Acoustics
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Module 6- Lumped Parameter Modelling of Transducers
Lecture 5
Radiation Impedance

So we have talked about radiation impedance for a sphere let’s look at very quickly radiation impedance for an open tube.

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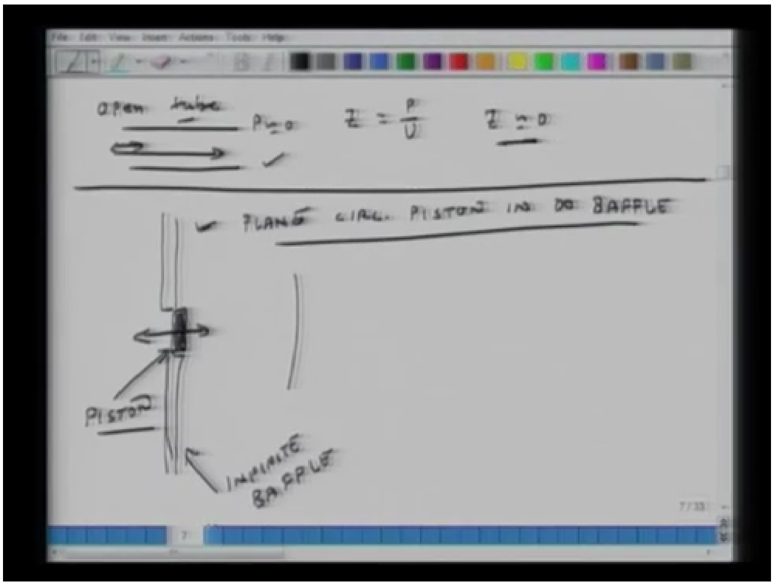

So ideally P is exactly it is equal to zero right, in reality you can say that it is approximately equal to zero because that changed to zero it happens over some finite distance. So this is an open tube, now my impedance I know is equal to P over U so if P is very is approximately equal to zero I can say that Z is approximately equal to zero also.

What does that physically mean? What is means is that when sound is travelling here then in case of sphere we saw that for a wide range of frequencies it is difficult to dissipate power into 3 A F from a spherical source because of that Xi factor and Xi becomes significantly larger in that case. You remember that discussion because of the power factor, (power fac) the value of power factor is significant at high frequencies and also at low frequencies in case of a pulsating sphere.

So at low frequencies you need very high energy to pump small amounts of energy into free air and at also high frequencies you need large amounts of input energy to pump, but in case of an open tube give that Z is approximately equal to zero it is relatively easier to pump energy at all frequencies into free air. So if I have a vibrating piston here it is relatively less difficult or easier to pump energy into free air, so that is it. So essentially when we construct systems which have to produce sound then you want to ensure them the radiating impedance of that system is as low as possible and also the overall power factor of that system.

The closer it is to you know Si being equal to zero the closer is it equal to that value the better it is the more efficient existence, if you want to have stronger invasion of sound which is the inverse problem, then you will like to construct systems which are in the other direction. So that you will like to increase the radiation impedance because that will essentially mean that less energy will get dumped into the environment and will be heard by individuals.

So they are several topologies through which sound gets propagated into the air, the very common topology is that you have an infinite baffle or a wall you can call and then here I have uhh diaphragm or a flat surface which is moving back and forth, especially in some of the older sound systems you would see very big boxes which will be having one speaker which is moving back and forth. Essentially it acts like as an infinite baffle because that stiffness of the box is very low.

So this is one topology another topology which we have talked about is (())(4:24) which is pulsating growing and contracting that also emits energy into air, this is another third topology right and a fourth topology could be a plane circular piston without any baffle so this is one , then how does it emit sound? So there are different approaches. So what will talk is a little bit more about this particular topology what we call plane circular piston in infinite baffle. So this is my infinite baffle and this is my plane circular piston.

The radius of this piston could be R knot, so again what we are trying to get an understanding is that as his piston is moving back and forth it has a certain volume velocity it generates some pressure, how does all that energy gets dumbed into here and that is quantified through this number called radiating impedance. So this problem you can develop some fine element models to some numerical experiments and after good amount of computational modelling people have come up with really standard values so for a plane circular piston in infinite baffle the radiation impedance looks something like this.

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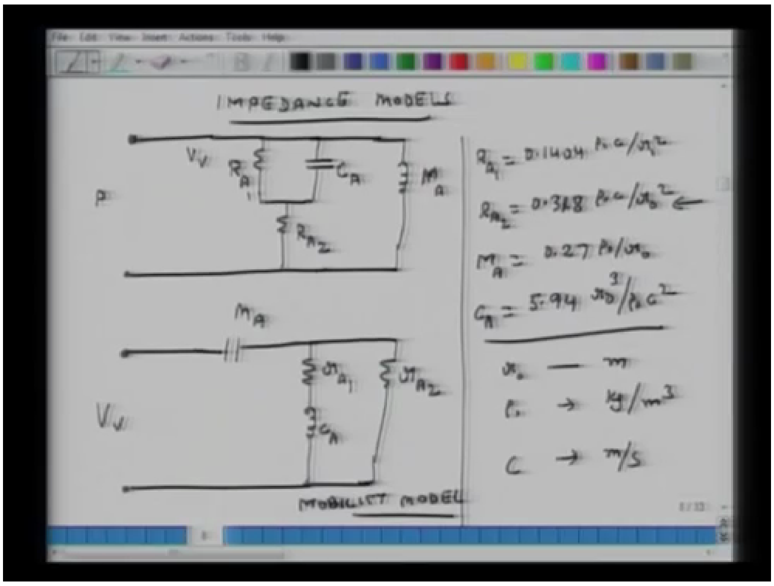

It is a bunch of (cap) one capacitor, two resistors in series and an inductor and this is for impedance model or mobility model the circuit looks. Now that you understand the idea of a dual it looks something like this, this is M A ok and the values of these elements have been computed and so (your) the value of R A 1 is 0.1404 times rho knot C over R knot square, R A 2 is my stylus so R A 1 is 0.141 rho C over R knot square R A 2 is this particular expression M A is 0.27 rho knot over R knot and C A another constant is 5.94 R knot Q over rho knot C square and because we are talking about specific constants 0.14 you know.

So these relations are good to the extent we are in SI unit, so R knot has to be in meters, rho knot C has to be rho knot is density so it is Kg over cubic meters, C which is velocity of air has to be meters per second. Otherwise these relations have to be adjusted and R A 1 lower case R A 1 and lower case R A 2 are basically just inverse steps of upper case values. So if you have a pulsating membrane fixed in a big on a big surface and air from back side is disconnected with air from the other (side) from your room then if you use this model then it will fairly accurately capture the radiation impedance of that membrane.

So what we will do is we will look at it and so this is mobility model. Now what we will do is we will see whatever the characteristics of this particular model at low frequencies and at high frequencies, what does this mean physically?

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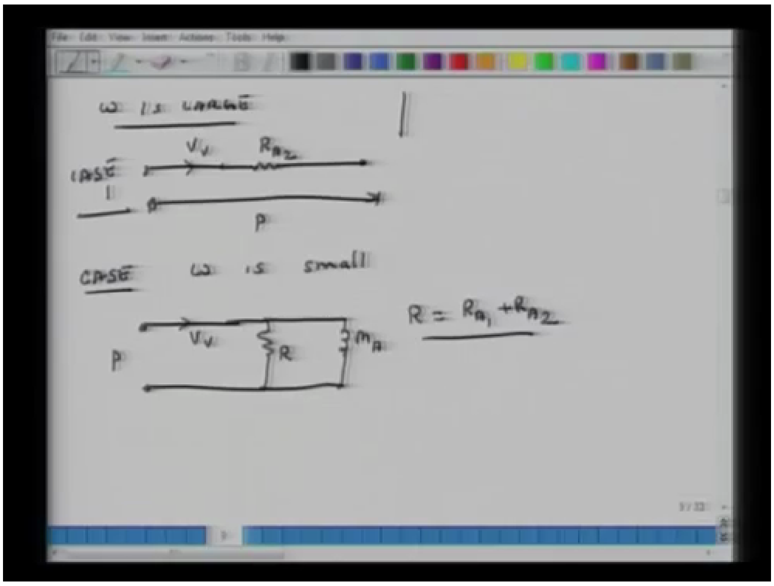

So if I am in very high frequency land when omega is large will let’s see what is going to happen. So when omega is large M A becomes extremely larger so very little current goes through here so I can drop this out. So the current has choice between going through R A 1 or through this capacitive element.

At very high frequencies the impedance offered by capacitor will become very low so current will prefer to go from C A and then it goes through R A 2 so my equivalent circuit at very high frequencies becomes so my this is my potential difference P and the current is volume velocity and this is R A 2 so what this shows is that when I have high frequencies, a piston mounted on an infinite baffle behaves like a purely resistive circuit. So the first thing is that the inefficiencies associated with that power factor they become minimize at high frequencies, I don’t have any inductive or capacitive element which alter the phase that is one thing.

The other thing is that in the near field again what this shows is that it the transmission of sound happens in kind of like a beam because this impedance is something very similar in nature qualitative nature to plane waves as they are moving through tubes open tubes. If you remember the impedance in a open tube is purely resistive thing and it does not attenuate in the strength as you move forward the strength of sound as it moves in an infinite long tube it does not decay with travelling.

What this shows is the it is the same thing happening here also. So what that means is that at high frequencies the behaviour of sound is like a beam it travels like a ray of light at high frequencies, this is what it means physically. Now offcourse when you go away from the source when you are in far field then it again starts to radiate but for a fairly god amount of distance sound moves like beam at high frequencies this is the physical.

So that is case one, case two happens what if omega is low, omega is small, so what happens?

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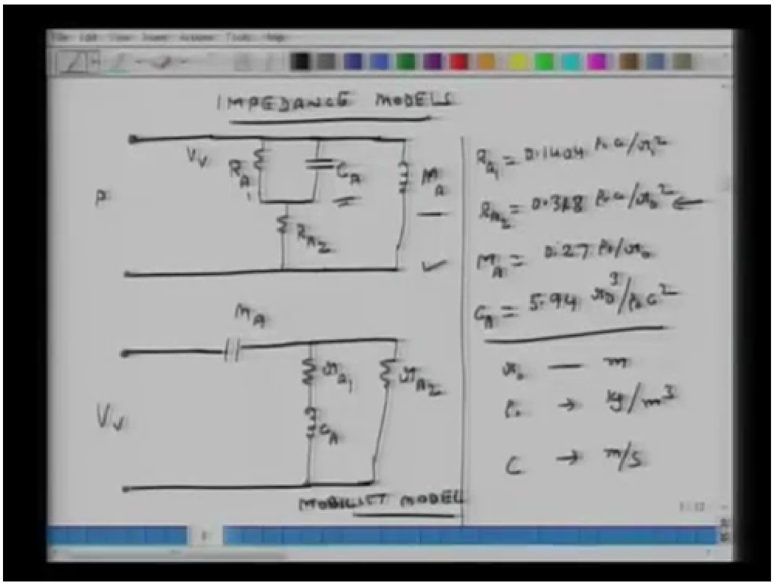

Let’s locate this circuit again when omega is small I can fairly easily drop this term out C A right, so I have what I have left what I am left with is M A and RA 1 and R A 2. At extremely low frequencies between R A 1 and R A 2 and this mass it will still prefer the inductive part. So in a little more general sense I can say that my circuit will be something like this. So for extremely low frequencies current will still lie to go through M A which means that for very low frequencies the dissipation of power through an infinite baffle will be very less.

Energy will go into the system at it will not get dissipated, so it is very difficult to dissipate for very low frequency sound into free air in infinite baffle. All what you are seeing is that different approaches of dissipating low frequency sound into medium is a not really is efficiently doing the same thing efficiently it is not easy. Similarly attenuating low frequency noise is not easy. So what we will do is we will yeah so you understand why I dropped out M A right at high frequencies.

Now at high frequencies capacitance the impedance offered by capacitance will be virtually zero because it is one over S times C A, it will be there but at high (frequen) impact of that, yes this respect to R A, what, I mean I can keep on increasing the frequency and I will hit a number where it will be with small enough compared to, so what that crossover point is will depend on the ratio of R A 2 and C A right. We have to figure that out. But at high frequency CA you can ignore it and replace it by a short.

Student: can you repeat the point that you mention about attenuation of low frequency times and the other equivalent time making the sound at high frequency.

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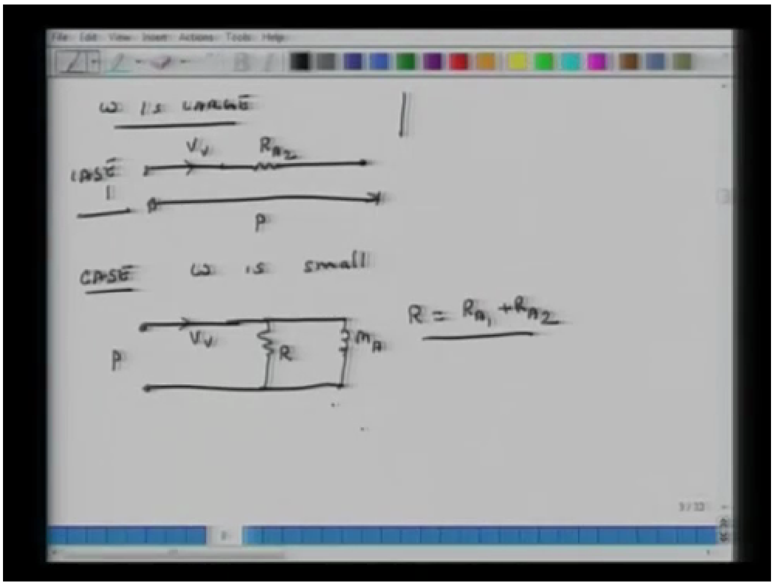

Yeah what we have seen is that when we try to dissipate sound through some of these sources into free air what we are seeing again here is that, that very low frequencies M A dominates all the current goes through M A which means that the dissipation of power is less. So all what happens is energy goes (into) back and forth into the system. We had seen earlier that the average power dissipated in general is essentially dependent on the R term if very a low energy is going through R the energy dissipated into the sound into the, into free air will be very less that is what I am saying.

So what we will also do is here, we will compare the efficiency of this system for low frequency with a spherical source, is it, is the spherical source easy to dissipate power into free air or is it easier to do it using an infinite baffle what will be, that is what we will do.

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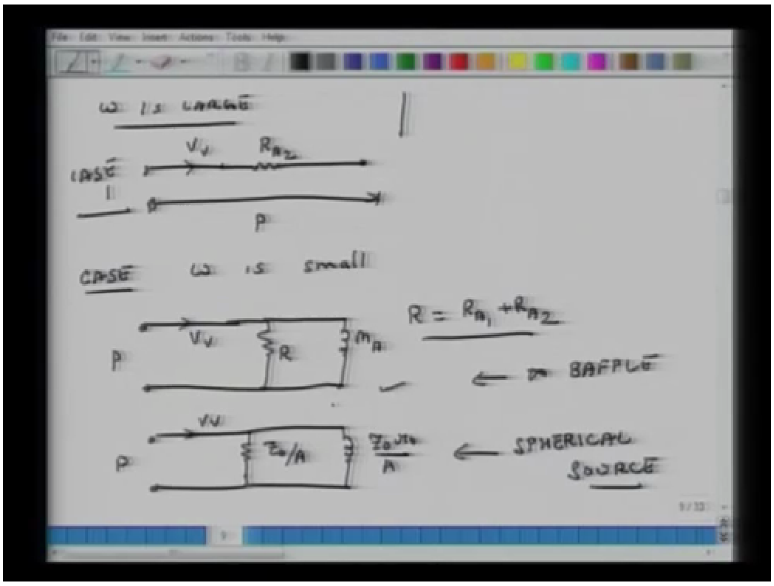

So this is for infinite baffle, now for a spherical source we had develop a relation something like this, the impedance was, this is the radiation impedance for spherical source ok.

So if the size of the source both the sources are same that is R rho is same, then look at let’s look at the numbers. So will construct a small matrix.

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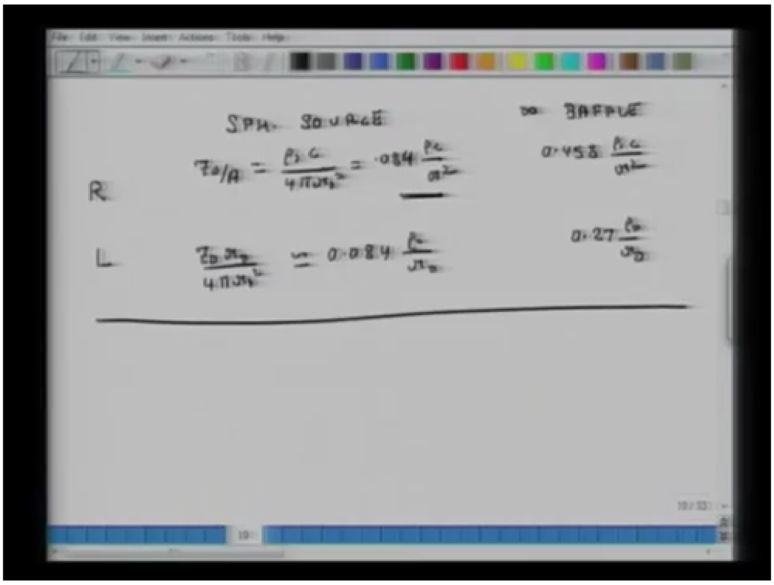

Spherical source and other column could be infinite baffle and what I will do is I will list R and L so again the circuits look very similar you have a resistor and inductor here in parallel same thing in case of a spherical source. All we will be doing is comparing the magnitudes of these elements and see how they look like.

So in case of spherical source the resistive value was Z knot over A that equals over 4 Pie R knot square is 0.84 rho C over R square that is my number 0.84. In case of infinite baffle it is 0.458 rho knot C over R square ok. The inductive value will be Z knot R knot over 4 Pie R knot square and if I do the math it comes approximately to 0.84 rho knot over R knot I just assume Pi to be 3, so I get some 0.084 maybe 0.083. In this case it comes to 0.27 rho knot over R knot ok.

So what do you see? When you look at these numbers, that the impedance offered by a spherical source is substantially less for the same size of the object substantially less than that offered by source of sound mounted on a baffle. So spherical source by itself is not a very efficient way to disseminate sound into hear at low frequencies and an infinite baffle is even poorer approach of disseminating sound at low frequencies this is the because you R is less your L is less or is way to source your baffle numbers are significantly higher, five times.

So we have talked about spherical sources we have talked about radiation impedance of a tube at the end of it, so now we will very quickly consider radiation impedance for a plane circular disc in free air if it moves as approach.

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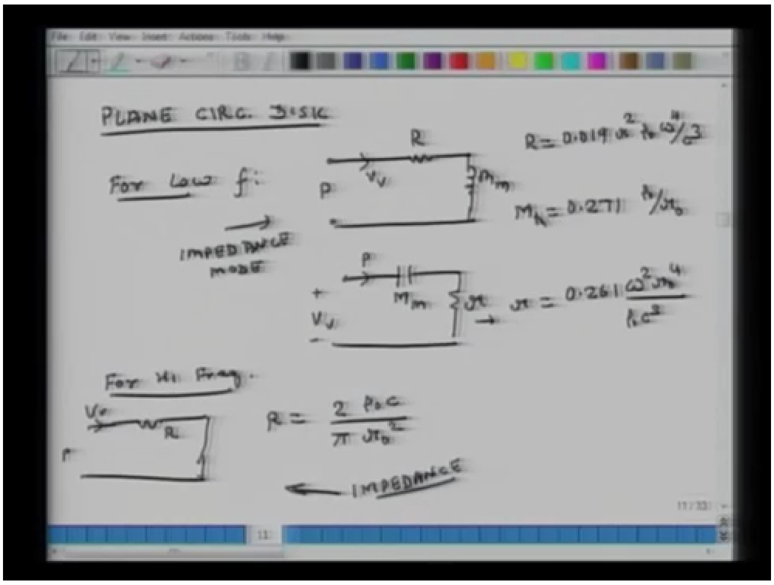

So for low frequencies I very quickly gave the impedance circuit looks something like this. So this is my impedance model so my cross variable is pressure here my through variable is pressure and a cross variable is volume velocity and R is 0.019, R square times density times omega over C cube and M1 oh I am sorry, MM equals 0.271 times rho knot over R knot.

And in this place R equals 0.261 omega square R knot 4 over rho knot C cube. You see that things start getting more and more complicated. So we will find out more exhaustibly some of different geometries and what each of these geometries have for their radiation impedance in a book like (())(24:40) which is the best like best of this course reference test and for high frequencies the circuit looks simply like this where R equals 2 rho knot C over Pi R knot square. And this is, this is valid for which model impedance or mobility? Which one? Impedance.

So I have to use for high frequencies something similar for a mobility what will I do in that case? R will get replaced by its inverse, ok. So what we will do is one example then see how radiation impedance improves R prediction of some of that (())(26:03) properties of systems.

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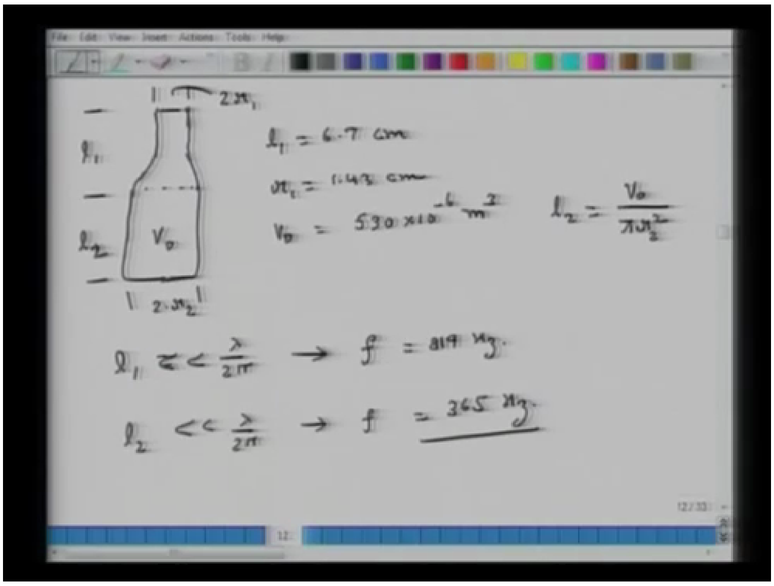

So let's consider a bottle this is actually a physical bottle which I measured so it has two parts so badly it is bottle, so there is a volume here this length is L1 let's called it L2 this length is L1 its mouth has a diameter of two times R1 right and the volume has a diameter of two times R2.

So what I would like to know is, what is the natural frequency of this bottle? This is their volume is like stiffness member and the open tube on top of it is like a mass right, so it is kind of a mass spring system and I am I like to see how good my model is in terms of predicting. So will put some numbers L1 equals 6.7 cm, R1 equals 1.43 centimetres volume of the lower portion is 530 times 10 to the power of minus 6 cubic meter.

I mean I can also find L2 is P knot over Pie R 2 square. So first thing I will check is that is my all these lengths significantly smaller than what number, lambda over 2 Pie right, so L1 has to be less than lambda over 2 Pie which gives me uhh frequency number corresponding to 819 hertz is everyone sure how to jump lambda to F right. So what this means is that I am significantly below in frequency from 819 hertz then if I approximate my cube which is L1 long as acoustic mass (())()28:53 that is what it mean.

Similarly L2 has to be less than lambda over 2 Pie which means frequency equals 365 hertz. So if using this mass spring approach if my resonance exceeds 365 hertz then my model is not valid basically that is what that means. Now will find what is the frequency of this system.

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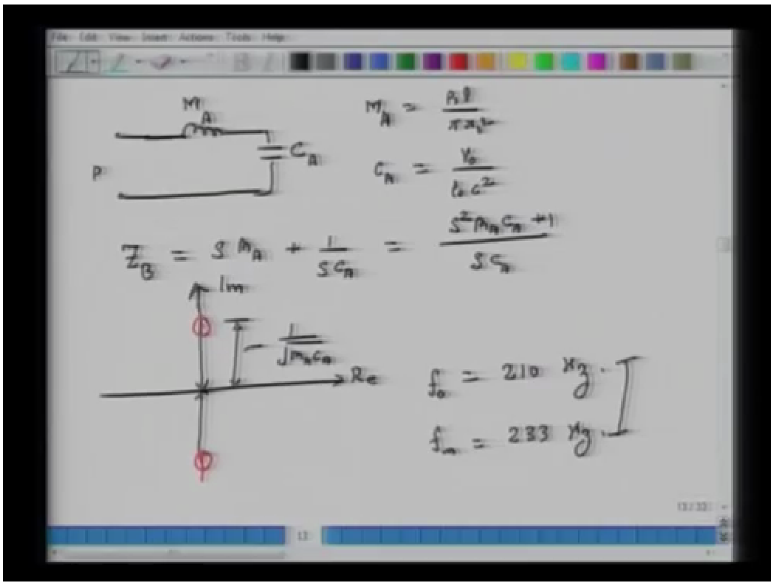

So if I construct an impedance model pressure that is my M M C, no I am sorry it is acoustics mass so M A we have seen from earlier lectures and discussions is rho knot L over Pie R knot square and my capacitance value is or acoustics stiffness is V knot over rho knot C square, so my Z bottle is S times M A plus 1 over S times C A.

So if I plot its poles and zeros I have a pole at zero and yeah a pole at origin, a pole at origin and I have two zeros symmetrically along the imaginary axis and this value is 1 over MA CA, when I do the math and I plug in all the numbers my first estimate on frequency resonance F knot comes to be 210 hertz and when I actually did the measurement so F M equals to 33 hertz they should have shown this. So this is clearly it is off and in terms of uhh estimation on frequencies they shouldn't be off by this much of an amount, frequency estimates come fairly close to reality.

So next we will see how we can improve our model to get a better and closer estimate. So what we will do is I went to this book (())(32:26) and I said ok there is this tube I have just modelled it as a pure mass element but there is some radiation this is a tube but it maybe seeing some radiation impedance here because it is dissipating sound into free air. So what is that value? So once I do that my refine circuit becomes something like this.

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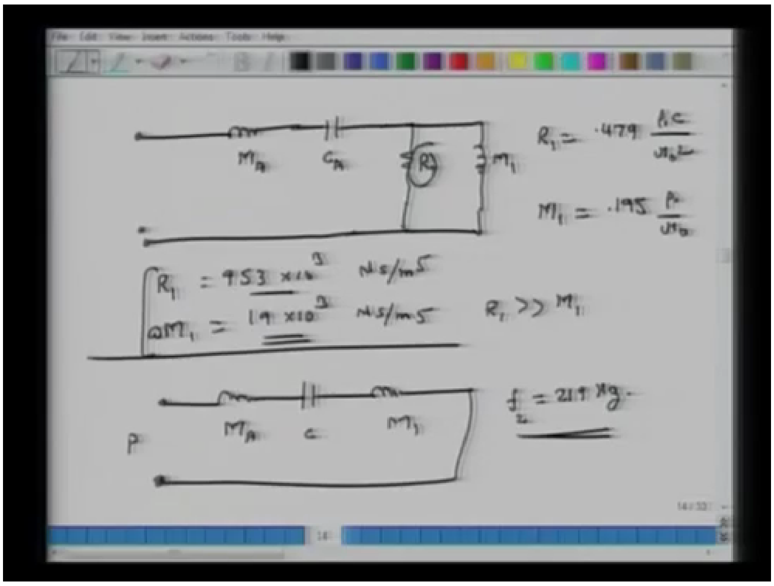

So I still I have my this is M A which we have already calculated this is C A and then I put so the extra elements based on more rigorous computational modelling are these terms, R1 and M1, where R1 is 0.479 rho C over R knot square and M1 is 0.195 rho knot over R knot. So putting the values of R and everything in this I get R1 equals 953 times ten to the power of 3 Newton second over meter 5 and M1 is 19 times 10 to the power of 3. So for low frequencies clearly R1 is very large compared to M1 ok. So I have to make correction here. This is omega M1, omega M1 for I just choose a small number 119 hertz where I said 190 is fairly close to 230.

So I wanted to get an estimate that in the neighbourhood of resonance which term is dominating R1 or omega M1, so what I am seeing is that in the neighbourhood of resonance this is very small this is very large these are in parallel so what that means is I can draw R1 out at around 192 hundred hertz. So then my approximation of this circuit becomes M A, M1, C I still have a pressure so I get these three elements, is everyone comfortable with why I dropped the R term. So I know M1 I know M A I know C so my second I try to get different estimates so my second frequency estimate natural frequency estimate came to be, what was it? 219 hertz.

So I was able to improve my estimate some 210 by another 10 hertz of so, so that is how close I got to that (())(36:07) after that I did not bother, so what this shows is that incorporation of radiation impedance in your actual mud will help you get a better picture of how the circuit I going to behave because the outside space does impede lower sound as sound iminates from vibrating member or a cube or whatever so some of these elements have to be captured in your circuit analysis. So that is all I wanted to talk about today.

Starting next lectures will start talking about more detail analysis of the whole speakers, microphones and see what they show us