Acoustics
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Module 6- Lumped Parameter Modelling of Transducers
Lecture 4
Radiation Impedance

So in the last class we had started talking about this idea called radiation impedance. So any acoustic element then it radiates sound into air free air or atmosphere, there is an impedance offered by the larger atmosphere onto that acoustic element and we had developed analytically a relation for a pulsating sphere, what its radiation impedance would be. We have not developed any analytical relationship or other acoustic elements that say you have a membrane moving back and forth which is mounted on a flat surface.

What is the impedance which that membrane sees as it radiated into free air? We have not developed relations for those types of elements. So what will talk today about will be will talk a little bit more about radiation impedance of a pulsating sphere after that we will talk a (())(1:16) and will talk about a concept called duals which is relevant in context of radiation impedance particularly and in general about the these electrical mechanical networks and then will go back to radiation impedance and see for other elements this other acoustic elements. So that is what we trying to cover today.

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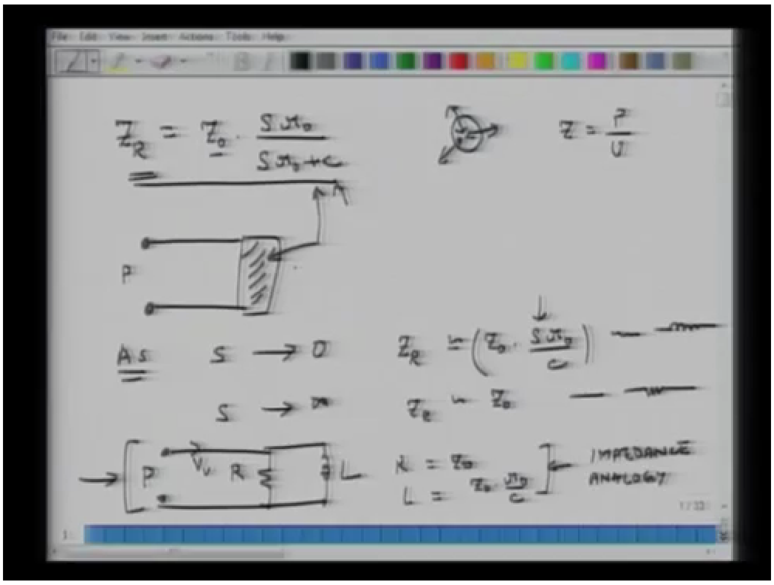

So in the last class we had developed a relation for radiation impedance of a sphere and the acoustic impedance look something like Z R and that was Z knot times S R knot over S R knot plus C this is for the pulsating sphere so expanding and contracting back and forth uniformly in all directions and you remember impedance in general is essentially pressure over velocity and context of radiation impedance is also talked about par square area rest of like that. So the question we had posed in the last class was that if I have to if I have this impedance in a circuit in my acoustic circuit and if I have to break it up into small resistors, capacitors mass elements inductors, how will I do that?

In other words if I have a pressure applied here and I have a black box which can constitute a group of different (acous) resistive inductive and capacitive elements, then how will they network look like so that the impedance offered by this network is equal to this, that is what we are trying to figure out because we know that then we can put those resistors inductors capacitors in the whole circuit and we can solve.

So what you see here is Z R equals Z knot, Z knot is a real constant number rho knot times C. So Z R equals Z knot times S R 0 over S R knot plus C and as so let's see how this Z R behaves as S goes to zero, S remember it is J times omega, so as S goes to zero Z R approximates to Z knot times S R knot over C right. So at low frequencies the radiation impedance of a pulsating sphere looks like a that offered by a of an inductor right because there is an S term and everything else is constant.

So this mimics like an inductor, inductor in the context that Z equals P over U right so I am in the impedance anility land, also as S goes to infinity that is as my frequencies are very large Z R approximates to Z knot which is independent of natural frequency. So at higher frequencies the radiation impedance looks more like a resistance. We know two things that if I have to break this into electrical elements it is composed of an inductor an a resistor. So the only question is whether it is in a parallel or in a series right.

So if it was in a series the form would look something different, so clearly it is a in parallel so I construct a circuit and I put a value R here put the value L here and this is my V V , so if I use the impedance analogy the value of R would be Z knot and value of L would be Z knot times R knot over C. What is R knot? R knot is the radius of the pulsating sphere it is pulsating sphere we are talking about remember. Here R knot is radius of the pulsating sphere.

So this is the electrical circuit which mimics the behaviour of the radiating impedance when a pulsating sphere is pushing sound into larger atmosphere and just you can double check to cross check if you put these and you try to compute the overall Z you will find that, that number comes same as what you have developed here. So this is for impedance analogy. If I move to a mobility analogy when we have talked that my inductors become capacitors R becomes 1 over R and so on and so forth. So my mobility analogy will be something like this.

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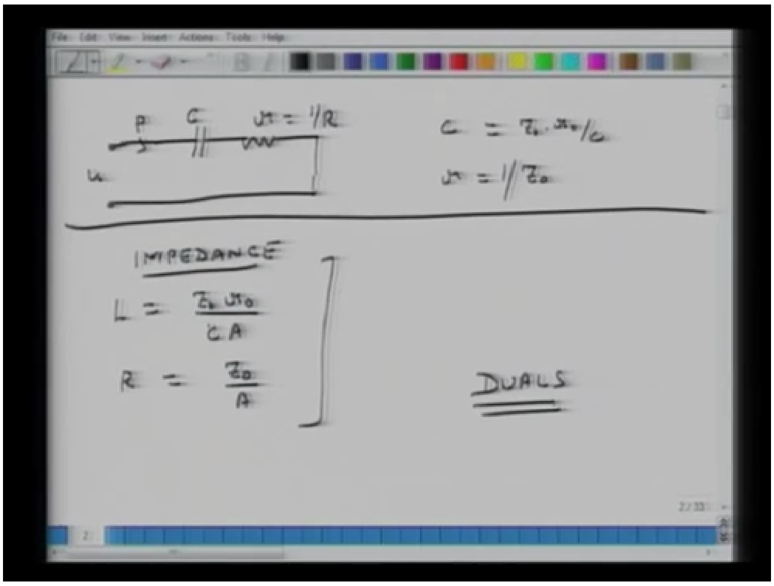

So this is R equals 1 over R this is capacitance, my across variable is V V through variable is special. So you (remember) you see that so here C the value of C is same as value of R which we had set Z knot times R knot over C and the value of R is 1 over R I think I made a small correction is due so here it is actually velocity so I have to adjust if I have to go to here. So if I use volume velocity and the pressure then impedance analogy my L which is the inductive part is basically Z knot R knot over.

What is the transformation ratio? A, and my resistance is Z knot over A. We see two things here that when we move from mobility to impedance and impedance to mobility capacitors we have seen this earlier also capacitors become inductor, inductors become capacitors, R becomes 1 over R so on and so forth. But one thing which we did not very explicitly talk about is that if there are some elements which are in parallel then when we transform it to the other analogy rig become come in series.

So that is what we are going to talk about and that is the context in which we introduced this term called dual. So we will talk about duals.

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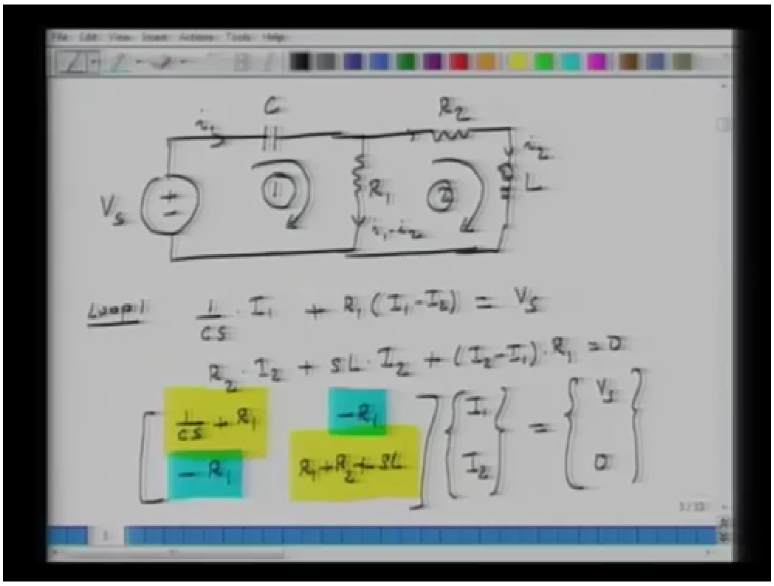

So let’s consider simple circuit I have a voltage source V S which is connected to a bunch of elements electrical elements. So I have a resistance here R1 another resistance R2 inductor L capacitor C the current going here is I1 the current going in this loop is I2, so the current in this loop will be I1 minus (I2) in a vectorial sense. So I have one loop here this is one loop and this is another loop so what we do here is because I have an external voltage source I will try to solve this problem using (())(11:39) voltage conservation laws.

So I will apply that law for loop 1 and I will apply that law for loop 2. So I will get two different two equations I have two variables I1 and I2 so I can solve. So for loop 1 what do I see, so first I move to frequency space by mapping this into their impedances so C becomes gets mapped as 1 over C S, L gets mapped as L times S right, so once I moved there then for loop 1 my equation is going to be 1 over C times S times I1 ok.

So that and then this part I have R1 times I1 minus I2 and this entire thing equals V S and then for loop 2 I have R2 times I2 plus S L times I2 plus so here I take I2 minus I1 times R1 equals zero, ok and now I re-arrange this, and just construct a simple matrix so my unknown variables go into a vector unknown variables are I1 and I2 on the other side of the equation I have my known variable V S and zero and here I have 1 over C S plus R1 minus R1, R1 and here you have R1 plus R2 plus S times L ok.

Look at this matrix equation carefully, so you have this part what is this physically it is 1 over C S plus R1 so it is essentially the overall impedance of first loop right, this same thing here also R1 plus R2 plus S L is the overall impedance of the second loop ok and these two terms this is like a finite element equation not strictly like a finite element equation this are coupling coefficients. So minus R1 in the first equation and minus R1 in the second equation is the common impedance which is common to loop 1 and loop 2.

So remember this because then when we go to the another example you will see very good similarity. So you have constructed a set of equations and then you can now invert this matrix and solve for I1 and I2.

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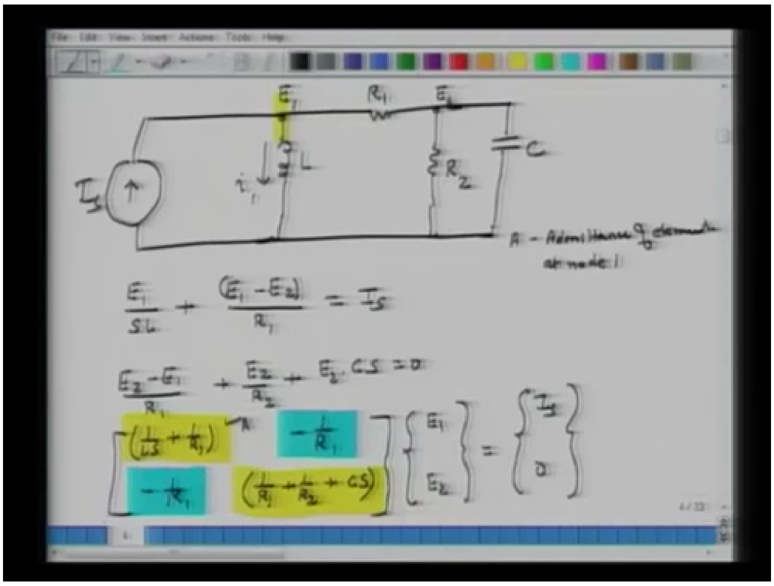

So now we consider another, here instead of a voltage source I have a current source IS ok and the circuit looks something like this, so the value of this inductance is L this is R1 this is R2 these are resistors then I have a capacitor. In this I have a current source so instead of using (()))(16:17) voltage conservation potential conservation law, I will use the other one current conservation.

So I have two nodes here E1 so let's say the potential at node 1 is E1 and this is another node E2, the potential at this node is E1 potential at second node is E2. So using conservation of charge I develop relations again I will have I have two unknowns E1 and E2 if I know E1 and E2 then I can compute everything so I have to figure out what is E1 and E2 I know what is IS, L, R1, R2 and C so the current going so the current at going coming into node at this first node will be same as current leaving this node that is essentially what I am trying to do.

So my first equation becomes E1 over S L which is essentially I1 ok plus E1 minus E2 which is the potential difference across the resistive element R1 so the current going through that will be that whole thing divided by R1 that equals what will come on the right side, IS, similarly the second equation becomes something like this, E2 minus E1 over R1 plus E2 over R2 plus E2 times C S equals 0, right. So now I re-arrange some of these terms I construct an out them in a mere matrix form.

So my unknowns will go in a vector E1 and E2 on the right side I put my known quantities IS and zero and my first term in the matrix is 1 over L S plus 1 over R1 then I have a 1 over R2 negative again I get negative 1 over R2 then I get here bar no bar R1 plus 1 over R2 plus C times S. So again we have a matrix of different equations two equations. So what is the first term in this matrix? It is 1 over L S plus 1 over R1, physically what that means is it is the total admittance at this note right, admittance is like 1 over yeah, 1 over R1.

So this term let's call this term A, A is the admittance of elements at node 1, admittance is your 1 over impedance similarly this term which corresponds to node 2 is admittance of all the electrical elements which are coming in at node 2 and this term minus 1 over R1 is the admittance of the term which is common which connects physically connects node 1 and node 2, that is one thing. Other thing is now let's look at the values of these terms, so what you are seeing here is just let's compare so you have 1 over C S plus R1 right.

That becomes 1 over L S plus 1 over R1. Similarly you have R1 plus R2 plus S times L, here what you are getting is 1 over R1 plus 1 over R2 plus C times S. So what is what essentially you are seeing is that A dal in circuits look similar but they are not identical the matrix equation look similar, they are not identical but if you compare these two circuits the first circuit where we had a voltage source and the other one we had a (electric) current source.

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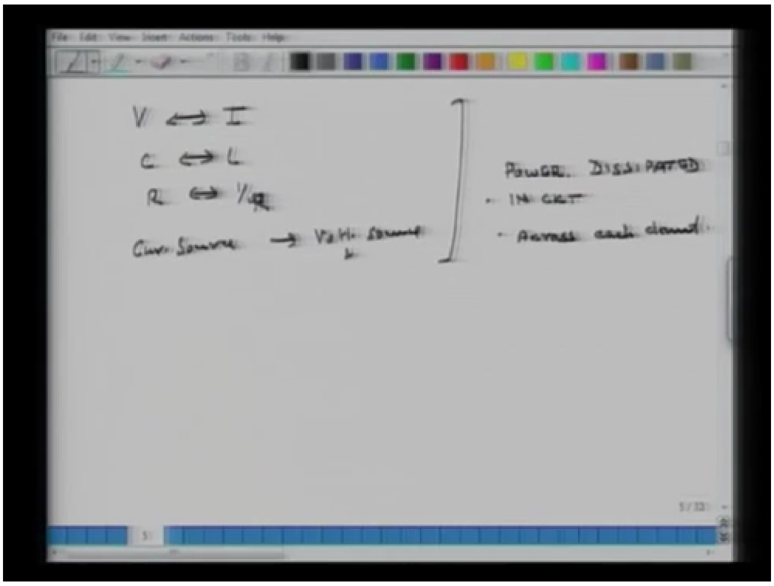

Then V transforms to I and back and forth, C transforms to L and back and forth, R transforms to 1 over R and back and forth, series elements which are in series they go in parallel and vice versa, elements which are in series they become go in parallel, element which are in parallel the given (())(22:53) like R1 and R2 and then you see here so R1 and both are parallel to R2. What else? Yeah a current source transforms to voltage source and back and forth, and one thing which remains constant in this is power dissipated in the circuit.

So power dissipated in circuit does not change also power dissipated in every network and across each element that also does not change. So circuits like the two which we saw they are called one is called circuit A is called dual of circuit and vice versa. Every electrical circuit which is composed of these three types of elements your R L and C you can have its another thing to notice that I have a matrix of equations only I have just the equations constants If I am just looking at this equation and I do not know the circuit then it is fairly straight forward for me to draw an actual circuit of the whole system.

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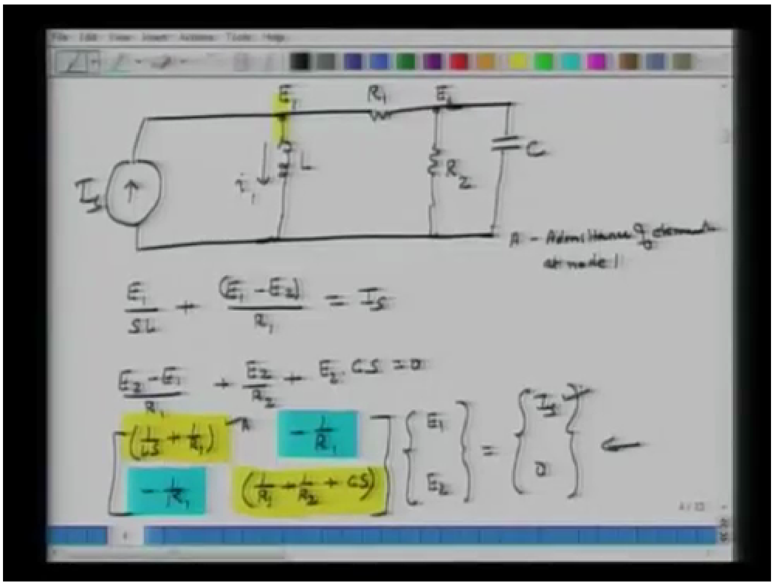

How will I do that? When I look at the circuit I will say oh there is a current source here so I will put a current source then I will say it is a 2 by 2 matrix so there are two potential nodes in the systems so I will put two dots. Then I will say what is the impedance of the two nodes which are connected to each other? So I will say that the admittance when I look at these blue boxes is 1 over R1, so I connect them by a resistor of value R1 then I say at node 1 I have two elements an inductor and a resistor and they are in parallel.

So using that kind of a thought process I can construct a circuit if I just know the equations of the overall system. If I know the system I can develop equation if I know the equations I can construct the overall network. So will very quickly do one example for illustration.

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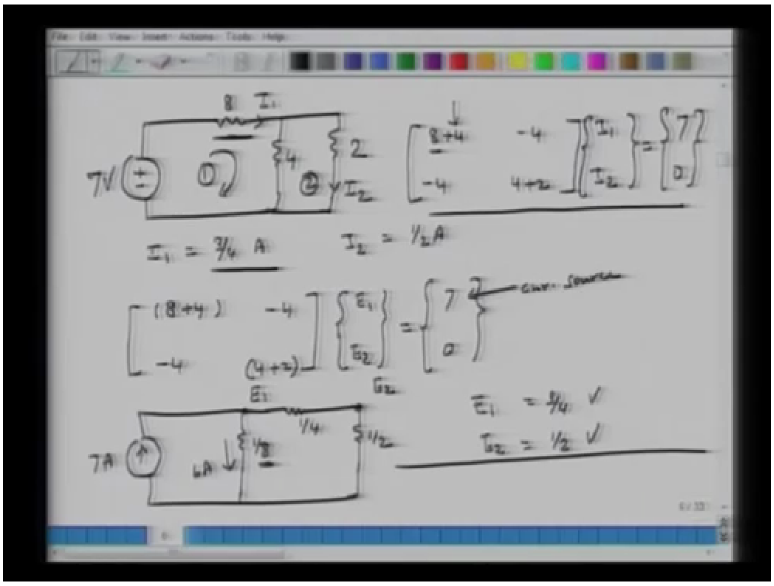

So let’s say I gives me 7 volts and this is a purely resistive circuit so this is 8 ohms the value of resistance here is 4 value of this resistance is 2. So the equations so I have two loops loop1 and this is loop 2 let’s say the current going in here is I1 and the current going here is I2.

So my equations become 8 plus 4 minus 4 minus 4 4 plus 2 times I1 I2 equals my external voltage is 7 and 0, so everyone can see the equation, solving this is get I1 equals three fourth of amperes I2 is half an amp, given this can we construct a dual? So my first step will be that I write the equation of dual. So what will go in this vector? V1 and V2 or E1 and E2 ok, what will go in the on the right side it will be seven but this is a current source, zero, what will be the first element of this matrix? It is remember it is admittance so it is one the resistance value will be 1 over 8 but the admittance will be 8 right, so it will still be 8 plus 4.

You see this, here these elements are impedance values admittance is 1 over that so this is a resistance of 8 ohms which will transform to a resistance of 1 over 8 ohms in the dual but the admittance of that 1 over 8 ohms will be (8 ohms) 8, right, so it will still remain 8. So 8 plus 4 minus 4 minus 4 4 plus 2, so now we construct a ranjan you got it? So now we construct that circuit so my first thing highest that there is a current source 7 amp current source.

How many nodes does this (sevel) element have? 2, so first one is at a potential E1 second guy is at a potential E2. What is the element which is connecting E1 and E2? 1 over 4 ohm resistor, the other thing is 4 and 2 so 1 over 2 and then on the first node I have 8 and 4 two elements each of amp impedance 1 over 8 and 1 over 4. So that is my equivalent circuit. So the current here in (across) going through 8 ohm resistance is three fourth time, what is the voltage of E1 voltage at E1? What is the potential at E1? No E1 will be 3 by 4.

E2 is half volts, what is the potential drop across this resistor? 8 ohm, 8 times 3 by 4 right, how much is that? 6, so voltage drop across (6) 8 ohm resistance is 6 volts. So the question is, how much is current going through one 8th this resistance? 6 amps, and son and so on, so let’s very quickly find the power dissipated. So in the first circuit power dissipated will be 8 times 3 over 4, 8 square times 3 over 4 plus 2 square no half square times yeah

Student: sir from the voltage diagram sir it should be 1 over R times, sir it is A plus volt in the voltage when there is a voltage source and sir just now we have seen that R1 becomes 1 by R1.

Yes so we have this, so a little jumbled up here. So what I will do is I will erase this now so that we can find the power dissipated on this page itself rather than going back and forth.

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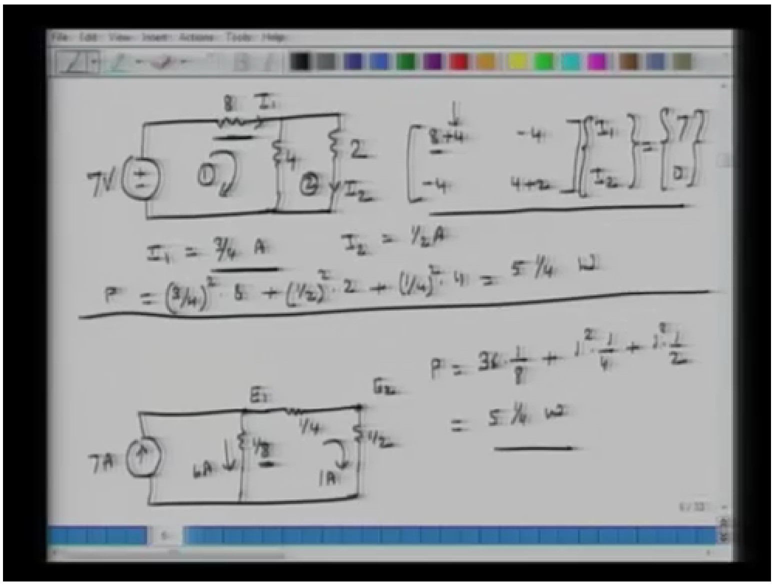

So power dissipated here in the first circuit is 3 over 4 square times 8 plus half square times 2 and what is the current going through the 4 ohm resistance? What the value of the current going through 4 ohm? One fourth, so everyone sees that the current going through 4 ohm resistance is (())(32:37) times 4, so if I do all this correctly my total heat is 5 or 5.25 ohm ok.

And in the second case so here my current is 6 amps and here my current is 1 amp, so my total power in this case is 36 times 1 over 8 to the first resistor plus I square times 1 over 4 plus 1 square times 1 over 2 and that again comes to 5, so these duals are good to known, why? Because in your real complex circuit analysis you maybe you may find that oh it is easier to move from mobility to impedance or impedance to mobility and so on and so forth. So when you are transforming one model into the other model these some of these laws have to be preserved.

So it is just not that capacitors become inductors R becomes 1 over R but also series goes into parallel and so on and (so forth). So it was for that reason we transform this particular network when we move to mobility model so the R became 1 over R, L became C but because they were in parallel they move became they went into series, so that is the context I wanted to (produ) introduce that a very sound (engineering) you go from parallel to series and so on and so forth.