**Acoustics**
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**Module 6**
**Lumped Parameter Modelling of Transducers**
**Lecture 09 Vibro-meter, seismometer,**
**Accelerometer, Shaker table**

So in the last lecture what we had covered was a little bit about microphones we had gone through a range of microphones classified them, we also went into details of two specific types of microphones pressure mics and pressure gradient mics, and explained how each of these mics works and under what circumstances each of these two different categories of mics are appropriate for usage we will continue on that journey.

Today what essentially will cover today is a explore couple of other instruments which are similar to mics but they help us measure vibrations or accelerations even though they operate on a very similar principles, principle as that for microphones following that what we will do is we will do some numerical calculations and manipulations on a term called microphones sensitivity and also on the decibel scales.

How do you add up decibels subtract decibels and so on and so forth. So that is what we will cover today essentially what we are doing is we started with physical acoustics now what we are doing is trying to figure out the electro acoustics part we covered loudspeakers now we are covering microphones which helps us measure sound levels and then we will later part of the course what we will go for is controlling sound .

Dampling it or reducing the sound levels and that relates to not only physical acoustics but also a little bit about psycho acoustics, so what I am going to start with is an instrument called seismometer and like a microphone seismometer.

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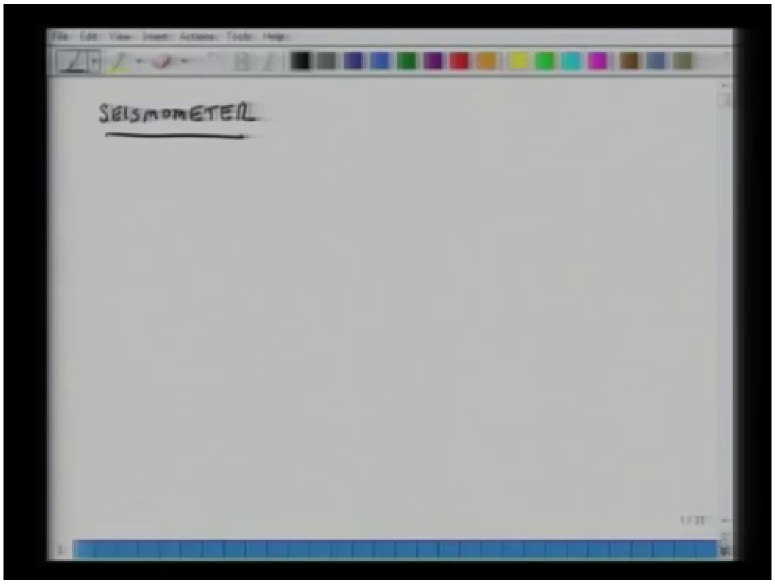


So like a very much like a microphone what it has is it has a moving coil we do not call voice coil in this case because it does not sense sound but it senses vibrations. And this coil is embedded in a magnet structure so as the coil moves up and down it current gets induced and we measure voltage across the coil and we sense vibrations so the way it is is that.

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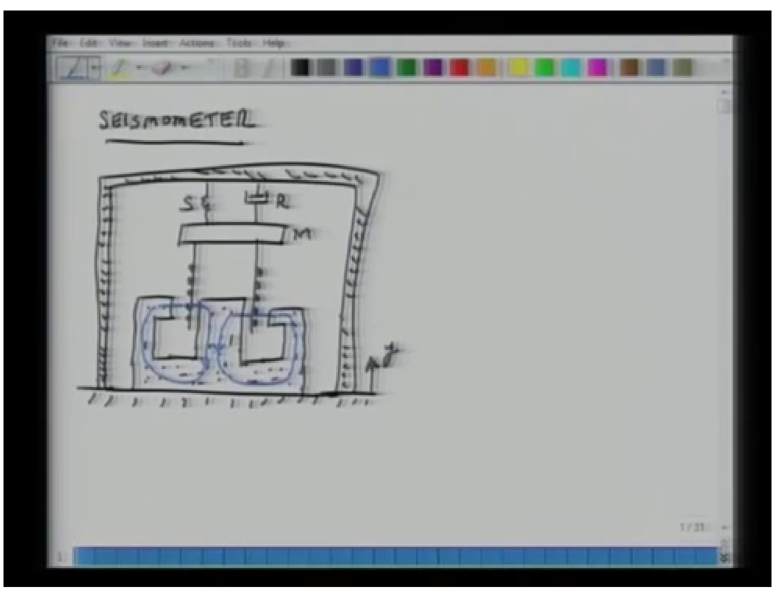

Let us consider if this is a structure and it is rigidly connected to ground and let say the vibrations in the ground are y the construction of seismometer is something like this so I have a motor structure and within this motor structure I have a voice coil.

Not voice coil just a electrical coil, this coil is connected to a mass let's call it m and the mass is connected to the outside frame by a spring of stiffness s and we could also some dampening and let's called this dampening coefficient is r or resistance, so what you have here is that in through this motor structure magnetic flux cuts I means this is how magnetic flux moves and cuts the conductor which is bound around the bobbin called the so that is the coil.

And that induces motion in the coil and because of vibrations coil moves related to this magnet and that induces voltage and we connect it to a external voltmeter and sense the vibrations the mass can have its own motions and let's call it x an x may not be the surely same as y because of these stiffness elements which may not be infinite in number so if s is infinity then x equals y, otherwise (s) x may not be same as y.

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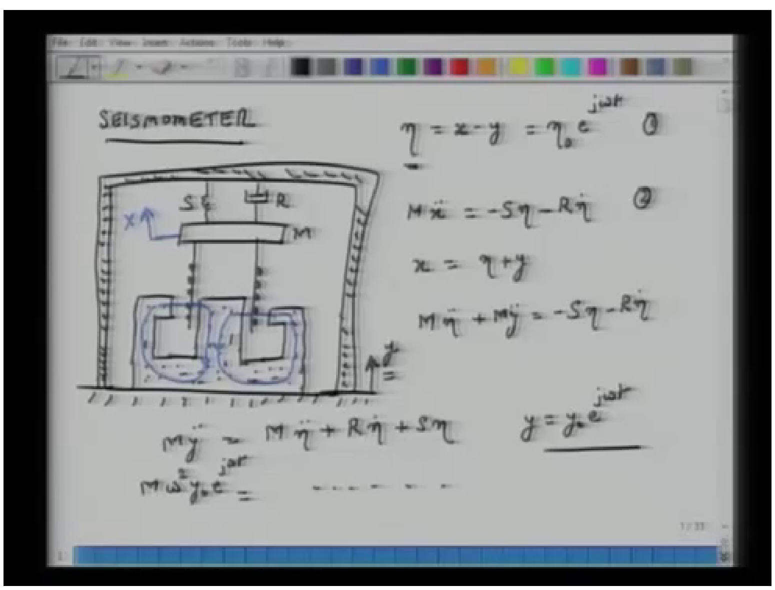


Let's define a term neta equals x minus y it could be of this form ej omega t times a constant eta o the question is that if I can measure this term through voltage because that's what the voltage will get translated I mean is a reflection of this x minus y how can I figure out the value of y I am interested in how much the structure is moving so that's what we will do from equation of motion we know that mx double dot x times acceleration.

Equals minus x times the relative displacement or the compression or tension in the spring minus the viscous force and from one we can write x equals eta plus y. So if I put that back into two what I get is m eta double dot plus m by double dot equals minus s eta minus r eta dot, I can rearrange this entire equation and write it as my double dot equals mass times eta double dot plus the dampening force plus the force due to the spring.

And if I have y of the form y equals y not ej omega t then what I get is basically my left side becomes m omega square y not ej omega t equals this entire term right, now in reality y does not have to have this exponential form it can be any complex signal which would have any shape but we know from fourier transformation principles that we can break up any complicated signal into a fourier series.

So if I have a solution for this I can use that understanding to decompose any incoming excitation into a sum of different wave forms. This is my equation I will label it 3.

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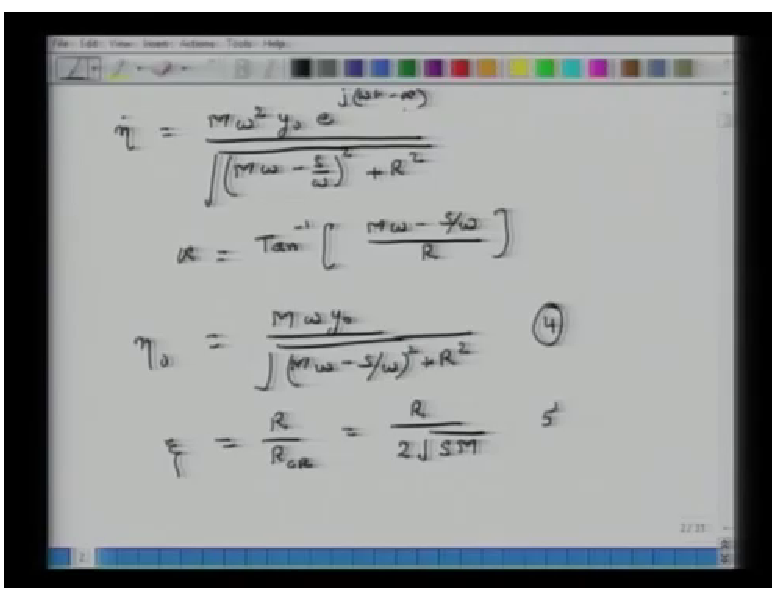


And if I do a little bit of math on this I am skipping a few specific mathematical steps operations what I get is relative velocity eta not equals m omega square y not ej omega t minus alpha over m omega minus s over plus r square and this entire thing is under the square root sign.

And what I have done is here is I have introduced a new term alpha and basically alpha is tan inverse a magnitude which is this term is this expression eta not equals m omega y not times m omega minus omega square plus r square we call this 4, so now I introduce another term and eta equals r over r critical r critical is my critical dampening frequency and this is essentially twice of underscore mass times stiffness.

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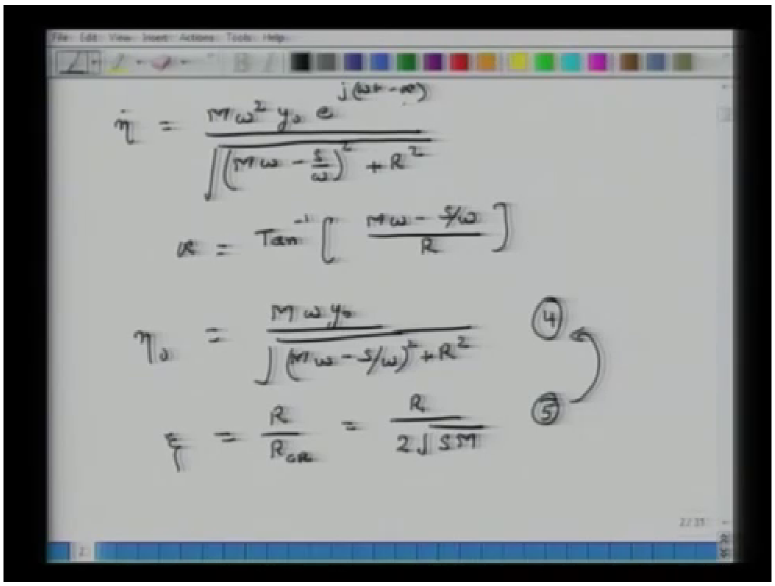

So if I plugged this into 4.

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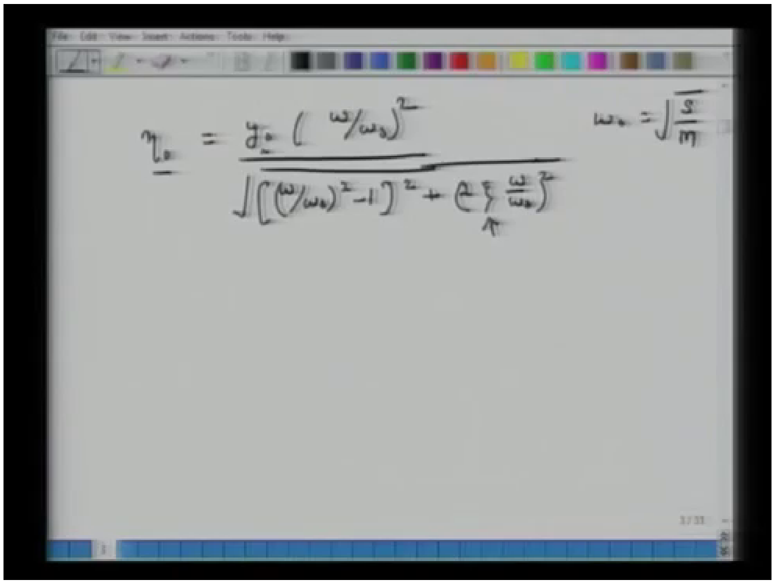

What I get is eta not equals y not times omega over omega not square divided by omega over omega not square minus one plus two eta times omega over omega not whole square so I understand that I have skipped a few mathematical operations.

So if you remember in one of the very early lectures we had said that this critical dampening that is associated with this r critical. Omega not is essentially stiffness over mass whole thing under square root ok, so what I have here is an expression of eta and y not in terms of omega right, and this factor which relates to dampening.

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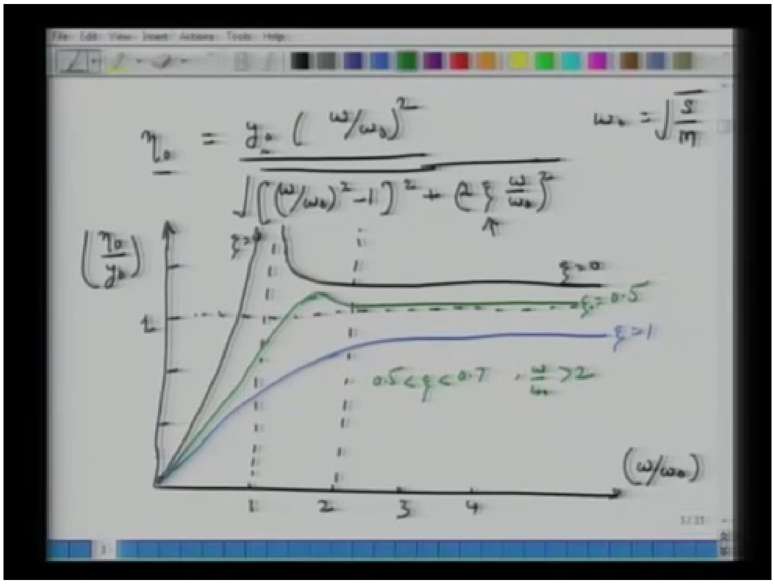


So what I will do is I will now plot this relation so what this is telling is how the magnitude of eta and y not are related right, so what I will do is I will construct some plots on my x axis I have this parameter omega over omega not.

And on my y axis I have eta not over y not and this is a linear scale so all the curves for and what I will do is I will construct different curves for different values of eta this term so for (all these) all the curves start form origin because when y not is zero then eta not is zero identically, so let say this is one so this is a very standard curves and what the construct the curves look like is this is eta equals zero and then it again comes down.

And has some sort of an asymptotic relationship as omega goes to infinity so this is still zero so it blows up at omega over omega not equals one I will draw two more curves this is eta equals one and I will draw third curve it goes up and then starts become flat something like this and again this is eta equals 0.5 so for value of eta equals 0.7 8 it will lie somewhere between green and blue colors right.

So what we see here is that eta if it is in this range 0.7 to 0.5 essentially we get a fairly flat frequency response once my omega divided by omega not is above somewhere two and omega over omega not two I get a flat frequency response so this seismometer will have essentially have a flat frequency response for if I said the dampening coefficient this value of r in such a way that eta equals 0.7 in that range and if my omega over omega not exceeds one.

Or (in the) actually two then my response curve will be fairly flat.

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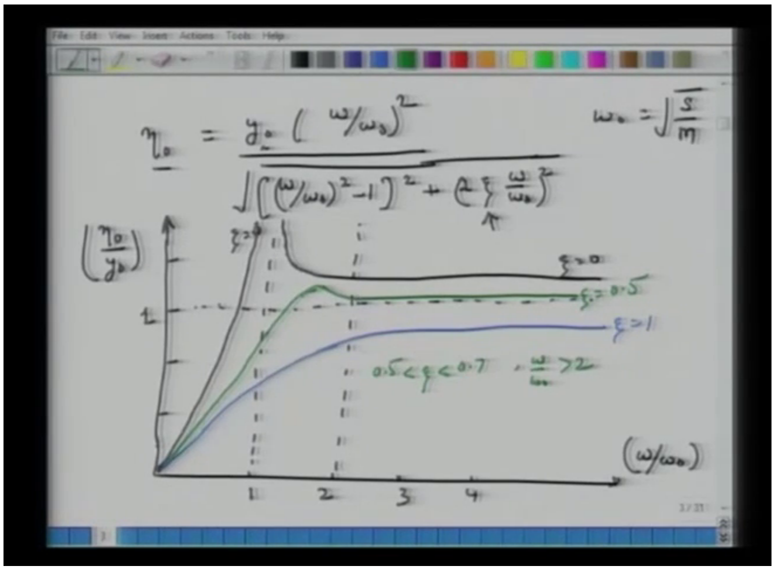


So that is the magnitude part what else are we missing there could be a phase also there is a phase relationship and ideally we would like that whatever excitation is coming in which is (y not) y it is in phase with whatever we are measuring which is eta right, otherwise for each particular frequency we will have to correct we will have to fix a specific phase correction factor.

So you want ideally we would like to have constant I mean zero phase difference for all frequencies or if we don't have zero phase difference when as a second best solution it should be constant, right?

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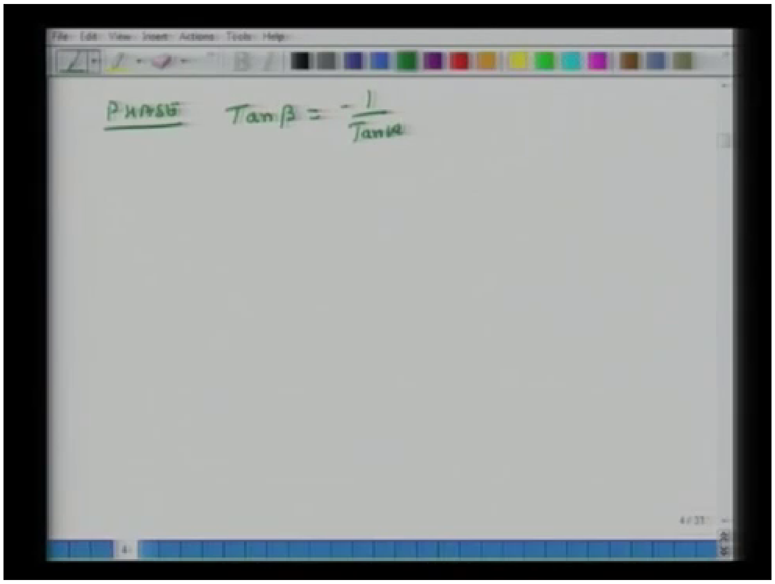


So what we will now do is we will plot the phase value it's tan eta equals minus 1 over tan alpha.

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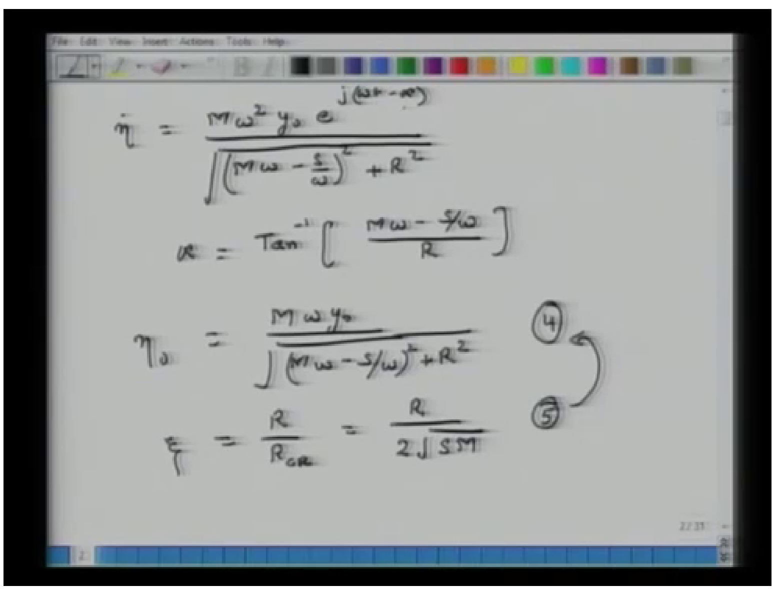

But even before that this relation what we see is a phase difference between eta dot and y is alpha right.

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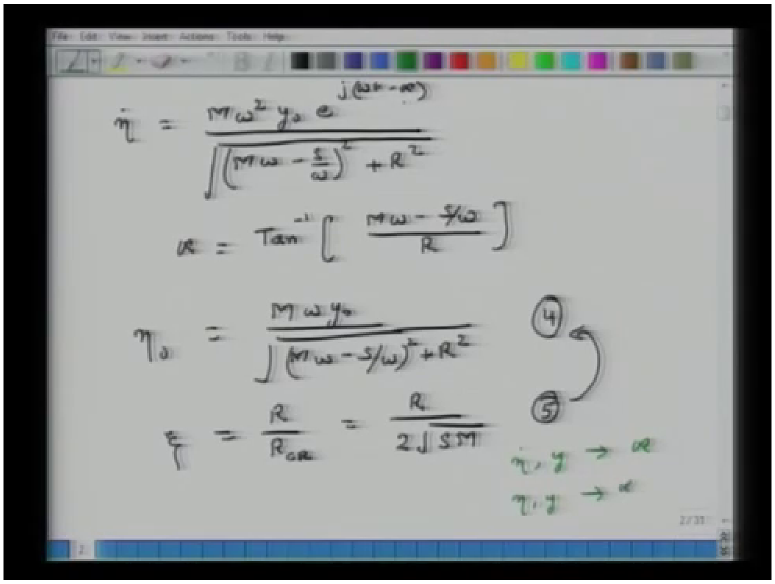

(So the phase diff) So between eta dot and y I have phase difference of alpha and if I integrate eta dot I get eta and then between eta and y my phase difference is alpha plus pi over two, right?

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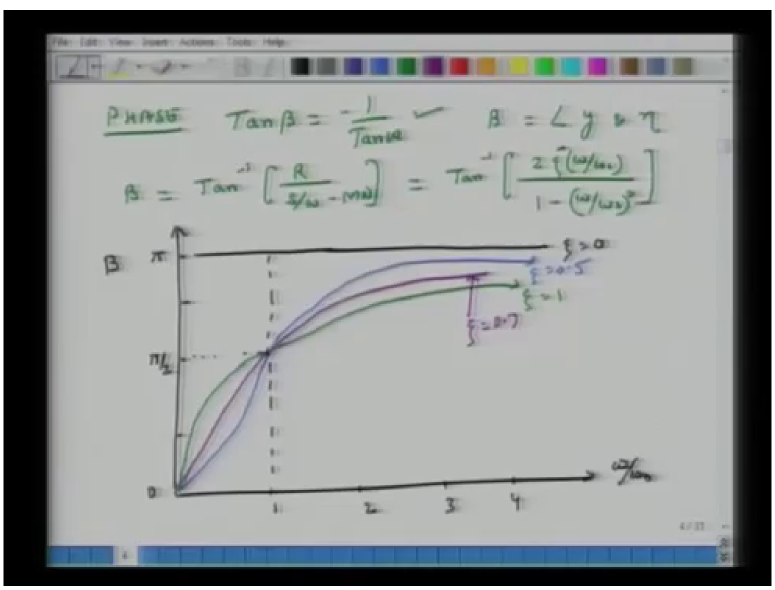


Given that we define this term beta equals phase difference between y and eta. So eta so beta equals tan inverse r over s divided by omega minus m excuse me times omega and this is tan inverse to eta omega over omega not divided by one minus omega over omega not the entire thing square. So now what I will do is I will plot beta as I vary beta for different values of omega.

So on my horizontal axis I have normalized frequency and on my vertical axis I have phase difference between the motion of the rigid, the mass and the motion of the structure pi, pi over two, zero one two three four so when this thing is zero eta then beta is basically pi and then for other values I have an inflection point or.

So this is eta equals 0.5 and this is eta equals one and somewhere in the middle, eta equals 0.7. So again I mean by looking at this particular curve of magnitude we have said that what this curve shows is that I can reliably measure the relationship between eta and y is pretty much a fixed constant for eta equals 0.5, 0.6, 0.7 in that range and also for omegas which exceed omega not by a factor of two. What this (factor) curve shows is that.

Let say if I pick up eta anywhere between 0.5 and 0.7 I have a fixed phase difference which is a little less than pei and if I keep my omega divided by omega not above a ratio of two then that fixed constant is same for all frequency based on this I know how to calculate the phase difference between source excitation and whatever the instrumental measuring and also I know how to translate that voltage into the actual displacement.

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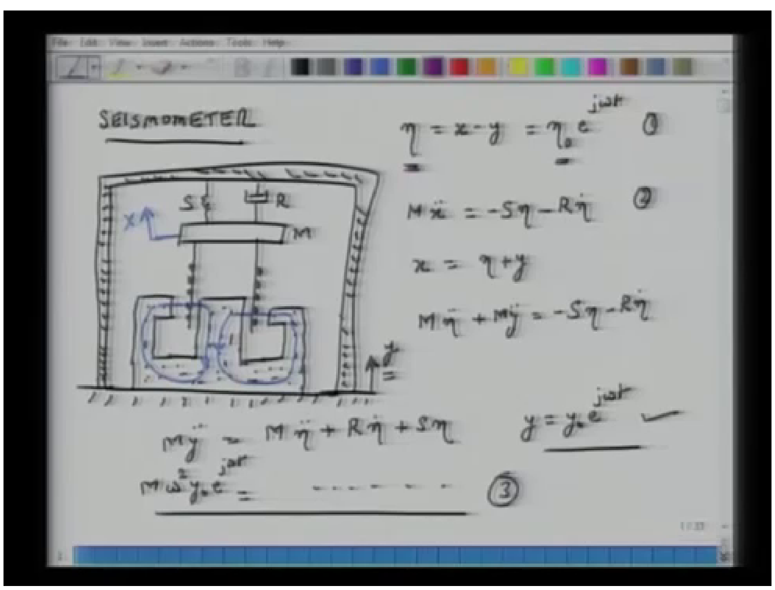

One thing in this picture what I have not shown is how is eta connected to voltage that relationship has not been expressly defined but in our earlier classes we have shown that bl term connects the velocity with the voltage.

So we can use similar transformation relationships and figure out develop the entire electro mechanical model what (he have) we have talked about is just the mechanical portion. So that was seismometer what we will talk about is now how the same device, the same device could be also used in as an accelerometer and it helps us measure accelerations.

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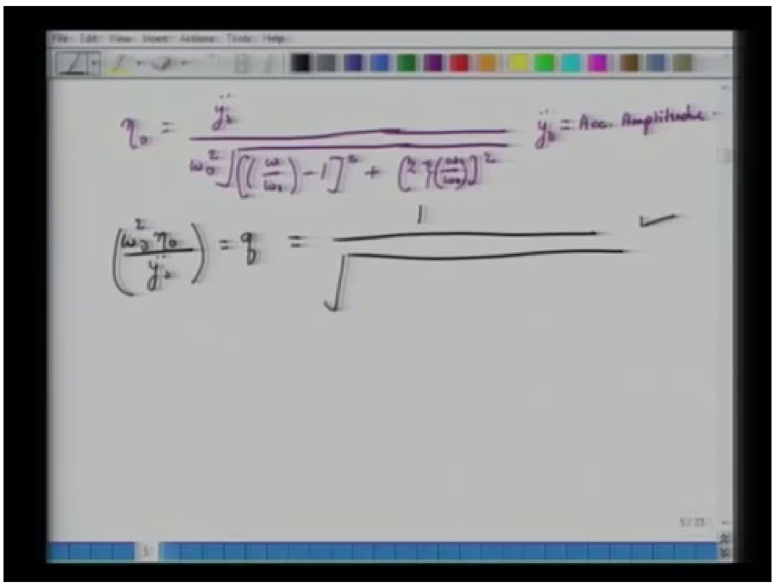


Eta not equals y not second derivative and I will define what that term means divided by omega square over omega over omega square, this entire thing is squared plus two eta omega over omega not and this entire thing is also squared. What y not is double dot is acceleration amplitude ok.

So it the amplitude but of acceleration so I can rearrange this equation in such a way that I get a term omega not square divided by y not double dot and this I call and lot of people call this qs mobility and (the un) you know the radical sign the terms are same I don't have to repeat. So now if I plot this equation q one the vertical axis versus normalized frequency and this is what I get.

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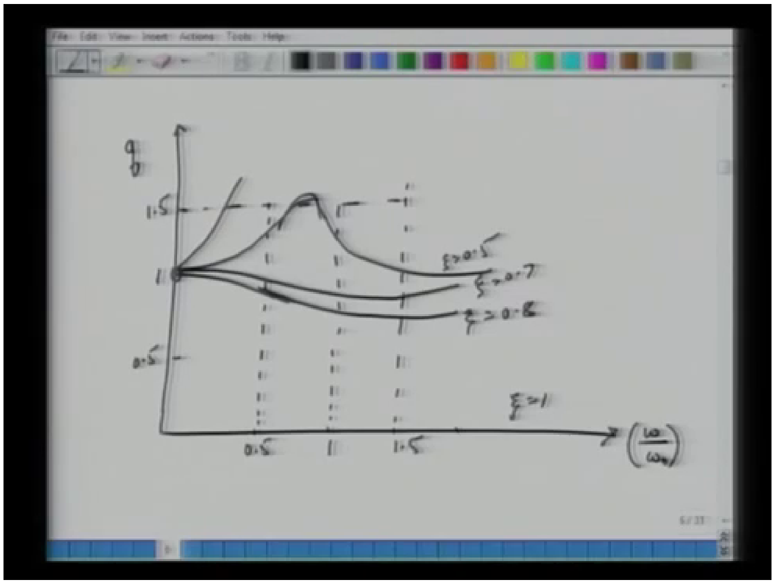


So what I am plotting here is q on the horizontal axis I have normalized frequency omega over omega not so that's my q equals one 0.5, 1.5 and my normalized angular frequency is 00.5, 1, 1.5 and so on and so forth so in this case all the curves starts from when omega is zero q is identically 1, so all curves starts from here. Ya you can do that but what you will measure is the eta not right through the volt meter and that you have to convert it into acceleration.

So what we are trying to do here is essentially see again where is the response curve lie that's all we are trying, so this is one curve then for eta equals one something like this and then it becomes flatter 0.7 and then becomes even more flat at and then if I (increa) reduce my eta further then it becomes something like this ok.

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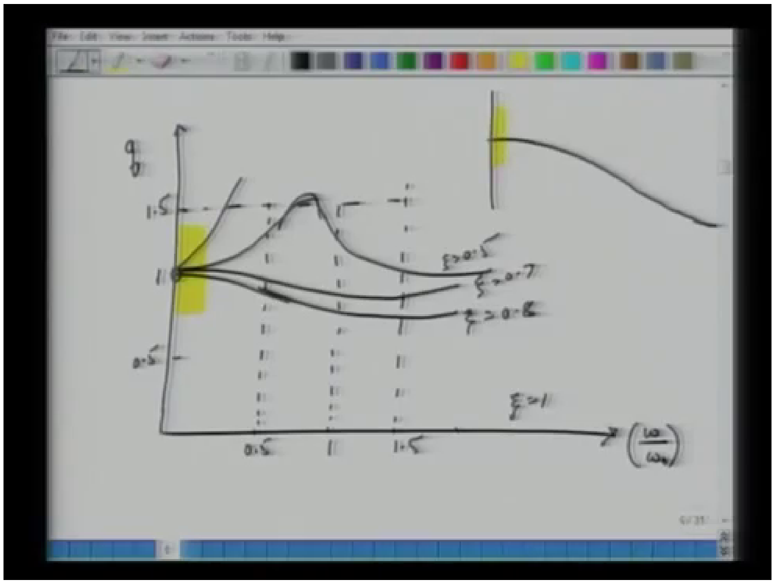

So what this shows is essentially that the operating range for the accelerometer so the operating range for the accelerometer is a very narrow band. It’s a narrow band, so if you want to make your accelerometer measure large very high frequencies you have to increase the stiffness very significantly omega not has to go, right?

That is what a lot of accelerometers do they try to increase the stiffness as I is possible and they do it by using solids like piezo electric crystals they act as stiffness members also so they act stiffness and they also generate a voltage  to both the functions.

They act as a stiffness members and they also generate voltage across themselves, so this is a better looking curve should be something like this, something like that does not become very asymptotic ok, but it is still flatter here in this direction. Now the problem with this so the challenge is that if I have to measure at 100,000 hertz the acceleration then I have to maybe increase my omega not up to may be 10 million or a million something like that.

Because it has to be very small I can do that my making  by reducing the size of the structure you know if I reduce the size of the structure the stiffness starts increasing significantly so I can do that but then challenge becomes that how do I make sure that the signals are larger enough so that they can be measured here we assuming that I can measure any amount of voltage it could be mille volts or micro volts or nano volts.

But my sensitivity also starts decaying very fast so that is a challenge and a lot times what they do is that within the accelerometer itself or very close to the accelerometer they have some amplifiers charge amplifiers so some devices, so whatever is the signal which is being generated it gets immediately amplified so that it can be measured reliably because otherwise if that device amplification device is par from the location of measuring.

It may get corrupted by the time it reaches the amplifier so we mentioned about sensitivity and so what we will do is we will define what is sensitivity.

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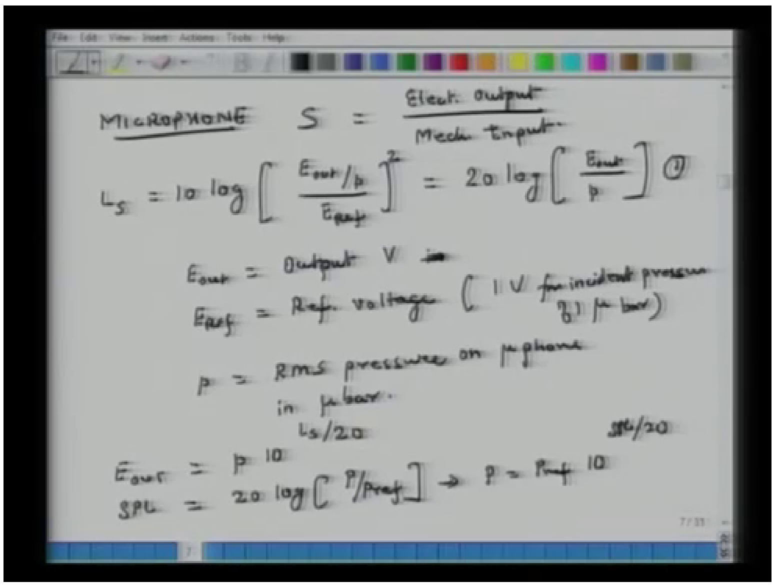


So for a microphone we want a flat frequency response which essentially means that for a large range of frequencies the response curve is flat you know and we also want that it should be a sensitive enough, if it has very low sensitivity and very flat frequency response it may still have limit reduce.

Because then it starts between difficult to measure the signals and data's, so if you go to small microphones very tiny small pressure type of microphones they have very flat frequency response but their sensitivity is very low if you make the microphones large because of this lambda over two pei factor their frequency range becomes of a limited width but their sensitivity goes up.

So it's an interplay my sensitivity let say s is electrical output divided by mechanical input which is your pressure in this case right, there is a term called microphone sensitivity level ls microphone sensitivity level and that is defined by this term ten logarithm and base ten times not times I mean logarithm of e out over p divided by e ref and I can make this 20 log e out over p and I will explain that how I made that jump.

When the two became once I remove this two becomes 20 but I also eliminated some here e ref so e out is output voltage from the instrument ok e ref is reference voltage and typically it is one volt for if my incident pressure is one micro bar one bar is ten to power of 5 pascals which is one atmosphere, one microbar is 0.1 pascals so e ref is reference voltage and it's about like one volt for incident pressure of one micro bar.

So because of that e ref goes away and then p is rms pressure it is not p pressure rms pressure on microphones in microbars so it not in passive but it isn't microbars. From this relation I get e out equals pressure times ten to the power of ls divided by 20 we also know that spl sound pressure level is 20 logarithm of p divided by p ref, so which gives me p equals p ref times 10 to the power of spl divided by 20.

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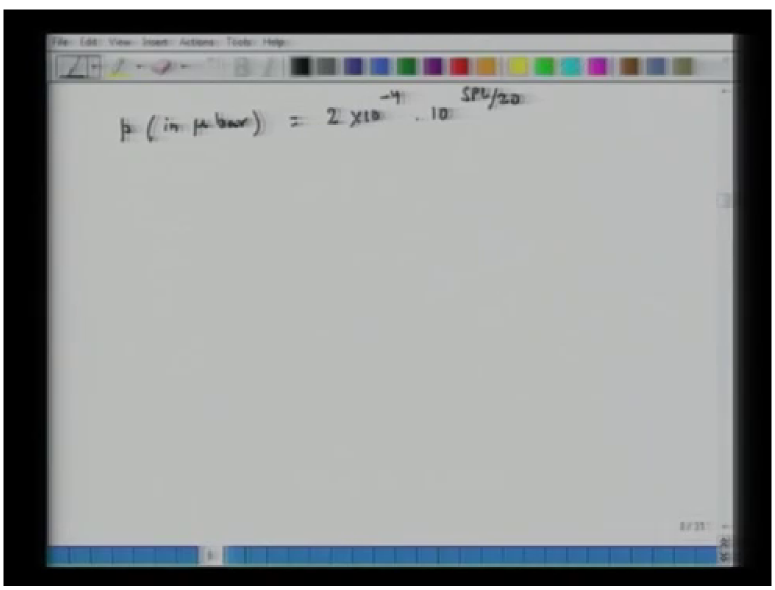

And my p ref it's industry standard is two times 10 to the power minus 5 pascals so from this what we get is p and I will make it explicit in microbars equals two times 10 to the power of minus 4 times 10 to the power of spl over 20.

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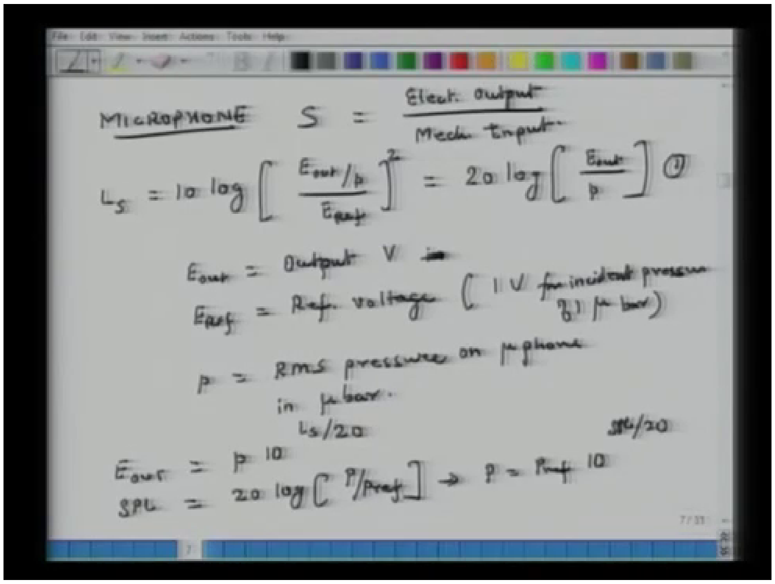


And if I plug this back into the original equation, is this 10 also yes but then it also gets multiplied by 10, no it is because we are doing in microbar so because of, so there is a factor of 10 right so that because of that reason it becomes two to the power 10 minus 4.

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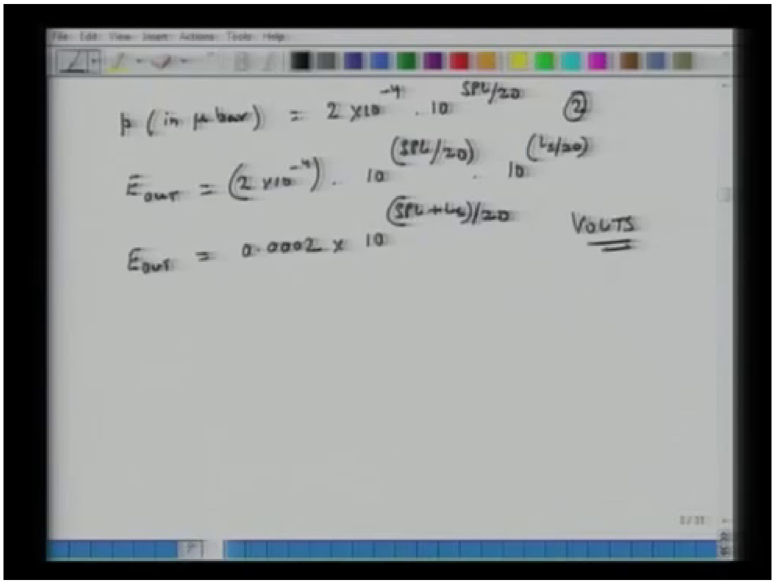


Let’s call that equation two and this is my equation one so from one and two what I get is e out equals two times 10 to the power of minus 4 times 10 spl over 20 times 10 to the power of ls over 20 or e out equals 0.0002 times 10 to the power of spl plus ls over 20 volts.

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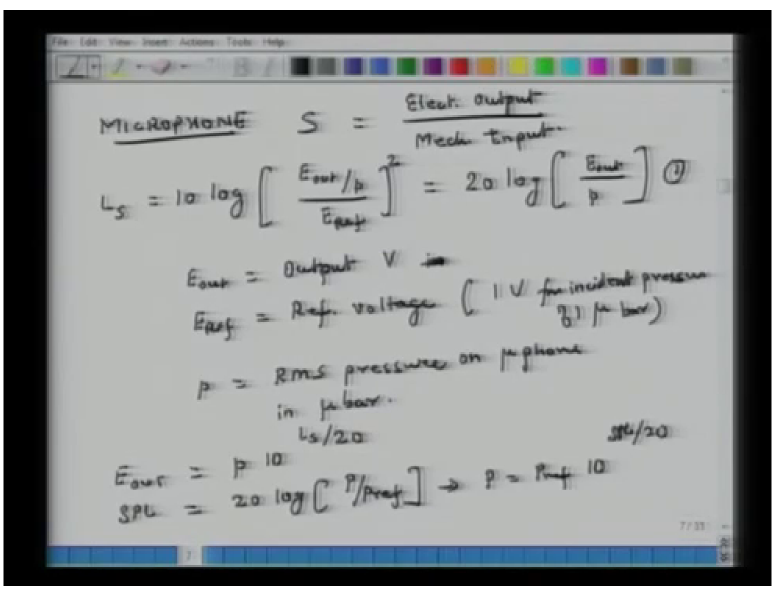


And one thing I omitted earlier was that the unit of this is decibel voltage over microbar they are not of the same decibel voltage over microbar.

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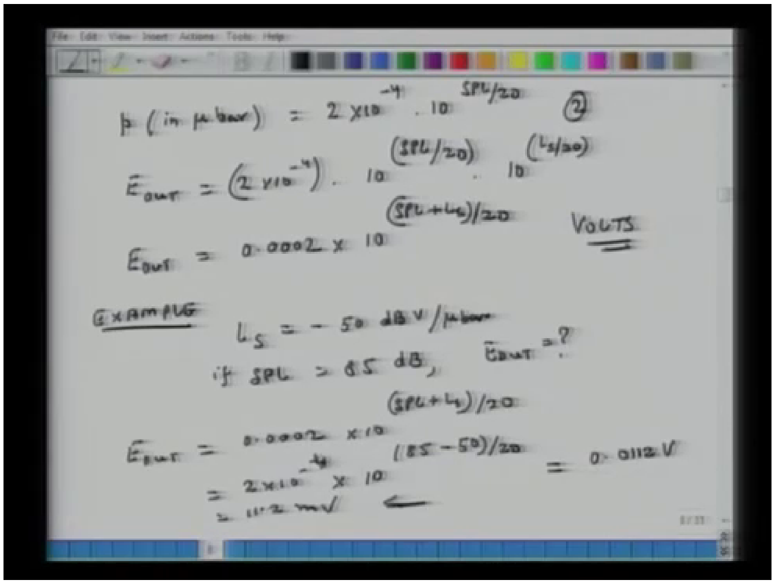


So we will do an example so let's consider microphone so this is a microphone which has a sensitivity ls of minus 50 decibel voltage for each microbar and the question is that what will be the output of this microphone when my sp less 85 db's so if spl equals 85 decibels what is e out so I just essentially used this relation e out equals 0.0002 times 10 to the power of spl plus ls divided by 20. And what I get is two times 10 to the power of minus 4 times 10 to the power of 85 minus 50 divided by 20.

And if I solve this I get 0.0112 volts, 11.2 mille volts, but essentially what this process tells you is that you will know if you have some idea or sound pressure level is going to be 120 db's at the peak level and the minimum sound pressure level is going to be let say 40 db's and if you want to measure it you should be able to figure out what will be the output voltage from that and (when) will my instrument be able to actually measure that voltage or not that is what (())(36:07).

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Yes but this point is, that is true so the answer to that question is to your question is e over e ref comes non dimensional right and p is in microbars which is again a number, p is in microbars, so it becomes a non dimensional quantity, so a typical sensitivity for large microphones is something like half a microvolt for each microbar.

Or you can convert it into a minus 125 decibels for each microbar so that is on the low end and then on the high end u it goes up to three mille volts for each microbar other thing we will know talk about is weighting and this relates to this esteem of psycho acoustics so let say I am playing a particular instrument at a very high frequency and I produce 100 db's from that instrument now we use another instrument let say guitar or tabla and it produces sound.

At 100 db spl but it is at low frequency my ear even though the spl at different frequencies is the same my ear will perceive the loudness of higher frequency it's different from that which is coming from a lower frequency this is how we have heard and this start so 40 hertz 100 db spl will be perceived it's not different but it will be perceived to have different loudness compared to say 1 kilo hertz compared to 15 kilo hertz and so on and so forth.

The other thing is that this difference in perception for different frequencies becomes more amplified if my over sound loudness level is low so if I am playing the whole thing at let say 85 db spl different frequencies then that difference in perception will be much of a much larger magnitude compare to if I am playing the  everything at high dbs. So one perception is related to frequency band.

And the other relate frequency perception is related to overall loudness level, so this is how ear works so in a lot of measurements which relate to human ear  if you are trying to reduce the overall noise in a hearing machine, you know machine shop where you are generating all sorts of frequencies what people try to do is that they weight different frequencies by different numbers so that ear based on whatever is the difference which is being perceived by the ear.

Because the aim is that the ear  should perceive the overall sound level is gone down right, you want to make it quite, quite means that ear is sensing that everything is become quiter if you reduce the frequencies by the same amount not the frequencies by the amplitude by the same amount the ear will not necessarily perceive that all frequencies have gone down by the same amount.

Because some frequencies it will perceive are still louder compared to others so that is what is what relates to this idea of bating and also loud spl level yes so that is what these three curves tell you

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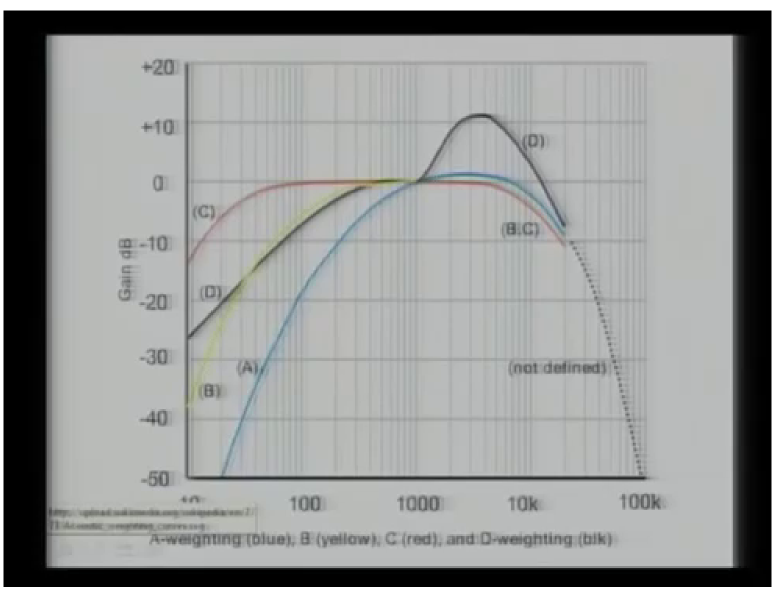

so you have this a bating level is in blue ok then you have this yellow curve which is b weight level and then you have this c weighting level which is in red, so a weighting curve basically is used or sound levels in general which are less than 55 decibels.

Then your b is used for 55 to 85 db's and c is used for 85 db's in excess of 85 db's and there is also a d weighting level and z weighting level z is not shown here, we will talk about that little later but it's important to know that in industry when you measure sound people say o what kind of weighting level you used a, b, c. So you should be able to know what is being talked about, now the most prevalent weighting level in industry today is a.

Because as you see in case of c which is in red it's pretty much flat across a very wide spectrum from let say 30, 40 hertz 3, 4, (watt) 5,000 hertz it's pretty much flat so a is the most widely practiced weighting level and b also has fallen into disuse it's used in very specific cases for some aircraft in its estimation and these curves have been developed based on how ear perceives sound at different frequencies at different levels.

And based on that on some statistical samples people have looked these curves and now they are they are the industry standard and a lot of environmental agencies say in India or in United States or in Europe specially in force that if you are trying to solve noise related problems you have to weight your measurements based on some of these curves so that is the purpose, you also see that there is d weighting level which is in black.

And what this is for is that if you have a bunch of not a bunch if you have a random noise where noise is at all frequencies then the loudness perception of that random noise is different than the noise which is around 6 kilo hertz, at 6 kilo hertz the loudness perception is at a higher level compared to random noise which is a broadband noise. So that is why you have this peak at around 6 kilo hertz otherwise it's again more less flat.

I wanted you to get expose to some of these curves and you can also get charge for you know reescus could be translated in chart industries under tables you can use them and weight your sound pressure level measurements accordingly also in a lot of sound systems you may have seen this loudness control knob right, there is a loudness control knob to some sound systems it's not same as volume knob loudness.

And that also it does the same thing if you are playing it at a higher level it uses some of these weighting curves through enhance certain frequencies and enhance not so much or not to the same extent some other frequencies and if you play it at a lower level in this loudness knob  does something which relates to a weighting level, yes something like that, so we will come to the last portion how do you add and subtract this bill levels.

So if I have an 85 db sound level coming from one speaker and the source is generating 100 db sound level spl what will be the overall spl will it be 185 so ya (so you will) because it's logarithm you cannot just add them out.

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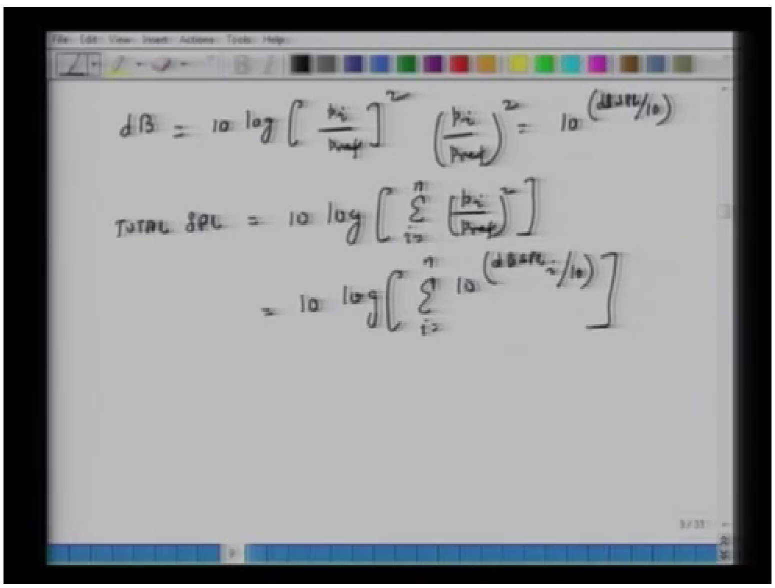


So we know that db equals 10 log pi over p ref right, so pi over p ref equals 10 to the power of db spl over 10, so if I have a bunch of decibel sources then total spl I can write is 10 log summation of pi over p ref square equals 10 log so this is fairly straight forward right.

A question here could be here what I am doing is I am basically using this 10 log relationship but (why) what I could also do is I could remove this two and make this 20 right, will that be an appropriate thing because I will get a different number will that be right or (will that not) will that be incorrect which one you will only have one correct answer.

So is this the right way to do it or is that way where you eliminate this  power of two and put that into this (20) on the other side of the equation  will that be the right approach and why would that be the case, but mathematically you can do this decibel if you remember in our original definition is a ratio of energies it's not ratio of pressures or ratio of velocities it's ratio of energies right or a power.

An energy is directly proportional to v square or pressure square or displacement, square of displacement and so on and so forth so I am basically when I am adding decibels I am adding up energies total energies, I am not adding a pressures so that is why eliminating power of others power number I am putting it on the other side of the equation will be invalid because that does not help me add energy levels.

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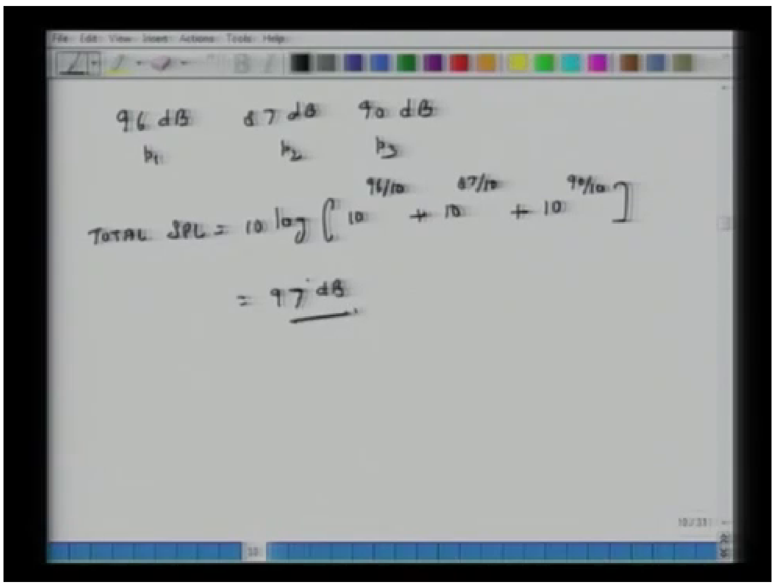


So we will do an example let say I have three sources 96 db's 87 db and 90 db p one p two p three so what is the total spl so total spl equals 10 log of 10 to the power of 96 over 10 plus 10 87 over 10 plus 10 to the power of 90 over 10 and that gives me 97 db so it just an increment of 1 db.

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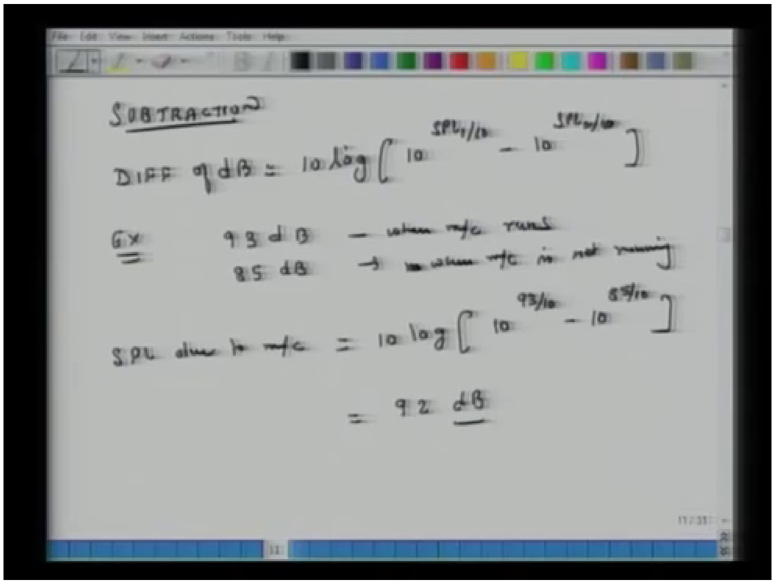


Similarly if I have to do subtraction then difference of db's to decibels is 10 times logarithm of 10 to the power of spl one over 10 minus 10 to the power of spl two over 10, so suppose so we will do another example let say we have a machine shop a large machine shop and there some ambient noise and then there is some also noise coming when a particular milling machine is running.

So when we run the machine then the total spl level is let say 93 db's then machine runs and when the machine is not running let say that total db spl is 85 db's not running so a logical question could be what is the overall contribution of the machine right. to the overall noise level, so spl due to machine is 10 log 10 to the power of 93 over 10 minus 10 times 10 to the power of 85 over 10 and this gives me 92 db's.

So these are very I mean it does not become very efferent right away on a very I mean prompt answer would be what 8 db's but the numbers are significantly (())(50:54)

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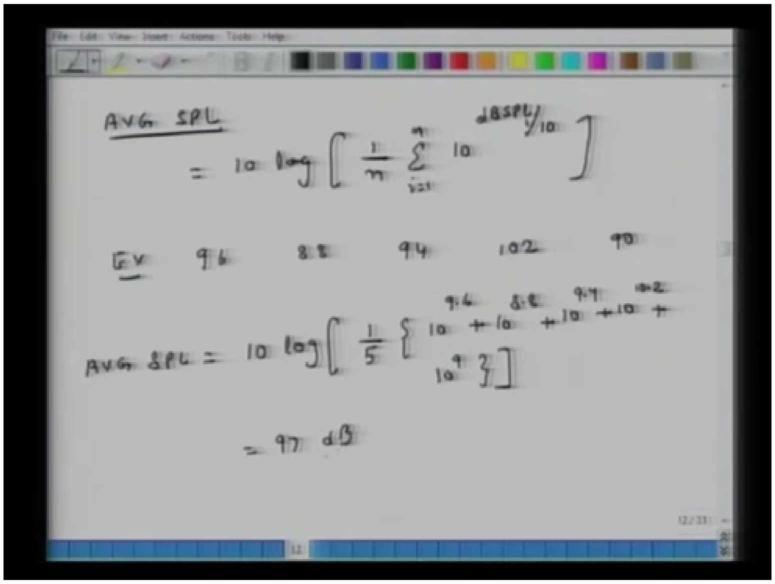


Another example so if I have this is the most common usage that you take a measurement in any experiment. It's value is v1 then you take 4, 5 same type of measurement v1, v2, v3 you want to be sure that the measurement is right right and these measurements will be different each time you do an experiment measurement will be different. So what is the average value now in linear systems we just add v1 plus v2 plus v3 the entire thing divided by number of measurements but here averaging requires a little bit different mathematical operations.

So average spl equals 10 to the power of log one over n, I equals 1 to n 10 to the power of db spl over 10 I value so it's 10 to the power of 10 times logarithm of this entire expression so another example I will do is let say my five measurements are 96 db's 88 db's 94 db's 102 db 90 db then my average spl equals 10 to the power of log so 10 times logarithm of one over 5 10 to the power of 9.6 plus 10 to the power of 8.8 plus 10 to the power 9.4 plus 10 to the power of 10.2 plus 10 to the power of 9 and that gives me 97 decibels.

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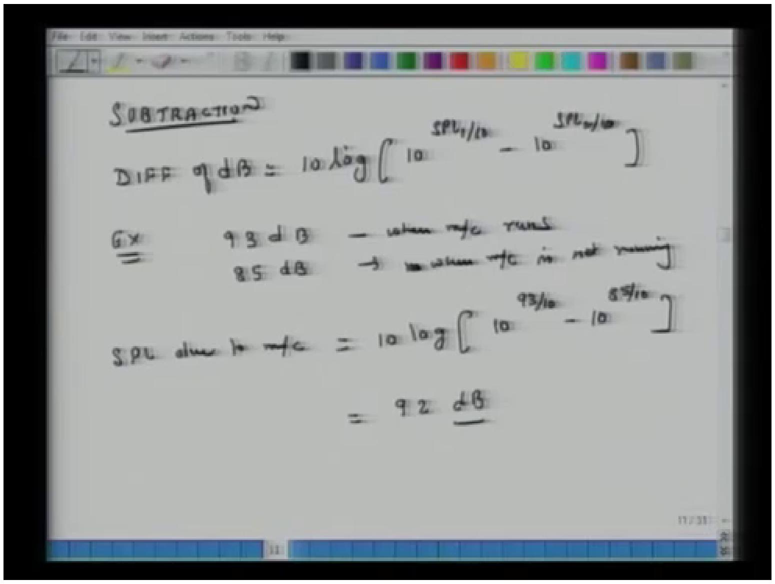


So each of these operations are important they have practical usage their case is where you all be expected to subtract contribution of source a from source b, there will be cases where you left take averages there will be still some other cases we left to add them up and we have to be aware that when you are manipulating decibels when the rules of the game are little different.

The last point I wanted to cover very briefly are two terms co related sound and un co related sound so a co related sound is essentially when you have a bunch of sources and each source is generating sound but you precisely know the phase relationships for each frequencies coming from different all of these sources and also at specific time if source one is generating f1 then source two is also generating f1 frequency.

So f3 is also generating f1 frequency (then they have) then the sound coming from all these different sources and the phase difference is also same and also the phase difference due to the variation of location is also not significant then you can add up there pressures in practical applications most of the sources of sounds which we have they are not co related, so you run a machine here, a person is talking a bird is chirping there is no co relation between these two sounds.

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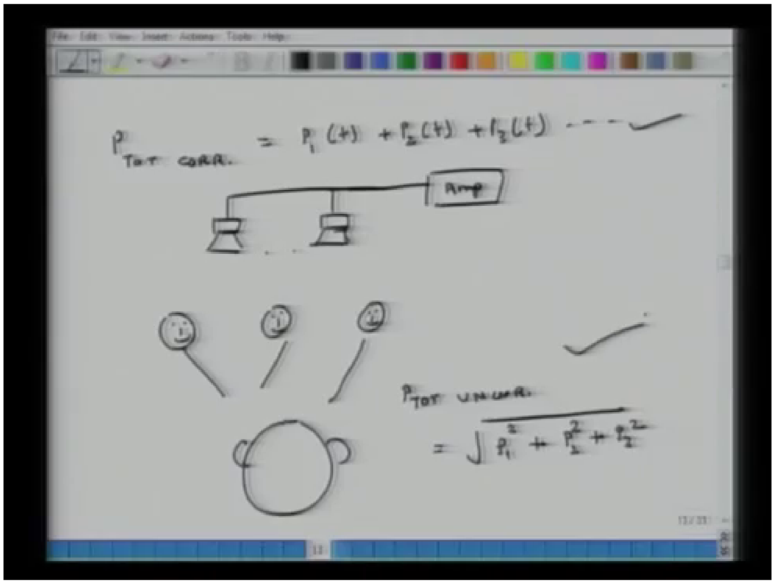


So you have a co related sound p total co related is essentially p one times t plus p two times t plus p three times t and so on and so forth a good example of this could be I have one speaker I have another speaker and both are having the same frequency response curves and they are being fed from the same amplifier the same signal and also the distance between these two is not significantly large.

In context of the wave lengths which these speakers are generating then I can use this relation to add up the pressures if I have un co related sound I could have one speaker or actually a human being so I could have three human beings they are generating sound they are talking and the person who is listening is situated a little away and in this case p total un co related equals p one it's a root square root of the squares of sums.

So here we don't have any information on the phase so again while we are doing measurements we have to understand are we measuring co related sources or are we measuring un co related sources and be aware what is it that we are trying to do when we are summing up signals that's all I wanted to cover for today and thank you very much.