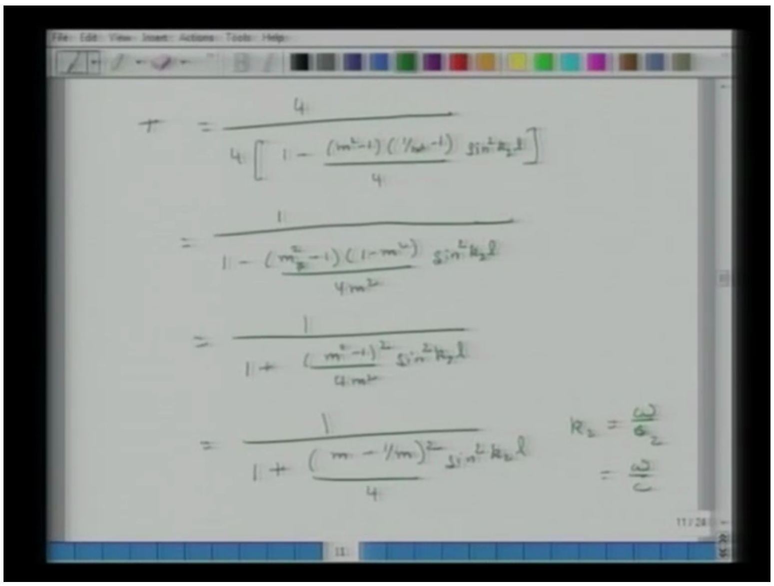

So now I put these values of r's in first relation which is the relation for transmission factor and what I get is P equals 4 divided by 4, 1 minus m square minus 1, 1 over m square minus 1 over 4 sine square k2 L and then what I get is 1 minus m square minus 1, 1 minus m square divided by 4 m square sine square k2 L.

I can sort this out as 1 over 1 plus square minus 1 whole square over 4 m square sine square k2 L and then I further modify it as 1 plus m minus 1 over m square divided by 4 sine square k2 L and k2 is again omega over C2 and what we found is that C2 is same as C1 as same as C3, it is essentially omega over C.

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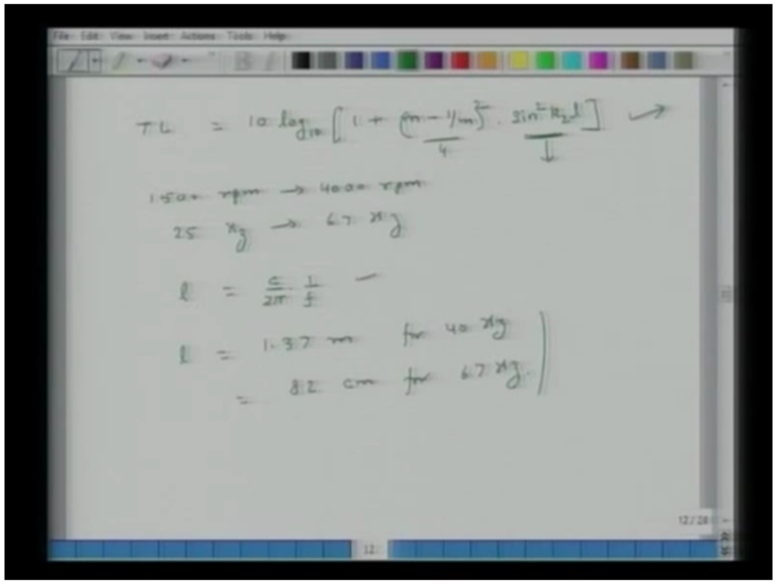

So if I use this particular relation and then find transmission loss in decibels, then my relation for transmission loss is 10 log to the base 10, 1 plus m minus 1 over m whole square divided by 4 times sine square k2 L, so in cars what is the typical RPM at which the engine moves around 1500 it starts you know most of the times it sets out 1500.

So range is 1500 RPM to what is 4000 let us say RPM that is 25 hertz to 67 hertz. So if I have to maximize PL, I have to improve this factor m minus 1 over m, I can make it as high as possible and then I also have to make sine square k2 L maximum and the maximum value it can assume is 1. For 25 hertz so L is basically C over 2 pi times 1 over f, I have to make k2 L equal to 90 degree basically so if I get this and this is there so that translates to L equals 1.37 meters for 40 hertz and it is 82 centimeter for 67 hertz. So if I know what type of what frequency I am trying to minimize then based on that I have to choose the right value of L that is what m.

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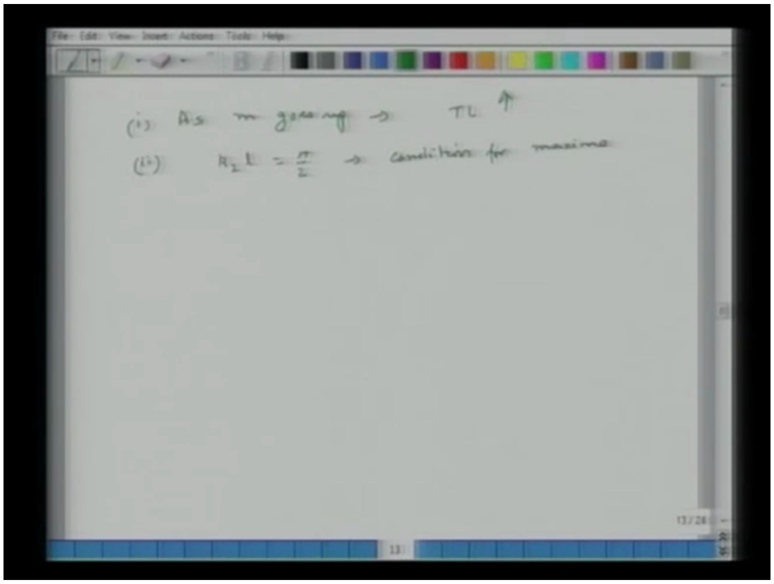

Couple of other things as m goes up, so does TL, so if you have a bigger expansion chamber which has a much bigger area compare to A2 A1, A2 over A1 if I maximize it, my transmission loss will be maximized and the other thing we have already mentioned is that if k2 L equals pi over 2 then again this is the condition for maxima.

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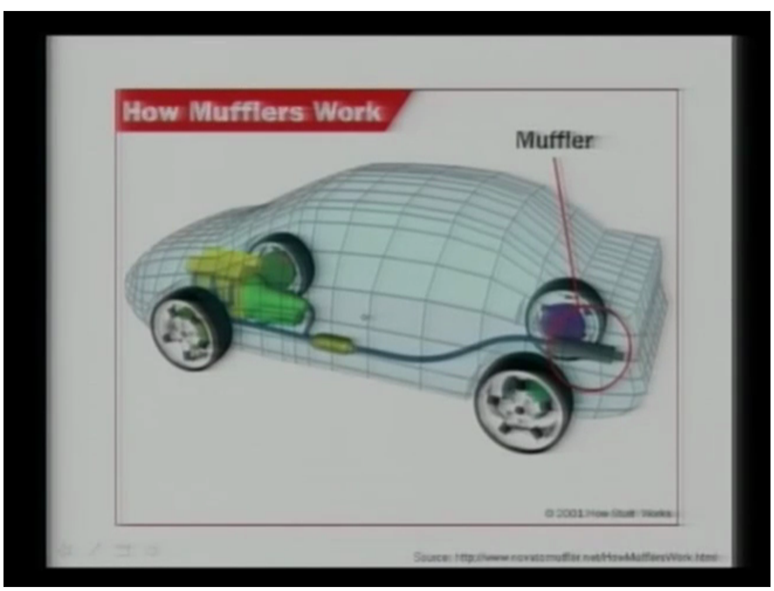

What I will do is I will show you a few slides so this is one picture I pulled from internet that the source (())(33:25) muffler.net and what you are seeing here is from the engine you have pipe then it comes goes to a yellow box which is what they call a catalytic converter where they reduce the poisonous content of the gases by to some catalytic process and then you have a long pipe and then that is your muffler.

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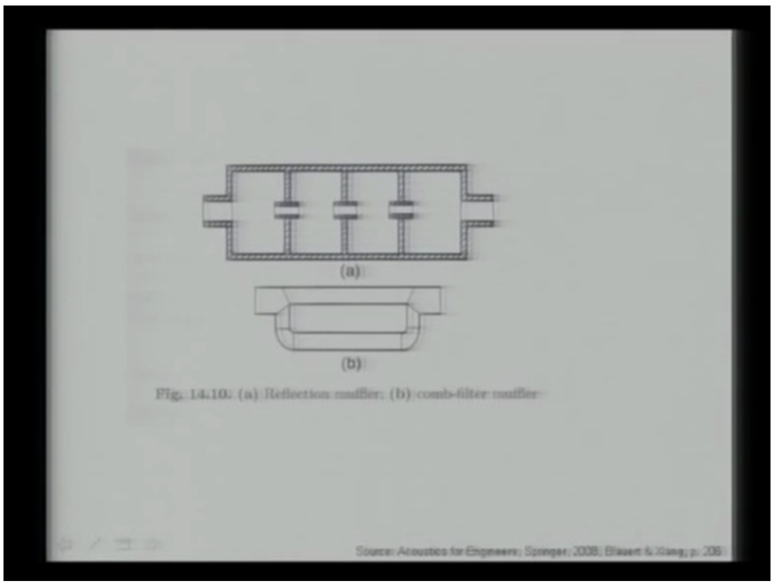

This is another one so in the analysis which we did that muffler is tuned to a particular frequency right, so as I shift from that frequency slowly the transmission loss will not be as great as I would like it to be, if I am very far away and when sine k2 L equals 0 then that will (())(34:13) loss. So in this figure which I pulled from this Acoustics for Engineers book, you have the upper picture A, 14.10 A you have a series of chambers and each chamber is tuned for a particular frequency so you can tune it accordingly.

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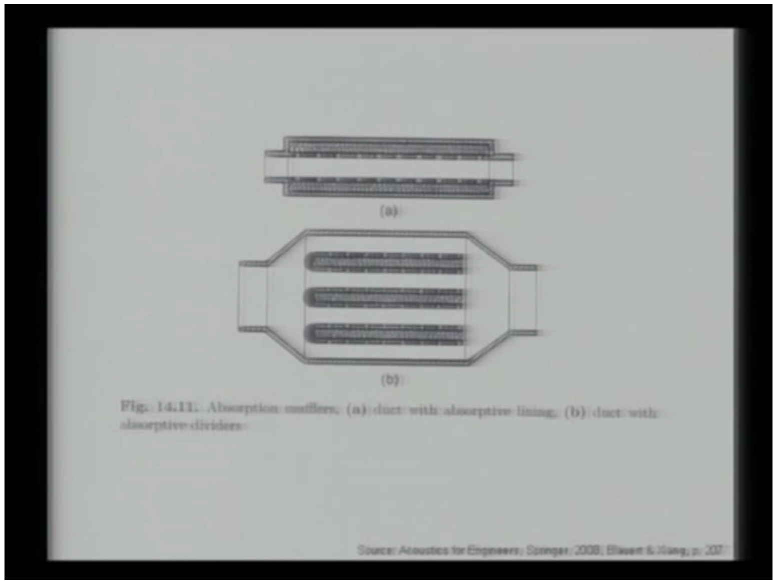

Now this is the picture, two types of mufflers where the reliance is not that much on expansion or reactive mufflers but these are dissipative one so you have lining inside the tube which absorb sound and in the second one what you have is segregated so the absorption area in the expansion chamber gets enlarged and that enlarged area surface is coated with absorptive material, so that absorbs sound. So here B is here like a hybrid you have an expansion chamber and also absorptive dividers are there.

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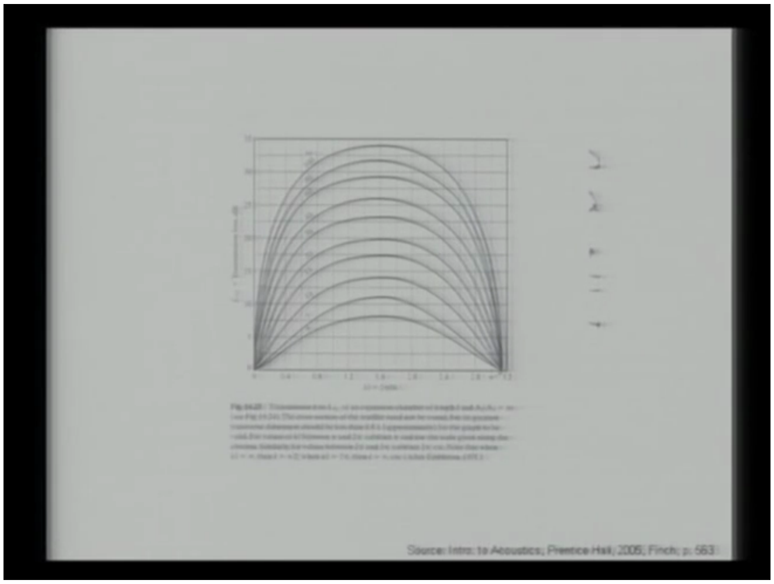

This is the probably the last one so what this picture shows is that how my transmission loss changes as kL changes so kL starts from 0 goes upto pi and then for different values of m, I keep on increasing my transmission losses as my m becomes higher and higher and again at 90 degrees for a given value of m, my transmission loss is maximum.

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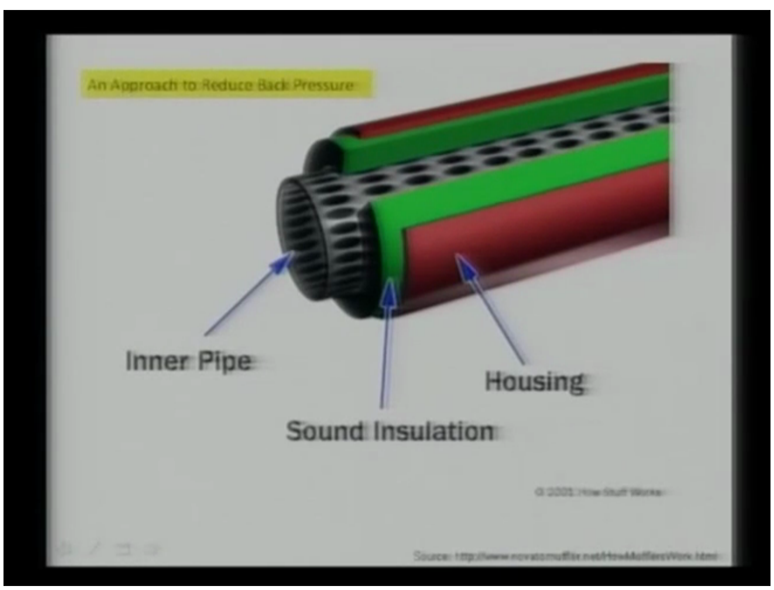

Last one, so one limitation of these reactive mufflers is that they generate a back pressure and what that does is that it has an impact on the engines efficiency if you have a very high back pressure then the engine efficiency goes. So in lot of cars which are used on race tracks where they do not care that much about noise pollution but are more worried about speed they tend not to use so much of reactive type of mufflers rather they tend to use have more reliance on absorption as the muffling agent.

So this is a particular type of a muffler and the name is called cherry bomb, I do not know why they call it cherry bomb muffler but where you have is an inner pipe which has a lot of holes in it and that is covered with a thick sound insulating layer which is basically glass wool another absorptive material and then outside of that you have this red sheet, it is with (())(36:55) so this is another one.

So what we will do is now we will go back and do a little bit more about mufflers, so we have talked about reactive type of mufflers and if also if we want to understand how absorption plays a role in terms of absorbing sound then we already have the necessary background based on previous class lectures how to figure out how much sound is absorbed through the walls in a muffler, right. Because if we know the alpha, absorption co-efficient of the material and if we know S then we can figure out how much sound is getting absorbed.

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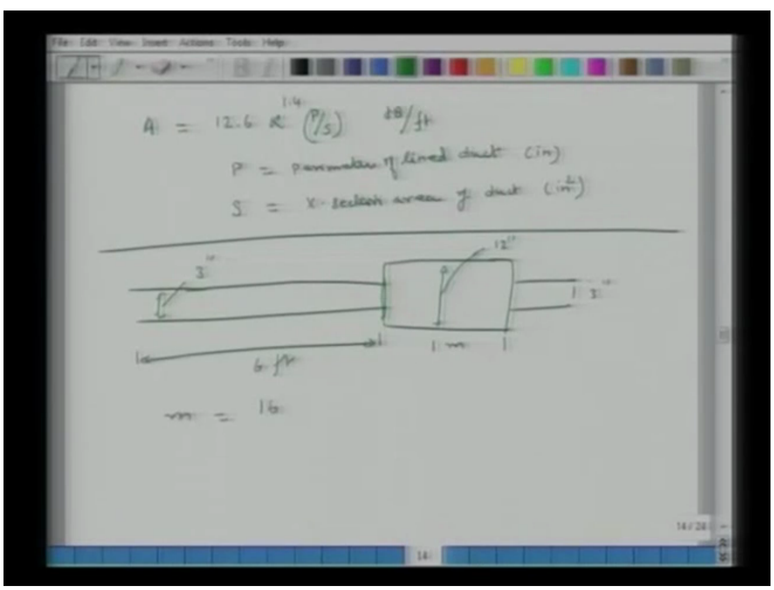

So what (())(37:32) had done was he developed this relation, he is A is attenuation is 12.6 alpha 1.4 P over S and this is attenuation and decibels per feet, alpha is again sound absorption co-efficient. P is not pressure, P is here perimeter of lined duct okay and this is in inches so you can convert it into meters and then S equals cross section area of duct so it is in inch square. So it is prevail to figure out how to reframe this particular relation in SI units.

So what we will do is we will do an example, so let us say I have a long pipe after it comes out of the catalytic converter then I have an expansion box so the expansion chamber and then sound gets out. I was looking at some of our cars and one car I found at a length of about 2 meters this one 6 feet and the diameter of this was about 3 inches and this diameter was about 12 inches and again this diameter is again 3 inches this length was about a meter so I can convert into. So m is what 16 right, m is 16.

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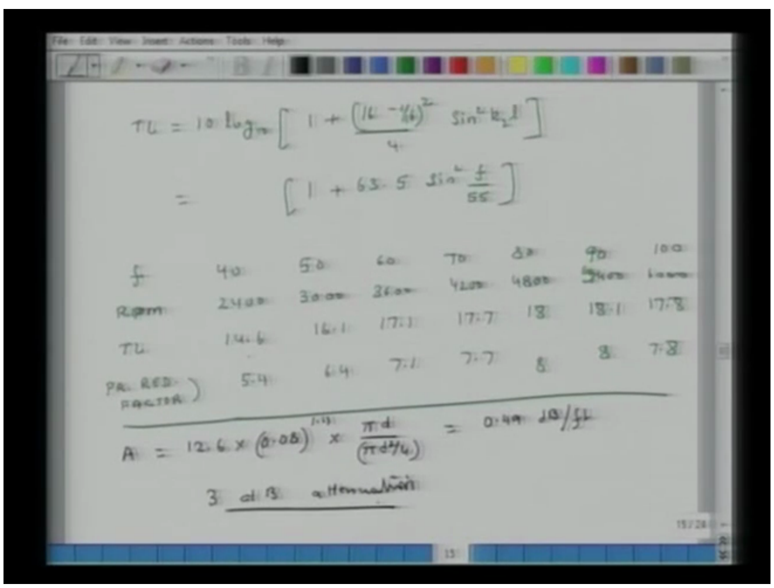

TL equals 10 log 10 1 plus 16 minus 1 over 16 square over 4 sine square k2 L and that is in this box I have is 1 plus 63.5 sine square f over 55, right it the map and in terms of frequency what this comes from. So now I will construct a matrix so frequency could be 40, 50, 60, 70, 80, 90 and 100 hertz, so in terms of engine RPM this is something we see on the screen on the dashboard is 2400, 3000, 3600, 4200, 4800, 5400 and this is 6000. Transmission loss in decibels I calculated is 14.6, 16.1, 17.1, 17.7, 18, 18.1, 17.8

So it starts going after 80-90 hertz, now if I want to see how much less noisy it becomes in terms of what is the reduction in pressure so I say pressure reduction factor, so in this case it goes down by a factor of 5.4, 6.4 what will be the value for 18 dB 8, 6 dB is twice 7.1, 7.7, 8, 7.8 we get significant reduction in pressure based on this so now if you below the car if you have more space then you can make the value of m larger or you do not necessarily have to make it round it could be a rectangular shape wherever you have more space so that your cross sectional area is larger, m is basically cross sectional area, ratio of cross sectional area.

So if I want further reduction in sound I can increase my m and I can also check, see where my maximum noise is coming from what frequency and based on that I can change the value of L. So I am getting 14 to 18 dBs of transmission loss then I said okay for comparison purposes let us say that if I line this entire tube, this some sort of highly acoustic damping material, what kind of an impact it has.

So in that case A equals 12.6 times and I assume alpha of 0.8 which is fairly high times pi d is the perimeter pi d square over 4 is my area so if I do the math what I get is 0.49 dB per feet, so this is roughly 6 feet so total absorption, attenuation will be of how much 3 dB attenuation, so over 6 feet there will be 3 dB attenuation by absorption. So the point being that absorptive mufflers are not that efficient, their response is much wider in terms of frequency band but they are not broadband but they absorb their absorptive capacity is not as great as compared to reactive mufflers.

So most of the reliance when you are trying to reduce the noise coming from engine is on reactive nature of system so that is what I wanted to cover in the context of mufflers that another application you have covered. Now today in another 15-20 minutes and in the next lecture we will talk a little bit about how noise get generated and some very broad approaches to which noise has managed of course one way is to mufflers but I wanted to cover it in a more philosophical way how noise is managed.

And noise can come through several sources there could be sources, there is some noise happening outside the room I do not want it to get inside the room, there is noise from the fan, noise from the ducts, noise from electronics equipment but even units which are supposed to make good sound they also generate noise and I you have a loudspeaker and I send it a pulse of

40 hertz, it will not be always the case that will only generate 40 hertz content it will also generate noise and most of the that type of noise comes through non-linearity in the junction so by a very simple example we will today see how non-linearity generates noise content.

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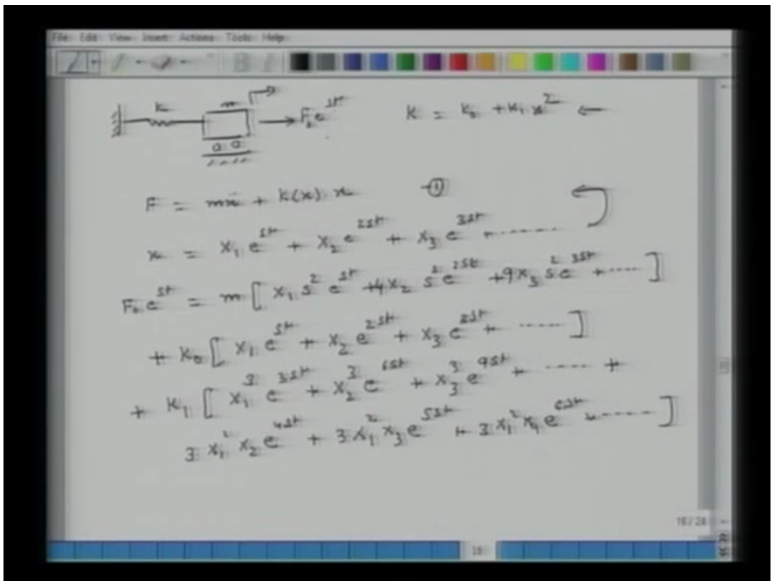

And for that we will have a just simple spring mass system so mass is m this a and this is F not e st and if k is linear then the relationship between F and x will be linear but if k is non-linear then there will be noise generated, so let us say k equals k not plus k1 x square this very simple model has direct application in transducer.

So you have force which is being generated by which part of the loudspeaker, the voice coil that is generating force on the membrane so that is you F not e st then there is moving mass of the loudspeaker which is m and then there is stiffness, I have eliminated damping here for simplicity process and this stiffness now if that stiffness is non-linear as in this case then what I am trying to understand is that if I excited by a frequency S what happens to x?

If x has only S frequency content then there is no noise but if it has something in addition then there is noise so the equation of motion is what mx double dot plus k where k is a function of x times x so we say that let me be assume a solution for x, x1 e st plus x2 e 2 st plus x3 e 3 st. So you can take an infinite series and we plug this equation in the equation of motion so my equation of motion becomes f not e st equals and then I am going to collect different terms so mass times x1 s square e st plus x2 s square e2 st plus x3 e I m sorry, S square.

Student: 2x square 3x square

2 st ?

Student: 4 x2 s square into st plus 9

Mass times accelerating so when I difference, you are saying here it should be 4 st?

Student: 4 x2, 4 into x2, co-efficient of x2

Oh yeah and 9, e 3 st that is mass plus K not x1 e st plus x2 e 2 st plus x3 e 3 st plus there is an infinite number of plus here plus K1 x1 cube e 3 st plus x2 cube e what, 6 st plus x 3 cube e 9 st plus and then they will be cross coupling terms, so 3 x1 square x2 e4 st plus 3 x1 square x3 e5 st plus 3 x1 square x4 e 6 st so I get an infinite number of terms here.

So if this equation has to hold true then all the terms associated with e st when I take their co-efficient they have to vanish they have to become 0 all the terms associates with e2 st their co-efficient collectively they have to vanish and so on and so forth, so that is what we will do.

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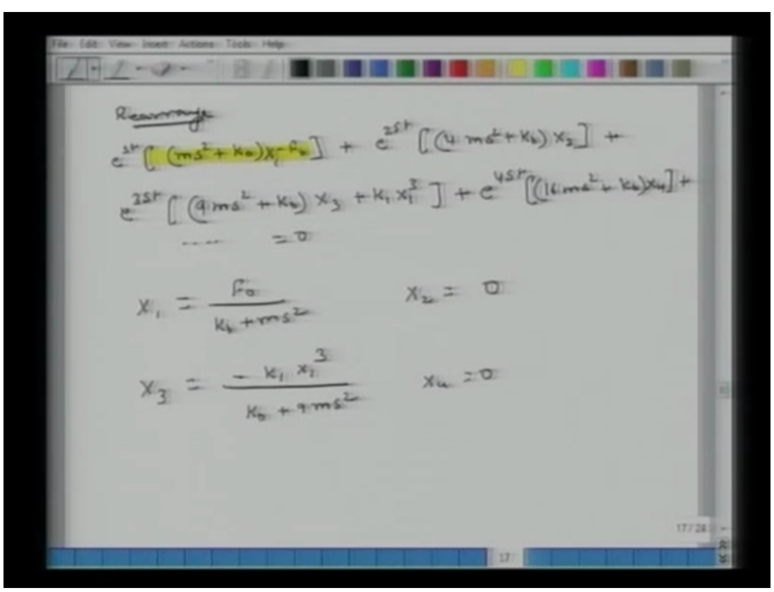

So once we rearrange we get we collect terms in terms of e st, ms square plus k not minus f not then e 2 st 4 ms square plus k not x 2 plus e3 st 9 ms square plus k not x3 plus k1 x1 cube plus e 4 st 16 ms square plus k not x4 plus, right.

So from this guy I write my first relation that how they has to be next one here in the first one x1 so x1 equals f not over k not plus m s square what is x2, x2 is 0, x3 is minus k1 x1 cube over k not plus 9 ms square, x4, so what you are seeing is that just because you had one cubic term in the stiffness you are generating infinite number of extra frequencies for x and that is all noise.

So this all the terms associated with x2, x3, x4 they are all noise so in reality the system is not only linear in the sense that its dependent on x square but it is also dependent on x cube, x n all powers of x. Stiffness by itself is dependent only on cubic you know that the dependence of stiffness on x is only at 0th order, square, fourth term, fifth term and so on and so forth.

But they are the terms damping that has other dependencies then this also dependent on frequency, properties of the material changes frequency. So things are not very nice and linear and once they are not nice and linear then you get 2nd order, 3rd order, 4th order, 5th order and so on and so forth harmonics, as long as your displacements are small, the magnitude of these harmonics is small compared to the base to the input but if your displacements are large for instance if x is large then k1 also starts (())(54:30) important and once that happens then noise gets generated by the system also.

So non-linearity is a very big source of noise in system so as we are engineering system and if we want to manage noise then one important consideration has to be that our system has to be more or less linear in the operating.