Acoustics
Professor Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
Module 6- Lumped Parameter Modelling of Transducers
Lecture 3
Transformers, Radiation impedance, and Helmholtz resonator

Good afternoon. So in the last class we had talked about several acoustics elements we had developed relationships which were analogous or resistors, inductors and capacitors something similar to these electrical elements we had developed acoustics elements acoustic mass, acoustic inductor or compliance and then an acoustic resistance where we left in the last class was that we still have not developed some elements which bridge the gap between electrical and mechanical, so in mechanical we have elements which behave very similar to electrical elements and then (mechan) and then in acoustic rim we have elements which behave very similar to mechanical elements.

But how do you convert from electrical jump from electrical side to mechanical side and mechanical side to acoustic side that piece is missing. Those specific elements we call coupling elements so will talk about those today, coupling elements.

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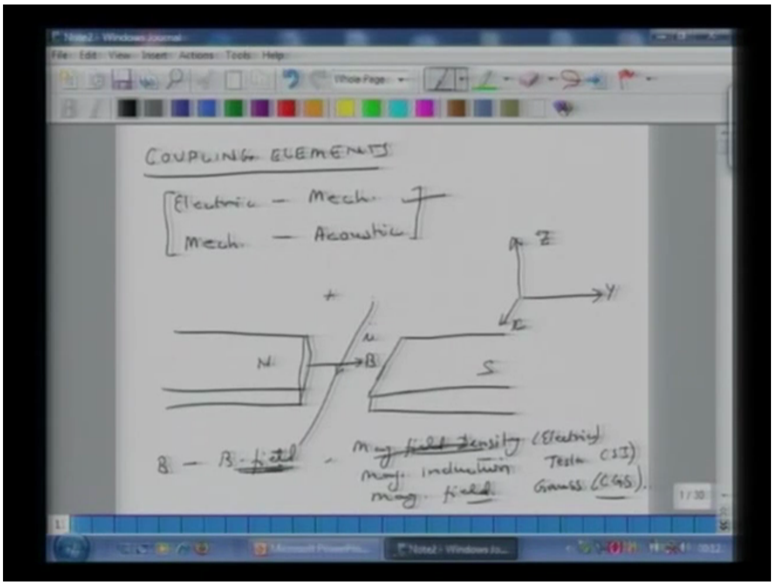

And these could exists between electrical and mechanical rims, electrical to mechanical or they can exists from mechanical to acoustic, similarly they can exists between mechanical and Piezo and so on and so forth.

So in today's lecture we will cover basically these two coupling elements and these are essentially transformers say so they transform one form of energy into another form of energy or transducers. So will start with electrical to mechanical coupling. So lets consider the magnet and this is its north pole and then I have the south pole this is my south pole and there is a gap between the two poles and what you have is a conductor passing through it and lets assume that the current it is having it is which is flowing through this is I and there is a potential difference across the conductor.

And because there is a north pole and a south pole you have magnet there is this magnetic field which we label as B and this is my axis system X Y Z. So this B has several names so some people call it B field others especially electrical engineers they call it magnetic field density electrical engineers call it, I think a B field is typically used by physics people. There are still others who call it magnetic induction, magnetic induction and then there is one more name in prevalence it is called magnetic field.

So notice it is magnetic field density but some people also call it magnetic and the units for this are Tesla in SI system and Gaus in CGA system. So just some nomenclature.

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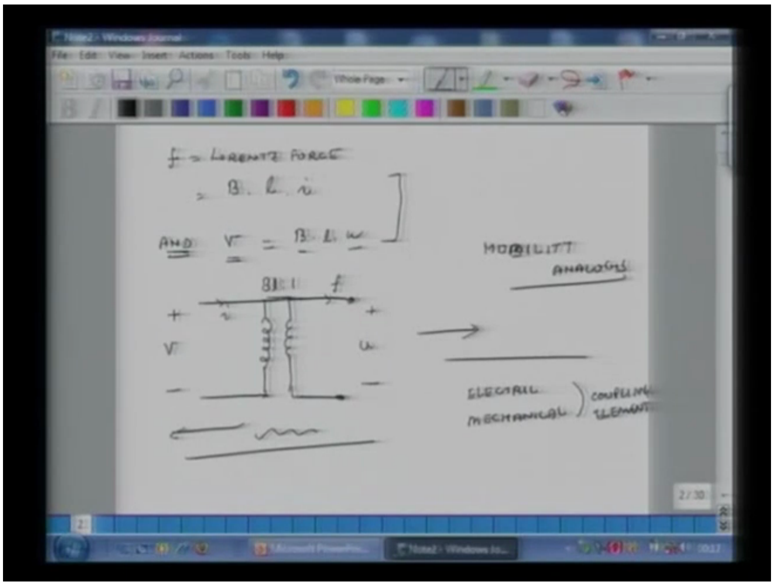

So we know from laws of electrical engineering that force, so when you have a conductor through which current is flowing and that current is cutting across a B field then this conductor experiences a force from laws of electromagnetics we know that, and that force which is called Lorentz force equals B times L where L is the length of the conductor times current.

And the potential difference across this conductor would be voltage equals V times L times velocity. So this is all standard electromagnetic and your V is your back EMF and U is velocity, B is magnetic field density, L is the length. So from these two relations if I have an electrical circuit and if it is coupled to a mechanical circuit through these what you can call it a motor which converts electrical energy into motion then I can make transformer something like this, so I have voltage we applied across this thing there is a current going through this and this voltage gets converted into velocity, and my current is getting converted into force.

And the turn ratio voltage equals B L times U, so my turn ratio is B L, so I can on this side put all sorts of inductors and capacitors and resistors and I can have as complex as an electrical circuit I can have and here I can have in this on this side whatever number of mechanical elements which are there in real system and if I have to compute force if I introduced this element this particular coupling element then I am able to convert voltage into force and I into I am sorry voltage into velocity and current into force, for mobility analog.

If I have impedance analogy then I have the same equivalence but my turn ratio will become 1 is to B L, you can do the math also in that situation my voltage will be the through variable and my current will be the across variable and similarly on the mechanical side force will be the across variable and velocity will be the through variable. So this establishes and equivalence between electrical and mechanical rims. So this is like a coupling, A element.

So current goes in U also excited with the voltage it gets converted into motion through this transformation and then that motion later gets converted into sound. So now will develop a coupling element which joins acoustic and mechanical networks.

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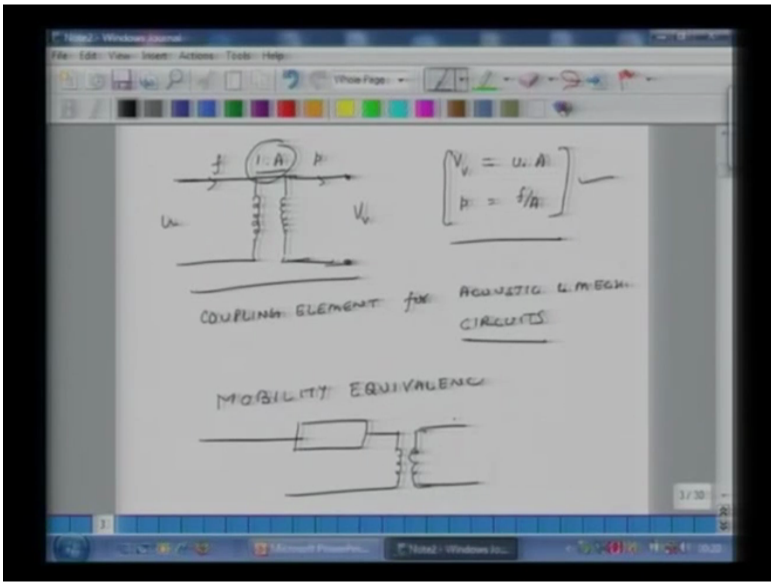

So again I have a transformer and in (mo) if I am using the mobility system of equivalence then my across variable is velocity or actually yeah and my through variable is force on the mechanical side and on the acoustic side my across variable is volume velocity and my through variable is pressure.

So from here you can see that V V we know that V V volume velocity is essentially velocity times area, area of what? Whatever membrane or you know surface which is pulsating is the area of that surface and it is actually normal to the direction of motion because volume velocity is a dot product of area and velocity, also we know that pressure equals force over area so again you have this so my turn ratio will be related to area and so your turn ratio is 1 is to A in this case. So this is my coupling element for acoustic and mechanical circuits and this particular ratio is valid for mobility equivalence.

If my equivalence is of impedance type then again these relations are still holding but my this ratio will become A is to 1 rather than 1 is to A. So this is the last piece in the whole puzzle. So now we know how to construct an electrical circuit so I can have an electrical impedance and then I can have a transformer if that current is getting yes

Student: the termination will be same, just we are changing the through variable and across variable then ratio will be then also come on to be

Turn ratio becomes inverted so when you do the actual math you will see that if it becomes 1 instead of 1 is to A

Student: sir why F? F is always (())(11:40) times A, now whether F comes pressure through variable then also it is.

In electrical transformers V times I.

Student: so ratio of across variable is turn ratio and ratio through variable is inversely to

Yes, because V times A is voltage times I on one side of the transformer is same as voltages time on the other side. So if you switch those then your turn ratio becomes inverted

Student: sir like here if you switch F and U sir F will come as across variable and U will become the through variable and the same side also V will become through variable and will become the cross variable.

So when you have a step up transformer where voltage is going up right your N1 and N2 have N2 has to be more than N1 right, and then vice versa. So if you use will explain it a little later maybe after the class we can go over it in a little bit more detail, but believe me this ratio gets inverted and will show it very clearly mathematically I have wanted to cover couple of other things but we will capture it explain it more detail.

So what we have talked about is the coupling elements and that essentially in a very general sense complete the picture. So you now have equivalences between electrical, mechanical and acoustic parts of the entire circuit, you can use similar philosophy if you have a Piezo-electric transducer and develop similar equivalences and so you can develop similar coupling elements between Piezo and mechanical or Piezo and electrical rims.

So the idea of showing all this exercise at this detail level is one to help you understand how these inter-disciplinary circuits are constructed and be if you run into some transduction processes then you have atleast the thought process which you can us to develop similar equivalent variables in other rim of engine.

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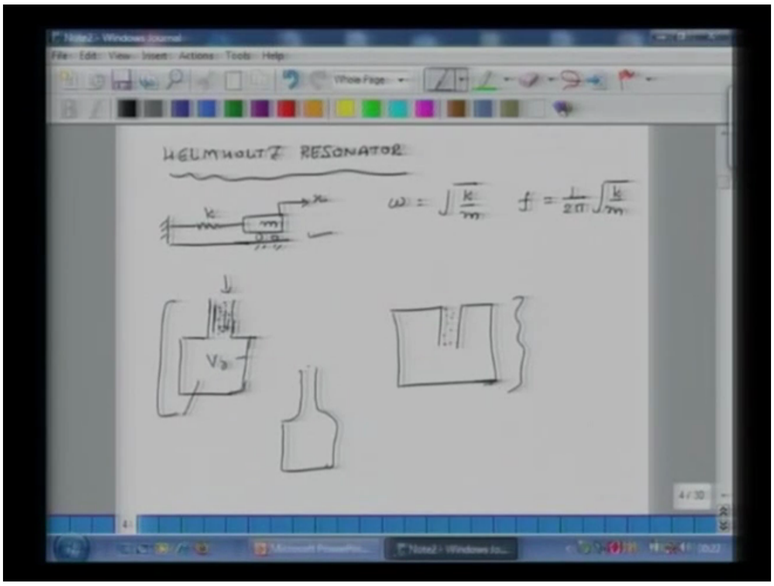

So now will talk about something called a Helmholtz resonator right, so in mechanical word you have a spring and there is a mass this is spring and if it moves on a frictionless surface and if I perturbed it by small distance X then it oscillates at a natural frequency omega equals Q over M or the frequency will be 1 over 2 Pie times Q over M. you can have similar acoustic mass acoustic compliance systems in one degree of freedom in area of acoustics that is called a Helmholtz resonator that particular device.

Physically what it looks like is in a very general sense you will have a volume and you will have a short tube and in mobility analogy we saw not mobility this volume we know it acts as a spring physically and we know that the air in this tube it acts as a rigid mass or an acoustic mass we talked about it at plane. So again you have a mass and you have a spring here and this has the tendency to resonate by itself. So if you have you know one of these soft drink bottles and they have a little bit like this and you blow over on top of it will hear some sound coming out that is essentially the natural resonance frequency of this bottle.

Maybe in one of the exercise will ask you to calculate what is that natural frequency and essentially what is happening is so again very similar to a mass and a spring system if I push the mass this spring gets compressed and lets say that is my steady state condition then I release the force on it so then the mass moves out but then it because of inertia it goes above and beyond the neutral position. So then the spring goes into tension so an at a point when mass stops moving this spring pulls it back then the mass comes back and again it exceeds the neutral position it compresses the spring and it goes back and forth.

Similarly if you press a bottle or a device something like this if you press the air in it and after time you release that pressure all of a sudden, the air the mass in this acoustic mass in this part will move back and forth and in theory it will move back and forth for infinite amount of time at a natural frequency which we will calculate because you have a spring here and you have mass here. So some other shapes of Helmholtz resonator would be like this, so this is again you have an acoustic mass element and here you have a spring.

So if I want to understand this and solve its solve for its natural frequency I can if I use mobility analogy.

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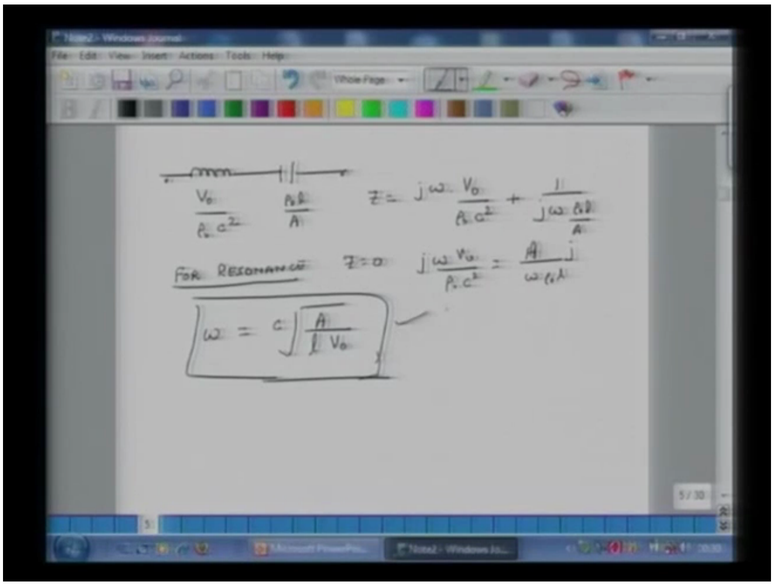

Then, this is my inductance which acts like a spring and the value of this is rho knot C square and this is my capacitance which is behaves similar to a mass and the value of this as we developed in the last classes rho knot L over A, where A is the cross section of this tube cross sectional area of this tube.

This is my L and cross section is (rho) A, so total impedance between these two points is J omega times V knot over rho knot C square plus 1 over J omega rho knot L over A. Now for resonance Z equals 0, in this case for resonance condition so J omega V knot over C square equals 1 over I can bring my A up J omega rho knot L minus and then I can eliminate the negative sign by bringing J up in the numerator and now if I solve for omega essentially what I get is omega equals C times cross sectional area of the cube divided by length of the cube times V knot ok.

I can use this Helmholtz resonator in a lot of ways as it application will talk couple of application about it. The first application where it is used a lot is in sound systems. So we will draw two boxes.

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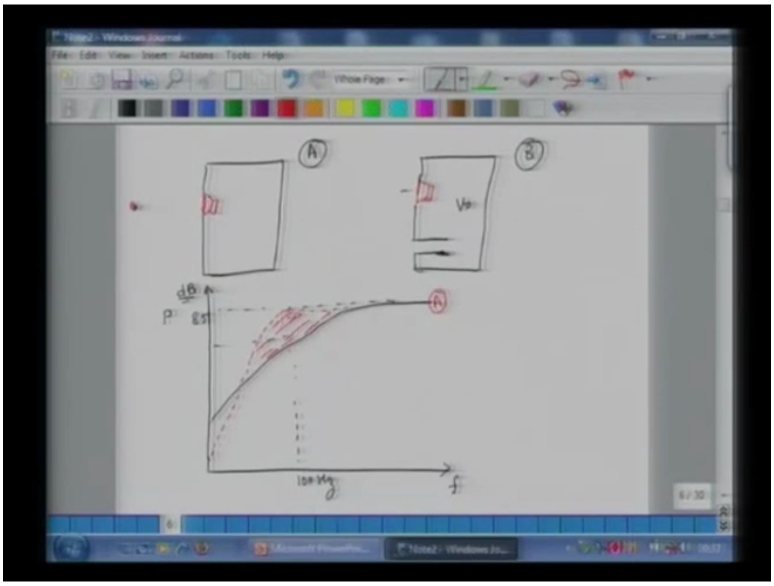

So I can have a box A (excuse me) and it has speaker mounted here and it is a sealed box. A lot of traditional boxes you have a speaker and just sealed, you may have seen it your homes also, and you try to measure the pressure at this location over a frequency range and then another system you can have is again a sealed box, with Helmholtz resonator and we will see what it does.

So I still have exactly the same transducer but here I have a port also or a tube ok when I plot the frequency response for system A and this is my system B typically when you do the actual measurement and in later parts of this course you will actually develop relations which will show so if I am plotting pressure at this location S P L and I am plotting it against let say frequency, it will be something like this and just to put some numbers lets say this is 85 and this is all decibels and this could be 100 hertz and note that this is a decibel scale ok.

So a drop of 6 dB is a big change 6 dB means like twice the energy content is two times less in the whole system. So if this drop is 6 dB essentially what it says is that 100 hertz at maybe a higher frequency it is producing 85 dB but at 100 hertz it is producing 6 dB or even less than 6 dB from that 85 dB threshold which is not a good thing which means that low frequency, base frequency which people like to hear to more above more. They are not being produced at an equal level.

So to solve this problem what people do is they introduce Helmholtz resonator and it is a very inexpensive solution they just introduce the tube. So you have a volume here and you have a tube a port. So you have a spring mass system and what it does is that because of the (reso) and they tune the length of the port and volume in such a way that the resonance is below some number lets say if I want to improve performance at 100 hertz or 80 hertz then I tune this in such a way that my resonance frequency is at around 80 hertz.

If I want to improve the performance at 80 hertz, so when I do that my this transducer is producing S P L something similar to that but my resonator is producing a high tend output at 80 hertz wherever I am tuning it to be, so for on 80 hertz wherever whatever is the tuning frequency and in the neighborhood of that I get a little bit of extra push so I get improve my base performance. So once you have so this is A and once I put a Helmholtz resonator (I am sorry) so this was something like this once I have a resonator it could be something like this.

So I have increased my frequency content the magnitude frequency in this range and on the negative side you have a steeper slope the on the low frequency end so you start killing lower frequencies then at certain threshold much more steeply, but you are able to get some extra base in this area. So lot of sound product companies they use these pipes so you have a sealed volume they put the pipe sometimes they put two pipes to solve to add extra content for specific frequency, so this is first step.

Another application is in musical instrument, so will show see two pictures.

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This is one of the original Helmholtz resonators constructed my Mr. Helmholtz in around 1850's ok. So you have volume and you have a little bit of a tube here. Now look at this was 1850's.

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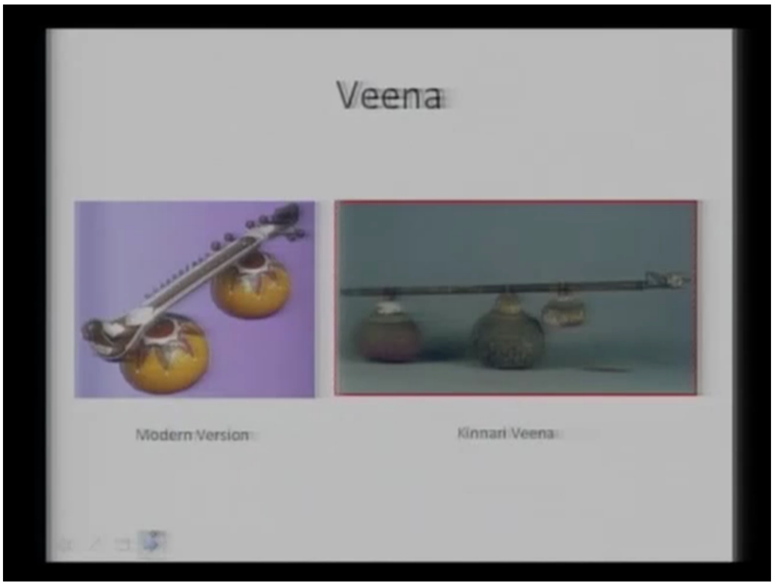

This is the picture of Veena so you have two pictures of Veena and these round big spheres are essentially some hallow either pumpkins or lauki you know that gourd so in traditionally they would hallow it out and they will have a small tube going up and this they have been using it over atleast last 2000 years because people in ancient drawings also people have found pictures like this.

So people have been using Helmholtz resonators prior to birth of Mr. Helmholtz very long time. Other thing is this is another Veena called Kinnari Veena and it has three resonators, so again these resonators increase the performance at specific frequencies right. So in this Kinnari Veena it just happens that this is a picture of a Veena which is sitting in a museum in France it is an instrument which was made something like an 18, 19 centuries and some French sound engineers and scientists what they found was that they found the tuning frequency of this little pumpkin and also the larger pumpkin and they found that the ratio is 2.

So they are off by one of it, so even in earlier times people had a very clear understanding of octave this is the proof it sitting there in a museum you can go and measure it if you want. People had a clear understanding of octave and also has figured out how to tune so that the natural resonance of one horizonator is twice or half of the other resonator. So this is so again in a lot of musical instruments in guitar there is a hallow box below the string.

Now you will not see an explicit tube in a guitar but we will talk about it a later that hallow box also kind of acts as a resonator. So that is another application of the same resonator. Third application is in air boxes so in engines earlier air used to go to a carburetor and there is used to mixed with petrol fuel and then it used to be, nowadays people are using less of carburetors and more of air boxes and there also the aim is that if they can figure out somehow as a way to smartly suck air more efficiently into the system then it requires less energy.

So again they use concepts rooted in the idea of a resonator and they suck air. Another example is an will so some math about (())(31:25). Architectural acoustics, so you have a big hall and someone is playing music or speaking and if the walls in the hall are parallel then you can have standing waves because they are parallel and the wavelength of those standing waves will depend on the distance between the two walls. So we saw earlier a two fixed structures they could be standing width.

So people try to eliminate those standing waves because those standing waves are not easy to hear a by using resonators. So in the first three examples in case of a loudspeaker in musical instruments and air suction we used resonator to enhance the frequency performance for certain pitches for certain frequency. In case of architectural acoustics they use the same tool to kill specific frequency and will do a little bit of math around it and see how it done their complesion.

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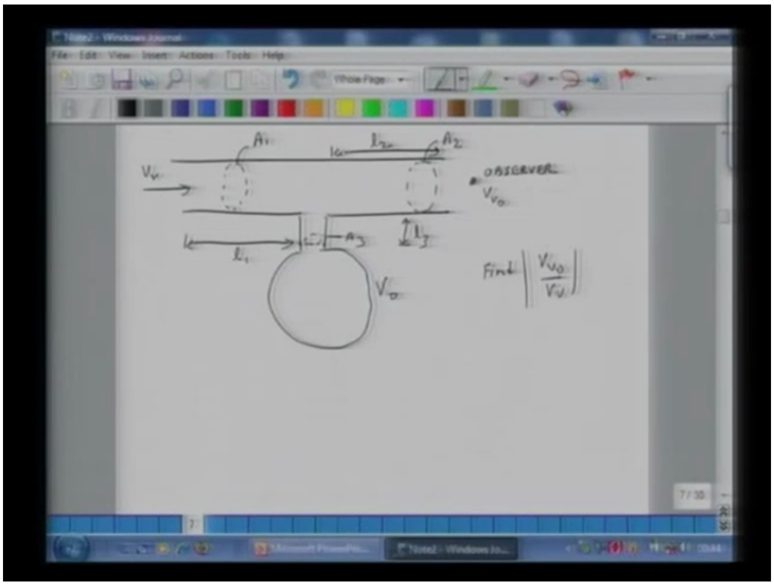

So let’s say that we have a long tube and the cross section area here is A1 here it is A2 and here the cross section area A3, also this length is L1 this length is L2 and this length L3 and then I have a big volume here not necessarily big a volume here V knot and I am sending signals through this side lets say the volume velocity is V V and then there is a guy sitting here observer and he is measuring V,V O, so this combination of volume and this small tube on top of this circle is your resonator right.

So the question is find V V O over V V, so we draw an acoustic circuit for this, so in this case because V V happens to be a through variable will use impedance analogy.

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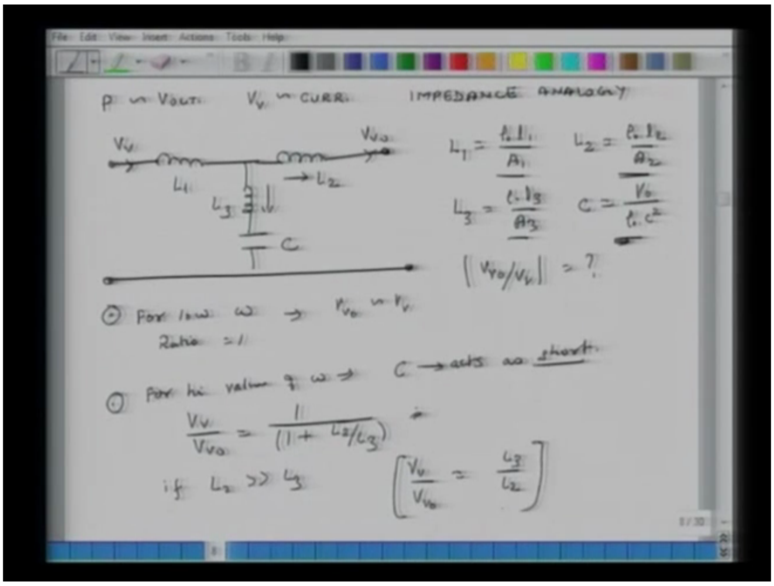

So my pressure is analogous to voltage and V V is analogous to current. This is impedance approach impedance analogy so now I construct a circuit, so essentially I have two tubes short tubes here another short tube here a volume here. My volume in impedance analogy will be modeled as a capacitor and my tubes will be modeled as inductors.

So let’s say this inductance is L1 L2 L3 and this capacitance is C. So my L1 equals rho knot times length of the first element over A1, L2 is rho knot L2 over A2, L3 is rho knot L3 over A3 and capacitance is volume over rho knot C square ok. So through variable as we talk is V V and here I am measuring V V knot. So the question is what is V V knot over V V? Magnitude of that ok. So at a very subjective level when the frequencies are very low how will V V gets split along this two paths? V V will split at this junction some of it will go through L2 some of it will go through L3.

But when frequencies are very low what do you think will be the magnitude of current going through this circuit? L3 C it will be virtually zero. Because impedance offered by C will be very high. So for very low omega V V knot is approximately equal to V V so my ratio is, is one. For high values of omega they will be some (curve) ok so at very high they will be virtually zero current but what we are interested is what will be the ratio of V V knot over V V? So whatever little current is going how will it get split in the two directions? That is the question.

C will be acting as a short circuit so it will split based on the ratio of L2 and L3 so C kind of acts as short so whatever V V is going in it will split like this. The influence of capacitance will be virtually negligible I am just ignoring it and if L2 is fairly large L2 is very large compare to L3 remember L2 and L3 are inductances which are these numbers they are not lengthwise, so if L2 is very large (compare) then V V knot over, V V over V V knot is essentially L3 over L2, and finally what will happen at resonance point? So you have this circuit.

What happens when the there is a resonance in this part of the circuit, impedance goes to zero so what happens to V V knot? V V knot goes to zero.

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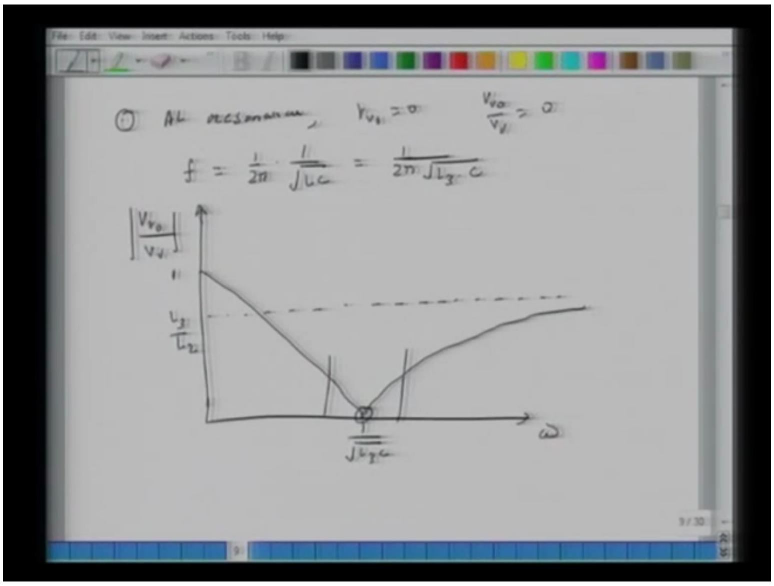

So at resonance V V knot equals zero so V V knot over V V is zero and the value of this resonance frequency will be 1 over 2 Pie times 1 over L C that is 1 over 2 Pie square root of L3 times C. So I at a very course level I plot this transfer function magnitude, I am having here frequency and here I am having or lets plot it omega. So here I have omega and here I have V V over V V.

So at low frequencies this value is one at resonance which is 1 over L3 C this value comes to zero and for the condition this condition if L2 is very large then large compared to L3 this should have been V V knot over V V this is an asymptotic value will be L3 over L2. So my transfer function will look something like this, so in all others examples of the resonator I was using the resonator to boost the performance at specific frequency. Here I am using the same device to kill specific frequencies in this case something in this band I am able to attenuate the magnitude.

So resonance can do all sorts of things.

Student: sir in the actual hall the it is the wall are designed that the architectural this thing is designed to kill this frequency so

This is used to be used especially in earlier times because people didn't have clear understanding of how standing waves are interacting, what is the damping coefficients of different materials at different frequencies nowadays there is a more higher emphasis on the shape of the room also the damping material being used, how the speakers are placed in the auditorium, where should people who are generating music or sound they should sit, all that.

The reliance is not that much on resonators but the point what I am trying to make is you can use the same resonator in totally opposing you know totally opposing or different purposes. You can boost frequency response or you can kill frequency (response).

Student: just I am trying to say that the system in the hall sir, it should also be tuned to produce this frequency so that it can be killed with this.

No uhh the tuning of the room will depend as I said, suppose there are two parallel walls and they are separated by some meter lengths so it is tuned to have standing wave which is related to that dimension, so it is in that sense it is tune so specific frequency. So what you want is that those standing waves they should not be heard loudly by the people sitting in the auditorium or in that listening space. So that is the consequence of the geometry of the room, the natural mode. So will do one very quick one more example, this is not a Helmholtz resonator but just for practice purposes.

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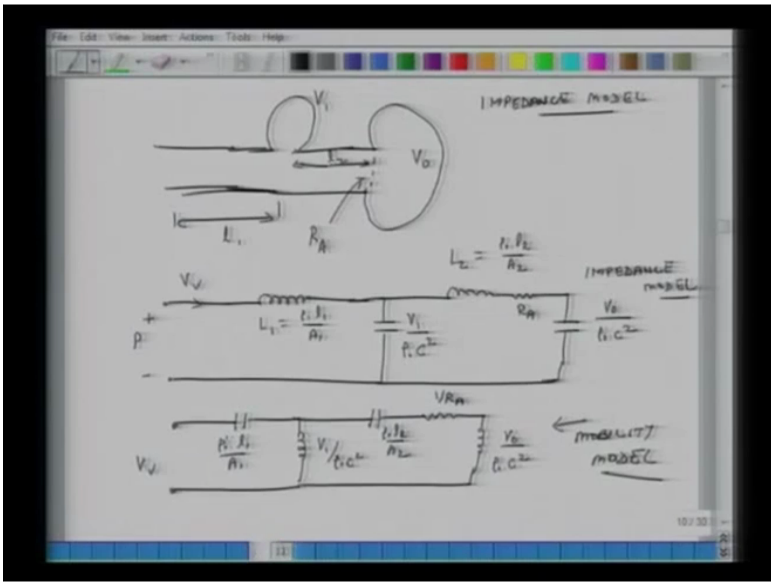

So I have a tube connected to a volume pardon my very poor drawing and this is connected to another sack and here you have an acoustic resistance R A in mobility the resistance will be 1 over R A, this is all this is right now impedance model. Here the volume is V knot this is V1 and this I call L1 and this length L2.

So will very quickly construct an acoustic circuit using impedance analogy so my across variable is pressure through variable is volume velocity it goes through the first tube which acts like an inductor such that L1 equals rho knot L1 over A1 and then I have an acoustic capacitor here or a string element which is V1 over rho knot C square then I have another acoustic inductance of value L2 such that, that being equal to rho knot L2 over A2 and then I have finally one more, so I should have put resistance here and if I do a mobility model, so this is my impedance model.

So in my mobility model I have volume velocity inductors get replaced by capacitors and vice versa the values do not change and the resistor is the same but its value becomes inverted. So and this is rho knot L1 over A1 and this is rho knot L2 over A2. This is mobility model. So in a lot of these circuits just wanted to recap couple of very simple but important points, you will see combinations of inductors and capacitors.

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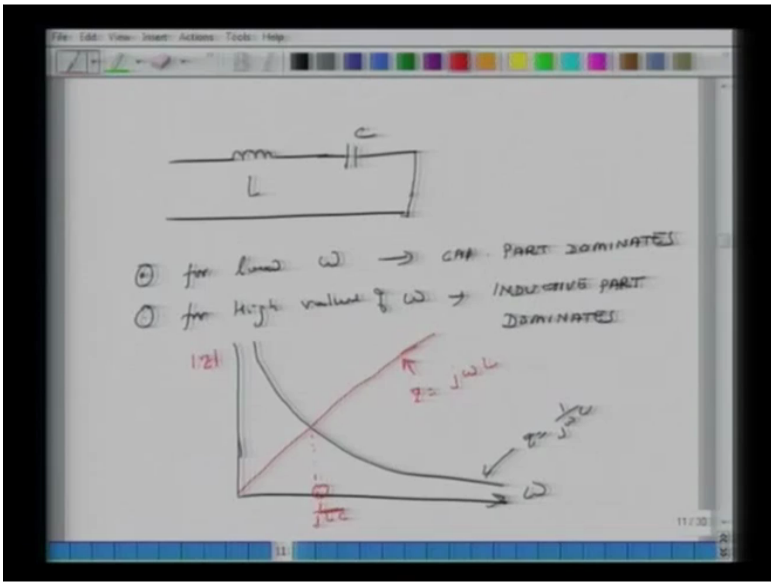

So you can have a capacitor of value L I am sorry an inductor of value L and a capacitor of value C, and when you see you should start thinking how is this thing going to behave? So again for low values of omega it will behave purely as a open circuit and the capacitive part will dominate for low frequencies the capacitive part will dominate for extremely low values it will become an open circuit will go towards open circuit but capacitive part will dominate and for high values of omega inductive part dominates.

So if I plot the impedance and this is my omega the capacitive component will be infinite at zero and it will fall like this. So here Z equals 1 over J omega C and the inductive component will grow frequently in here Z equals J omega L I am just plotting the (mag) this is magnitude. So again I am just plotting the magnitude, so this is the cross over point and the value at which these two equal is 1 over square root of L C and the last point I would like to make here is all this approach where we are having lumped parameters doesn't matter whether it is electrical rim, mechanical rim or acoustic rim they work to the extent that your frequencies are less than compare to lambda over 2 Pie.

The lengths of the elements are less than lambda over 2 Pie the length of the element is less than lambda over 2. So if in an electrical rim you are starting to violate some of this condition you will not be able to accurately model the electrical circuit using lumped elements because then they become continuous systems as we saw earlier in several lectures back in transmission line as we were developing the transmission line capacitors stop behaving as lumped capacitors when at very when the wave lengths violet this fundamental condition.

The same thing is true for mechanical and same thing is true for acoustic rim also. As we are solving some of these equations and constructing these complex circuit we have to be cautious that in every single rim are we sticking to some of those guidelines or not. So that is what I wanted talk today in context of Helmholtz resonator and a little bit about overall electrical circuit. Now I wanted to introduce this notion of acoustic radiation impedance, radiation impedance what is it? An electrical circuit lets say an amplifier and it is receiving electrical energy.

You know how to convert that electrical energy and model that conversion process to mechanical energy we saw this whole transformation process. Then this mechanical energy gets converted into sound which is generated by a loudspeaker and we know how that conversion is happening in and we have developed elements lumped elements for short tubes, close tubes, short volumes and acoustic resistor.

Once the sound leaves that box or sound system whatever and it reaches my ear there is some impedance in that travel process also that is called radiation. So far we have not captured that if I have to accurately model the entire circuit from source to the listening point of the observation point that is one last piece of puzzle which is left and that is radiation and the radiation impedance will change depending on, if you have a sphere which is pulsating back and forth it will have one type of radiation impedance.

If I have a sound source mounted on infinite wall the radiation impedance will be different. If it is a sound source just moving back and forth maybe for some frequencies low frequencies the person at the other end may not hear anything because as a piston is moving forward it is creating positive pressure on the front surface and on the back surface it is creating suction, so once those two waves add up and they reach your ear you will hear very little sound.

So all that is about radiation impedance and that depends on the distance between the listener and the source, how the sound source is mounted, what is the boundary condition at the source point, the geometry of the source and several other variables. What we will start today is radiation of a pulsating sphere because we have developed some relations in past, how sounds propagates as it eminates from a simple spherical small source, we have developed some relations and then in subsequent lectures will couple will do some more example of different types of radiation. So will start from the definition of instantaneous power.

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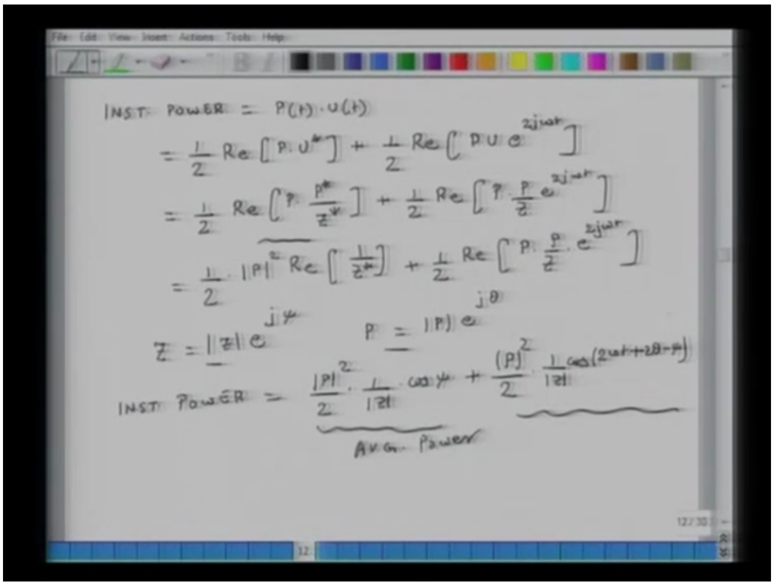

So instantaneous power we had talked about it earlier is instantaneous voltage times instantaneous current and in acoustic rim will be instantaneous pressure times instantaneous velocity, so it is P T times U T and then in some of the earlier lectures we have developed this relations such that this is equal to P times U star plus real component of T E 2 J omega T and then the U star is essentially complex conjugate of pressure over complex conjugate of Z that is Z star plus half real P and then again U is P over Z P Q J omega T.

This one I simplify so I get half times magnitude of pressure star real of 1 over Z star plus half real E times E over Z times E 2 J omega T. In one of the earlier lectures we had said that Z which is the impedance could be a magnitude times exponent J Si, J Si was related to this power factor notion also pressure will have a magnitude which is a real positive number times exponent of J theta, so if I put these two relations in my above equations I get instantaneous power equal magnitude of pressure whole square times and real of 1 over Z star would be times 1 over Z times cosine Si plus times 1 over magnitude of Z times cosine 2 omega T plus 2 theta minus Si E square over 2 yeah.

Also so now this is what we had defined as average power and this is the fluctuating portion and there is a Si component I mean this influence of Si also. So when I plot this and this is to a certain extent a recap of some of the earlier work.

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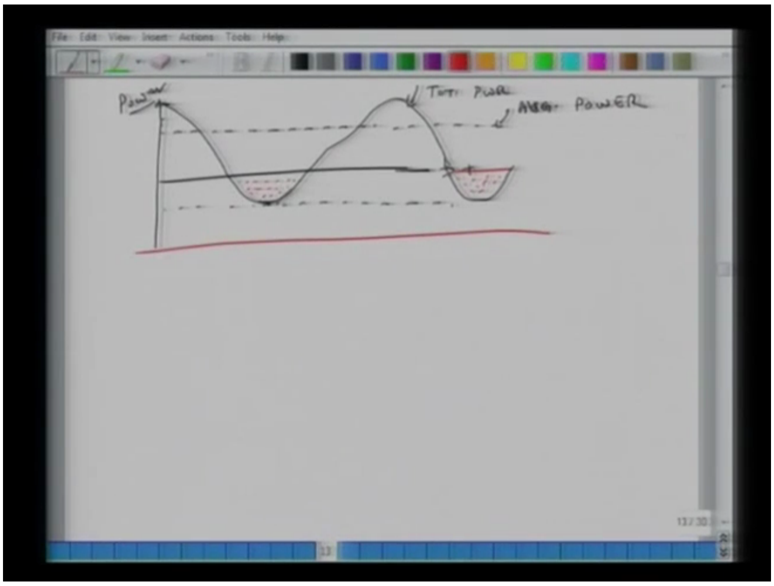

For Si not being equal to zero this is my time axis and that is my power the total power will be something like this. Where my average power is going to be positive non-zero entity and this is my total power and this area below the axis is essentially the energy which is getting dumped back into the system.

Also we had in the last class or one of the earlier lectures talked about that (becou) because this is not symmetric along the horizontal axis the time axis what it means is that it will heat more than the north (pole) the normal current which should have we needed if Cos Si was equal to 1. Because there is lack of symmetry along the horizontal axis it will consume more power the case where Cos Si would have been identity 1 that is if Si was equal to 0. This was a recap of some of the step which we have done.

So now remembering this will look at a spherical (radiate).

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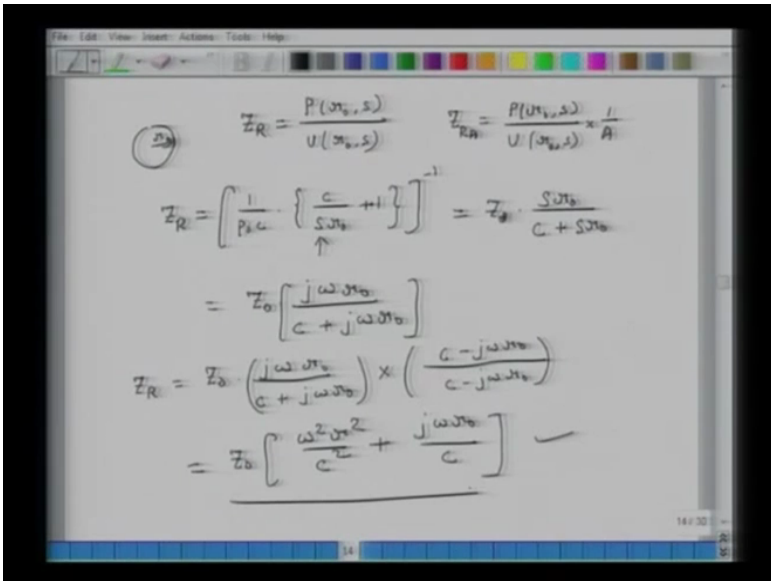

So I have a small sphere we had developed its radiating impedance to be R0 S over U R0 S, what this means is, that this is the radiating impedance of small spherical source of radiating of radius R knot on the surface of itself. So this is what the body will seek as its impedance on its surface. If you move away from the surface this R knot will grow will become whatever R value is given.

So my Z R A which is acoustic impedance is essentially Z R over area so that is E R knot S over U R knot S times 1 over area and remember S is equal to S equals J omega Z R we had seen was 1 over rho knot C times C over S R knot plus 1 the whole thing invert. For a purely resistive circuit the what would be the value of Si power factor? Si would be zero. You see here that there is an S component here which means that the impedance offered by a pulsating sphere is not purely resistive.

So whenever you have a pulsating sphere it will by nature of it presence introduce a non-zero Si in the entire circuit which means that at the amplifier level it will draw more than optimal value of current and it will require more than optimal value of voltage. So we are introducing just because by virtue of that it is a spherical surface radiating in free space you are introducing a non-zero value of Si and maybe to fix that at the electrical level you can produce some capacitors or you know to rectify that.

This is Z knot if I inverted Z knot is rho knot times C times S R knot over C plus S R knot and if I replace S by J omega I get Z knot times J omega R knot over C plus J omega R knot.

What we do here is we try to rationalize this term so I get basically what I do is Z knot times J omega R knot over C plus J omega R knot times I multiply and divide by Z minus J omega R knot and I will divide it and what I get is if I do simplifications omega square R square over C square plus J omega R knot over C. So I will know plot this, I will lot its real component and also its imaginary but before I do that.

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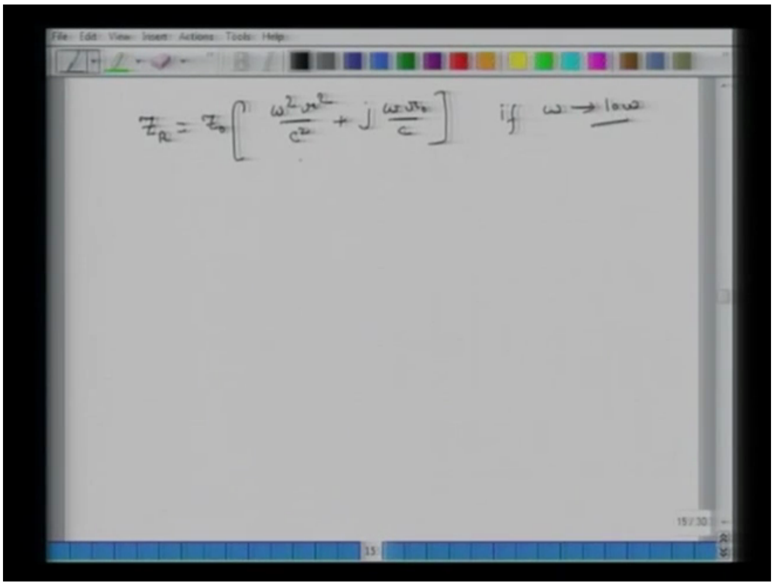

So I will rewrite it Z R equals Z knot omega square R square over C square, did I a take J omega not over C? We are assuming, why?

Student: Sir if do the exact maths there is a omega square R knot square term in the denominator

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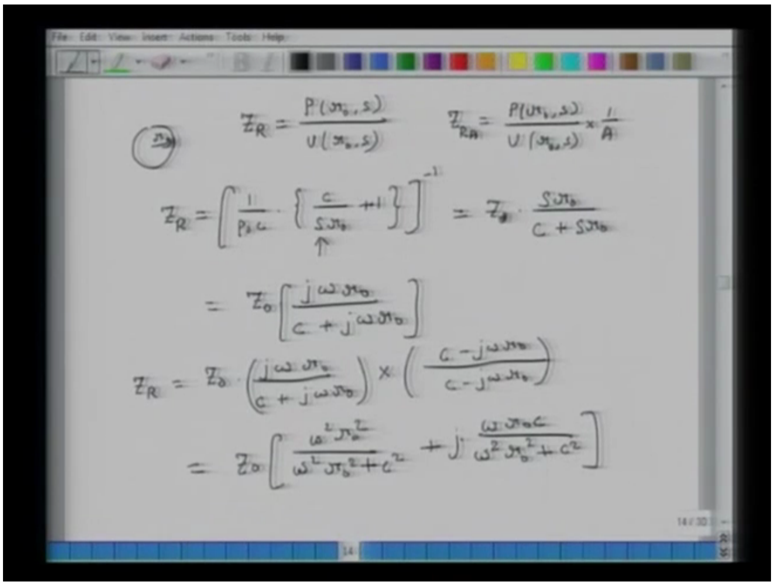

So this is Z knot and this is omega square R square R knot square over omega square R knot square plus C square plus J times omega R knot C over omega square R square plus C square yeah, that was the simplification I have done it. If omega is low when this is my relation and if omega is large.

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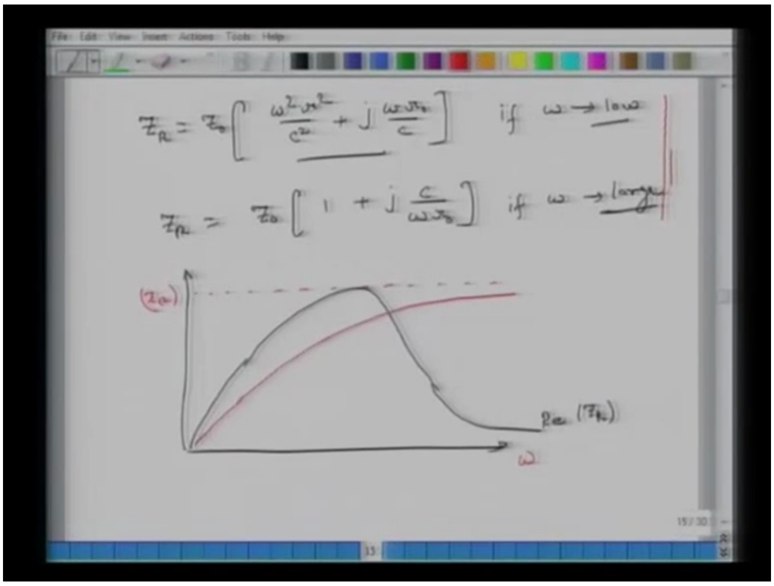

Then I get Z R equals Z knot 1 plus J C over omega R knot regardless with omega small or omega is large you have complex imaginary components in the entire term.

Except for very low frequencies what you see is except for very low frequencies it becomes purely resistive in nature and at extremely high frequencies again it becomes purely resistive in nature but in the intermediate range it has a reactive component and what that means is that most of the at most of the operating frequencies what these two approximation show is that Si will remain non-zero so power factor will be non-zero uhh will be not equal to exactly equal to 1 for most of the frequencies we are talking about.

So I will just plot, so I have here what I will plot is this is my real component of Z R and my imaginary component will be something like it will be (())(66:19) so what this shows is again this is a plot of different components for Z R real and imaginary this is my omega axis and what we have shown here is that regardless except in very extreme conditions extremely high frequencies or extremely low frequencies in most of the range my spherical sources are fairly reactive in nature which means that if I have to produce significant amount of dB S P L from these sources the spherical source it is a difficult thing to do.

Because they eat a lot of power at the amplifier level that is the implementation. So if you are design a complex system and if you are thinking of using a spherical source then you have to think little bit more carefully how you develop. So that is all we want we will be covering today in the next lecture what we will talk about is.

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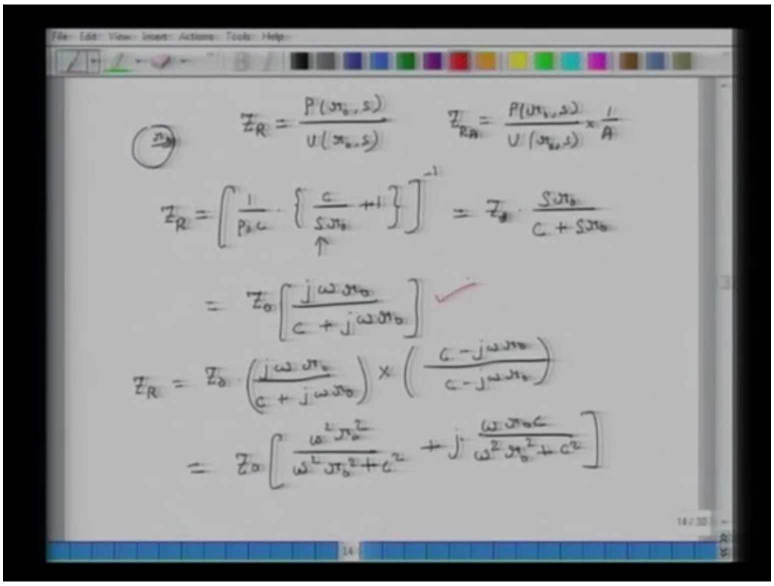

So this is my Z R I still have not approximated this into a set of resistors inductors and capacitors in the acoustic word. So I have if I have electrical mechanical and acoustic I have all that conversion but I still to have till so far not converted this into some lumped systems which I can construct in the acoustic rim. So that is what we will do for a spherical source in the next class and then we will also consider couple of (())(67:59.3).