Acoustics
By Professor Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
Module 7
Sound in Public Spaces and Noise Management
Lecture 2
Sound in 3-D rooms

So in the last class we had started talking about natural modes of rooms we have started with one dimensional room with very very long ducts and we had talked about 2 D rooms and I think we also 3 D rooms, we did not cover rooms ok so we will recap a little bit of that and we will extend the same analogy to 3 D rooms today and we will also introduce a slightly new concepts also

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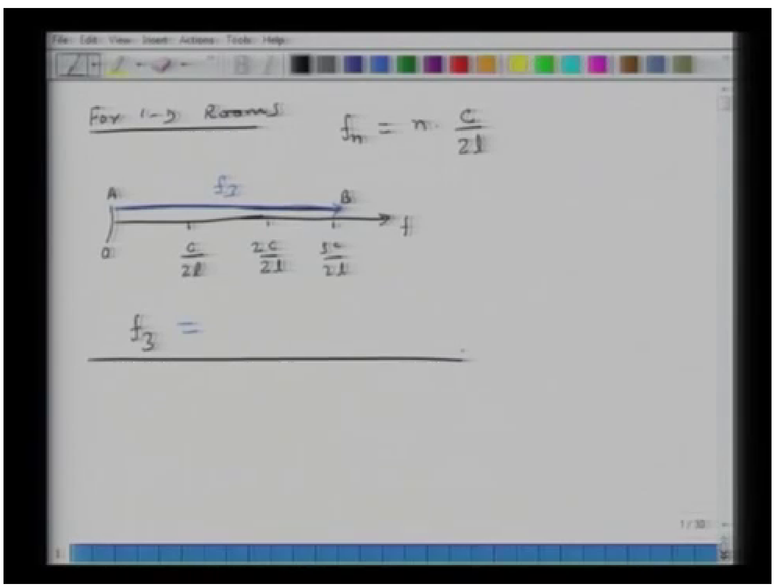

So for 1 D rooms we had said that the natural mode and the associated natural frequency. Let's say it is fn is basically n times c over 2l where l is the length of the room c is velocity of sound and n is the as an integer number and I can represent this in a vector form like this so this is my frequency this is my origin zero so the existence of a node is after every period a distance or unit distance where a unit distance is c over 2l long, so every c over 2l I have a node f3 for instance will represent, this is f3.

So the third natural frequency of this room will be f3 where f3 could be is defined as frequency associate with 3c over 2l.

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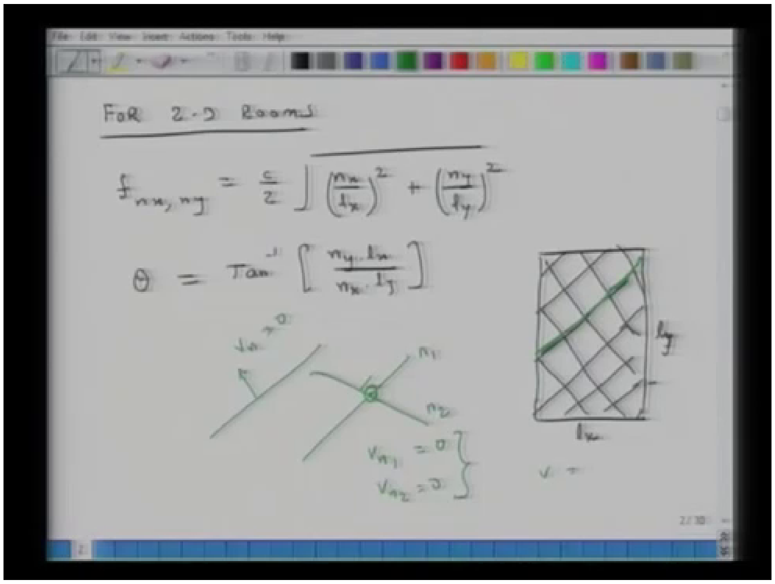

In terms of wave lengthy each of these units is lambda over 2 in the direction of the length so that is for one dimensional so now we will extend similar third process for 2 D rooms so in case of 2 D rooms the frequency is associated with an x direction parameter and a y direction parameter so fnx and y equals c over 2 nx over lx square plus ny over ly square.

And we had also shown that the angle of incidence of nx ny'th mode or natural frequency of the room has to be Tan inverse of ny times lx over nx times ly and based on this understanding we had shown some natural modes of rooms let say I have a room its again it's a two dimensional room its dimension in y direction is ly dimension in x direction is lx then there could be a criss cross pattern of nodes and criss cross pattern of nodal lines in the room.

And the boundary condition the number of nodes in y direction is essentially ny the number of nodes in x direction along the length of the room is nx so that is the physical interpretation also on each line let say I consider this particular line I am just highlighting it making it thick so along this line my normal velocity vn is zero and that is basically because the boundary condition at the point where this line hits the wall.

If I have one node line going in this direction and another node line going in this direction then these two mines the velocity condition will be vn 1 equals zero let's say this is line 1 with a normal n 1 and vn 2 equals zero for this line which has a normal n 2 these lines don't have to be naturally at right angles to each other because essentially these are reflections so at point of intersection this is by velocity condition vn1 is same as vn2 is equal to zero.

If the angle of intersection of these two lines is 90 degrees identically then the total velocity is zero at the nth point of intersection we had developed this kind of a concept for a 1 D room we will represent similarly in 2 D space the frequency of a two dimensional room and we do it in this way.

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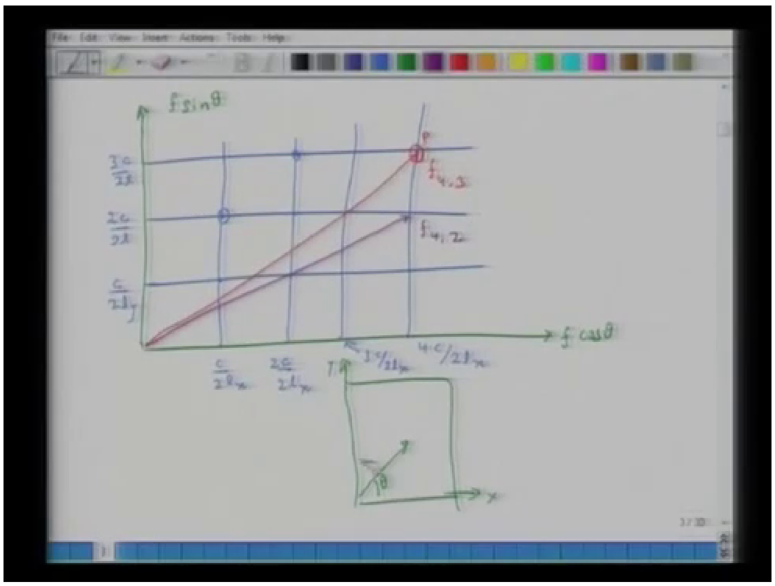

So I have nx and a y coordinate system on the horizontal axis I am plotting f of co sin theta n in the vertical direction I am plotting fy sin of theta.

Where theta as we had explained in earlier class is the direction of the incident wave this is my x axis in a room this is my y axis in a room it has the angle of incidence so just as in this case we had put a unit length c over 2l if I extend this analogy in 2 dimensional space what will be the unit along the length direction x direction c over 2 lx right and in the vertical direction c over 2 ly 3 or 4 lines and you will have to bare with my not such a good drawing ok.

So this is c over 2lx where lx is the length in x direction for the room c over 2 ly times 2 this is 4c over 2l x either should be x and this is 3c over 2lx similarly in vertical direction I have c over 2ly two times c over 2ly three times c over 2ly and so on and so forth so the natural frequency of a room will be any intersection point of these two of the grids, so these could be a natural frequency, this could be a natural frequency, this could be a natural frequency.

This is not a natural frequency this point it's not a natural frequency (07:35 it is just a intersection) so for instance if I consider this point p then this is f 1, 2, 3, 4 in the x direction 4, 3 similarly and the natural frequency could be f 4,2

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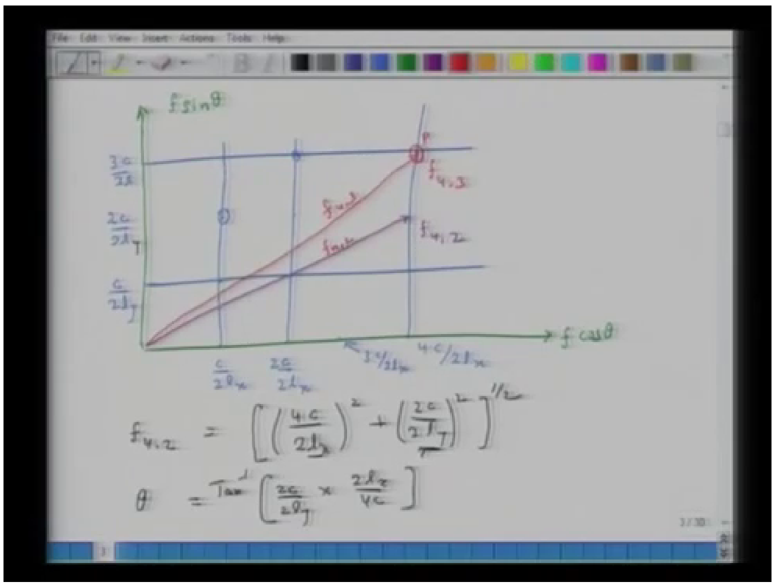

So for 4,2 I can write the relation as 4 times c over 2l basically the length of this vector is the magnitude of the frequency so it is 4 times c over 2lx whole square plus two times c over 2ly whole square the square root of the whole thing.

And the angle of incidence associated with 4,2 which has to be maintain to create this mode in the room is theta and that is essentially Tan inverse of this component divided by this component so it's 2c over 2ly times 2 lx over 4c I take the inverse tangent of (09:20 b) so once again this is my f42 this is f no I am sorry it is 43 and this is f42 so essentially we have extended this kind of a representation to two dimensional space for 2 D rooms.

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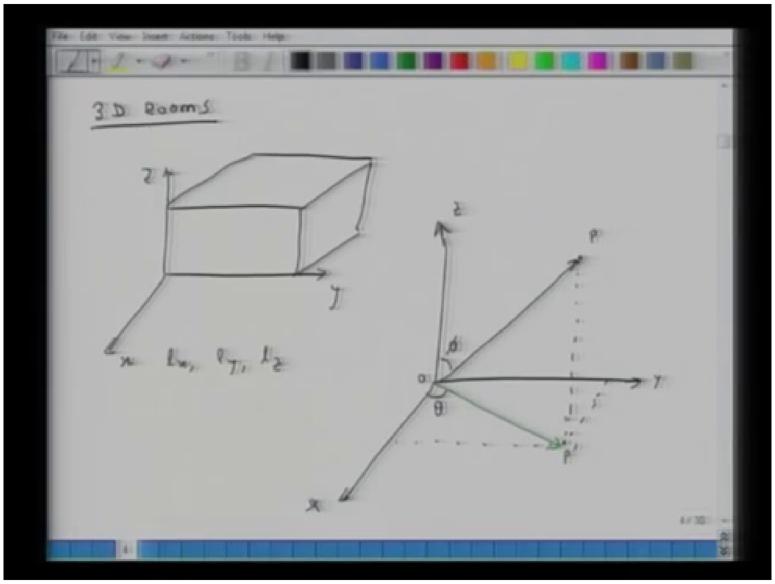

So now we take the next jump and we go to three dimensional 3 D rooms so you can have a room, it could have a height width and length again these are rooms with parallel walls all this analysis is assuming that the walls are parallel so this could be my x axis, y axis, z axis the dimensions along x y and z are lx, ly, lz so if I draw a three dimensional coordinate system and put a point in it right so I have a point p this is my z axis, y, x.

I make an angle phi with respect to the z axis of this vector and in the xy plane the projection of this vector p vector op is op prime and op prime makes an angle theta with respect to x axis.

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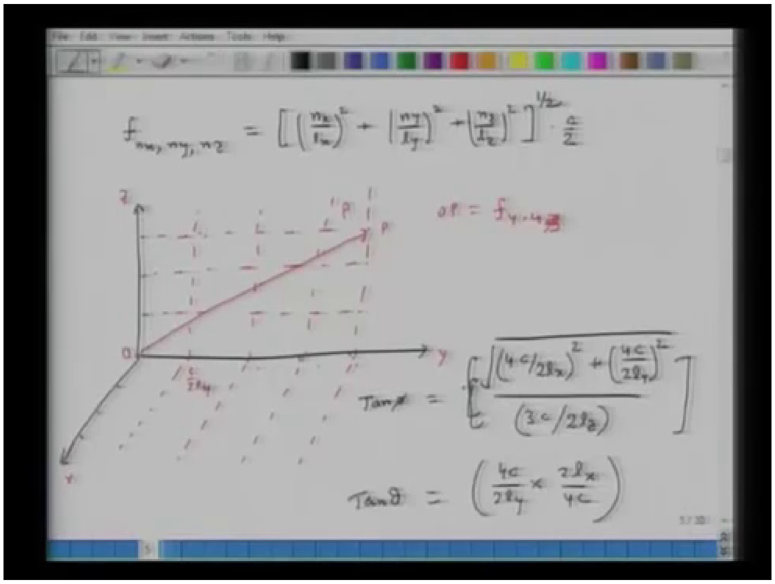

So this is standard 3 D cartridge in geometry such a system what will be the natural frequency for a room if I use similar approach is basically you add one more term here right and nz over lz whole square, so f nx, ny, nz equals nx over lx.

Square ny over ly square nz over lz square the whole thing we take a square root of it times c over and the way we express this frequency in two dimensional space for 2 D rooms we will use a similar approach to figure out the value of theta and c in 3 dimensional space for 3 D actual rooms so this room is more or less like a 3 dimensional room, it has a little bit of projection non rectangular structure there but otherwise more or less 3 D room.

So what I do is now I represent this fnxny in 3 D space so this could be a point p and if this point p is located at an intersection point of c over 2lx, c over 2ly, c over 2lz lines then it is a node and it's a valid frequency right, similarly like we developed for 2 D so it has to reside (at the node points) at the grid point of (these parallel) these lines it has to lie at the intersection of these lines, so in this case I have to extend it a little bit.

So my vector op equals f what will it be? This is my y axis, x axis, z axis so this is c over 2lx, ly twice thrice 4 and it could be 4 4 and in the z direction I have 3, zero over I have to draw a better picture to sow it in 3 dimensional space it appears but the point is that it is sitting somewhere which is 4 times c over 2lx along the x axis 4 times c over 2ly along the y axis and 3 times c over 2lz in the z direction that is the location.

So maybe you can draw a better picture and but that is what I am trying to show and from this kind of a representation I can figure out Tan theta so Tan c is Mayank can you help with what is the value of Tan c in this case, well I mean Tan c ok, in the z direction it's 3 times c over 2lg and in the projection of it in the xy plane will be, in the xy plane yes so it will be in the denominator it will be 3c over 2lz right and in the numerator it will be 4c over 2lx plus 4c over 2ly.

This end I think right and Tan theta is basically 4c over 2ly times 2lx over 4c so you have a process and an approach through which you can locate the physical nodes of different frequencies for a given 3 D rectangular room you find out it's coordinate and for each frequency in a very methodical way you can say therefore this is a node, this is a node, this is a node so at the nodal lines my pressure is minimum or maximum.

At points of where these three intersections are happening, it's maximize and my velocity is minimized, these node locations vary for with respect to each frequency so the challenge is that if there is a no room where they are lots of nodes big band of frequencies what does that mean in terms of perception of (17:49 someone) as person moves from one place to other he will see big changes in the volume related to specific frequencies.

So his understanding of sound or his perception of sound will change from place to place as he moves from one place to other place, so that is not a good room to have right because some place he will see a higher level of spl when he moves to an entire node location where he will have a lower level of sound then he moves to another one and so on and so forth so we have to figure out how to construct a room which does not have this challenge.

So that actually is a segment to the next concept which is related to all these approaches we have developed to figure out what is the location of a node in a room.

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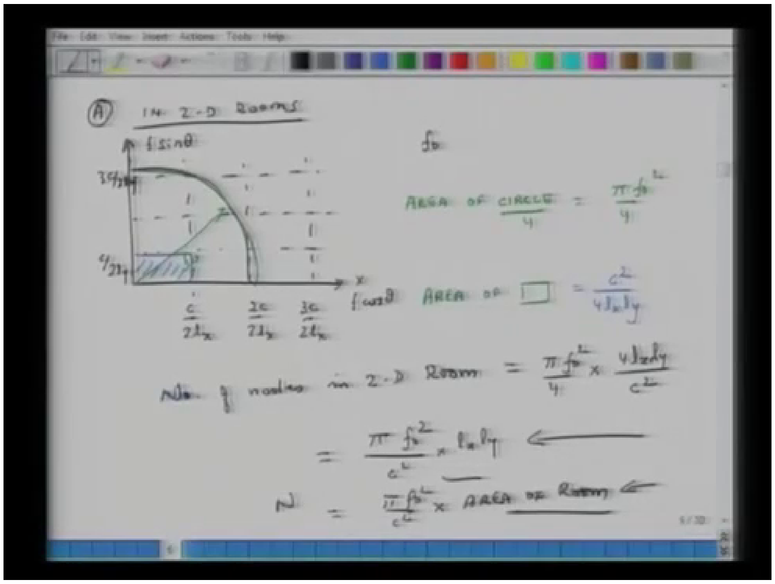

So now what we will try to do is we will try to quantify how many nodes are there in a room ok, so we will develop that for a 2 D room and then we will extend it for a 3 D room so in 2 D rooms, I will draw the same xy system my x is same as f co sin theta and this is f sin theta.

And then I am drawing a grid so each of this is c over 2lx 2c over 2lx 2c over 2lx 3c over 2lx and here I have c over 2ly this is 3c over 2ly and so on and so forth, want to find the number of nodes in this 2d room associated with a frequency f not then what I do is I draw a quarter of circle with the radius f not, so whatever number of nodes are there inside this circle or one forth of a circle that is the number of nodes this room is going to have right.

Area of circle actually one forth area of circle is pei f not square over 4 and area of one of these guys these rectangles is what so number of node is how much I just think that ratio of these two so pei f not square over 4 times 4lx ly over c square what do I get pei f not square over c square times lx ly and this is pei f not square over c square times area of room I can make this generalization to area of room only if it's a rectangular room.

Actually this is a more fundamental relation lx ly so what this says is that n is dependent on the square of frequency, it represents how many rectangles are there in this c circle frequency circle, yes so this is a very broad estimate because you will see that are higher frequencies this number of nodes run into millions so you can omit 2, 3, 10 you know but it's a fairly good representation so at discretization of a container's function so what this says is that.

Number of nodes is depend on the square of frequency as frequency doubles up number of node becomes 4 times that is one thing and obviously if you have a larger room you will have larger number of nodes, if you have a smaller room you will have smaller number of nodes, this c over 2lf, No the tightness of one node to other closeness of one node to other will depend on the wave length, lx and ly are independent but what you have here is f not over c which is wave length.

If you look at wave length then f not over c is the wave length. So the spacing of the nodes will not change form one large room to small room, this is a representation this is the actual relation you have to look at this relation when it's totality what this says is if you have big area of the room you have large number of nodes, what this says is f not over c is basically lambda if you have to relate it, visualize it in terms of spacing of the room.

You can do an experiment yourself numerical experiment that what is the spacing of room whether (if this) if you have a room this big and then you make the half as big as it, at the end of the day everything ends up in terms of functions of lambda because you have a node at the room away from that as I move inwards in a 1D tube after a distance lambda over 2 I have another node then after another lambda over 2 I have another node and so on and so forth. So the spacing you are not changing what is changing is how many nodes you have in 1D room

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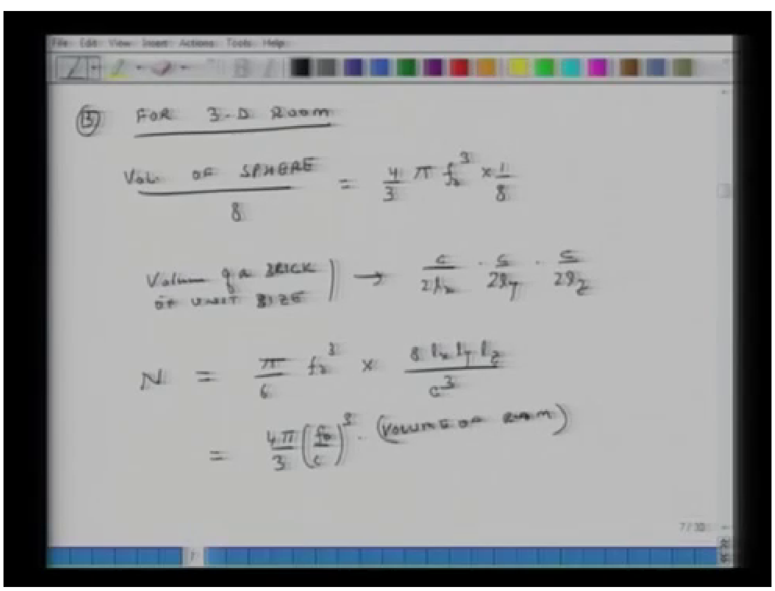

So for 3D rooms what will we do we use the same approach for a 3 D room so instead of area of a circle what do we compute (24:48 one'th areas) is what will it 4 by 3 times pei f not q over 8, area space is 4 by 3 right volume and volume of a brick I can call it of unit size in that xyz coordinate system where each dimension is c over 2lx c over 2ly c over 2lz.

This is c over 2lz times c over 2ly times c over 2lz ok so number of nodes n is basically I take the ratio of these two (25:49 guys) what that gives me is pei over 6 f not q into 8 lh ly lz over cq which gives me 4 pei over 3 f not over cq times volume of room.

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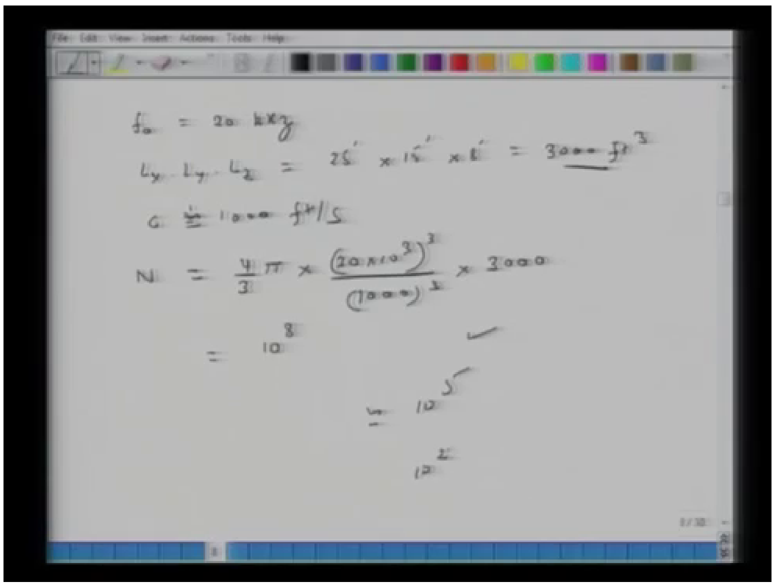

So in the 3D room the number of nodes explodes as frequency goes up if I increase my frequency by a factor of 10 I have (hundred) one thousand as many nodes.

In a 1 D room it just is linearly independent in 2 D rooms it goes up in a squarish way in 3 D rooms there is a cubic dependent so we will do a very small example let's say my f not is 20 kilo hertz that is the highest frequency which a human being can hear and let say lx times ly times lz is 25 feet times 15 feet times 8 feet so this gives me 3000 cubic feet and c is I making a very crude approximation one thousand feet per second.

So I can either convert it to si or this is cubic and when I do a cq I get feet cube per second cube so it will so n is 4 over 3 pei times 20 times 10 to the power of 3 cube over 1000 cube times z room, z room is 3000 so this gives me 10 to the power of 8 where 100 million nodes in a room at 2000 hertz if I bring this down to 2000 this number will become approximately equal to 10 to the power of 5, If I bring this down to 200 then this goes 100 nodes.

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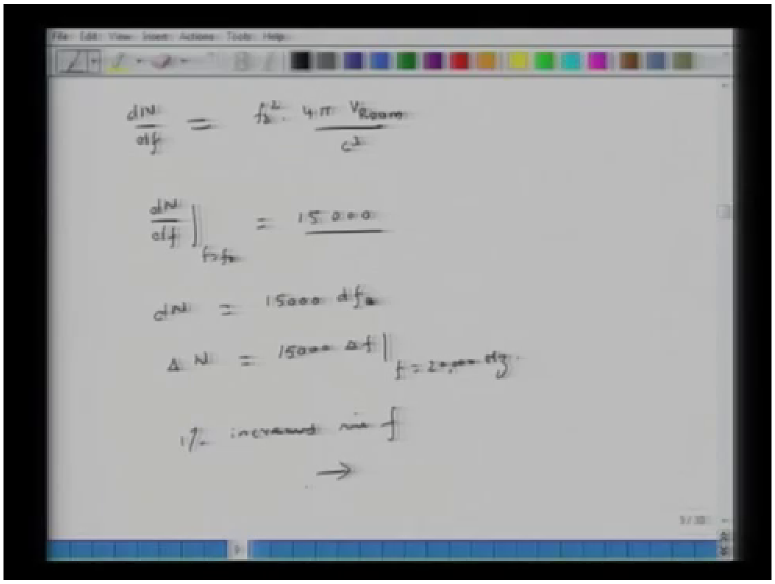

So for low frequencies you will hardly hear change in the sound as you move from one location to another but at higher frequencies at very high frequencies you will hardly hear the change in frequencies also the spacing will be very time, wherever your ear can perceive (because you have ears typically you) not typically always we hear from two ears so if the frequency the spacing is very tight just may hear something this may hear something and the brain will kind of average it.

And give you a kind of a mean or an rms value but if this spacing from this ear to ear is less than the spacing of the node then you may hear changes in pressure, for very big auditoriums there will be nodes for base as well yes they are bigger issues and very big auditoriums which is we will talk about it maybe in next lectures reverberation and echoes but yes for large auditoriums you will have change in perception of sound for low frequencies.

But for most of these rooms of this size that is not initial, so very simple math but it shows you a lot if I do dn over df then I get f not square times 4 pei z room over cq so again let's do a very quick example for this example for these data my dn over df at f equals f not comes to be 15,000 ok, so dn equals 15,000 df so if I make a one person change in frequency as change in n where (n is sma) delta n is a small change is basically 15,000 times delta f not or actually f.

At f equals 20 kilo hertz.

So what that means is that at 20 kilo hertz if I increase again just for appreciation if I increase my frequency from 20 kilo hertz to 20,001 I get 15,000 extra nodes in the room of the dimension, if I increase my frequency by 1 percent you know increase in frequency then extra number of nodes which I get is 3 million you can use this relation to calculate, so at higher frequencies very sensitive through changes in frequency in number of nodes changes very rapidly.

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So what we will do now is a very simple experiment so I have a very basic speaker and what I will do is from the computer I will generate 2 frequencies the first frequency will be 125 hertz and what you can do is that when the sound is coming you can close one of your ears and you can move your ear like this that whatever and see if you can sense the change in sound pressure level.

Then I will stop that and I will play another frequency at 3 kilo hertz or 2500 hertz and then try to figure out whether you can sense changes in sound pressure level ok, the other thing is that if I play the sound directly from my computer it’s a very clean sound but looks like there is some electronic issue so even if I am not playing it and when I run it there is some background noise static noise so you can ignore that, volume from laptop is but for purpose of this experiment.

I have checked it this noise will not lead you to wrong conclusions so what I am going to do is when you maximize this and play the ok so that was 125 and hold your thoughts so now I will change the frequency and this is now 2500 hertz so what did you sensed did you see what was the difference when you play between 125 and 2500 hertz, so what we will do is for this room this is like a rectangular 3 D room kind of there is some extra non rectangularity here.

What we will do is complete the number of nodes and see because of reflections in everything at the end of the say you will get all the modes which will be rectangular what this room has is lx equals 7.06 meters ly equals 7.06 meters lz equals 3.12 meters ok so my v room is 155.5 meter cube so if I compute the number of nodes in this room is 4 pei fq over cq 3 times z room and if I do the math I got to calculate it today after long number of years.

I get 1.586 times 10 to the power of minus 5 fq so for a 125 hertz frequency I get 1.586 times 10 to the power of minus 5 into 125 cube 31 nodes and for n 2500 basically I can just multiply this by 8000 and I will get the same number right so it's about 2.48 10 to the power 5 equals 2.48 lakh nodes or 248000 lambda 125 hertz c over 125 is 2.76 meters right so lambda at 125 hertz divided by 2 this is the spacing of nodes in an approximate sense.

Because it also has to depend on theta which we are not talking about here is 1.4 and lambda for 2500 is 0.138 meters so lambda over lambda 2500 over 2 is 6.9cm, so the first thing which you may have sensed which Abhishek mentioned was that for 2500 hertz tone he had to move is head very little and it's directly related to the fact that lambda over 2 is just 7 centimeter.

About this much, I was browsing the net and there is some website. Where they have a lot of frequency tones mp3 file so I just downloaded two frequency and the Shitej found some tools which I have not used where you can actually now input frequency and it will give you that particular tone so again to Shitej's point what he was saying earlier for low frequencies 100 hertz 60 hertz 40 hertz actually I tried playing 40 hertz on this bit it will not it was not producing sufficient I could not listen 40 hertz sound on this.

Because it's too small where you have seen that for 40 hertz to have a significant test skill level first is resonance point has to be low which we saw earlier probably its resonance point is maybe 90 80 100 hertz and second thing is it has to move a lot or it pistonic area has to be large it could not generate also I tried playing 125 hertz on this computer and I could not hear anything just my computer itself so you can generate the frequency but you may not be able to hear it.

Unless you have the right speaker the perception of low frequencies does not change significantly in rooms of this sides or even a larger room because you have to physically move a long distance from one place to other to measure the distance which brings me,

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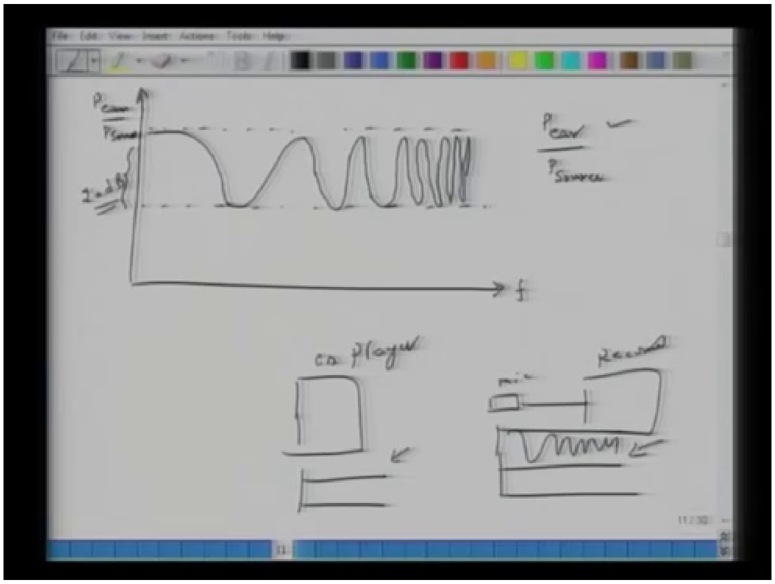

So to the next point that if I am going to plot the pressure at my ear right I am sitting in a chair in a room someone is speaking or some music is coming out or some noise is getting generated.

So my ear is hearing something and that is p ear and the source is generating some noise or sound content that is p source so if I plot p ear over p source and in the horizontal axis I am just varying frequency because of some of these effects which you just now experienced my plot will look something like this right, as I move from one node to other node there will be variation in the pressure and in a lot of cases this is about 20 decibels let say this is 20db.

So the maxima and minima are of by about 20db let say and that is basically because if the person is not moving then for different frequencies will have here different spl levels and there will be some frequencies where he will hear the minimum there will be some frequencies where he will hear the maximum and that spacing will get tighter and tighter as frequency goes up, we have to draw an exact but basically what I am trying to show here it’s not (40:36 sinus pattern).

Right it will be wavy because it’s a continuous function it won’t be jumping think about this, this is p ear you can also replace it by microphone and a p source could be a radio or a tape recorder or a cd player so you have a cd player with speakers and everything and you have a microphone and then you take from the mike and then you record it that sound this cd player could be producing a flat frequency response and what the mike is going to record.

Will be something like this then you play this sound again and you hear it then the overall threshold so the first thing is in the first step this will be of flat frequency response this will be a (41: 37wegly) frequency response, even though this is producing sound faithfully mike is not recording it faithfully.

Because there are echoes, there are reflections there is interference and because of all that the mike is recording something totally different.

When you are measuring sound this is very important you may conclude based on this data that your source is producing this kind of a sound but that may not necessarily with the case that is why in reality lot of times people make measurements and an quake chambers where there are no reflections but anyway going back so you are recording this

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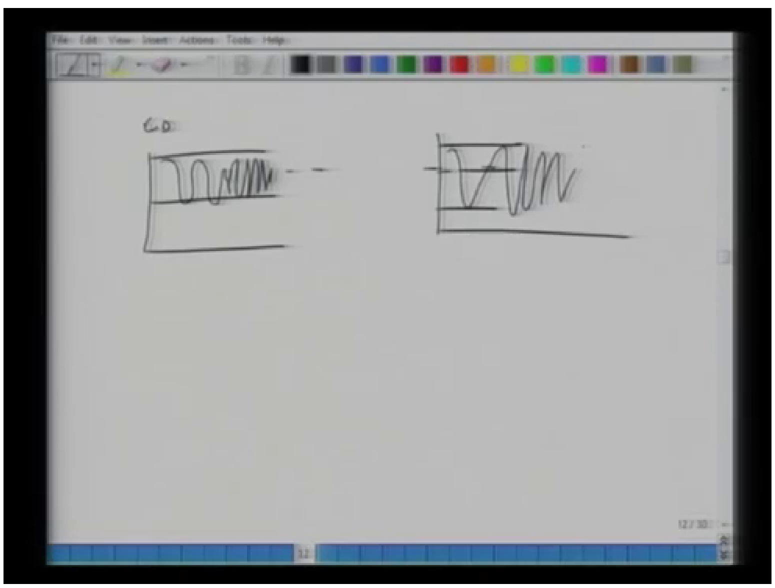

So in next step your cd is producing this sound because I am feeding that recording back to the cd, so it’s producing this and then the mike will record from here it will this threshold will go down right the top will remain same but this threshold will go down so it will become like this.

Do you understand what I am saying so each time you record an original sound track the threshold at certain frequencies will keep on going down and down and down, that is the reason whenever you are trying to duplicate sounds a lot of times.

It's not directly from what is being heard through a sound system rather (than) rather what is preferred is that you just directly take the electronic output in terms of voltages and recorded if you want to replicate but if you reproduce sound it will also capture you will also be capturing the back of reflections and echoes and that will destroy the original sound track totally if you do this iteration several time and the more you do the more corrupted.

Your overall sound recording becomes and it could be for enjoying purposes it could be for entertainment and all that so that is something I wanted to share with you so that's all for today in the next lecture we will try to capture we will try to explain a little bit about mufflers which is a little bit of a jump from we are doing today in several lectures back in time we had talked about Helm holds resonators right we are using a volume and a tube.

We could eliminate specific frequencies or not specific frequency a is specific frequency or a narrow band located around a specific frequency so that is Helm holds resonators what we will talk about is little bit about our mufflers which are used in a lot of automobiles or all the automobiles, scooters, motorcycles, cars, trucks how do they work and then we will also start introducing the impact of echoes, what is a reverberant feel in rooms.

How can you reduce the impact of reflections in rooms, so that's what we will cover in next couple of lectures that's all I want to talk about thanks?