Acoustics
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Module 06 Lecture 07
Lumped Parameter Modelling of Transducers
Solution of Loudspeaker model

So in the last class we had just begun analyzing the electrodynamics lump parameter model of an Acoustic Transducer.

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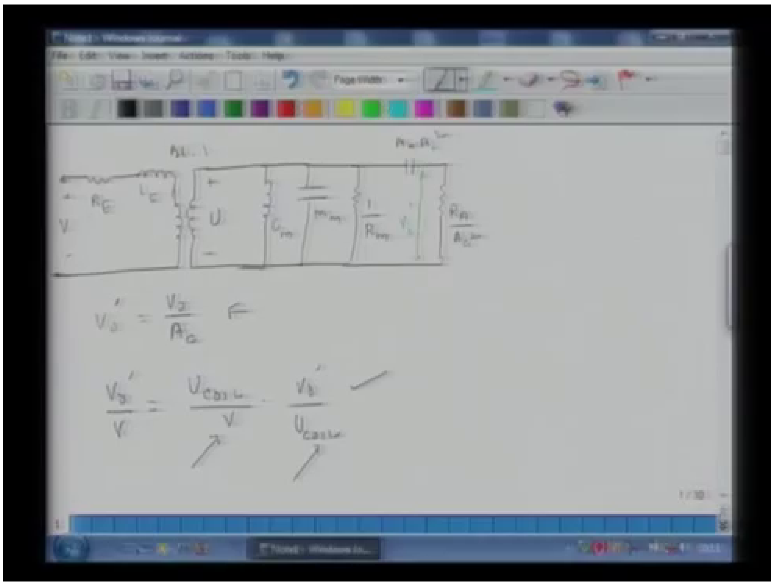

Where we had left in the last class was that we had identified some break points or resonance points specifically they work on electrical resonance between R e and L e and that was happening at around 1820 heads that is what we calculated then there was a resonance as a consequence of C m and M m and that was happening at 89 hertz.

And then an Acoustic resonance between because of M a and R a that was occurring at 717 hertz or something like that. The other thing we had done was that we had decomposed this transfer function V 0 prime over V 0 in terms of U coil over V times V 0 prime over U coil. So what we will do today is we will develop a bode plots for the first ratio U coil over V and see how it changes as frequency changes.

And then we will also develop a bode lot for second transfer function V 0 prime over U coil and then once we have these two bode plots together then we will add them up and see. How the overall transfer function of the overall transducer looks like. As we developed this relation we want to remember that an ideal transducer should have a very flat transfer function over a range of frequencies.

So the wider that flat area is from an ideal stand point the better it is listening purposes. The other feature of a good transducer would be that not only it should be flat but it should be as high as possible. What that means is 35 and a little amount of electrical energy it gets converted into more and more of sound. Typically the efficiency of very good Acoustics transducers does not exceed one or at the most if you are really smart 2 percent, so that should give you some overall perspective.

And the overall efficiency of a sound system or a transducer hardly exceeds 1 or 2 percent.

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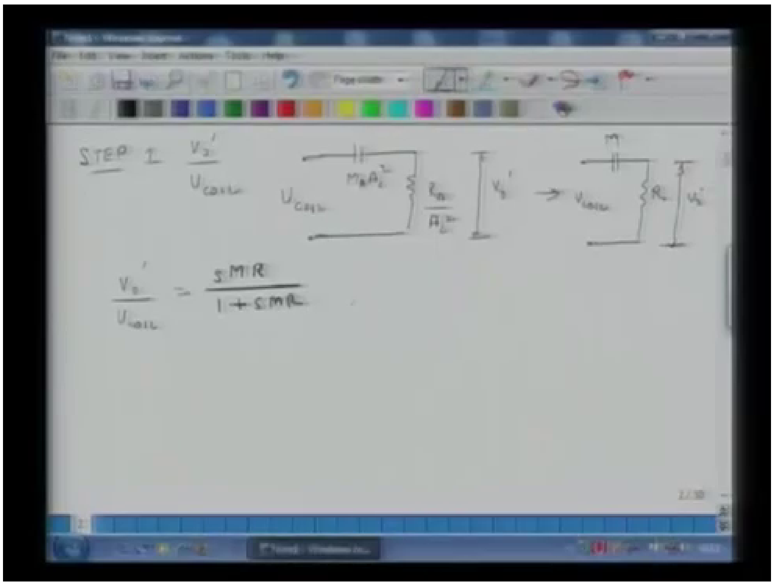

So we will start developing a relation for U coil over V V 0 time over U coil, so we will do this in two steps.

Step 1 is we are going to do V 0 prime over U coil, now when I look at this circuit my V 0 prime is the voltage across this Acoustic register R a over A c square and U coil is basically just the voltage across the transformer after the electrical circuit, right? And the current in the mechanical circuit is essentially force and the current in electrical circuit is obviously current.

So when I look at this circuit what I infer is that to get this ratio all have to consider is basically the element M a times A c square and R a over A c square, right? That is the only thing I have to worry about.

So the equivalent circuit is something like this. So this is M a times A c square this R a over A c square, my input voltage will be in this case U coil and what I will be measuring and what I am interested in is V 0 prime. So this just for sake of simplicity is something like this I just simplified the notations, right? And if I compute impedance if I compute the impedance and I try to find the ratio the relationship which I get is V 0 prime over U coil comes to this complex frequency times capacitance which I have noted as M times R over 1 plus S M r.

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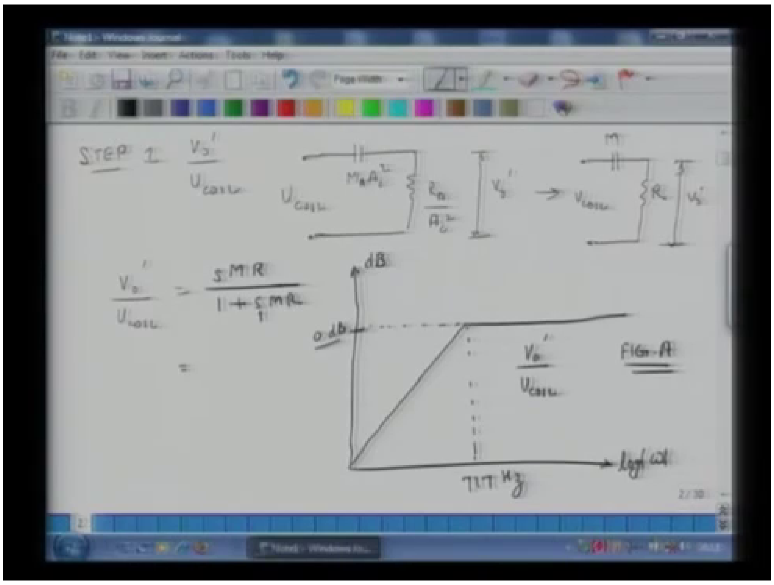

So now I construct a boot plot, so on my horizontal axis I have logarithm of omega and on my vertical axis I have decibels. Omega tends to 0 how will the bode plot look like as omega or that is s which is j omega. As omega goes towards 0 the low frequency asymptote, how will it look like. It will go through the origin and it will be a straight line with a positive slope of 20 decibels per decade.

And once it crosses a certain threshold then as omega goes towards infinity what happens this ratio essentially converges to 1, right at infinity. So log of infinity or log of 1 is 0, so the high frequency asymptote is something like this and this what is this value 0, 0 decibel.

What is this break point? This point, 1 over MR what was the 717 hertz we have calculated ok, so this is my bode plot for V 0 prime over U coil let us call this figure A. So we have finished developing a bode plot for the second portion which is this now we will do for this one.

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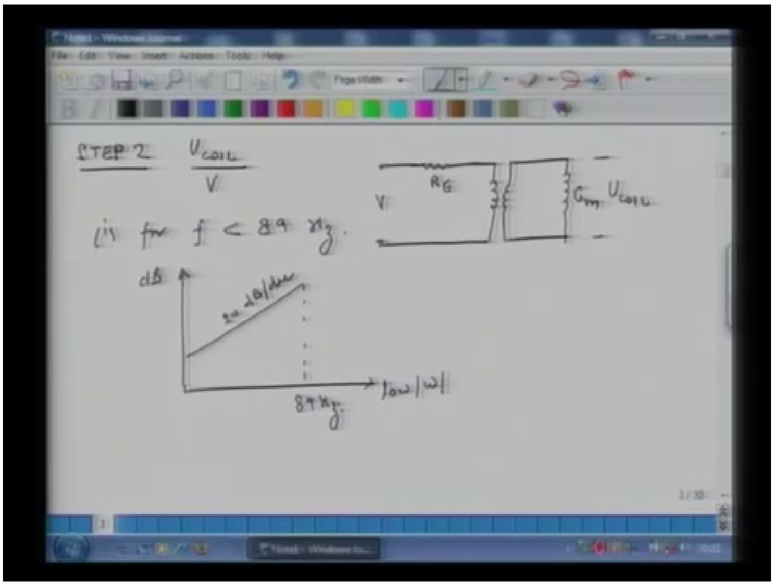

In the second step, step 2 what we are going to compute is bode plot for U coil over V. In the last class we had found that there was a resonance happening at 89 hertz, right? So below 89 hertz U coil over V is this my well V and this is U coil. Below 89 hertz what is going to happen is that most of the current is going to pass through C m the mechanical inductance. The (ind) impedance due to M m R m and all these things will be extremely high.

For s less than 89 hertz my equivalent circuit is going to look like C m and that is my U coil. Why did I neglect L e in the circuit I dropped out L e in the circuit on the electrical side I have not included L e, why have I done that? L e becomes important as we have calculated after 1820 hertz. Below 1820 R e is going to dominate, right? So I can neglect the impact of L e for f less than 89.

The bode plot in this case is going to look like, how will it look like? It will be a straight line slanting positively and the slope will be 20 db per decade. And this is 89 hertz ok. So now I have to go from 89 to what is the next break point 717 hertz, right?

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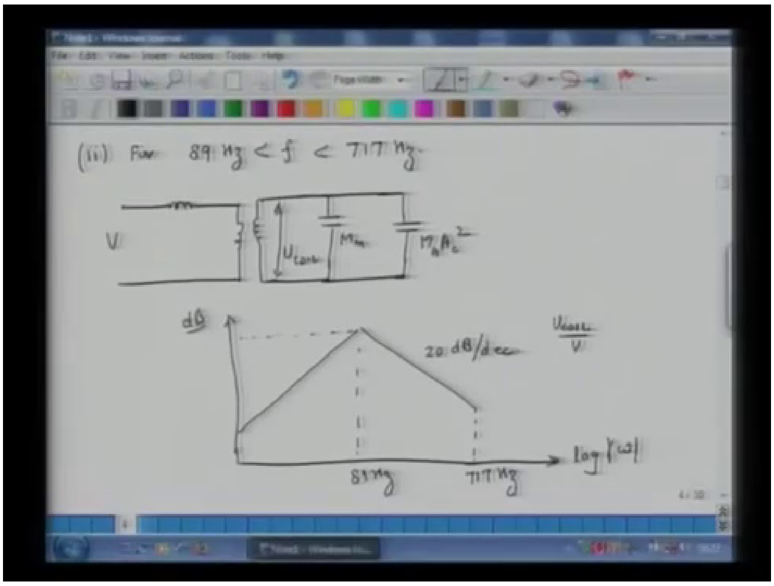

So for F 89 hertz (is larger than) is smaller than f and that is 717 hertz. So my electrical circuit is going to look like; so now I will have the capacitor M m and both these capacitors are going to (())(11:18) because very low current is going to pass through this inductor because I am above 89 hertz and the capacitance will become active. In an asymptotic like if you go individually like s and then M r and then.

So as S becomes infinite as S becomes infinite the numerator is becoming more and more as s tends to infinity numerator and denominator this becomes more and more equal. So the ratio goes to 1. Regardless of the values of M and R. So this is my reduced circuit for this frequency range 89 hertz to 717 hertz again this is V and this is U coil.

So my bode plot in this case. And as frequency goes higher and higher the impedance offered by these. So I get my this, this was my original light going up 89 hertz then I have a negative slope And the negative slope is 20 db per decade. And this is going up to 717 hertz. Again on the horizontal axis and plotting logarithm of omega another vertical axis I have decibels.

And this is still U coil over V, so we know response below 89 hertz response above 89 hertz upto 770 hertz. We could ask ourselves a question that what happens at 89 hertz specifically. So that is what we will once if we know the answer to that we will know this value. How many dbs at 89 hertz? So that is what you will find.

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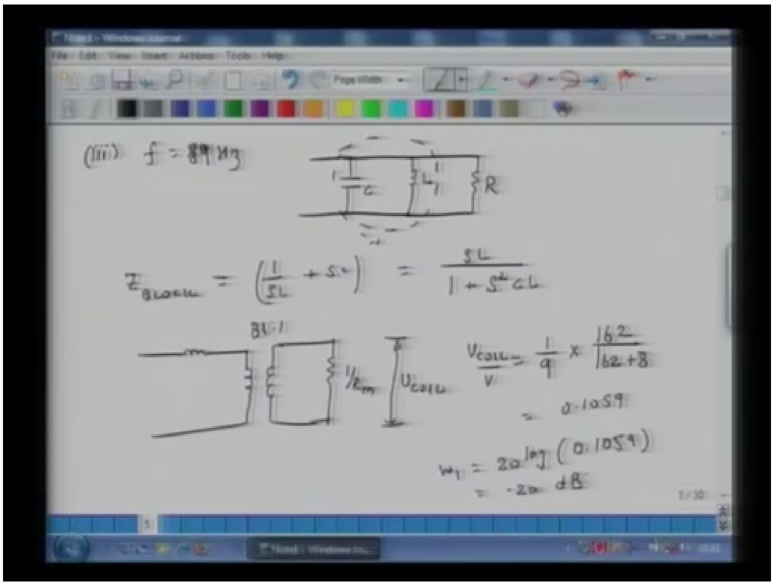

So what happens when f equals 89 hertz? So at 89 hertz on the other side above the beyond the transformer the circuit will look like there will be a capacitor there is an inductor which is M m capacitance is, yeah so I have capacitance of M m inductance of C m and resistance of 1 over R m. In the general sense I am just calling them L C R. And when L and C resonate and they are in parallel their overall impedance does it go to 0 or does it go to infinite when they are parallel.

Is it goes to infinite, we will see that, if we just find the impedance of this block Z block is 1 over S l plus S c inverse equals S l 1 plus S inverse C l, right? So at resonance the impedance offered by this entire block is infinite. Because 1 plus s square C l becomes 0, right. So no current is going at resonance when an inductor and capacitor they are in parallel and they resonate no current flows through them and all the current goes to the resistor. Our original question was that what is happening at 89 hertz? What that means is all current is going through the resistance.

So just at 89 hertz my equivalence circuit is for this one if I actually compute the value I get U coil over V equals 1 over 9 times 162 plus 8. And this basically plugging in the values which we had recorded earlier. So this comes to 0.1059

In decibels it becomes if this is W 1 then W 1 becomes 20 db 0.1059 log minus 20 degree above.

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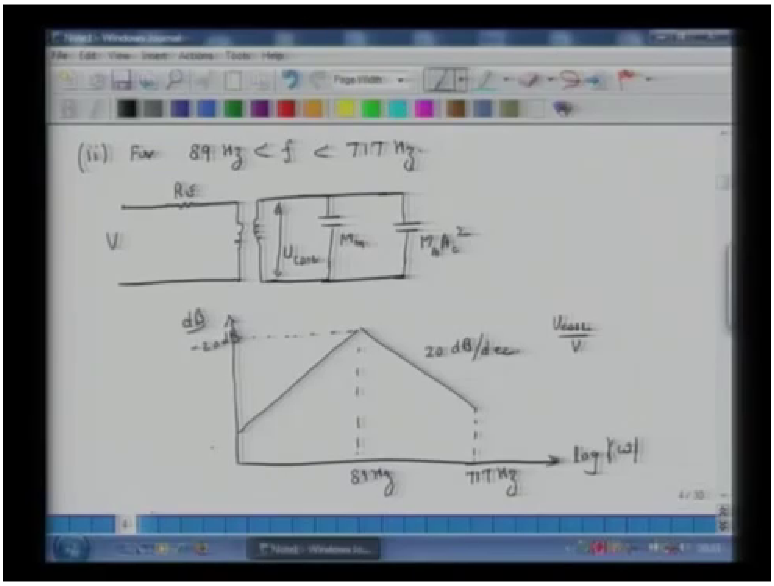

So I go back and modify this graph and I make this peak value minus 20 db. One note of correction in all these cases the value here is registered. Below 1820 hertz it is going to be a register because that is larger than the inductor and both are in series. So this also needs correction. Oh I apologize, so now we have figured out abode plot from 0 hertz at 89 hertz this value becomes minus 20 decibels. Beyond 89 hertz again it will start rolling off at a slope of minus 20 dbs per decade.

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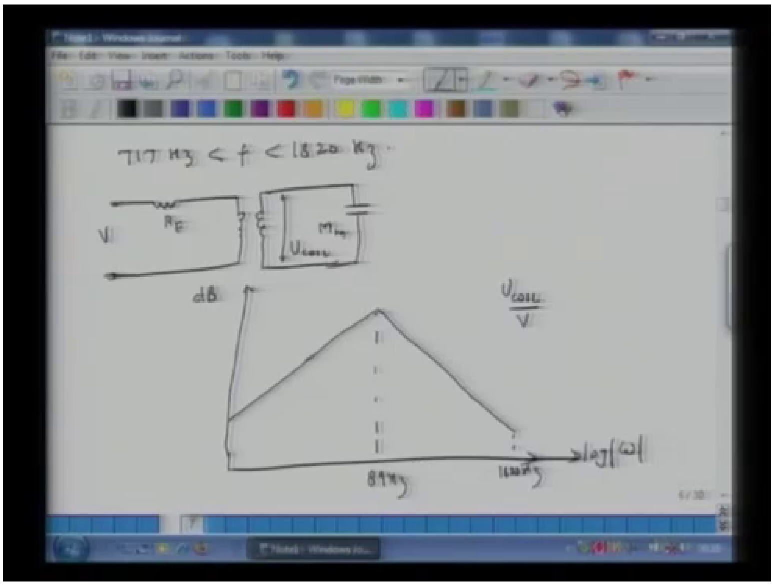

So now our next one is for the range 717 hertz to 1820 hertz. So again looking at making judgements which components include which components should not be included my equivalent circuit looks like this. The only thing I have is M m. This is my U coil is everyone clear by this is the case. The bode basically what this picture tells me is that the bode plot will not change its characteristics beyond 770 hertz also. The slope will continue to fall at the same rate the value.

So my modified bode plot becomes 89 hertz and it keeps on going till 1820 hertz. That is my omega this is in decibel and what I am taughting is U coil over V. This is upto 1820 hertz.

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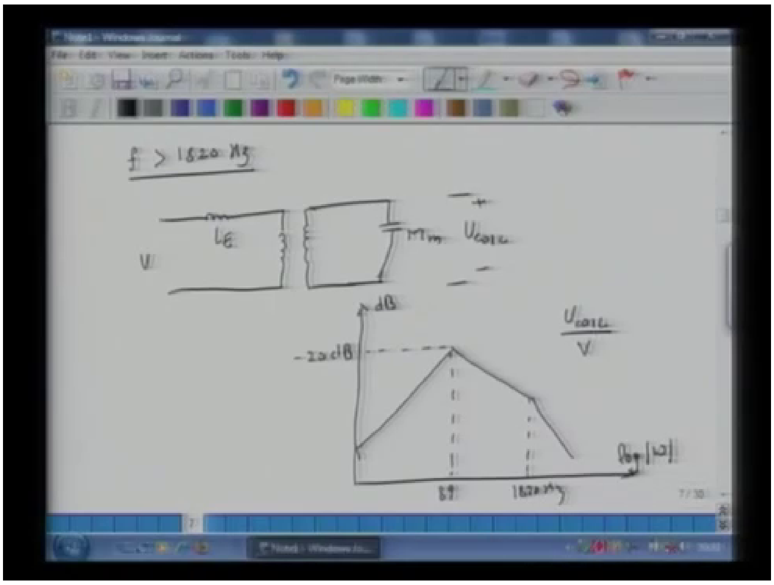

Finally for f greater than 1820 hertz I get so here I have a resistance and a capacitor above 1820 hertz what is the change going to happen and that too becomes active, the electrical inductor becomes high and the slope of this line will be what it will have a slope of minus 40 decibels. Now I have to earlier we had seen (())(20:38) capacitor and inductor cease which is the same case here, the slope becomes minus 40.

So I will go back and modify if I will make a new one clearly this figure is not the scale and this value is minus 20 decibels. So again this is U coil over V.

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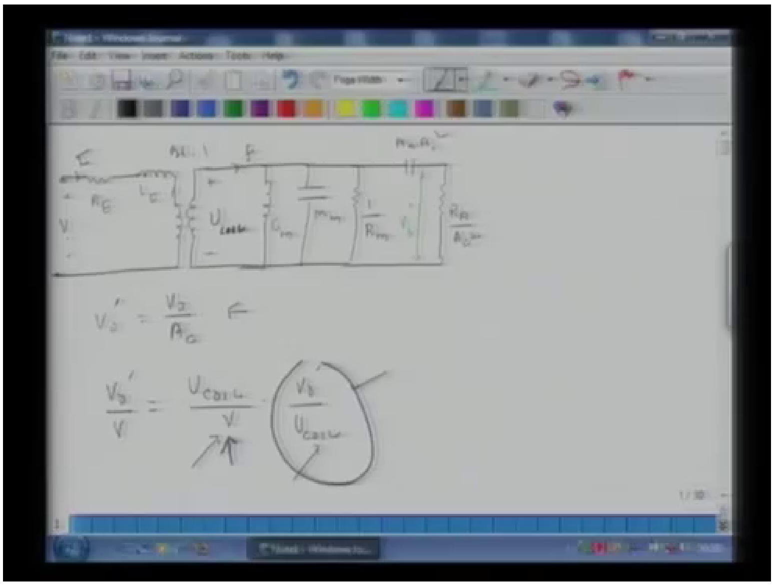

We started with the fairly not a very complex circuit but the fairly complex circuit but just by making identifying the break points decomposing our transfer function into small manageable ratios.

We have been able to plot the bode plots for all the important all the components which are going to play around. So now what I am going to do is I am going to bode plot for the entire function which is essentially a sum of those two bode plots.

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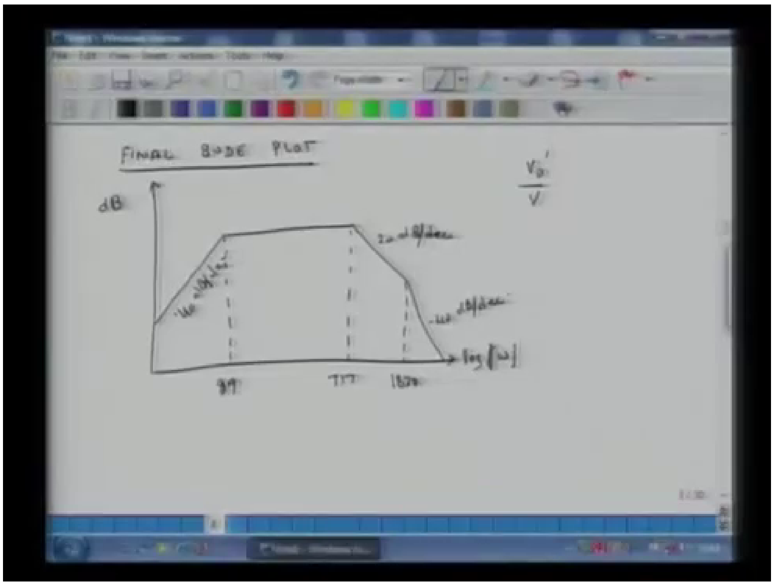

My final bode plot will look like something like this. So this final bode plot will be a plot of V 0 prime over V. What is this slope the slope of the first segment 40 degrees? And this is 20 decibels and this die is minus 40 decibels per decade. Looking at this picture what conclusions do you draw you should operate in a 89 to 717 hertz, that is the first thing that from 89 to 717 hertz you can use it and whatever is going in it will be faithfully reproduced at the same level output response will be same below of that response.

Now in this entire analysis I mean we have plotted it upto infinite hertz this range would be as high. But as we were developing the lumped parameter model of the entire system we had made certain assumptions. And the most central assumption was my size of the transducer or radiating area shall not exceed lambda over 2 (())(24:04), right?

So I have not done the calculation but the validity of this bode plot is only good to the extent that whatever let us say lambda over 2 pi comes to 1200 hertz I am not calculating but you can go and calculate what that number is, so this bode plot can be relied upon only upto that particular figures. That is the second thing the first one is that I have a flat frequency response from 89 to 717 hertz.

If everything else is good then I can rely I can use this in this way. The second thing is may have to restrict the usage of this bode plot at a cut off frequency which corresponds to lambda 125 to be have to find. The third thing in lump parameter model is specially from mechanical stand point is that I am assuming that the masses are rigid I have a diaphragm and it just moves like this.

Now in reality when you have any membrane or any structure it has it own modes it could be that the diaphragm starts breaking up meaning that it starts exhibiting its own modes at 300 hertz 400 hertz I do not know if that is the case then that is another word of caution that this particular bode plot has its validity as long as rigid masses remain rigid.

We have assumed that the diaphragm is infinitely stiff and it does not bend and twist as it moves in backend foot. So someone while this analysis is done at the more detailed and a refined level someone has to go and do a modal analysis of the diaphragm using some finite element methods and stuff like that and figure out what are the modes at which the diaphragm starts exhibiting its own, it is no longer rigid, so that is third thing.

What is (())(26:06) in what sense? No lump parameter model it gives you guideline, if because it is a relatively straight forward way just doing the diaphragm finding modes of diaphragm all it will tell you is that the diaphragm is going to have, so when you do.

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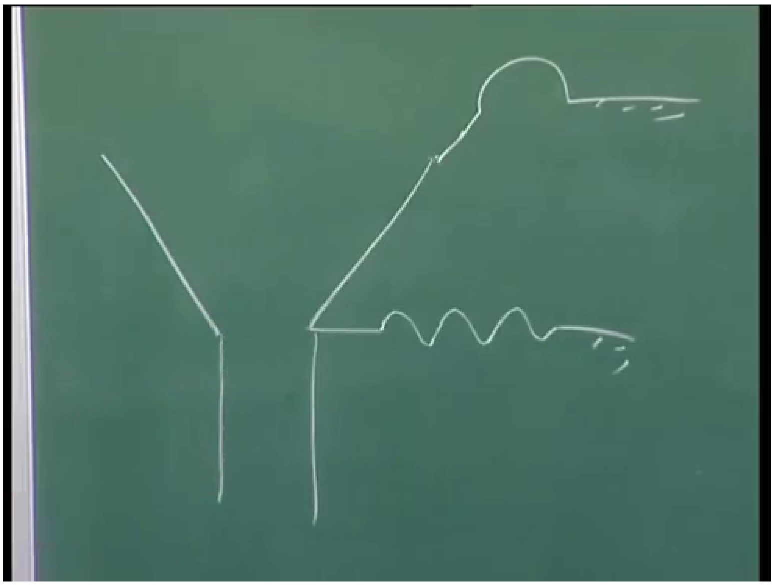

See this is a diaphragm right? And here you have a voice well I am exaggerating can anyone see this can you see it? So and there is a suspension here, so it is like a spring so when you do and there is also suspension here so when you do modal analysis of this using finite element method. You model you create elements on the spider you create elements on the surround on the cone. You have elements and then you do modal analysis.

Naturally the first mode which the finite element analysis show it will spit out the show will be what it will be equivalent to your mechanical resonance k over M right? So there is a K associated with these springy elements and then there is the overall mass. So the first mode will be let us say M 1 will be I do not know what we calculate K over M 75 hertz, right? So it will calculate it is 75.

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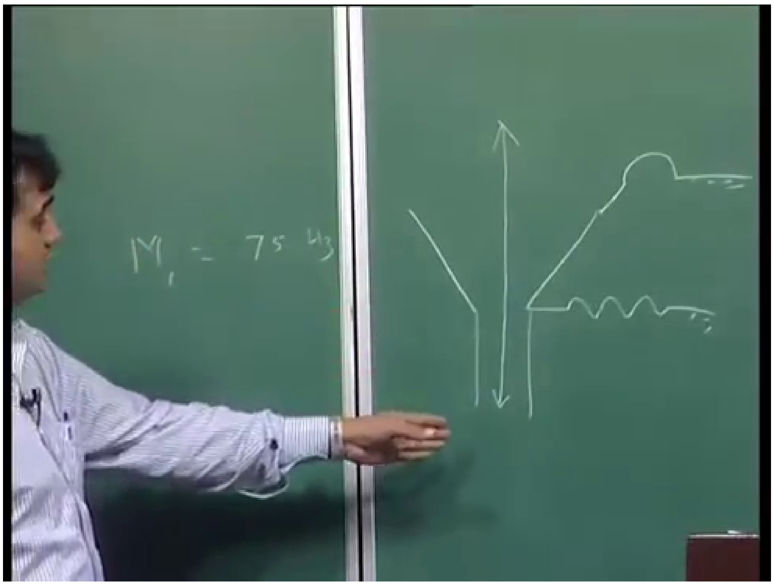

So what that means is that this entire structure and when you draw the mode shape you will see that finite element will predict that this entire thing moves up and down in the modal analysis. That is the pistonic rigid body motion that is fine and may be some other mode M 2 what it will do is that it starts instead of moving like that the cone also has a propensity in the structure to do this, right?

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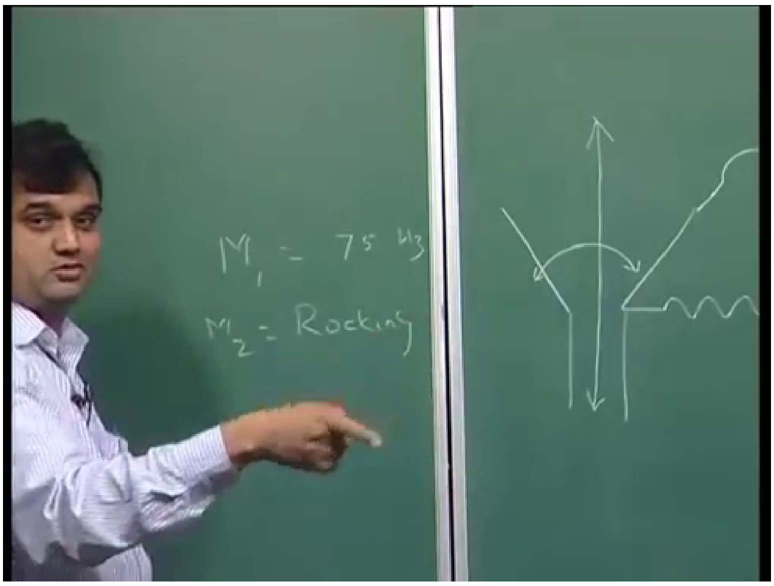

I mean it can move like this it can also do this because it is hinged here. But at higher frequency so this also that the, so this can do this, so this is called dropping mode. This is going too deep into transducer design we are not developing sound systems in this class since you are.

But still higher what starts happening is may be in the next class I will show you some pictures that this cone it no longer remains rigid.

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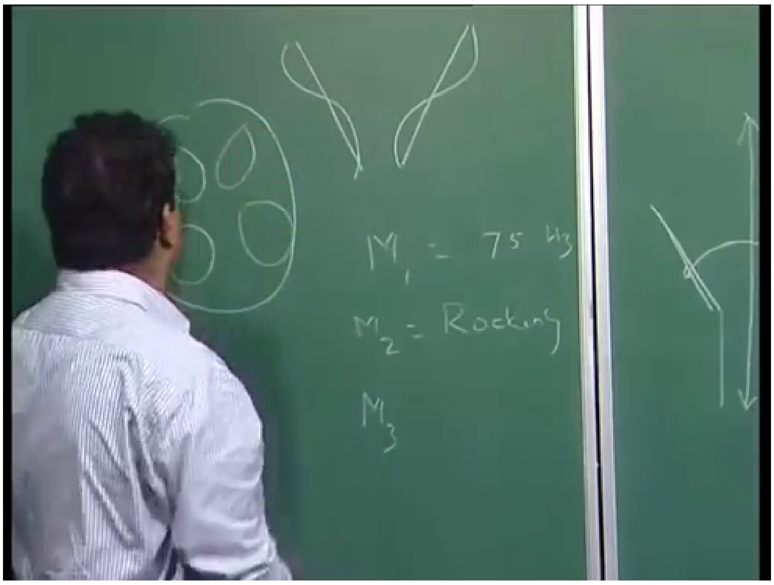

So it starts doing suppose this is a cone surface starts doing something like this. And then you have to listen 3D when you do a 3D picture with 3D if I look at it from the top side you will see something like this. So some part is coming out some part is going down and so on and so forth. What that means is the cone is no longer rigid. So at frequencies above that particular number.

The value or the faith which I will have in the predictions of my lumped parameter model will be limited. They may still be valid in a quantitative sense but they will still qualitative sense but they will have a limited validity. So but again we start typically from lumped parameter model because it gives you an overview how the system is going to behave.

And it is very easy to tweet specific parameters and adjust this flat range, right? If I want my flat range from let us say 40 hertz to 1000 hertz. Suppose the requirement is that I want my transducer particular transducer to start working from 40 hertz to 300 400 hertz. Then I can use this kind of approach to figure out what are the optimum parameters, how much is my moving match going to be.

Once I know that I can figure out the size and the thickness of my cone in different parts and I can design them. I know from this approach what is going to be the stiffness of my system. So then accordingly once I have that target number I can design the shape of my spider shape of the surround and figure out how to get that particular number.

It also tells me what is the value of D l going to be which has been designed the magnet. The same model also tells me what is going to be the optimum value for R e it helps me design the voice well.it also tells me what is going to be the optimum value of A c radiating area. And that essentially helps me understand how big my transducer has to be.

So there is a very strong value in using this approach in figuring out the overall design at the coarse level. And once I have figured out the coarse level in parting parameters of the design then I go part by part and design them at a more refined level. So that is the value of this system overall at the system level modelling the entire system.

So what we will do again in next 10-15 at the most 20 minutes is we will change play with the couple of parameters in this circuit and see how it impacts the overall response of the system. So that will again give you a flair the value of this particular way of modelling transducers. What we had assumed in this particular model was that the transducer is mounted in how is it mounted? Infinite.

So you have an infinitely large wall its mounted there the wall is rigid it does not move back and forth. And the speaker is radiating sound outside in the first you have played. Now in a real application you do not have infinite bandwidth. So in a lot of conventional system you may have seen big boxes where they mount these speakers. So what we will try to find out is what does that do to the performance.

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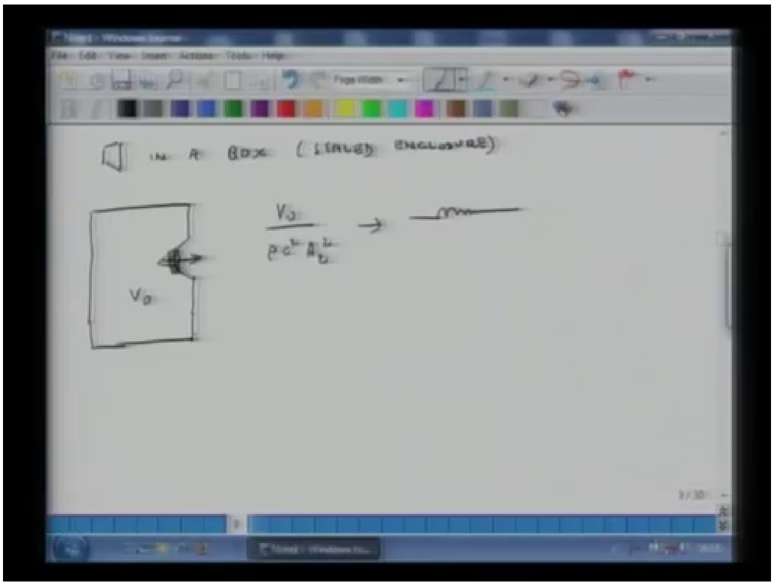

Transducer in a box a sealed enclosure, so I have a box, and this diaphragm is moving in a and let us say the volume of this box is V 0, physically we have talked about this earlier what is a volume to the taxes are string so I have this entire circuit and the stiffness of this string is V 0 over Rho c square. And if I have to remove that transformer I also put an A c square in the denominator.

So this is acting like a inductor. This inductor is going to be parallel will it be in parallel to the mechanical inductor or in series? to the mechanical inductor which is aspect it will be in parallel if you add two strings in series the overall stiffness does it go up or does it go down, it goes down right? If you have the two strings in parallel the overall stiffness goes up.

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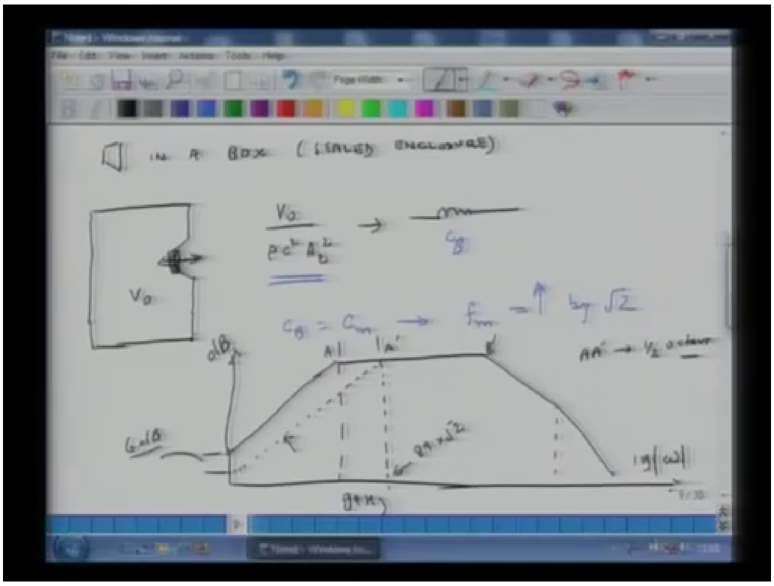

So what this box is doing is? It is going to increase the overall stiffness physically I am trying to explain all I have to do here is I have to add one more element. Let us erase something for so that I can put in more stuff. I will put one more compliance member and this will be C b compliance of the box (())(34:52) right?

When the compliance of the box is this value. To make my life simpler for analysis purpose I can assume that C b equals C m it makes things simpler what that does is that my F m mechanical resonance it goes up by how much basically if C b equals C m then I can add these two up, right C m and C b I can just make the overall thing is 2 C m.

My mechanical resonance will go up by factor of root 2. My overall system response will be this was my original curve 59 hertz. Basically what I will get is, I will get another parallel line this is 89 times root 2. Excuse me! 89 times root 2 that is this by omega log omega decibels. So the dark black line solid black line is the original line, this dotted line corresponds to the performance of the transducer in a box.

Whose stiffness is same as the stiffness of original transducer. So if that is the case then this gap AA time is opt by half octave. An octave is (())(36:56) root 2 times root 2 is this drop 6 decibel. So basically what you have done is once you out system in a box you have reduced the operating backup of the transducer optimal. So that has to understand why your conventional systems will come in boxes which are very large.

So that the additional stiffness exerted by the box on to the whole system is the lowest possible. If my system becomes larger and larger as A time move towards point B A will move towards B. So if I have to bring it closer to A then I have to keep on increasing the size of A. So that is why you know large (())(37:48)in the auditorium sometimes they have boxes as a room itself sometimes. I mean height of the room something comparable to height of the boxes that is the motivation.

We will do one of the one is what they call a portrait box. We have talked about this portrait box earlier in one of the earlier lectures will again revisited in context of (())(38:15) we have developed.

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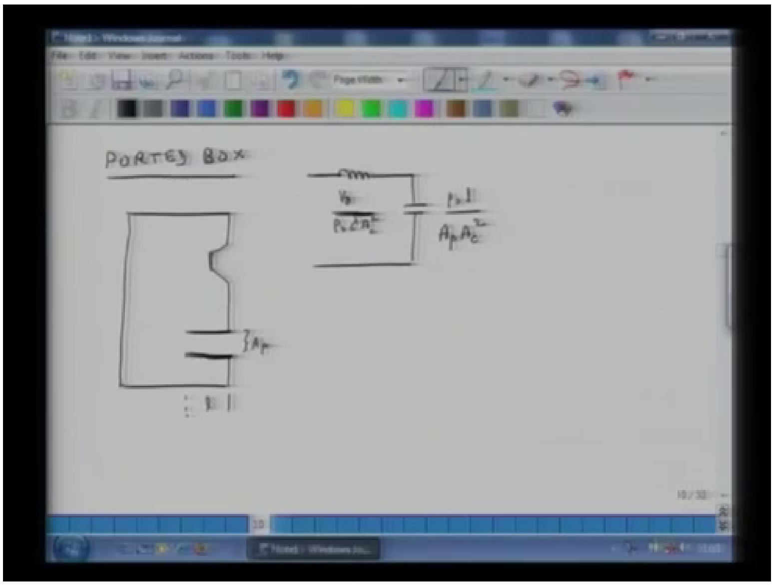

So I have my Portes box and then I have a court. So its length is L its area is A a , B. What this does is it acts on the elements to the circuit. (()(38:46). So it is V 0 over Rho 0 C square A c square. This is basically the stiffness of the box and then Rho 0 L over A c times A c square. This is the Acoustic mass which we have talked about earlier bode size will act as Acoustic masses.

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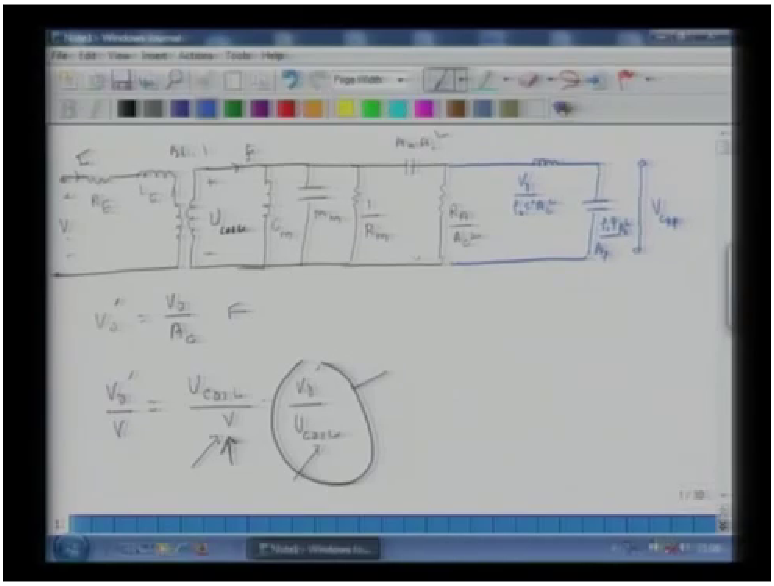

So in this circuit what I am going to do is, I am going to add this additional circuit this value the inductance is V 0 over Rho c square A c square and it is Rho 0 L over A c times A c square Ac square will be a numerator, ok it is an LC. How I am interested in 2 parameters 1 is whatever is the volume will be in physical sense sound out of the board that is 1 thing I am interested in.

The other thing in which I am interested is whatever what is respect to sound as it comes out of the speaker. There are two sources which are radiating energy so this is Voice coil call it D cap sound is coming out of the speaker that is one surface. Sound is also coming out of the port that is another surface, I am interested in both of them. And in a very qualitive sense we will try to see how things changed at resonance, below resonance, above resonance so on and so forth.

So the first thing is that this additional circuit has its own (())(41:04), so it resonates which is there in blue because the capacitor and inductor (are in para) are in series. There overall impedance goes down to 0. When they are parallel there impedance should strengthen. So when that happens almost all current goes through this route. So at resonance of this L c circuit you will have a peak in terms of dbs cube but there will be can have narrow band around the resonance itself.

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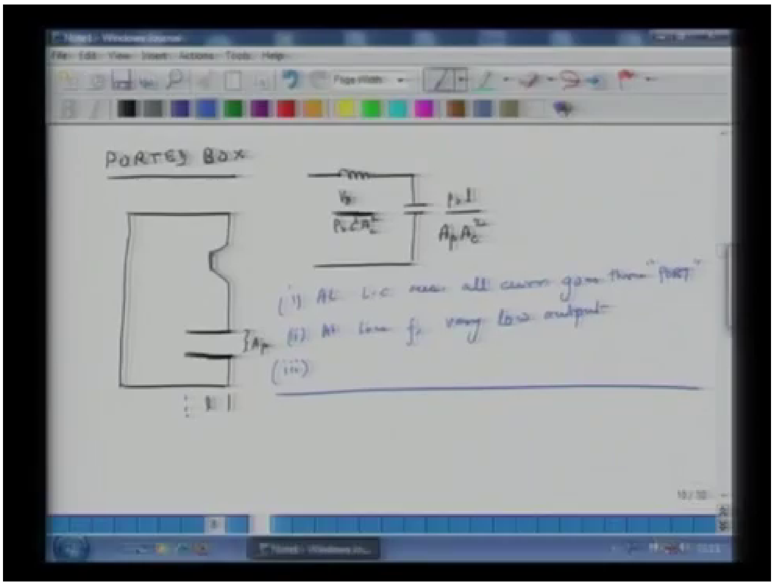

So one at L c resonance all current goes through ports I am putting port in parenthesis because I am using mixing current with port. Second think about it physically what happens when frequencies are extremely low. When you have very low frequencies meaning below mechanical resonance.

When the speaker is moving out there will be a vacuum which will get created in the low pressure condition which will get created in the box. And air will come in from the port and it will go in. You have a suction condition happening when the speaker is moving out. So the transducer is trying to generate a first to pressure and port is trying to generate just the inverse of it. And both these things will almost cancel out each other at low frequencies.

So the output at low frequencies will be negligible. Point about frequency is important because at higher frequencies the time it will be required the energy effects it will become important enough so that the time required for air to come in and compensate for the pressure will not be larger. At high frequencies this will not happen, but at low frequency this will happen.

At low frequencies very low output what this case has happened with all source of (()) (43:17). What is low frequency depends on the system parameters but if it is not specific to this particular transducer but it is too (())(43:29). Third at mechanical resonance, mechanical resonance implies C n and M m when they resonate the over head impedance we had seen was infinite.

Because C m and M m are in parallel what that means is physically that at that point the distance does not move much, even though current is going there is some hindrance then it will not move much if the impedance is infinity. So the piston does not move much. All the energy it gets channelized through the port. All the energy gets channelized through the port.

So the volume velocity at mechanical resonance of the port is very high. So you can design the system in such a way that the at resonance the volume the contribution of port is high at specific frequencies where the transducer itself is not contributing. And at the frequencies where port does not much your transducer plays an important role.

So lot of so what this system does is its performance is still not as great as that of an infinite raffle but in reality you cannot have infinite raffle and if you want to get closer and closer to infinite raffle you need very large boxes which is not practical. So keeping I have started using a lot of cases ports. Even in small sound systems boom boxes which you may have you may see a small tube you may have support is the volume inside and the support inside that is how it is physically.

The third example we will consider is increased mass. By mass I physically mean mass of the cone mass of Voice coil part of the mass of the these strings. What happens if I increase them without increasing or changing other engineering parameters specifically resistance, stiffness and all this. When you increase your mass your resonance is going to come down.

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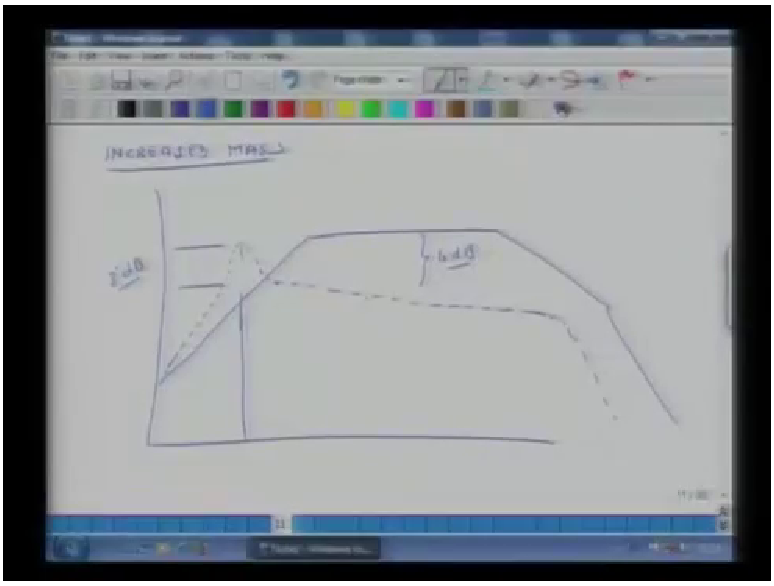

So that is my original curve when my mass close up then in theory my curve looks like this. What I get here is there data reduced resonance I get some extra decibels it is about if I increase my mass by if I double it then I get an enhancement of about 3 decibels which is significant. My overall broadband response goes down by 6 db but my operating width becomes a little larger.

The operating width becomes a little larger. So this is a good story as well as a bad story. A good story because my operating width has become a little larger so what I can do is that to get the same sound performance special level I can put in more power and I can get more energy more sound energy for a wider band. The bad thing that I have to put in more power.

This is what I wanted to cover in today's lecture I also wanted to quickly recap on an assignment problem which I had given in one of the earlier assignments.

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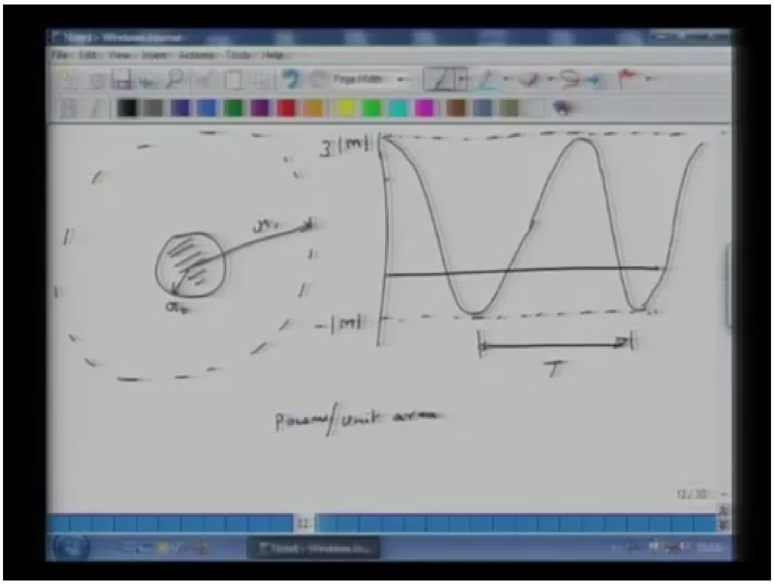

And the question was I think that merits are little bit explanation. The question was that if I have a valve of radius R 0 if this was in assignment 5 and it is pulsating uniformly and its emission energy such that at radius R 1.

The total pressure not pressure power output is something like this, right. This corresponds to T (())(48:44) What I am plotting here is power per unit area. This value is 3 times positive number m this value is negative m. And one of the questions was that if I have this graph what is the phase difference between pressure outward going pressure and velocity wise pressure.

What is the phase difference phase angle of the outward going pressure So they were some questions on some that particular this particular part of the problem I just wanted to cancel that. If I start this what is the average power in this case m times 4 pi r square, 4 pi r square because this is power per unit area. Some people (())(49:36).

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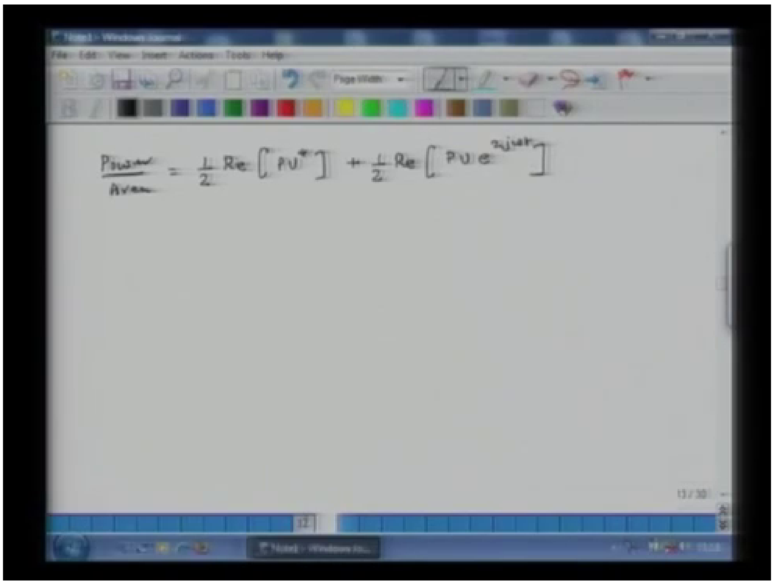

We know that power per unit area we have developed this relation is half a real user plus half a real [p u e 2j omega t] Now before I go more into detail I just wanted to just by looking at this picture we can make some inferences that given that there is minus 1 this graph of our power is not symmetric. If it was perfectly symmetric then the average power will be 0, right.

What that means is that it will not have resistivity element. This is a combination of some resistive element and induction or capacitance in the overall system. And we know that for a point source or a source of finite size as it emits radially into far field. There is a real component in its impedance and there is a component.

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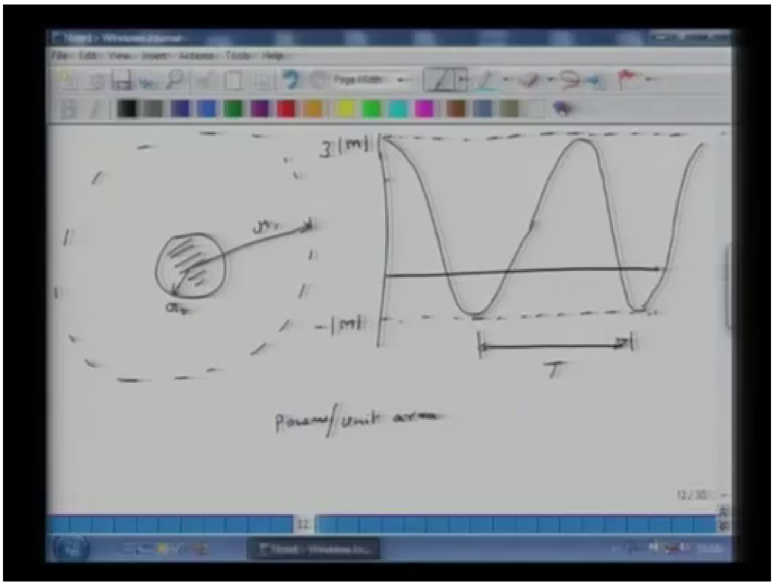

This A symmetric function is a consequence of dependence of real and imaginary components. So this picture should give us some idea how to figure out what is the phase relation if it was perfectly symmetric then the phase would have been either minus 90 or plus 90, right. If it was everything was 0 or positive then the phase would have been 0. Because it is the only resistive.

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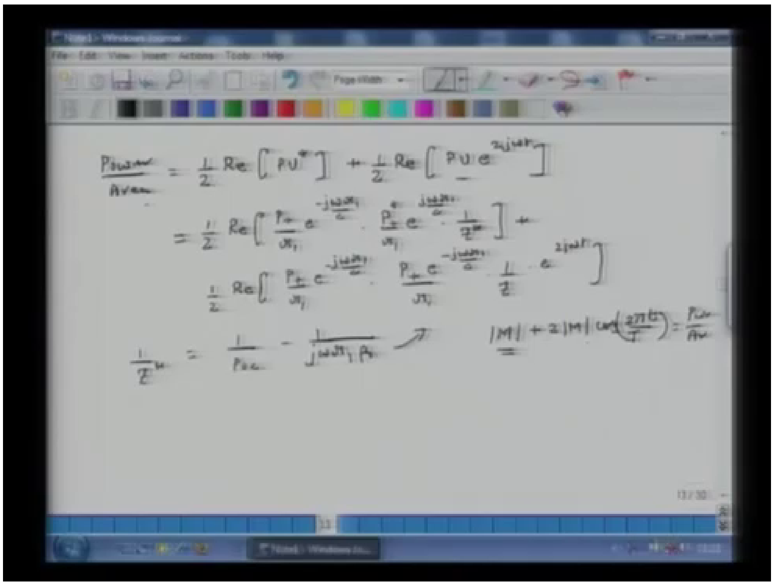

The expectation is to figure out what is the phase difference and finding it actual number is done by information provided. So this is I develop this further real p plus over R 1 e minus j omega 1 over C that is my P times u star. So that is p plus star over R 1 e j omega R 1 over C times 1 over Z star, right plus half.

So now I will expand the second component this one is e plus over R 1 e minus j omega R 1 over C times U. So it is e plus e minus j omega R 1 over C over R 1 u is basically p over Z, right. So I 1 over Z then I have e 2 j omega d 1 over Z star is 1 over what is that relation Rho 0 C minus 1 over j omega R 1 Rho 0, 1 over Z is this entire thing there is no negative sign but there is a positive sign.

So Z star is just basically I put a minus so plug this up here I know that power over area is also the other thing is the 3 m, 2 m, 2 m this is also power over A v right? From this graph the average power is m which is and the consumed component is 2 m cosin 2 pi 2 pi over t.

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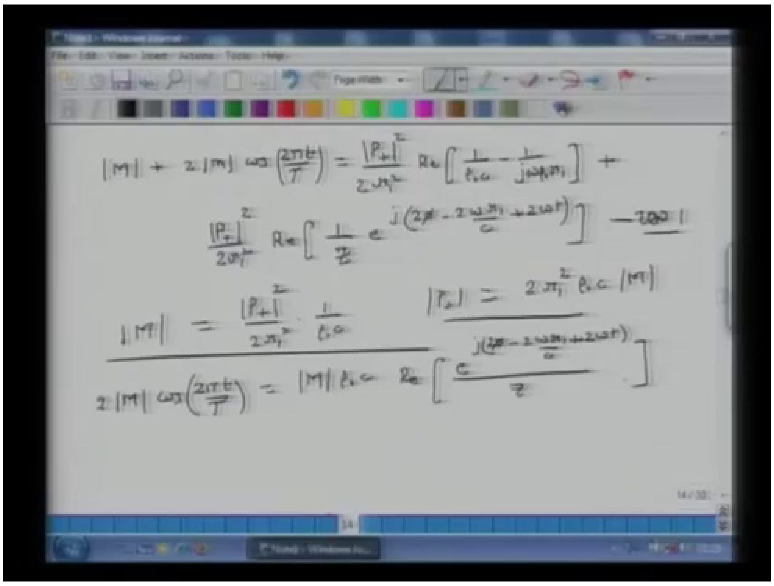

So I put all these in this original relation and what I get is m plus 2m cosine 2 pi t over t equals my first component which is real pu star f.

So that gives me p plus over R 1 square 2 why do I do that because p plus times p plus (()) (54:18) it means modulus real 1 over Rho 0 C minus 1 over j omega Rho o R 1 plus now I am writing the transient component to R 1 square. Now before I expand I just wanted to make a small comment. I am assuming when I am developing this relation the transient proportion of I am assuming that Z is modulus times e j z I can assume that.

So with that assumption I get 1 over Z e j 2 e this is minus 2 omega R 1 over C plus 2 omega t, ok. T plus times p plus, second time does not contain star I am assuming ok yes. So as I said I am assuming that p plus is oh I am sorry t plus is what I get is, so I have 2 sides in the equation. Now what I will do is I will just equate the steady state part with the steady state part as the first step.

So let us call this equation 1. So comparing steady state parts I get I am just talking about the first part no sir the second time you have written, yes you are write. So comparing only the steady state parts on both sides I get times 1 over Rho 0 c ok. So my p plus becomes 2 R 1 square Rho 0 C times m. So now I use this equivalence to put it in transient and then equate the two transient portions sides.

So what I will get is using that exercise 2 m cosine 2 pi t over t equivalence, m R 1 square and 2 they get cancelled out. Rho 0 c real portion of this entire thing exponent j 2 t minus 2 omega r 1 over c plus 2 omega t over z.