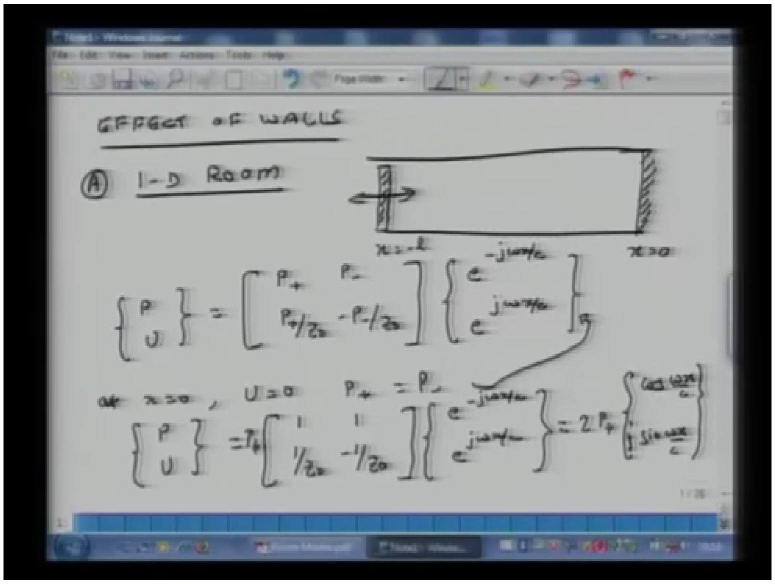
Acoustics
Professor Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
Module 7
Sound in Public Spaces and Noise Management
Lecture 1
Sound Propagation in Rooms, 1-D rooms, 2-D rooms

So starting today and in next couple of lectures what we will be touching upon is how does do a sound propagate in rooms into a real rooms , So far we have tried to understand propagation of sound in narrow long tubes but what happens when sound isn't in actual room and closed space what happens what kinds of modes it has so that is something we try to understand and that will lead us into the next segment of this course architectural acoustics.

So what we will be doing starting from today is a little bit exposure to architectural acoustics how do you design spaces where its sounds good we will develop some equations and also criteria in terms of what means a good sound what does not mean a good sound in a architectural space, so what we will do is today explore effect of walls on sound then we will start with a 1 D room, a 1 D room is a very long room.

The length is significantly larger compared to the other two dimensions and also the wave propagating in it is a plain wave, so we will start with 1 D room then we will go to 2 D room and then in few time we will try to capture 3 D rooms today otherwise we will do it later and all we will be doing in next couple of lectures is effect of walls, we will not be considering effect of medium because as sound travels through the medium it also in reality gets damped.

(Refer Slide Time: 02:11)



Because here also has some dampening but we will be ignoring that effect, so this is effect of walls and the first thing is bar a is 1 D room ok so we can see there long tube one dimensional tube is we call a 1 D room and at x equals zero it sees perfect reflection the length of the room is l and what I am doing is I am exciting it through a piston at x equals minus l so we know from our earlier course work.

That pressure and velocity can be expressed as p plus p minus p plus over zee not p minus over zee not is an active sign here times an exponential form which captures the forward travelling wave e minus j omega x over c and then the reflected wave would be e j omega x over c and notice that in this I am ignoring the time portion of it so all this thing in reality gets multiplied by e j omega d. So we know that fx equal zero the boundary condition is that there is no velocity.

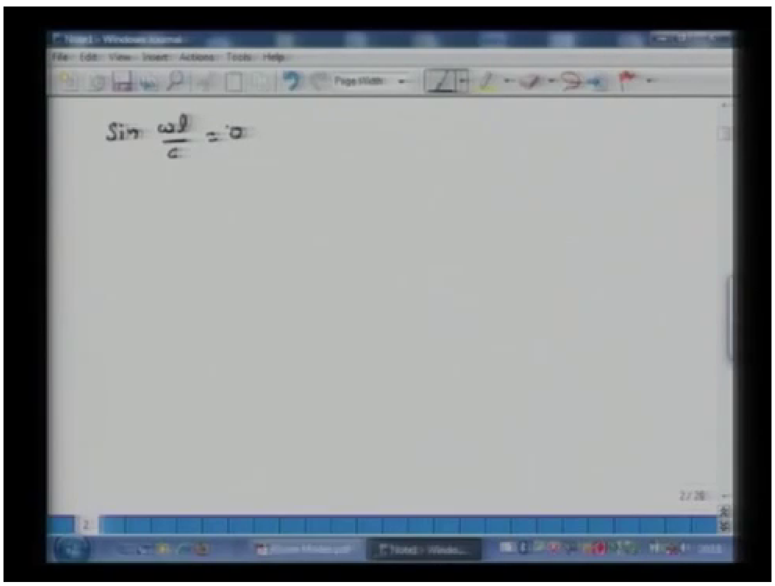
Q is zero so at x equals zero u equals zero and what that gives is e plus equals e minus so plugging this back into this relation I get p and u can be expressed as one one one over zee not minus one over zee not I have a p plus outside the matrix and e minus j omega x over c e j omega x over c ok and this if I expand and add up the terms what I get is 2 p plus times cos sin omega x over c and sin omega x over c minus this j here p plus.

Where in mind it can be a complex intuiting we have not assumed that p plus is a real or a complex intuiting. So if we have to figure out p plus we will pose the first boundary condition that at x equals zero u is zero and at x equals minus l we know what is the velocity. So once we impose that boundary condition on this particular equation I can get the value of p plus the eigen figure or what is the value of p plus.

Because this is the velocity which I know at x equals minus l I am inducing that velocity now consider case that after I have excited this piston for a while a few seconds then I bring the piston to a dead stop all of a sudden did, what happens in that case in conceptually what will happen is that the sound which is there in that 1 D room it will travel back and forth and it will get reflected from the walls on both hands and what is the behavior of that a phenomena.

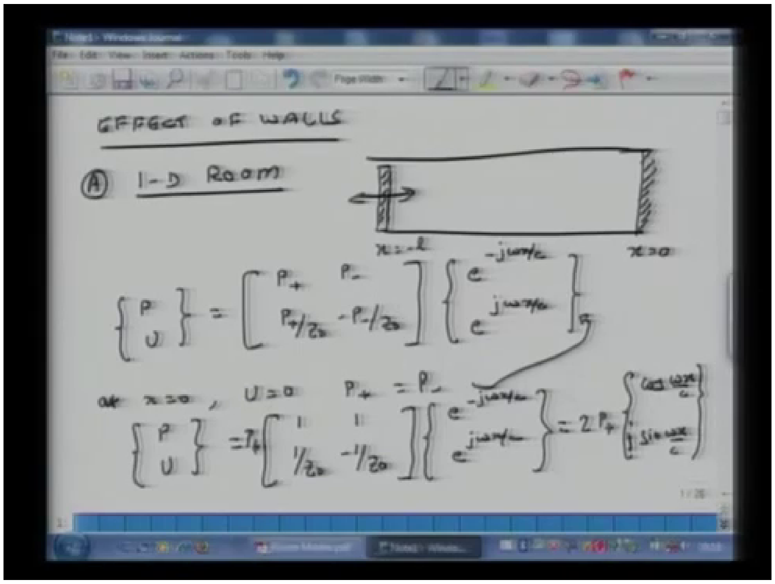
That is what we are going to explore now, so once I have brought this piston to a dead stop.

(Refer Slide Time: 06:07)



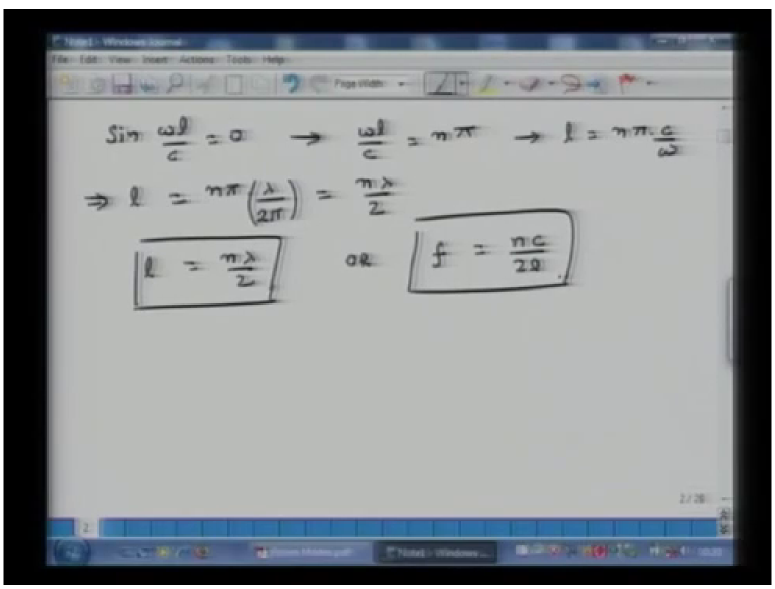
So the velocity at x equals minus l becomes zero, so then I can say that sin omega l over c equals zero, why do I say that?

(Refer Slide Time: 06:22)



Because here velocity is 2p plus times j sin omega x over c right so at x equals minus l this entire term has to become zero.

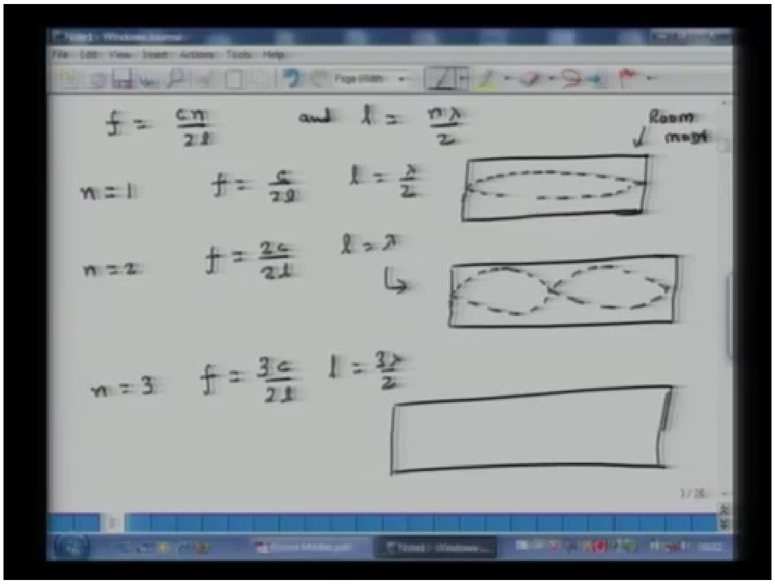
(Refer Slide Time: 06:38)



Which means sin omega l over c has to be zero, which means omega l over c has to be n times pei which means l equals n pei times.

Or l equals n pi times I know c over omega I can transform it to lambda over two pi and if I simplify it I get n lambda over two so one relation is l equals n lambda over two n equivalently I can also show that frequency equals n c over two l ok.

(Refer Slide Time: 07:39)

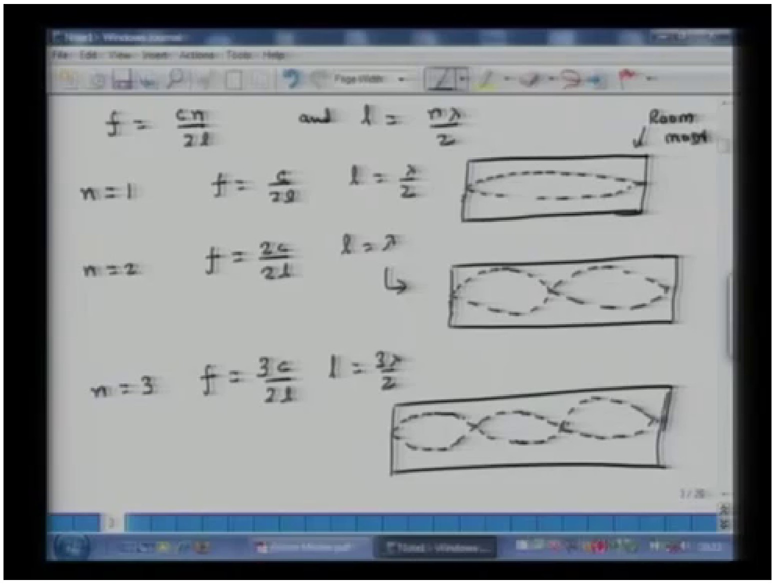


So f equals cn over two l and l equals n lambda over two so let's consider n equals one then my frequency equals c over two l and l equals lambda over two right.

So if my two tube or this 1 D room is a long room like this the velocity profile what that means is going to be something like this velocity is going to be zero and the wave form is going to be such that half of wave will cover the entire length of the room for n equals to f equals two c over two l, l equals lambda and in this case if I draw the room mode it will be something like this so what you have drawing here is the room mode associated the shape.

And what you are computing here are natural frequencies or room normal frequencies of the room so f is normal frequencies of that one dimensional room and associated with that is a mode shade which is called as room mode, for n equals three f equals 3c over 2l when l equals 3 lambda over 2, so we will talk about that I am going to come to that point let me just make this picture that's exactly the point what I am trying to get to.

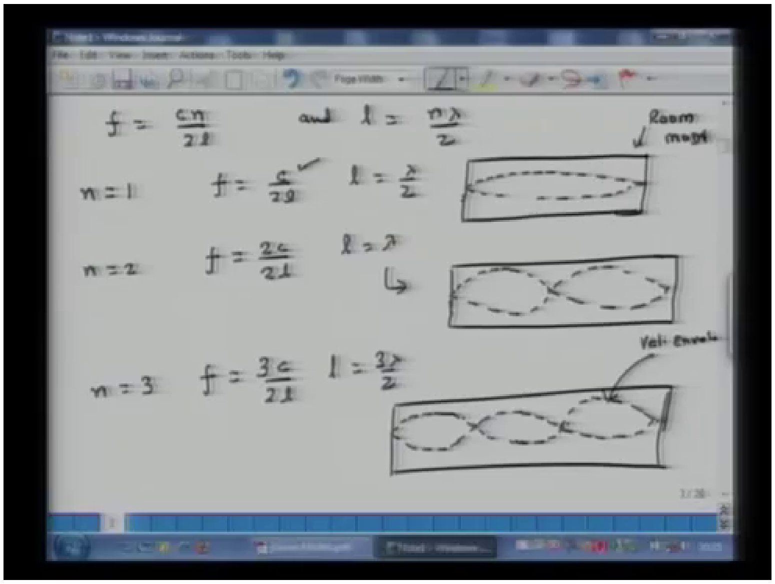
(Refer Slide Time: 10:10)



Because here excitation frequency could be anything like these frequencies have discrete values it could be let's say 100 hertz, 200 hertz, 300 hertz, 400hertz, but I can excited at 40 hertz or two hertz or hundred and one hertz so we will talk about that so all what we have shown here is that a long one dimensional room has certain specific natural frequencies associated with it this is what these pictures in this analysis is showing.

And these natural frequencies are dependent on the length of the room c is constant 345 meters per second so they are only dependent on the geometry of the room which is on length and associated with the length is a particular natural frequency and they are infinite in number because n theoretically you can run up to infinity. So you have an infinitive infinite number of but discrete frequencies and each frequency has a specific mode shake ok.

(Refer Slide Time: 11:10)



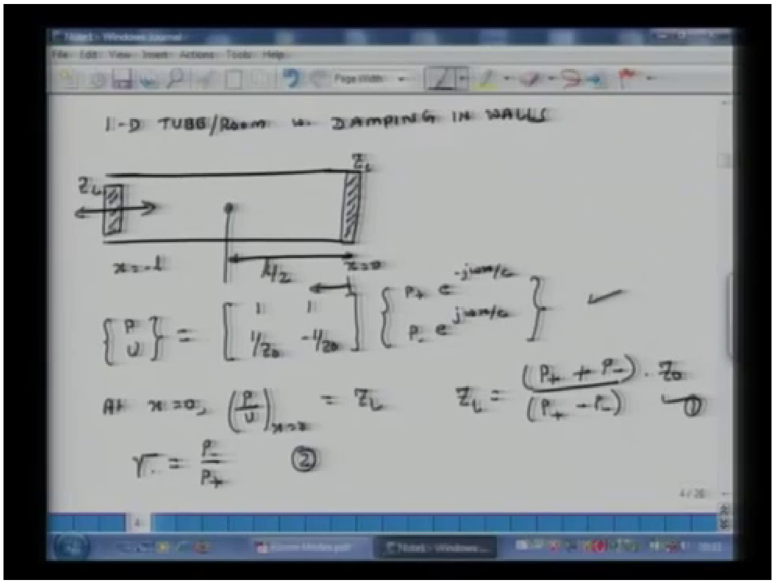
Another thing is so this is the mode and this is actually a velocity envelope, the velocity at any given point at this particular value of x it will not exceed the envelope. But as Mayank was saying that my excitation frequency could be anything, let say that which use a particular value of l such that f1 is hundred hertz then f2 will be two hundred hertz, f 3 could be three hundred and so it will be hundred two hundred three hundred four hundred so on and so forth.

But I could excite the room my piston could be initially getting excited at 45 hertz which is not an integral multiple of this so the question is what happens when the piston is exciting at specific frequencies which are not same as this. So we have two situations if the piston is generating frequency or was in this case, because now it has come to a dead stop. If it first generating frequencies which are coincident with the room modes.

Then you will have these type of standing modes in the room and in theory they will go on till infinite time because there is nothing which is dampening the vibrations in the room, if the frequencies of a piston (of) the frequency of the piston was not equal to the exciting No the natural frequency of the room then you can develop a Fourier solution which where these will be a natural frequencies and in terms of these natural frequencies.

You can develop a Fourier solution which will tell you the overall response of the room, does that answer your question, thank you, so in this analysis we did not include any damping it did not include damping in the air and we also assume that there was perfect reflection at both the walls.

(Refer Slide Time: 13:07)



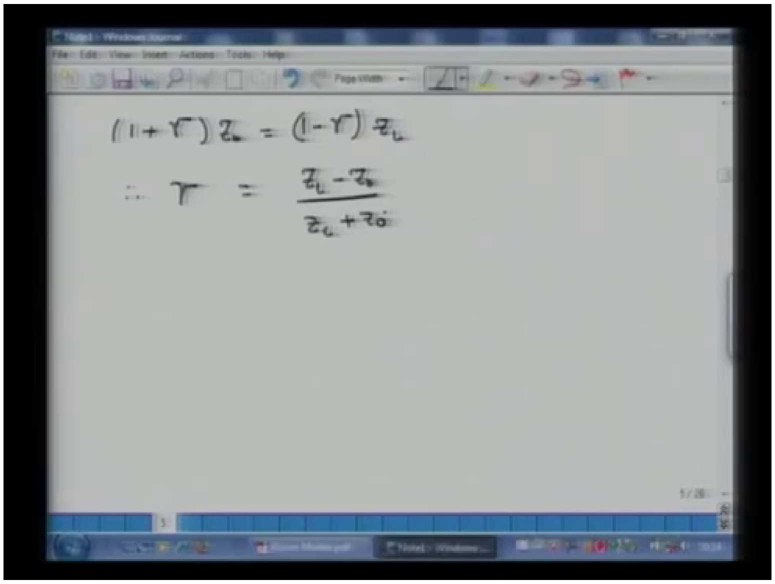
So our next step will be that 1 D tube or room with damping in walls again so even here we are not considering damping in the air we still assume it to be zero.

So I will draw another long room it has an impedance at the end of the room zee l the excitation is coming through a piston and here also the impedance is zee l, x is zero at the fixed end x equals minus l that's my access system, so the total length of this tube is l and what I am doing is I am putting a fresher microphone here at the distance l over 2, I am putting a pressure microphone and I am measuring pressure what happens.

So my pressure and velocity relationships are something like one one one over zee not minus one over zee not p plus e minus j omega x over c e minus ej omega x over c so till so far I am not imposed any boundary condition now we know that at x equals zero and impedance total impedance is what is the impedance that x equals zero zee l so p over u x equals zero equals to zee l so I used this to figure out the relationship between p plus and p minus.

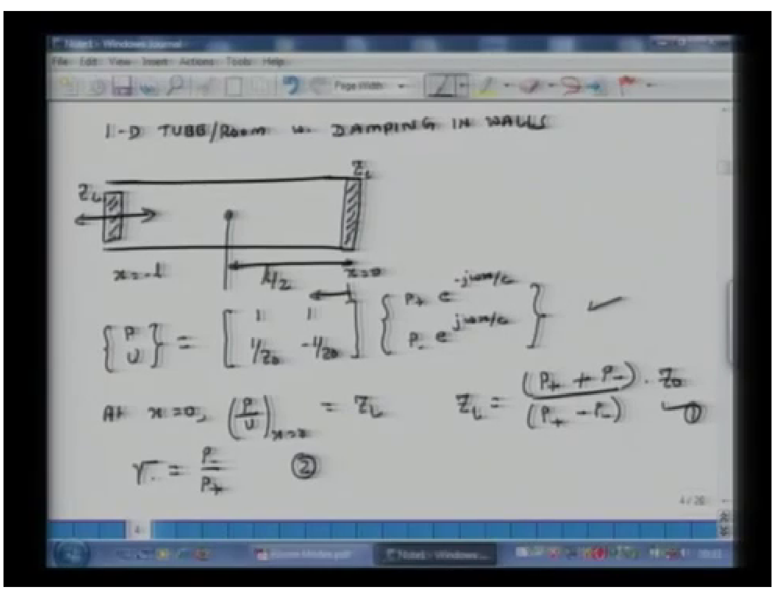
So essentially work that tells me is that zee l equals I put x equals zero in this relation so I get p plus plus p minus over p plus minus p minus times zee not ok so zee l equals p plus plus p minus in this entire relation and if I introduce that term which we had earlier defined a reflection coefficient gamma is essentially p minus over p plus so let's number this equation one and this is two.

(Refer Slide Time: 16:03)



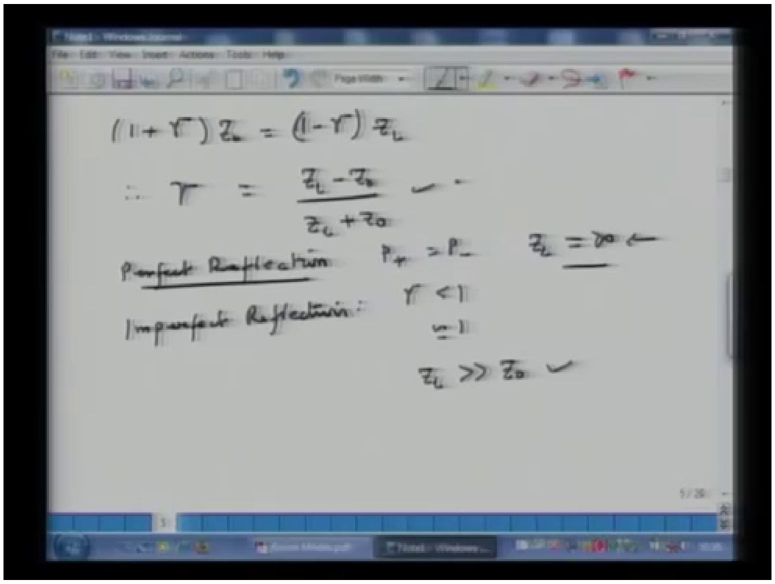
So if I introduce gamma in this expression for zee l. Is essentially what I get is one plus gamma zee not equals one minus gamma zee l and p plus get cancelled out ok so therefore and then I manipulate this and I get gamma equals zee l minus zee not over zee l plus zee not, question gamma minus one ya ya ya.

(Refer Slide Time: 16:47)



let's if I take p plus out this will be one and then minus ok. So this is my expression, this essentially is same expression which we have shown earlier for perfect reflection right p plus equals p minus what that means is.

(Refer Slide Time: 17:07)

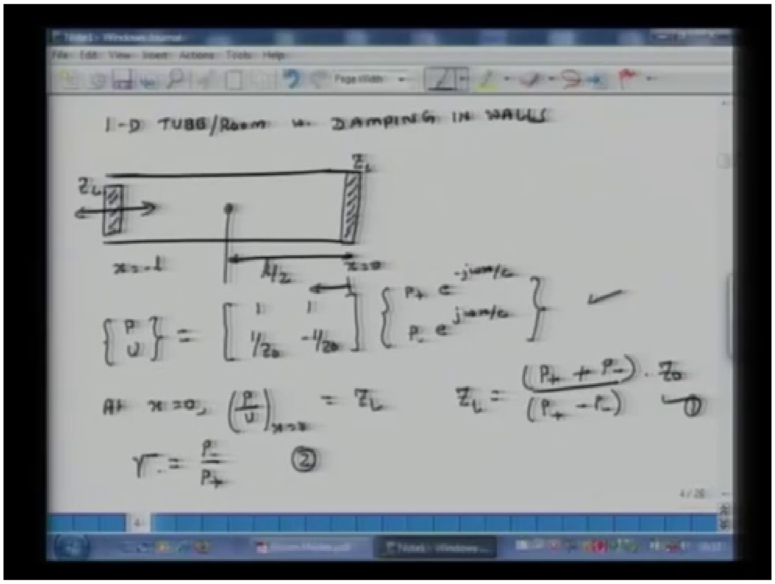


And that can happen p plus equals p minus which means gamma is one and that can happen only when z l equals infinity so were perfectly rigid surface z l is infinity for imperfect reflection gamma is less than one and for walls which are very stiff gamma is still less than one but almost equal to one and that is the case of a lot of structures where you have break walls or concrete walls or stone walls that is the type of structures you have.

Gamma is almost equal to one but it still a little less than one and in that case zee l is not infinity but zee l is extremely large compared to zee not you must introduce this relationship in this one you get gamma is almost equal to one physically what zee l equals infinity means is zee l is essentially at equals zero zee l is the ratio of pressure and velocity right, so zee l being infinity means that pressure is non zero but velocity is zero.

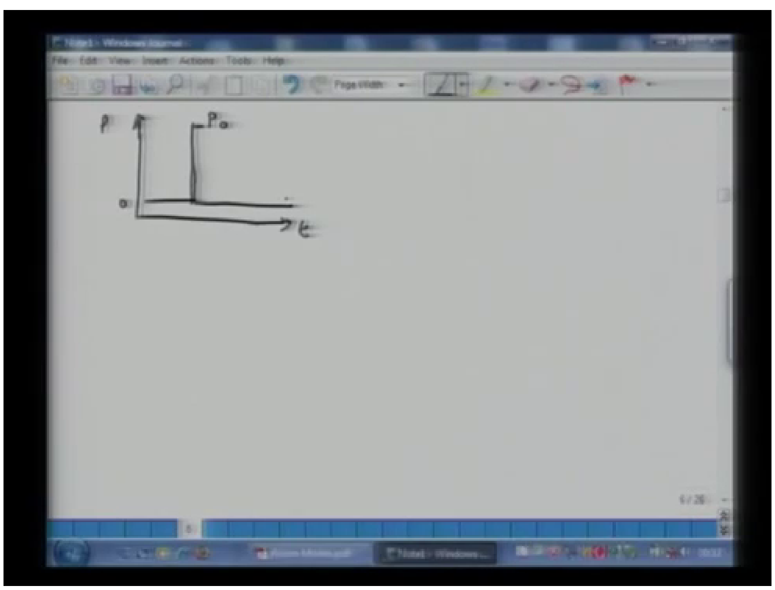
Because the wall is perfectly rigid so nothing is moving so velocity is zero so p over u becomes infinity that's what it means physically ok.

(Refer Slide Time: 18:33)



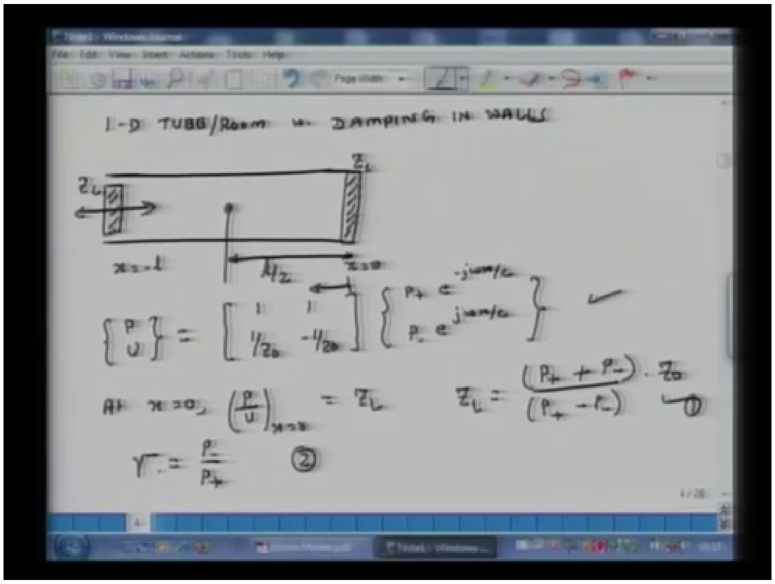
So now we will consider a scenario where zee l is very large compared to zee 0 and this is a thought expansion you think about it that instead of this piston which is getting excited continually what’s happening is that the piston at p equals sometime equals some initial value t not.

(Refer Slide Time: 18:57)



It generates an impulse pressure what is an impulse pressure? That the input signal is something like this t and then this is pressure so the input signal is zero to begin with and at some critical time at some specific time it shoots up to a particular value it is p not and then it drops again and it remains zero forever ok.

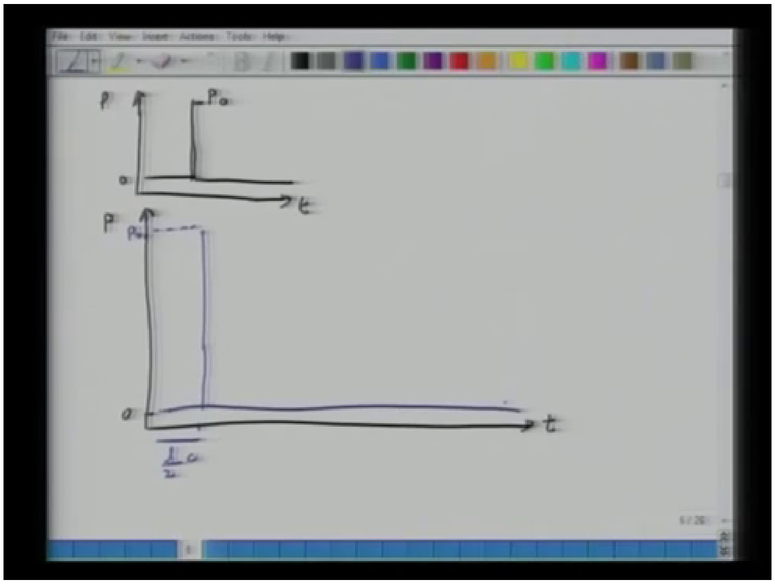
(Refer Slide Time: 19:22)



So in this tube what disguise is piston is doing is that it generates an impulse pressure at a specific moment and then it falls silent.

It does not vibrate anymore and what we are interested in figuring out is that the wave generated because of that impulse pressure how does it change with time as says by this particular microphone this is the only instrument we have in the setup so what we are interested in is that once this impulse pressure has been generated that x equals l what is the pressure sensed by the microphone which is located at x equals minus l over 2 with time.

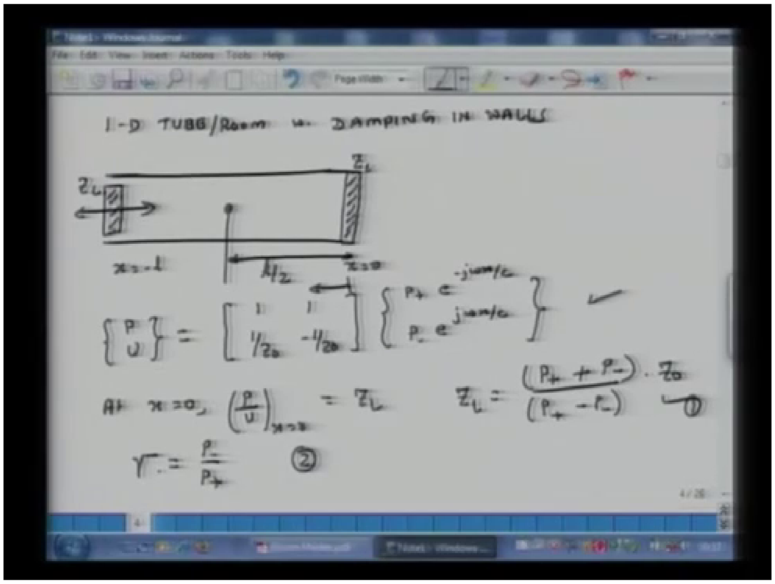
(Refer Slide Time: 20:06)



So this is how the pressure will look like so here what I am plotting time and here I am going to plot pressure. So just for clarity purposes my zero is little above x axis this is p equals zero ok this is l over 2c, l over 2c is the time it takes for the pressure to start from here and reach this point right, the distance is l over to the velocity of wave propagation is seen so the total time it takes to reach the microphone is l over 2c right.

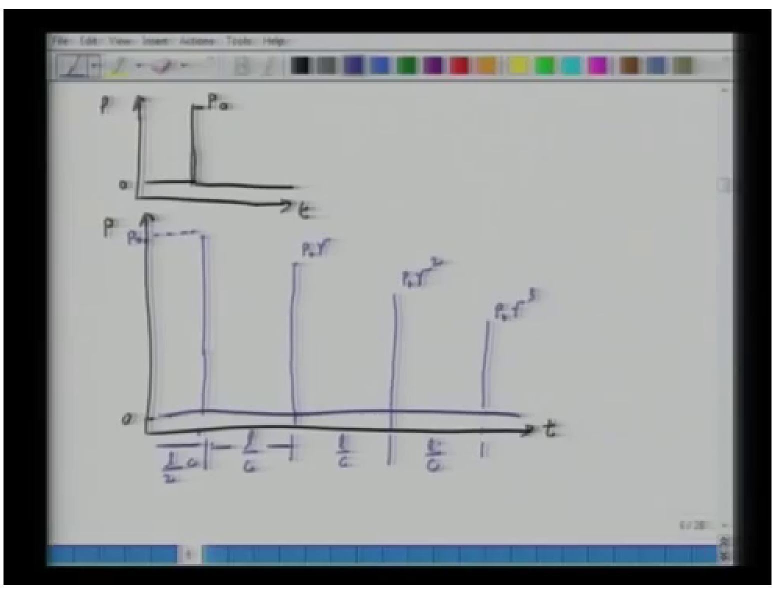
What will be the pressure sensed by the microphone at time l over 2c? so this is p not sorry so before that time the pressure microphone will sense zero pressure and that time it will sense p not and after that it will fall.

(Refer Slide Time: 21:15)



then the same pressure wave it will travels from here it will come to x equals zero location so it will travel a distance of l over to it will get reflected and come back to the microphone right.

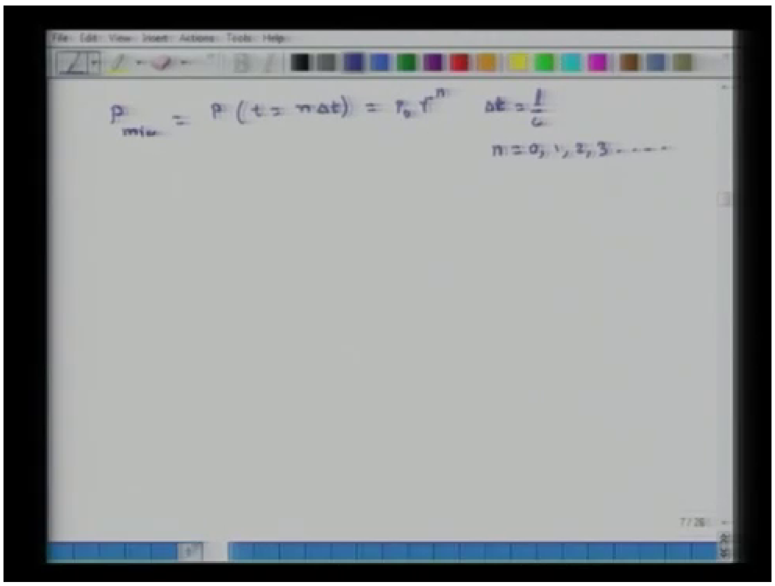
(Refer Slide Time: 21:31)



So my next time interval when the pressure will be sense by the microphone will be after the time interval l over c so this distance is l over c, what will be the pressure sensed by microphone at this time? Gamma times p not right because the reflection coefficient is so and gamma is a little less than one so this is p not times gamma then the (micro) wave will continue to travel in negative direction it will get reflected by the piston which also has a reflected coefficient of gamma.

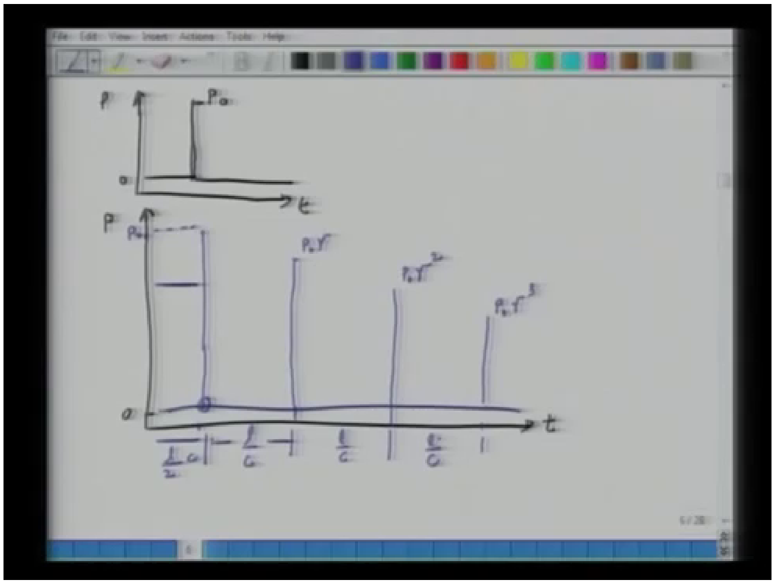
Because zee l is same so at after another l over c seconds the microphone will again sense pressure and this time the pressure will be gamma square and then p not gamma q and so on and so forth. so my wave will continually decay with time just for an impulse function.

(Refer Slide Time: 22:33)



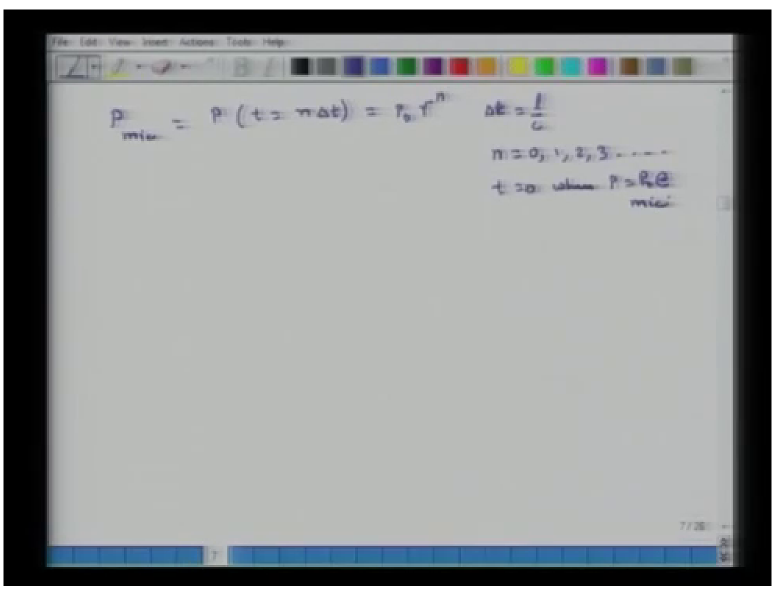
So we will develop some mathematics along so we can say that p microphone is p t equals n delta t, t is a function of t where t equals an integral number n times delta t will define delta t. Equals p not gamma n right where delta t equals l over c and n equals zero one two three and so on and so forth also.

(Refer Slide Time: 23:13)



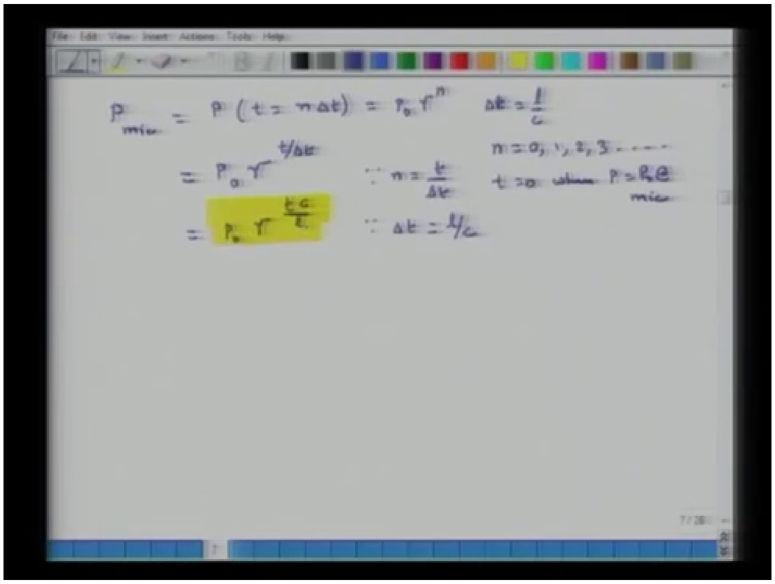
This equation is valid at t equals zero if you start measuring time from this point right if I measure time from this then in this zone this relation is not valid because they are is l over two factor but after t this point and so on and so forth.

(Refer Slide Time: 23:38)



This relation whole square, so my t equals zero is when p equals p not at microphone at mike.

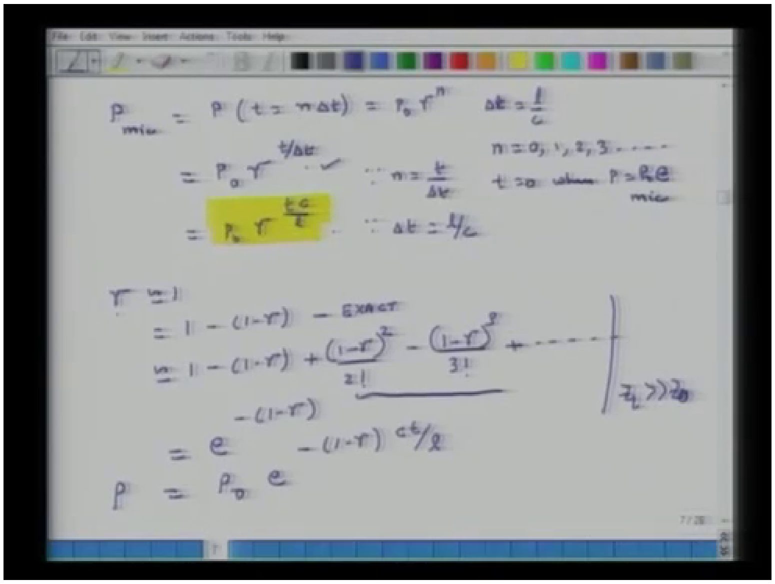
(Refer Slide Time: 23:50)



So my original position is when pressure equals p not at microphone that's my t equals zero position so I get p not gamma t over delta t because n equals t over delta t ok and which is p not gamma tc over l because delta t equals l over c in this relation what I have done is all of a sudden I have converted a discrete time function right to continuous time function, here is a big assumption I have made here.

It was discrete here it was dispute here it may not be, for an impulse function this may not be this may be more accurate this may be that much accurate but we will talk about this later.

(Refer Slide Time: 24:57)



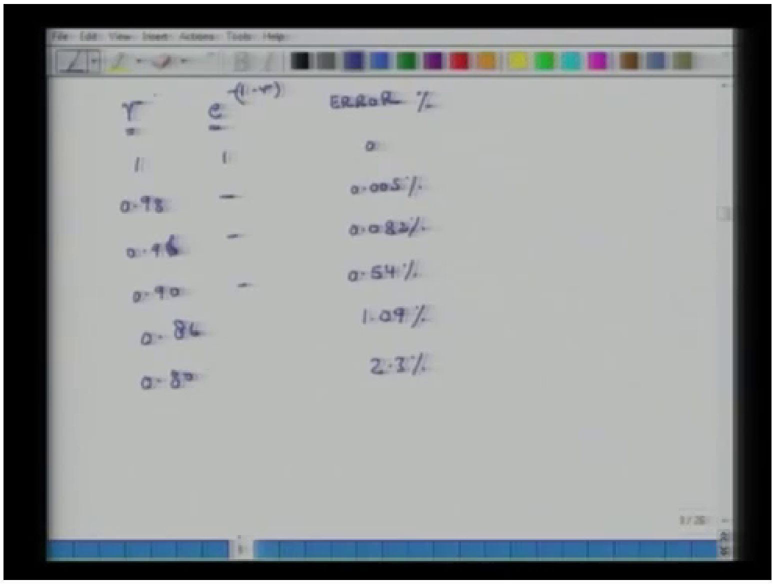
So now we say that gamma is approximately equal to one and this is based on reality most of the walls are fairly rigid and they are gamma is fairly close to one I can say that is same as one minus one minus gamma this is an exact relationship ok.

But then I can make a big approximation and we will explore the goodness of this approximation later I can say one minus one minus gamma plus one minus gamma square over two factorial minus one minus gamma q over three factorial plus so on and so forth if gamma is very (sma) is almost closed to zero than the contribution of all these terms will be very small these extra terms and when you look at this entire series you can say that this is essentially same as exponent.

And we will see the goodness of this approximation a little later so my p equals p not exponent minus one minus gamma ct over l again all this approximation is good to the extent zee l is very very large compare to zee not.

So what we have done is we have made a big jump we have approximated gamma as exponent of minus one minus gamma and we will see how good this approximation is so what I will do is I will just select different values of gamma.

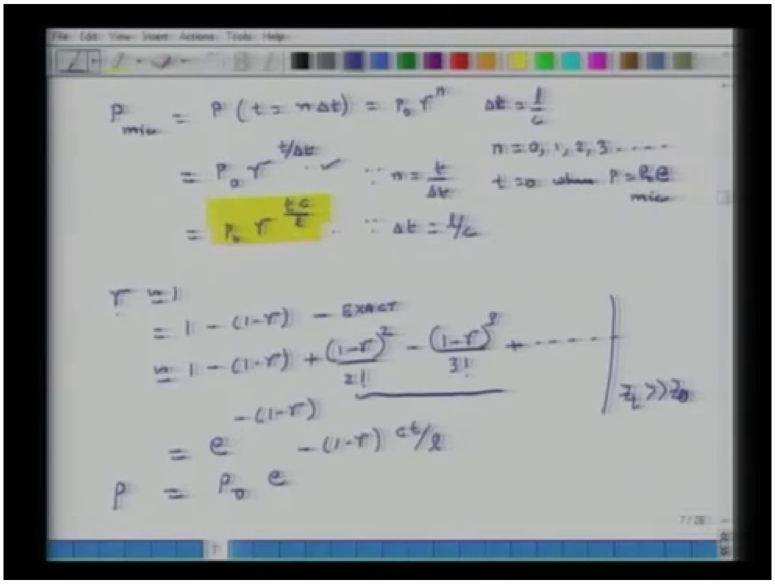
(Refer Slide Time: 26:46)



And then in another table I will take minus one minus gamma ok and then I will compute the error, error is basically this term minus this term divided by this term times hundred I am computing error in percent so when gamma is one disguise one error is zero gamma is 0.98 I am not going to list these values here error comes to 0.005 percent gamma is 0.94 96 my error is 0.082 percent this is in percent which is less than one tenth of a percent.

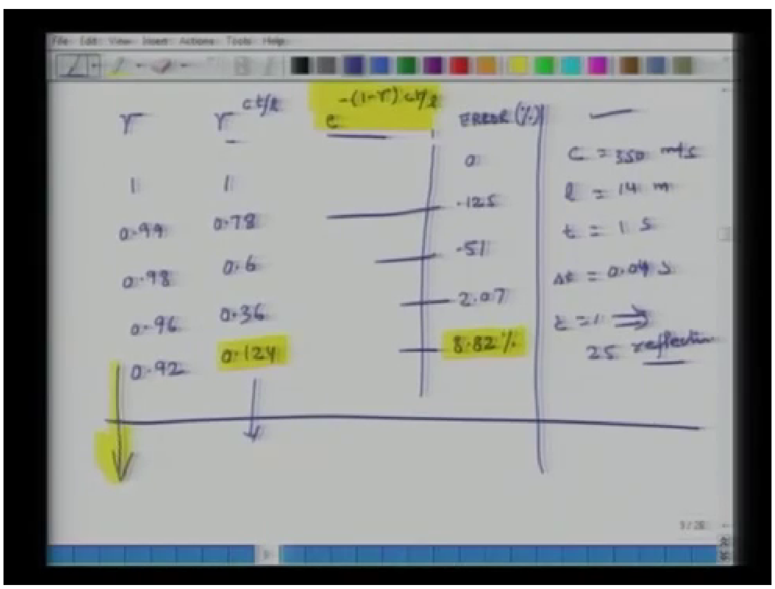
Gamma is 0.90 my error is 0.54 percent Gamma is 0.86 my error is 1.09 percent Gamma is 0.8 my error is 2.3 percent so this approximation is not that bad that’s what I am trying to show to the extent I am comparing the closeness of Gamma with respect e times minus one minus gamma.

(Refer Slide Time: 27:59)



But in the actual equation it’s not just e minus one minus gamma but this also time ct over l right so this ct over l may amplify the error also.

(Refer Slide Time: 28:20)



So l at very quickly did another table so what I am going to do here is I will pick different values of gamma I will compute gamma to the power of ct over l, I will compute this value with the exponential form and then I will compute the error in percent ok (28:41 unless less) three four values in this my c is 345 meters per second standard value actually for this one I assume 350 just to make my calculations simpler.

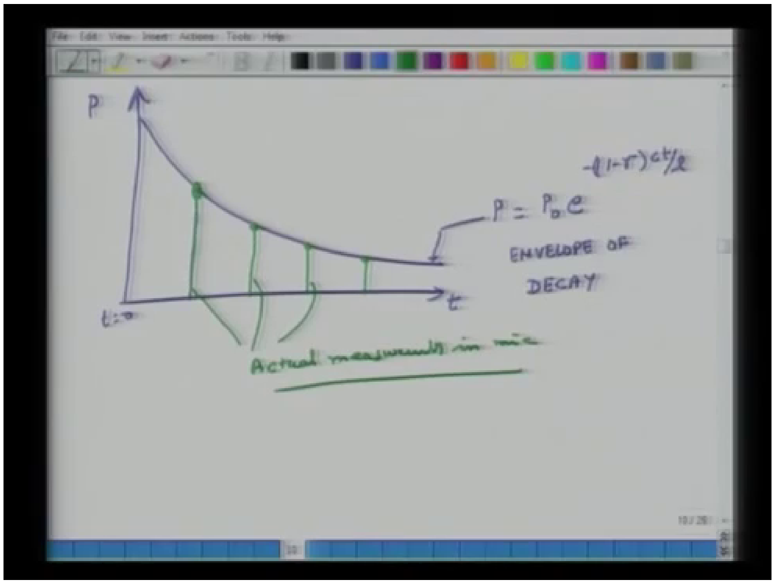
I assumed l to be 14 meters long and I took t to be one second before I construct this table I wanted to you to understand what does t equals one second mean so my delta t what is going to be my delta t, delta t is l over c right so what is it going to be 0.04 seconds fourteen over 350 0.04 seconds so d equals one second means 25 reflections right, that's a very big number of reflections so let see for 25 reflections so t equals one means 25 reflections.

It was one or two reflections so lot of reflections so I will just list value of Gamma one 0.99 0.98 0.96 0.92 and I am just going to side the errors actually this is one 0.78 0.6 0.36 0.124 and my error is going to be obviously zero in this case, in this case its 0.125 and this is again in percent here its 0.51 here its 2.07 and finally 8.82 percent so for 25 reflections this particular approximation you know this one is not that bad even for a high value of.

Or for a low value of gamma we say like 92 percent the error is still 8.8 percent which is not that bad, so this approximation fairly descent approximation now of course if gamma starts between very small and it's no longer close to one and or this ct over l parameters starts between very large then approximation will not be valid but in that case, see this after 25 reflections for a little high value of damping this is only 1.124 but that means is that.

The 25th reflection will be only 12 percent as strong as the original reflection so once you go down this scale ok if you go down this scale further these numbers become so small they actually start going to t1e minus 6 1e minus 12 1e minus something like so on and so forth then the comparison of those very small numbers with another set of extremely small numbers it becomes meaningless because essentially what the story tells you is everything is died down.

(Refer Slide Time: 32:05)

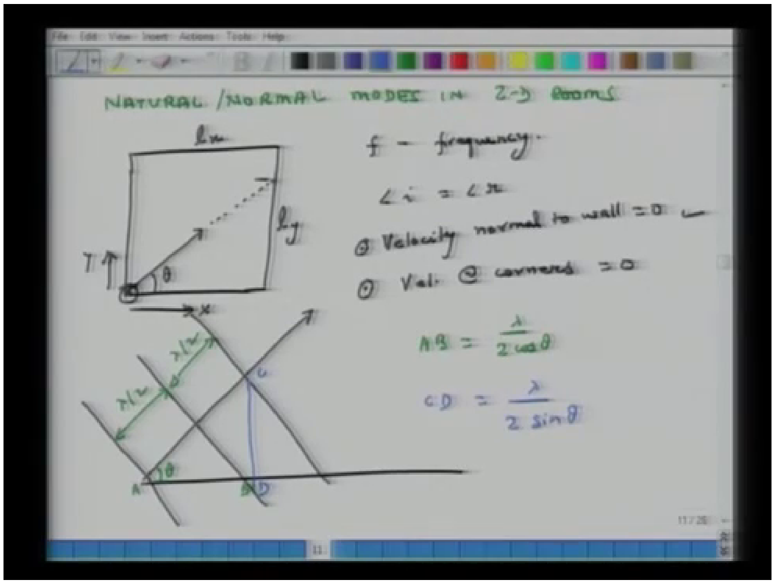


So this approximation is fairly good the plot of this expression so what I am going to plot here is time and here I am plotting p so it looks like something like this so what this is p equals p not exponent one minus gamma ct over l and this is called envelope of decay, what this shows is that the decay will follow this curve but the actual pressures and this is t equals zero the actual pressures will be like this for each reflection you left discrete pressure fluctuations in the mike.

But the pressures will be bounded by this particular envelope of decay these are actual measurements in mike, one last thing this particular envelope decay will shift if we change the location of the mike because we are just assumed that it was in center so wave going on the right side and wave on the left side they (33:45 tide) equal amount of time so it will change its shape and so will these discrete values also change if I shift the position of mike.

So position of the instrument is very important at least in this type of measurements where you are measuring the impact of reflections so this gives you some flavor of 1 D rooms right, one thing is that there is an exponential decay in the room if there is wave standing wave and the thing also each long room is associated with a bunch of standing modes and in terms of those standing modes you can decompose any set of externally imposed frequencies through for your expansions.

(Refer Slide Time: 34:32)



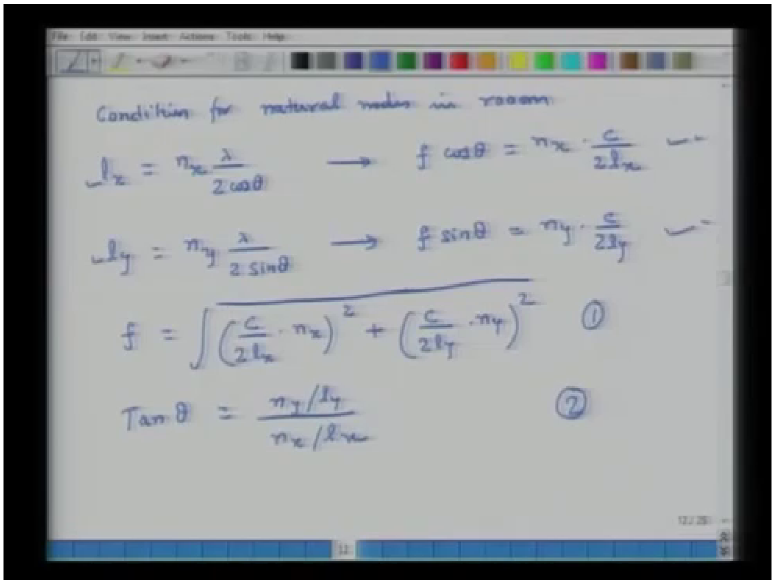
So now what we will do is we will develop something similar for two dimensional rooms we are still not talking about real rooms three dimensional rooms 2 D rooms we will talk about natural or normal modes in 2 D rooms ok, so let say I have a room it's length is lx and its other dimension is ly that's my x axis that's my y axis this is my origin and my incident wave is travelling like this at an angle theta with respect to the x axis.

That's of the frequency is f so as the wave travels and hits a wall it will get reflected owing Snail's law angle of incidence equals angle of reflection same law which we use in optics what will be the boundary condition along this wall? So velocity normal to wall equals zero so along this wall when it hits the velocity which will be normal with that is in this direction will die down the other thing will so that is one boundary condition.

And the other boundary which is essentially a consequence of this particular first statement is that velocity at corners equals zero because both the normal components have to be zero so we will draw another picture with this understanding so I draw my x direction this is my incident wave ok and I draw several lines so let's say this distance is lambda over 2 this is also a line parallel to lambda over 2 so just as we saw in case of 1 D room.

You have nodes at every interval of lambda over 2 you have a line of velocity nodes which are space by distance lambda over 2 distance now this angle is theta so ab equals lambda over 2 co sin theta and similarly I can draw vertical line cd, cd equals lambda over 2 sin theta ok.

(Refer Slide Time: 37:33)



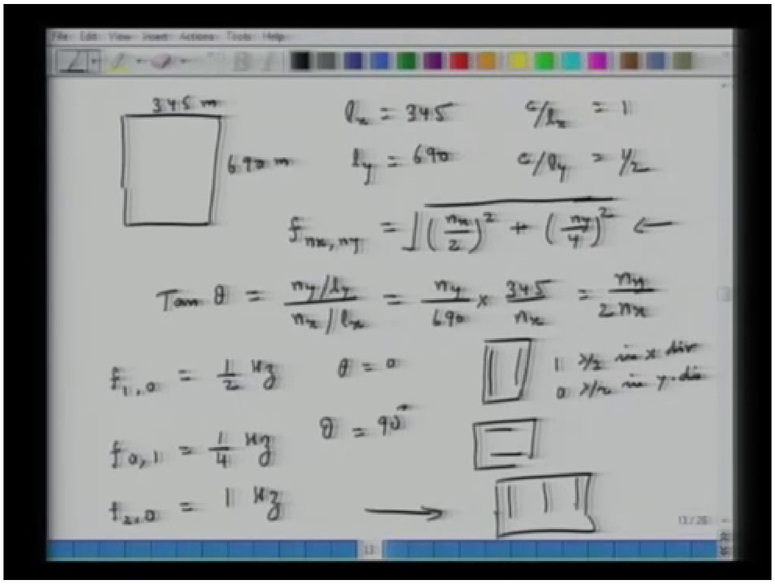
So using the analogy of a 1 D room I am just extending that analogy this is not an exact proof, standing that analogy condition for natural modes in room.

Is such that lx equals a number nx times over 2 co sin theta and ly equals another number ly and we will physically interpret what nx and ny mean 2sin theta what this gives me is f co sin theta equals nx times your 2lx and this gives me is f sin theta equals ny times your 2ly so if what this shows is that if lx which is a physical dimension of the room and ly they satisfy these conditions then f is the natural frequency of that room.

So I can also combine these two such that f equals c over 2 lx times nx penta thin square plus c over 2 ly times ny penta thin square and tan theta is ny over ly times nx over lx ok is that clear? So in a two dimensional room a normal mode depends on three parameters one is the physical dimensions of the room it is accu parameters length and width those are the two parameters and then it also depends on the orientation.

A mode is associated with the orientation which is theta unlike in case of a 1 D room ok

(Refer Slide Time: 39:50)



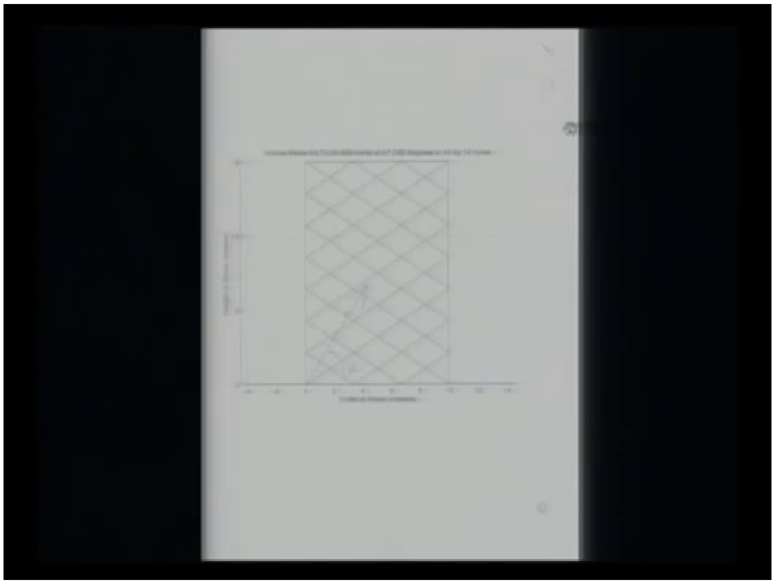
The other thing is that nx and ny they are integers so a room can have infinite modes associated with every angle but that does not mean that all frequencies are normal modes of a room, I mean it's an infinite set but it's not a complete set we will do some quick examples physical interpretation of these let's consider a room and it's a longish room.

So this is 690 meters and this is 345 meters ok so lx equals 345 ly equals 690 so we will use this relation c over 2 lx and c over 2 ly so c over 2 lx equals 1 no I am sorry, c over lx is 1 and c over ly equals 2 so f nx ny I should have the subscript here associated with nx and ny so f nx ny equals nx over 2 square, I am sorry you are right 2 ny over 2, no I think c over ly is half, yes and similarly tan theta equals ny over ly over nx over lx.

And that is ny over ly is 690 times 345 over lx is ny over nx ok so I now start putting values so I pick up a specific value of nx, pick up a specific value of ny and I construct a mode so f1 zero is I put nx in this relation so basically this is half hertz right, I put nx is one ny is zero it gives me half hertz and my theta is zero so my room mode looks like this it has one lambda over 2 in x direction and zero lambda over 2 in y direction ok.

What you think about f zero one so this is zero nx is zero ny is one so one forth of a hertz and theta equals 90 degrees and the mode looks like this another one f 2 zero so this is one hertz so here the mode looks like it has 2 lambda over 2 in the length direction zero lambda over 2 in the y direction ok so I will show you some other modes also.

(Refer Slide Time: 43:57)



So what you are seeing here in this picture is can you read may be its too small font.So this is the normal mode this is how the picture of a normal mode looks like for a room which is n meters in the length direction x direction 15 meters in the height or the width direction and the value of theta is 57.265 degrees and nx is three ny is seven and f37 is about 96 hertz.