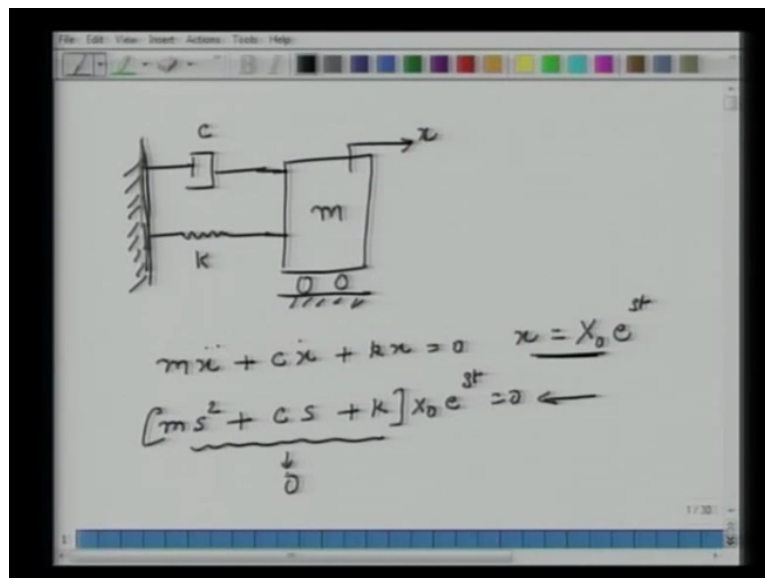


Acoustics
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Lecture 3
Module 1
Review: Poles and Zeros, Phase and
Magnitude Plots, Transfer Functions, Bode Plots

Good afternoon, so in today's class we will be capturing a few more review concepts and with this capture of review ideas once this is done then going forward we will move in formal acoustics area and we will start with one-dimensional wave equation but at least in today's class or maybe one more class we will be essentially doing a review of some of the concepts which we will be using frequently in the area of acoustics.

So the first concept I would like to talk about is having a very brief overview of what is an over damped system, what is a critical damp system and what is under damp system? And I will explain that in context of string mass damper system and hopefully with that illustration this term is over damped, critically damp and under damp will become clear.

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So let us consider a dashboard and then I also have a spring, so the dashboard viscosity is C spring constant or the stiffness member is K and both these elements they are connected to a rigid body mass which can move on a frictionless surface. So now let us assume that I am trying to perturb this by a small displacement x . So from previous classes we know that in such a situation the system is governed by differential equation of this type mx double dot

which is mass times acceleration plus viscous coefficient times velocity plus stiffness time displacement equals 0.

And to solve this equation if I assume that x is some constant times exponential of st where s could be a complex number and if I plug this in my differential equation what I get is $ms^2 + cs + k$ times x not est equals 0 and if this equation has to hold true at all times for all frequencies then the only natural conclusion we can draw is for this equation to hold true this term $ms^2 + cs + k$ should be 0.

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Handwritten mathematical derivation on a whiteboard:

$$ms^2 + cs + k = 0 \quad \text{Quadratic eqn in 's'}$$

$$s = \frac{-c \pm \sqrt{c^2 - 4km}}{2m}$$

$$s = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \quad S \rightarrow \text{COMPLEX FREQ.}$$

CASE I

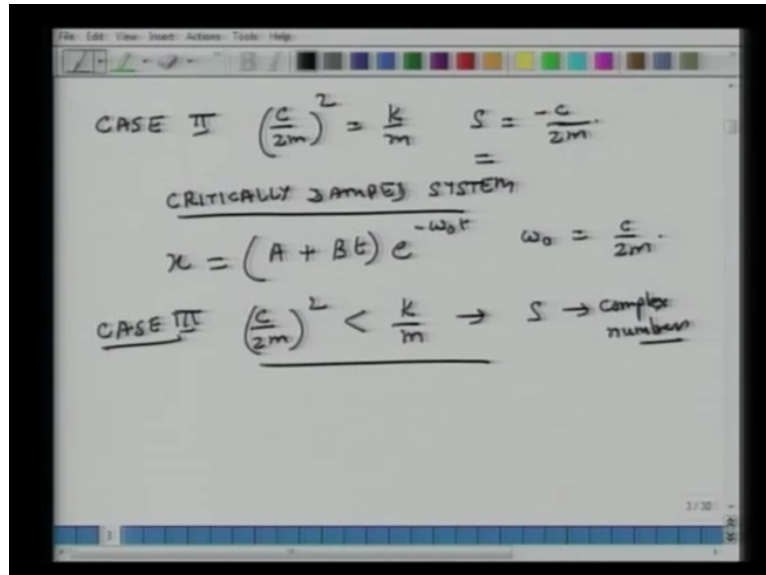
$$\left(\frac{c}{2m}\right)^2 > \frac{k}{m} \quad S \rightarrow \text{Purely real -ive number OVERDAMPED SYSTEM.}$$

What that means is, so I will write that $ms^2 + cs + k$ equals 0. Now we know what is m it is a physical quantity mass of the object, we know C it is again a physical quantity viscous coefficient of the damper and we know k which is the stiffness of the string. So this is a quadratic equation, it is a quadratic equation and S , so I solve for S , so S equals minus $2C$ I am sorry it is minus C plus minus and then I have C square minus $4km$ and then I divide the whole thing by twice of mass which is minus C over $2m$ plus minus and then I move mass under the square root sign.

So what I get is C over $2m$ square minus k over m , so that is my S and S we know we call S is complex frequency. So now we will come up with 3 scenarios, so first is case 1 and in this case this term is the term under the square root sign is larger than 0. In other words over $2m$ square is greater than k over m and when that happens what it essentially means is that S is a purely real negative number.

So there is no imaginary component to S and which in turn means that for a system where you have C over $2m$ square is greater than k over m there are no oscillations in the system and such a system where C over $2m$ whole square is larger than k over m such system is called over damped over damped system.

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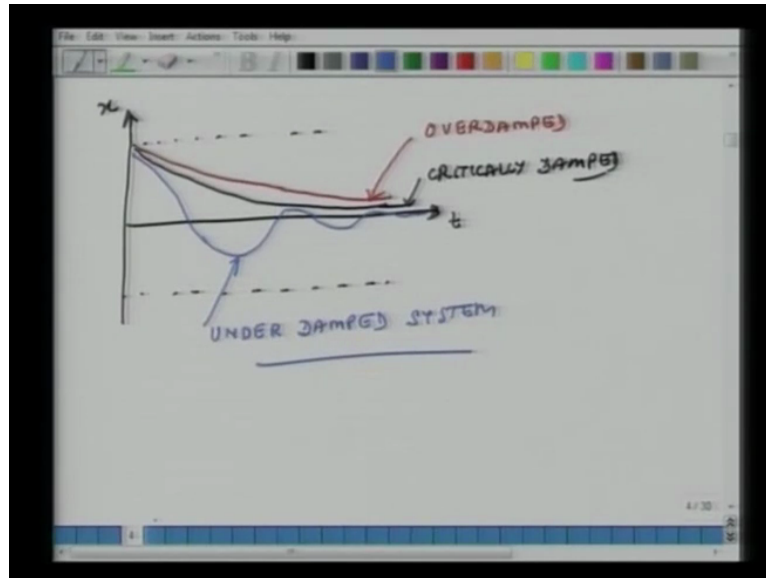


Then there is another case, so case 2 would be C over $2m$ whole square equals k over m . In that case my S will be minus C over $2m$. Again C is a real number, m is a real number, so S is a real number and it is also negative. So once again in this case there are no oscillations happening in the system and the system is called critically damped, critically damped system. A good example of critically damped system is sometimes in the doors which have dampers, so that the door closes silently without creating a lot of noise sometimes those dampers are tuned in such a way that the whole system is critically damped.

Some other examples where people use critical damping is these military guns where the barrel coils back when the system is being fired, so once you have charge coming out of the barrel the barrel recoils and that recoil phenomena is critically damped. So in critically damped system the response as you may have studied in your previous courses would be something of this nature $A + Bt$ time exponent of minus $\omega_0 t$ where ω_0 not equals C over $2m$ and then we have a third case, so it is case 3 where C over $2m$ whole square is less than k over m and in this case S is a complex entity, so it is a complex number and once you have a complex number what that also means is that the system is going to oscillate

because a part of a component of the complex frequency is imaginary in nature and that will that essentially implies that the system will oscillate.

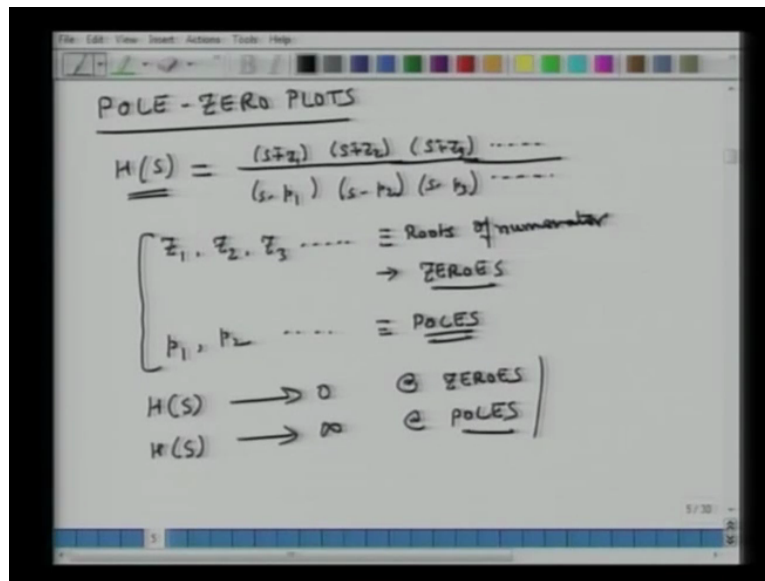
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So that is there, so let us plot some of these performances, so if I have a system and I am trying to plot its displacement if I perturb a string mass damper system single degree of freedom and if I perturb it and along time axis the horizontal axis is for time and the vertical axis is for displacement then a critically damped system would be the fastest to decay, so this is critically damped critically damped.

Then an over damped system will also decay fast but not as fast as a critically damped system, so it will be fast but it will be a little slower and note that in both these situations the oscillations are not happening, this is an over damped system and then finally you have an under damped system where there will be oscillations and the magnitude of these oscillations will slowly decrease with time in an exponential way. So this is my under damped system. So that covers the review of different types of damping over damp systems, under damped systems and critically damped system.

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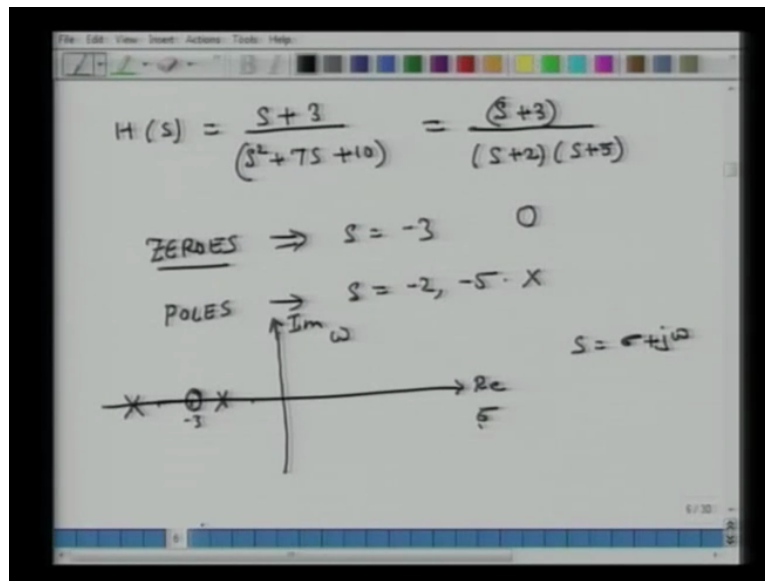
So now we will move on to the next concept in the class and this is what we call a pole zero plot, so what we are going to talk about? Pole zero plots, so any transfer function and we had talked about transfer function in some of the earlier classes. So if I have a system there is an input signal going into the system and then I am observing and I consider that observation as an out of the system and if I take the ratio of output and input and if a system is linear than that ratio is called a transfer function and this transfer function can depend on the National frequency not National frequency the frequency of the ingoing excitation and it will also depend on the element properties of the system.

So any transfer function can be its a ratio and because it's a ratio I can decompose it into roots of the numerator and roots of the denominator. So it could be S plus Z1, S plus Z2, S plus Z3 and so on and so forth depending on how complex the transfer function is and I can also decompose the denominator in terms of its roots. So S plus P1, S plus P2, S plus P3 and so on and so forth.

So here Z1, Z2, Z3 these are roots of numerator and they are called zeros, they are called zeros because at S equals Z1, Z2, Z3 the transfer function goes to an identically 0 value. Also the roots of denominator P1, P2, P3 they are called poles and at these values of S that is which is complex frequencies. So HS goes to 0, at zeroes and HS goes to infinity at poles.

Actually I made a small error here, actually these should be S minus Z1, S minus Z2, minus Z1, S minus Z2, S minus Z3 similarly here I should having S minus P1, S minus P2 and S minus P3.

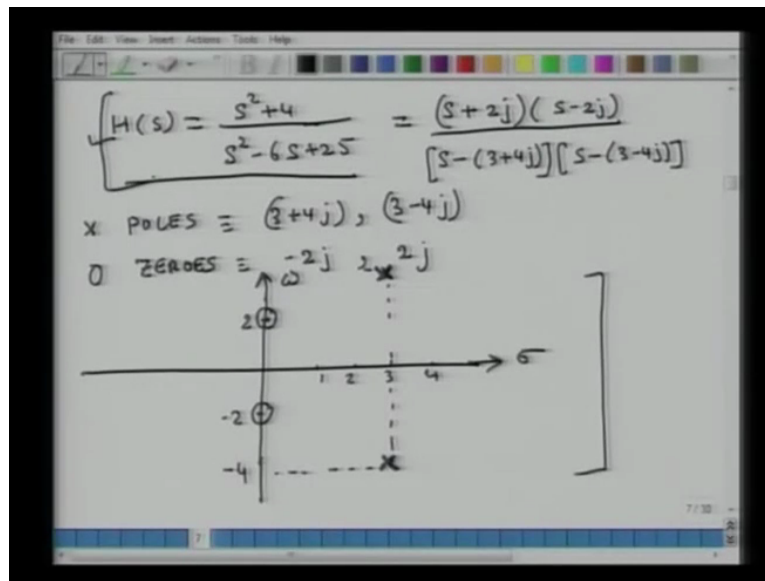
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So when I am, so let us do an example, so we will consider transfer function H which is function of natural which is function of complex frequency S and it is expressed as S plus 3 over S square plus $7S$ plus 10 . So this is S plus 3 and then I can make I factorise the denominator, so it becomes S plus 2 times S plus 5, so my zeroes are S equals minus 3 which comes from the numerator and the roots of the denominator will give us the poles.

So my poles are S equals minus 2 and minus 5, so if I plot this on a complex plane that is my real axis, this is my imaginary axis. So 1, 2, 3, 4, 5 and I will designate a 0 as a circle, so at industry I have a 0 and then I can designate a pole as an x. So at S equals minus 2 I have an x which is a pole and at S equals minus 5 I have another pole. So I know that S equals σ plus $j\omega$. So, on the real axis I am plotting σ and on the imaginary axis I am plotting ω .

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We will do another example and in this case the transfer function is $H(s) = \frac{s^2 + 4}{s^2 - 6s + 25}$, so I can decompose the numerator into its roots, so that is $s + 2j$ times $s - 2j$ and then the denominator is $s - 3 + 4j$ times $s - 3 - 4j$. So my poles which are designated as X are roots of the denominator, so it is $3 + 4j$ and the other pole is $3 - 4j$ and my zeroes designated as O are $-2j$ and $2j$.

So if I have to plot them on a complex plane that is my real component, this is my imaginary component, so my zeros are located along the imaginary axis plus and minus $2j$, my poles are located in the complex plane as I am going to just plot this now, so it is $3 + 4j$ and $3 - 4j$. So let us say this is 4, so I have a pole here and I have another pole at $3 - 4j$, minus 4 and then I have plus 4 on the positive side. So this is the representation of my poles and zeros for this transfer function $H(s)$.

So now that we have understood how to identify poles and zeros of transfer functions we want to go to the next step and which is plotting these transfer functions and I can plot the transfer function and it will have a phase component and the other component will be a magnitude component, so that is what we are going to do. The purpose of making these plots is that I have a graphical representation of how $H(s)$ is varying as ω is changing.

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The image shows a digital whiteboard with the following handwritten content:

$$H(s) = \frac{(s+3)}{(s^2+5s+10)} = \frac{s+3}{(s+2)(s+5)}$$

Now $s = j\omega$

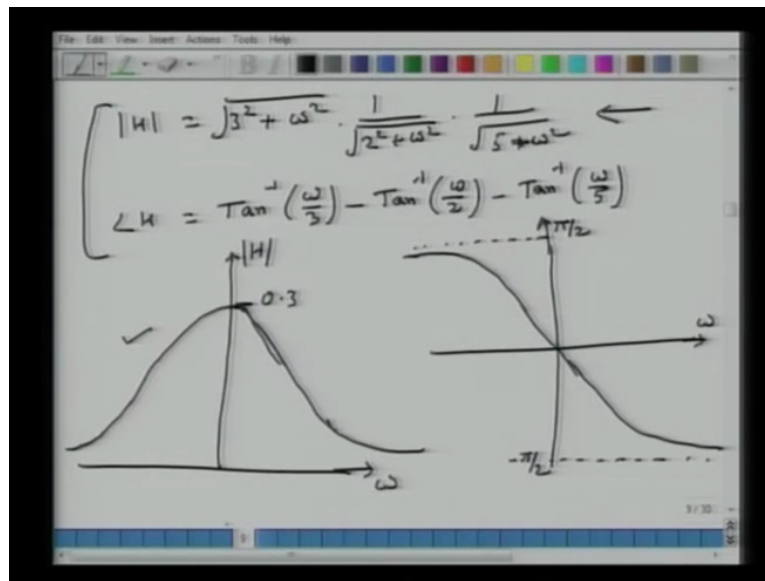
$$H(j\omega) = \frac{(3+j\omega)}{(2+j\omega)(5+j\omega)}$$
$$H(j\omega) = H_1 H_2 H_3$$
$$\begin{cases} H_1 = (3+j\omega) \\ H_2 = \frac{1}{2+j\omega} \\ H_3 = \frac{1}{5+j\omega} \end{cases}$$
$$\text{Mag } |H(j\omega)| = |H_1| \cdot |H_2| \cdot |H_3|$$
$$\angle H(j\omega) = \angle H_1 + \angle H_2 + \angle H_3$$

So typically, so let us say I have HS, so a phase plot or a magnitude plot will be a plot of H when S equals j omega. So I do not put a real part in S and I just plot H is a function of j omega. So my horizontal axis is going to be omega and my vertical axis is going to be the value of H the magnitude of H or the phase of H. So let us say H has equals S plus 3 over S square plus 5S plus 10.

So we had broken this earlier as this S plus 3 over S plus 2 times S plus 5. So now if I have to do a phase plot or a magnitude plot for H, I said S equals j omega. So then my H j omega becomes 3 plus omega j over 2 plus omega j times 5 plus omega j. Now what we see here is that H is a product of 3 individual small transfer functions, why is that? So I will just write down H1 times H2 times H3, where H1 is 3 plus omega j, H2 is 1 over 2 plus omega j and H3 is 1 over 5 plus omega j.

So and that is H j omega and we know from the mathematics of complex variables that if I have a relation for H in such a way then magnitude of H j omega is basically magnitude of H1 times magnitude of H2 times magnitude of H3 and also the phase of H j omega. So the phase of H j omega is basically phase of H1 plus phase of H2 plus phase of H3.

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So in this case the phase of, so again the magnitude of H is basically the magnitude of H1 and freshman is 3 plus omega j, so magnitude of H1 is 3 square plus omega square times magnitude of H2, so I will write down that times 1 over 2 square plus omega square the whole thing under square root and then similarly I have magnitude to write down magnitude of H3. So it is 1 over square root of 5 square plus omega square.

So now we have a relation and I can plot the magnitude component of H in terms of omega. Similarly the phase component I can plot and I know from complex variable theory that the phase of H is basically a summation of phases of H1, H2 and H3, so that is what I will do. So phase of H1 is Tan inverse omega over 3 minus Tan inverse omega over 2 minus Tan inverse omega over 5.

So I have now 2 relations I can use these 2 relations to develop a phase plot and a magnitude plot for H. So in a quantitative sense this is how the magnitude plot for this particular transfer function looks like. So what I am plotting is magnitude of H, on the horizontal axis I have omega and this function it peaks at 0 and the value of H at 0 is about 0.3 and then it asymptotically goes to 0 as omega tends to go to infinity and the phase plot again on my horizontal axis I am plotting omega and my vertical axis I am plotting phase angle.

And the phase plot looks something like this; these are asymptotes at pi over 2 and minus pi over 2 and my phase plot look something like this. So it is an anti-symmetric curve, symmetric along the vertical axis and the magnitude plot is the symmetric curve, symmetric being along the vertical axis. So this is how we can construct phase and magnitude plots for

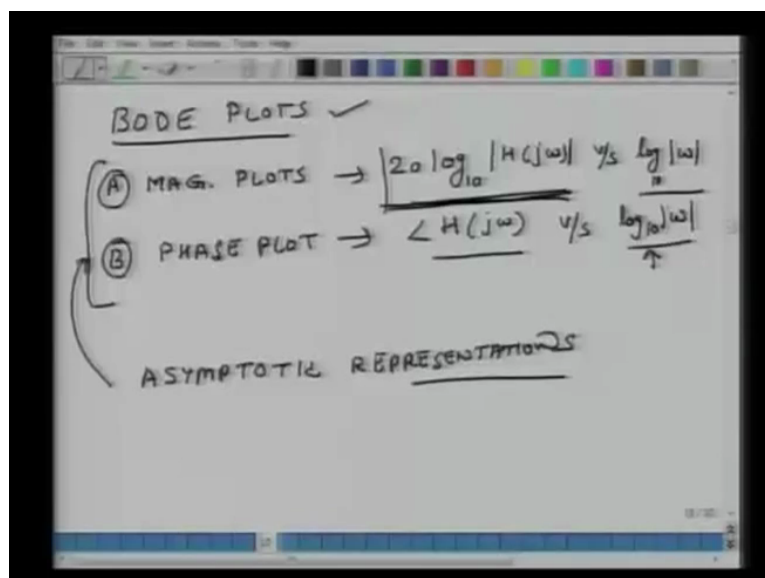
transfer functions by step 1 decomposing the whole transfer function into its poles and roots and this associated with those poles and goods.

And B if it is we are trying to plot magnitude then we find individual you know factors and multiplying them and if we are trying to find the phase then we find phase relations for individual factors and then just adding them up.

We will do another concept which is called a bode plot. So right now what we did was, we plotted H in on a linear axis, we plotted the magnitude of H on the linear axis, the horizontal axis was linear and also the vertical axis where magnitude was being plotted that was also linear and same was true for phase. Now what we have? Another concept and it is called a bode plot.

And bode plots are essentially graphical depictions of transfer functions when S equals j omega, so there are graphical depictions of transfer functions at S being equal to j omega and more specifically they are asymptotic behaviours. Further when we are plotting bode graphs then we essentially use logarithmic scales on horizontal and also the vertical axis and we are plotting the magnitude and when we are plotting phase the vertical axis remains a linear scale but the horizontal axis still is a log axis plots.

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So they can be 2 type of bode plots, one is magnitude plot and this is basically a plot of 20 logarithm on based 10 magnitude of H j omega was on the horizontal axis we have log of omega, again it is a log10 and the second bode plot is a phase plot and here we are plotting

the phase of H when S equals $j\omega$ and on the horizontal axis we have $\log_{10} \omega$ and these plots are basically plotted in such a way that we have asymptotic representations.

So essentially we are trying to plot asymptotic interior of magnitude and asymptotic behaviour of phase of H as ω is changing in case of the magnitude plot we are plotting asymptotic behaviour of H on a log scale on the vertical axis and also on the log scale on the horizontal axis. For phase the vertical axis is still linear but the horizontal axis is logged scale, the central point when we are plotting these bode plots is that we are focusing on asymptotic behaviour because these plots very quickly helps us to generate a curve which we can look and get a feeling of the approximate behaviour of the system.

And once we have an approximate feeling for the approximate behaviour of the system then we can use that information to validate our understanding of the model which we are trying to analyze. The other point which I wanted to make is that some thought as to why we are using 20 times logarithm of $|H(j\omega)|$ especially when we are plotting the magnitude plot. So this is because it is not directly a representation of the magnitude itself but rather it is a representation of the energy contained in the system.

So what do I mean by that, so let us try to understand a little bit more at a detailed level. So typically when we plot H in physical systems I have a signal going in and I have a signal coming out, so let us say I have an acoustic system it is getting excited by some signal let say it is an electrical signal. So I try to monitor that electric signal in terms of volt, so my input parameter is voltage.

And my output parameter in case it is a microphone or some other displacement sensor or velocity sensor at the end of the whole transduction process I end up measuring some more voltages. So my input is a voltage and my output in a lot of cases is voltage as well or in other cases it could be current and current or combination of voltage and current.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, it states $V^2 \propto \text{Energy}$ and shows a circuit diagram with a resistor R and current I , leading to the equation $\frac{V^2}{R}$. Below this, it states $I^2 \propto \text{Energy}$ and shows a circuit diagram with a resistor R and current I , leading to the equation $I^2 R$. The next line defines decibels as $\text{dB} = 10 \log_{10} [\text{Ratio of Energies}]$. This is followed by the derivation $\text{dB} = 10 \log_{10} \left[\left| \frac{V_2^2}{V_1^2} \right| \right]$, with a note $V_2, V_1 \rightarrow f(j\omega)$. The final line shows $\frac{\text{dB}}{\text{dB}} = 20 \log_{10} \left| \frac{V_2}{V_1} \right|$ and $\frac{\text{dB}}{\text{dB}} = 20 \log_{10} |H(j\omega)|$.

V Square which is square of voltage is essentially proportional to energy. For instance across a resistance if I have voltage V and the energy dissipated is V over R. Similarly I square is again proportional to energy in a general sense, again if I have the example of a resistance and the current going through it is I find the value of resistance is R and energy dissipated or power dissipated is I square R.

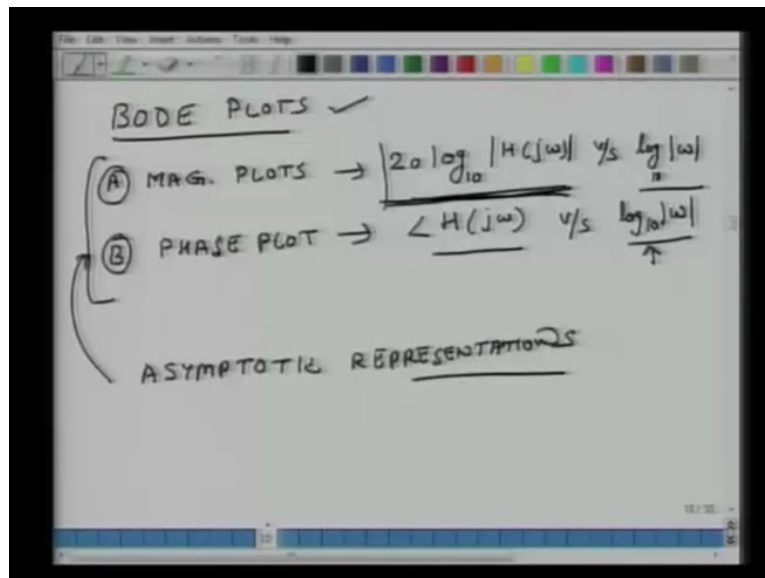
So when I am taking ratios, so let's call this a decibel and this is defined as 10 times logarithm of ratio of energies. So as energy is directly proportional to Vsquare and, energy is also directly proportional to I square, so at this point of time now that we have seen that energy is proportional to the square of voltage or energy is proportional to the square of current we introduce a term called decibels and decibel is essentially defined as 10 times log of ratio of energies.

The log is on log10 scale, so what that means is that it is essentially log10 and ratio of energies could be V2 square over V1 square and I have to take the magnitude of it as this is logged and I can similarly have a relation for current also. Now note that V2 and V1 are functions of j omega they can be functions of j omega. So when I have a square relationship, what this essentially means is that it is 20 log 10 of magnitudes of V2 over V1.

So when we are taking when we are plotting transform function, so they are also ratios of whatever is going in and whatever is coming out. So I have if I am converting transfer function on the decibels scale than what I get is decibels equals 20 log 10 of H which can be a function of j omega. So as we plot 20 log 10 j omega H j omega on of ys space.

So on a vertical axis we are plotting on a log scale and on the horizontal axis why we are plotting on a log scale is because typically the range of frequencies for acoustics problem they can start from 20 hertz or maybe even sometimes less than 20 hertz and then they can go up to tens of thousands of hertz, so it's a wide range, so it's much more convenient to use a log scale on the horizontal axis.

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So we were talking about Bode plots and we had said that these plots could be of 2 types' magnitude plots and phase plots. So now we will see how to plot, how to construct these plots?

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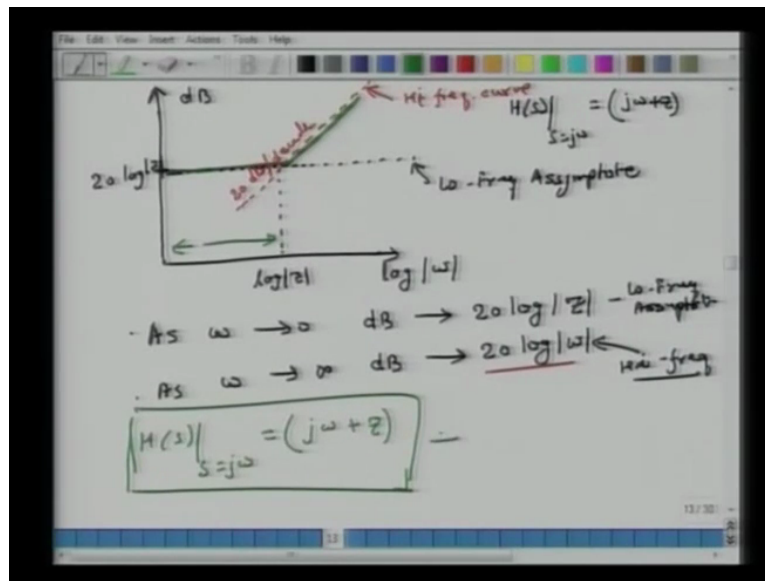
The image shows a whiteboard with handwritten mathematical notes. At the top, a transfer function is written as $H(s) = \frac{(s+z_1)(s+z_2)\dots}{(s+p_1)(s+p_2)\dots}$. Below this, a single factor is highlighted: $H(s) = (s+z)$. The next line shows the magnitude in decibels: $dB = 20 \log_{10} |H(j\omega)| = 20 \log_{10} |z+j\omega|$. The final two lines describe asymptotic behavior: "Asymptote as $\omega \rightarrow 0$: $20 \log |z|$ " and "Asymptote as $\omega \rightarrow \infty$: $20 \log |\omega|$ ".

So as we had discussed earlier any transfer function could be decomposed into its factors of the numerator and factors of the denominator, so let's do that, so this is a general transfer function and then on the denominator I have poles. So now what we will do is, we will as step one construct a bode plot for a transfer function which is made up of only a factor of the numerator and in another case it is only a factor of the denominator and see how it behaves and then we can try to develop an approach through which we can combine plots of different factors.

So let us consider a transfer function S plus Z , Z implying it is on the numerator and it relates to a 0 and I will construct a magnitude plot for this. So you have to construct a magnitude plot I have to convert this into decibels which is as we had explained earlier is $20 \log$ on scale 10 magnitude of HS where H equals j omega which is $20 \log 10 Z$ plus j omega and as we mentioned earlier we have to develop a plot which reflects the asymptotic response.

There will be 2 asymptotes for this relation, so asymptote as omega tends to 0 is $20 \log Z$ and asymptote as omega goes to infinity is $20 \log \omega$.

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So I will plot these, so my vertical axis as I had said is on a decibel scale, my horizontal axis is on a log scale, so I am plotting log omega on the horizontal scale, on the vertical scale I am plotting decibels and I am plotting decibels for the transfer function HS as S equals j omega equals j omega plus Z, we had seen that as omega goes to 0 the value of dB tends to 20 log Z and as omega goes to infinity the value of dB goes to 20 log omega.

So these are the 2 asymptotes, this is the low frequency asymptote and this is the high frequency asymptote. So the low frequency asymptote is conversing to a straight horizontal line 20 log Z. So let us say that value is 20 log Z and the low frequency asymptote is a horizontal line and that is my low frequency asymptote and my high frequency asymptote is 20 log omega and it cuts that the high frequency asymptote cuts the low frequency asymptote at omega such that log omega equals log Z.

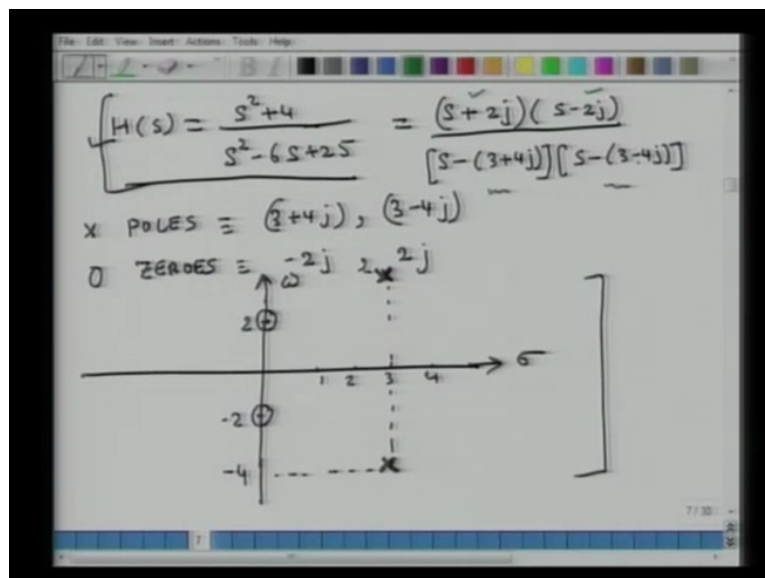
So at this line my high frequency asymptote cuts the (()) (44:06) also this, so this is my high frequency curve also the slope is 20dB per decade and we have explained earlier a decade is the space between 2 frequencies which are off by factor by 10. So when omega increases and let us say we are starting from omega equals omega 1 to omega equals omega 2 where omega 2 is 10 times omega 1 then this number goes up by in such a way that the slope of this line is 20 decibels per decade.

So this is my again to retread the black dotted line is my low frequency asymptote, the red dotted line is my high frequency asymptote. The asymptotes cut at an important point which corresponds to omega such that where omega equals Z or log omega equals log Z. So I have a

high frequency asymptote Red Line, low frequency asymptote backline and the combined line I will drop draw is the green line. So this is my total response curve.

So my response curve for transfer function $H(s)$ where s equals $j\omega$ equals $j\omega + Z$ where Z is the zero is this green line. What this line tells me is that as long as my ω is less than Z , so in this region a low frequency asymptote is going to dominate and when my ω exceeds Z then the high frequency asymptote is going to dominate. So my total response curve is this green line which is a combination of low and high frequency asymptote. So what I have drawn here is a response curve, a bode plot for magnitude of a very simple transfer function where transfer function is essentially $Z + j\omega$.

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So now we will draw another transfer function $H(j\omega)$ or actually we will start from S , so $H(s)$ equals 1 over S plus p . So remember we can decompose a very complex transfer function into a bunch of simple fact this S plus Z_1 , S plus Z_2 , S plus Z_3 which all go in the numerator and in the denominator we can decompose it as S plus P_1 times, S plus P_2 times, S plus P_3 and so on and so forth.

So what here I am doing is I am going to plot the bode plot or a very simple transfer function $H(s)$ which equals 1 over S plus P such that S equals $j\omega$, so this becomes $H(j\omega)$ equals 1 over t plus $j\omega$. So again I have to draw asymptotic curves because I have to develop an asymptotic response. So asymptote as ω tends to 0 gives me dB becomes $20 \log$ of this whole relation is $20 \log 1$ over P is basically minus $20 \log P$ and asymptote as

omega goes to infinity gives me in decibels if I have to do that is $20 \log 1 \text{ over } \omega$ equals minus $20 \log$.

So now if I have a transfer function which has only a pole then this is how the asymptotes of the transfer function are and now I will plot it I have a horizontal axis on which I am plotting logarithm, they are in mind these are all log to the base 10 omega and my vertical axis I am plotting decibels and am going down because both these are negative numbers. So my low frequency asymptote I will draw a black line, black dotted line this is my low frequency.

And the Y intercept of this is minus $20 \log P$ and the high frequency asymptote I will draw in red and this has a slope of minus 20dB per decade, again I will draw it as a red dotted line so, this is high frequency asymptote you can see asymptote, the slope is minus 20 decibels per decade and the combine response, so before I talk about combine response the intersection point corresponds to a value of omega and omega equals P, so this is log of P. So this is my intersection point.

So my combine response I will plot in green is for low frequencies below omega equals P. My low frequency asymptote is going to dominate and for high frequencies, for high frequencies the high frequency asymptote is going to dominate. So this is my combine curve for transfer function which is expressed as $H(S) \text{ equals } 1 \text{ over } S \text{ plus } P$. So once again I have a low frequency component which is this part and I have a high frequency asymptotic response which is this part.

The slope is minus 20dB per decade, the breakpoint corresponds to logarithm of P where P is this entity and the horizontal flat curve is basically a straight line in cutting the Y axis at minus $20 \log P$. So using this approach so we have seen that a complex transfer function can be expressed as a sum of as a bunch of products on the numerator and also a bunch of products in the denominator. Now we have seen how to construct plots Bode magnitude lots for each of these numerator components and also we have seen how to construct magnitude plots for each of these denominator components.

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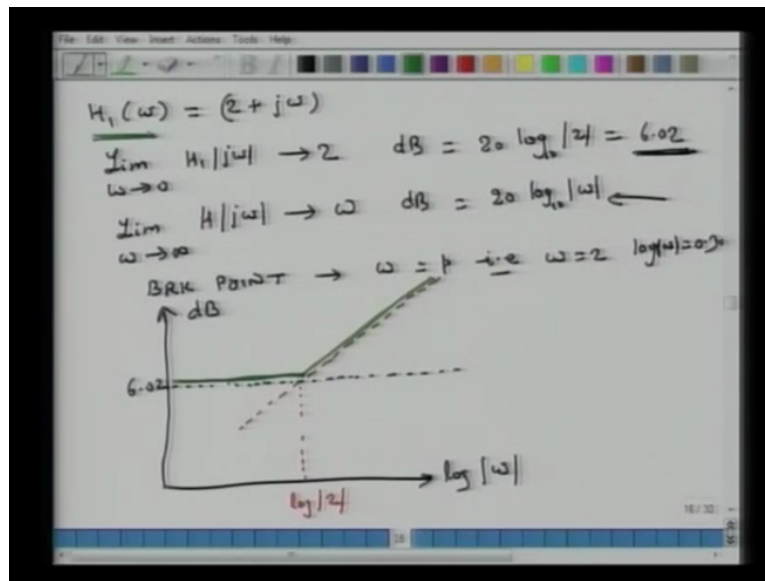
$$H(s) = \frac{s+2}{s^2+10s+109} = \frac{(s+2)}{(s+3+10j)(s+3-10j)}$$
$$s = j\omega$$
$$H(\omega) = \frac{(2+j\omega)}{(3+10j+j\omega)(3-10j+j\omega)}$$
$$= H_1(\omega) \cdot H_2(\omega) \cdot H_3(\omega)$$
$$H_1 = (2+j\omega) \quad H_2 = \frac{1}{(3+10j+j\omega)} \quad H_3 = \frac{1}{(3-10j+j\omega)}$$

So now the next step would be that you synthesise all these individual plot into one single plot and that will be the overall Bode magnitude plot for the whole transfer function. We will illustrate that through an actual example my transfer function let us say is HS equals S plus 2 times S square plus $10S$ plus 109 HS equal to, so now I factorize the denominator and what I get is S plus 2 over S plus 3 plus $10j$ and S plus 3 minus $10j$.

So now what I do is S equals $j\omega$, so my H which is a function of ω becomes 2 plus $j\omega$ over 3 plus $10j$ plus $j\omega$ times 3 minus $10j$ plus $j\omega$ and this I can write it as $H_1(\omega)$ times $H_2(\omega)$ times $H_3(\omega)$ where H_1 equals 2 plus $j\omega$, H_2 equals 1 over 3 plus $10j$ plus $j\omega$ and H_3 is 1 over 3 minus $10j$ plus $j\omega$. So I have expressed a complex transfer function such as H in terms of 3 simple transfer functions is H_1 , H_2 , H_3 .

My next step will be to construct Bode plots for H_1 , a bode plot and magnitude of H_2 and a bode plot and magnitude of H_3 and I will sum these bode plots of because when I take a logarithm of H is essentially a logarithm of H_1 , if I take a logarithm of H it is same as \log of H_1 plus \log of H_2 plus \log of H_3 . So it is a valid mathematical exercise to construct individual log plots for H_1 , H_2 , H_3 and then sum them up and that is my total bode plot for the entire system.

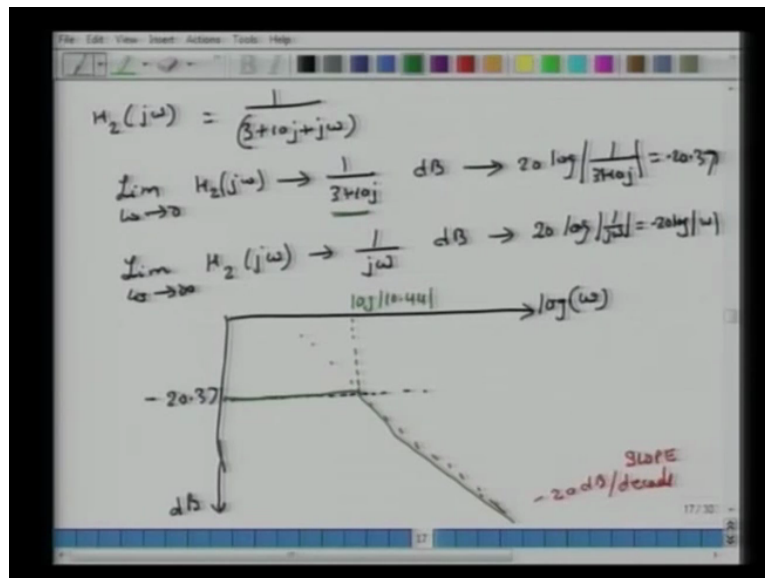
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So will construct the board plot for H1, so we know that H1 omega equals 2 plus j omega, so limit as omega goes to 0 and essentially developing an asymptotic response for H1 j omega is 2. So my decibels is 20 log 10 of 2 is 6 about 6.22 and then my limit as omega goes to infinite for H1 is basically omega. So my decibel is 20 log 10 of omega and my break point, so before I talk about breakpoint this is going to give me my low frequency asymptote, this will give me my high frequency asymptote and the breakpoint where these 2 asymptote will cut each other will be such that omega equals P that is omega equals 2 or log omega equals 0.301.

So now I plot might low frequency my bode lot for this function H1. So my low frequency asymptote I will do it in black, the Y intercept is 6.02 this is my log omega from the vertical scale I have decibels and my high frequency asymptote which has a slope of 20 decibels per decade is the red dotted line and this breakpoint corresponds to log of 2 and my overall curve for H1 will be this green line. So this is the plot for H1.

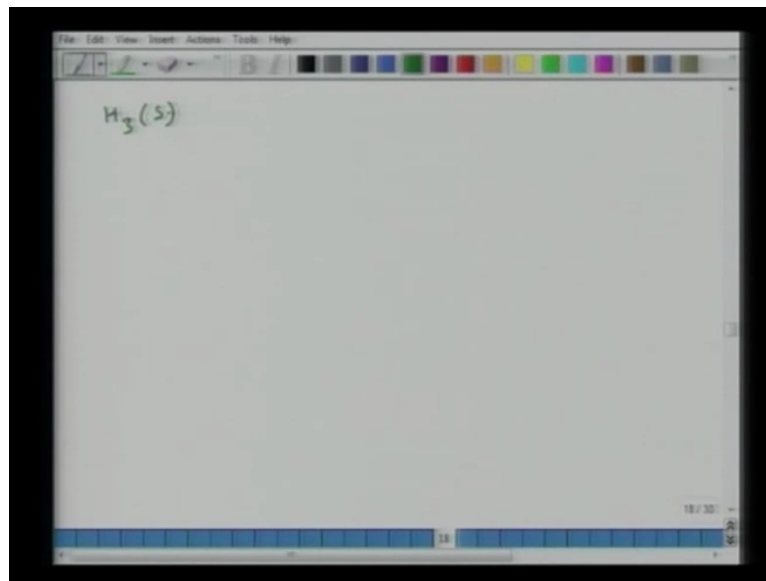
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Now I will draw a plot for H_2 , so H_2 we have seen in terms of $j\omega$ is this whole term 1 over $C + 10j + j\omega$. So it is 1 over $3 + 10j + j\omega$. So limit as ω goes to 0 for H_2 $j\omega$ is 1 over $3 + 10j$, so in decibels this term goes to $20 \log$ magnitude of 1 over $3 + 10j$ is basically 10.44. Basically I can what do is I can find $3^2 + 10^2$ that gives me 109 take it's square root find the log of it minus of that and I get, so this number is minus 20.37.

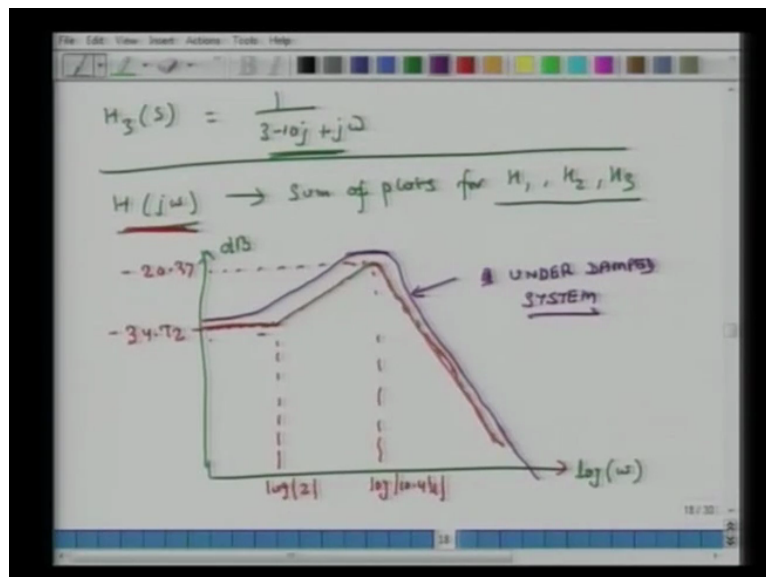
And then similarly limit as ω goes to in finite and this will give us a high frequency response is H_2 $j\omega$ that will go to 1 over $j\omega$, so my decibels will be basically $20 \log$ 1 over $j\omega$ is minus $20 \log$, so my bode plot by horizontal axis is $10 \log \omega$, vertical axis is in decibels, my low frequency line is going to be again a straight line cutting the Y axis that minus 20.37 and my high frequency asymptote is again straight but slanted line of slope minus 20 dB per decade. Slope is minus 20 dB per decade and my overall response is going to be this green line and the breakpoint is going to be such that this is \log of 10.44 where 10.44 is the magnitude as we had seen of $3 + 10j$.

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I can do the same exercise for H3S, so my H3 is essentially 1 over 3 minus 10j plus j omega, 1 over 3 minus 10j plus j omega and my bode plot will be identical. So this is my bode plot for H2j omega and when we do all the math we will find that the bode plot or magnitude for H3j omega will be same as this particular plot because the magnitude of the denominator does not change with the change in sign here.

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So I have constructed the plot for H2 and H3 and I have constructed the plot for H1 j omega. So my next step is to construct a final plot. So H of j omega which is essentially a sum of plots for H1, H2, H3 so if I have construct a bode magnitude plot for H it is essentially a sum

of bode magnitude plots for H_1 , H_2 and H_3 and that curve looks something like this. So initially I have a horizontal line then I have an upward sloping segment and then I have a downward sloping segment.

The break points my horizontal axis is $\log \omega$, my vertical axis is on the decibel scale the break points are at $\log 2$ the second breakpoint as you have expected is going to be $\log 10.44$ and the Y intercepts are going to be minus 34.72 and the other peak, this peak point is going to be minus 20.37 decibels. So this is my asymptotic response which corresponds to transfer function H which is a product of 3 individual transfer functions H_1 , H_2 and H_3 .

Now again this is an asymptotic response for the magnitude, the actual response is not the same as this dark red line, in this case where we actually plot the actual magnitude response that is shown here in the purple line. So asymptotically on this side the line converges to the Red Line, this is the asymptotic sponsor the actual response is something like this and at infinity the Red Line and the purple line converges and meet.

Another observation from this is that because the purple line which represents the actual system response is above the Red Line it happens to be an under damp system and we will explore this more in our later let us. So in this class what we have covered till so far is a little bit about what is and over damp, under damp and critical system then we went into the area of phase and magnitude plots.

You also understood what is the meaning of turns poles and zeros, poles being the roots of the numerator factors I am sorry zeros being the roots of the numerator factors and poles being the zero roots of denominators and then we constructed phase and magnitude plots and then we moved to bode plots and what we have covered today is how to construct a bode magnitude plot for a transfer function which is a multiple of several small simple transfer functions.

So that is what we have covered today, in the next class we will learn how to construct bode plots for the phase of the same transfer function, thank you.