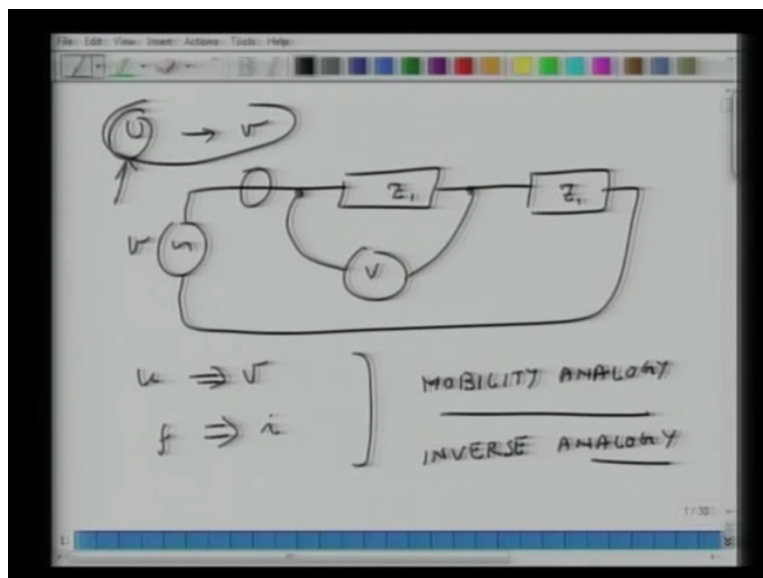


Acoustics
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Module 6- Lumped Parameter Modelling of Transducers
Lecture 2
Examples of electromechanical systems

So in the last class we had established equivalence between the electrical side and the mechanical side so we had mapped velocity onto voltage force onto current, compliance which is inverse of stiffness onto inductance and the moving mass of an object onto capacitance then we introduced a term which is inverse of damping called mechanical resistance and that we mapped to the electrical resistance.

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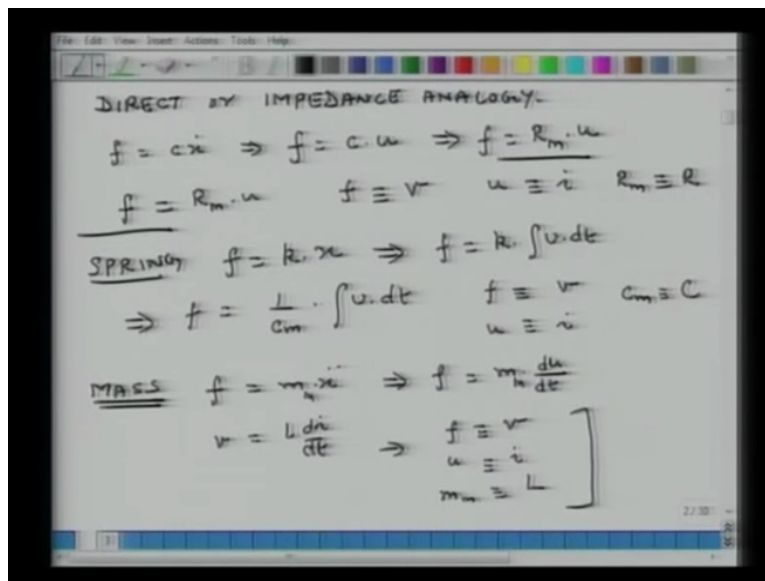


As we developed this analogy we had seen that we had mapped U which is the velocity to voltage. One thing in this context it maybe important to understand is that if I draw a circuit it could be any circuit I have a voltage source and there is an impedance there is another impedance and then I am closing the circuit. So here the impedance is Z_1 here the impedance is Z_2 it is actually physically possible for me to measure the voltage across this impedance by using a voltmeter and in doing so I don't have to cut the cable here. If I have to measure the current then I have to introduce an emitter here and then I have to actually physically cut the circuit.

Now there are newer current measuring devices which use magnetic fields but the least some of the less expensive ways is that you have to cut the thing and then measure current going through that circuit. Similarly when I have to measure this velocity I don't have to break the mechanical circuit if something is moving I can use it use the laser or some displacement meter to measure the velocity. If I have to measure force, which is analogous to current I have to again break the circuit, because force is something which passes through the strive of things in there.

So it is important to understand that, and that is another benefit of this analogy which we are using. So this analogy where we had equated U with not equated we had establish analogy between U and voltage and force and current and so on and so forth, is called mobility analogy. It is also called inverse analogy. There is another set of analogies which is called direct analogy or impedance analogy where these mappings are just the opposite. So we will very quickly go over that.

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So what we are going to talk about is direct or impedance analogy. So we will start very quickly with a dash part so force equals C X dot which gives me force equals C times U which is velocity which gives me force equals R M times U. So here it is in upper case, so R M is same as the damping coefficient. So from this I can say force equals R M times U and will see that and if I say that force is similar to voltage.

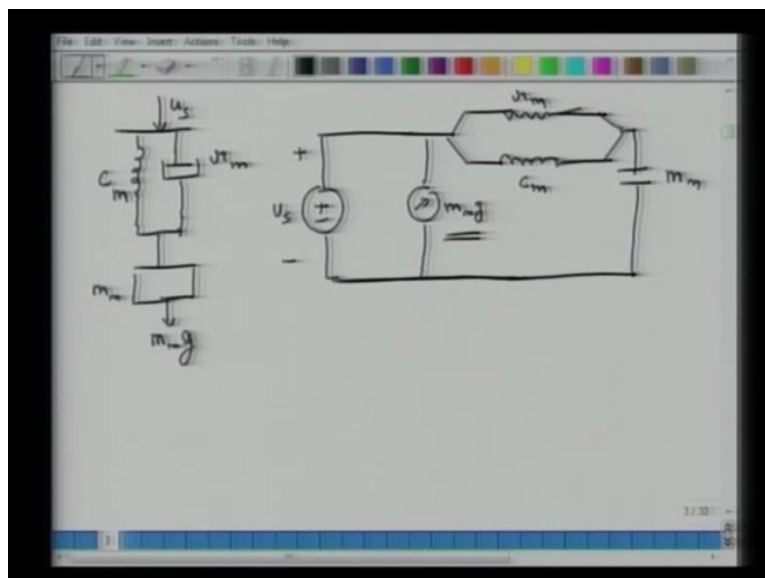
U is similar to current and my element variable which is R M or damping coefficient R M is similar to electrical resistance. So this is called dash part. I see that it is direct it is called

direct probably because you have force which is mapping onto voltage where volt is itself like a force type of a quantity, it relates to electromagnetic EMF you know electromagnetic offcourse.

It is I have no clear answer for that question but this is a terminology and we are just using it but I don't know exactly the history behind the nomenclature. So other one is a spring so force equals stiffness times displacement is equivalent to force equals stiffness times integral of $U \, dt$ so this same as equals one over $C \, M$ time s integral of $U \, dt$ where $C \, M$ is my capacitor, compliance so again force maps to voltage, velocity maps to current and my compliance maps to capacitance and then I go to mass and here force equals mass times acceleration that is force equals mass times first derivative of velocity over in time so I mean we has earlier used $M \, M$ that is moving mass.

So I will just used the same notation and I know that in electrical area voltage equals $L \, D \, I$ over $D \, T$ so again my force looks similar to voltage I looks similar to, velocity looks similar to current and my moving mass $M \, M$ looks similar to inductance. So this is my other analogy. Now if I do a four point transformer in this you will see that the turn ratio instead of P is to 1 some says 1 is to T . So will do couple of examples where we will just construct mechanical circuits not electromechanical (circuits) just mechanical circuits and, so that we develop some practice.

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Let's say I have a spring mass and a damper system like this so I have a spring and also it I have a dash part this is connected to a moving mass acted upon the force due to gravity so it

is M the force on it is M times G which is an external force and let's say I am exciting at this end U , here it is mechanical resistance is in lower case R and the compliance of this is C . so if I have to construct a equivalent electrical circuit then what I will do is I have a voltage source which is U then also I have a moving mass M and in some of the earlier lectures we had said that moving mass always refers to an inertial frame, which in electrical engineering translates to a ground line which is this.

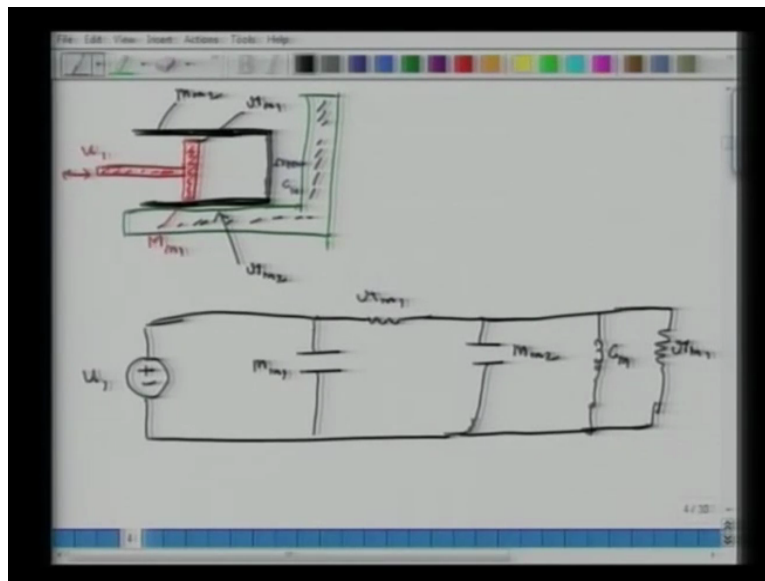
So I have I put a capacitance of value C then I have to ground it which is my reference and then the mass is connected in parallel to a spring and a dash part. So I have a resistance of value R and I have an inductor which mimics the performance of a spring and the value is C . So I am missing one thing in this picture, what is that? I have also gravity I have to put and the gravity is acting on the mass and that current flows through dash part and also the spring so I can put a current source also here and the value of that is M .

So in this case what I can do is initially when the thing is setup and it is having no excitation of U then I can solve using this current source of M times G and basically that will give me a D C of set in the whole displacement and velocity right and once I have that velocity will be zero because it is a quasi-statics situation but they will be the mass will move down a little bit then after sometime it will stabilize I can figure out what are the displacement of that then I apply an external voltage then again solve it.

So whatever my total displacement will be initial displacement which is a consequence of gravity plus the displacement due to this excitation velocity so that is how I can solve it. So what you are seeing here is that instead of writing long equations if I am able to develop a lumped you know system for lumped mass system for this entire mechanical network then fairly in a very straight forward way I can solve for complex systems. This is a simple system but I can also solve for complex systems.

So will look couple of more examples, let's look at another one where I am having two masses.

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So I have a base I have a piston let's say a cylinder and a piston and inside this I have a piston. The mass of piston is M_1 the mass of the cylinder is M_2 also I am exciting my piston by a velocity source U_1 it could be a this could be driven by a motor and the piston could go back and forth then I have a rigid frame in which this cylinder is on which this cylinder is placed and I have a spring so I have a rigid frame and here is a spring of compliance C_M .

The friction between the piston and the cylinder could be we designated as R_{M1} , and the friction between the cylinder and the frame on this surface is R_{M2} and this is ground plane so this is rigidly held. So if I have to develop a circuit for these again I have a velocity source which is like a voltage source generating velocity of U_1 , how many masses by having this? Two masses M_1 and M_2 .

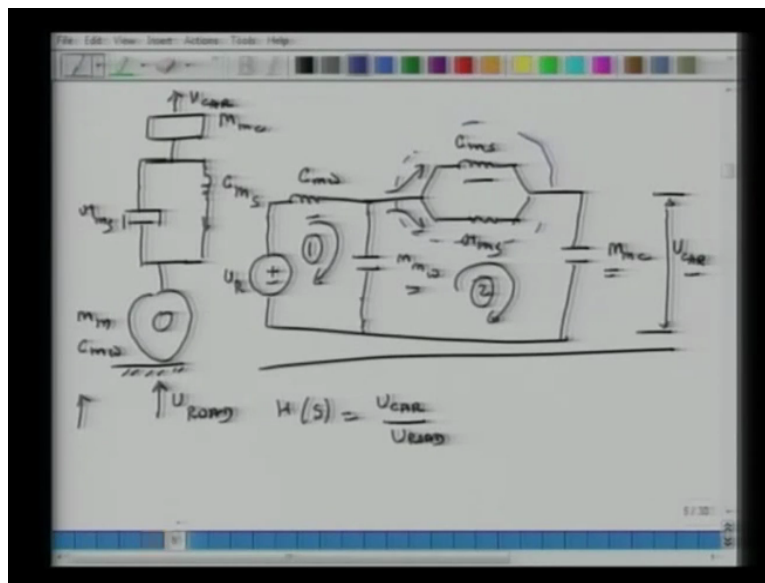
So I have two masses and I am using mobility analogy, this is M_1 , M_2 and I have to again ground the masses we have to provide an inertial frame to them, then my first mass is linked to the second mass through this and the interface between that I have mechanical resistance R_{M1} of course the first mass is also seen excitation through U_1 and the second mass and the ground are connected in parallel through two elements C_M and the resistance.

So I have an inductance of value C_M and then I have another resistance mechanical resistance value R_{M1} because whatever force which is being exerted by the second mass will be distributed amongst the compliance member and the friction member. So they are in parallel the force (vec) will split into F_1 and F_2 so those two members right. So you have to

like what (15:20) you have to think a little bit carefully how I have to construct those element but once you have the circuit network correct in fairly straight forward you solve these problems.

Will do one more example and then will move on and this case what we will do is the example of an automobile suspension. So typically you have a car four wheels on each wheel you have a spring and also a dash part a damper to kill the vibration and then on that structure you have the mass of the whole car distributed right. So if I just take one wheel plus one dash part plus one stiffness member then that system will see one fourth of the mass of the car.

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I have ground, then I have a wheel, wheel could have a mass let's call it M_w and ofcourse they face air inside the wheel also so it is acting as also a spring. So I have C_{ms} then the wheel is connected to a dash part and also it is connected to a spring. So let's call the spring stiffness to be C_{ms} and the damper resistance to be R_{ms} . So I had two springs in this case, one spring is inbuilt into the wheel itself by its very structure you can also argue that wheel also has some damping properties because it is made of rubber and all that.

You can also incorporate that typically the damping provided by the wheel itself is not significant so we can drop it and then on top of it, what do I have? The mass, of the car, how much mass? One fourth, so I put a mass here and I call it M_{car} . What I am interested in when I am doing this? Did if I have any disturbance in the vertical direction in this direction coming from the road. Suppose the car is going like that and you have a bump on the road so it will induce a velocity of U_{road} right or U_{road} you can all it.

And then what I am interested in? That the person whose seating here in the this $M M C$, how much of a velocity he sees? Ideally he would like that value is zero, right that is the ideal situation. So what you are interested in knowing is what is U_{car} ? Is everyone clear about this (terminology) $M M C$ is mass of the car one fourth U_{car} is velocity of the car where the passenger is sitting, $C M S$ is the compliance of the suspensions stiffness $R M S$ is damping mechanical resistance of the damper, $M M$ and $C M$ are the wheel of the mass and compliance of the mass.

So we will construct a mechanical circuit for this, so again this is my excitation which is acting like a voltage source I call it U_R and then I have two masses, the first mass is the mass of the wheel, so I make it $M M W$ I will connect it to ground, the second mass is mass of the car $M M C$ and I again ground it. There is a stiffness member between this mass and the excitation source which is $C M W$ ok. You could ask why is this $C M W$ not in parallel to this mass? So any thoughts on that? The compliance in the wheel, why is it not in parallel to the mass of the wheel itself? For that you have to see the physics of the problem.

So where is mass of the wheel? Most of the mass of the wheel is concentrated at the center where you have iron or steel and between that steel and the road you have that air getting compressed. So it is not in parallel it is in (series). So from here I have two mechanical elements in parallel $C M S$ and then these guys are connected through. So this value is $C M S$ and this value is $R M S$. What I am interested to know is, what is the velocity of this mass? So if I can figure out the voltage difference here which is U_{car} , for given a value of U_R if I can figure out what is U_{car} then I have solved the problem.

So when you are looking at this circuit think about it, for very low frequencies the impedance offered by an inductor is for all sets of frequencies is ωL , as frequency is very low ω is going towards zero the total impedance offered by an inductor is going towards zero. So for very low frequencies an inductor behaves like a short circuit right, as if frequency goes up ω goes up and the impedance offered by that inductor which are these compliance member this is compliance member, it keeps on going up significantly and at very high frequencies the impedance offer it becomes closer and closer to a open circuit.

For a capacitor it is just the inverse because the impedance is $1/\omega C$ so at low values of frequencies look it tries to become behave more and more like open circuit and that high values of the same thing it behaves more and more like short circuit

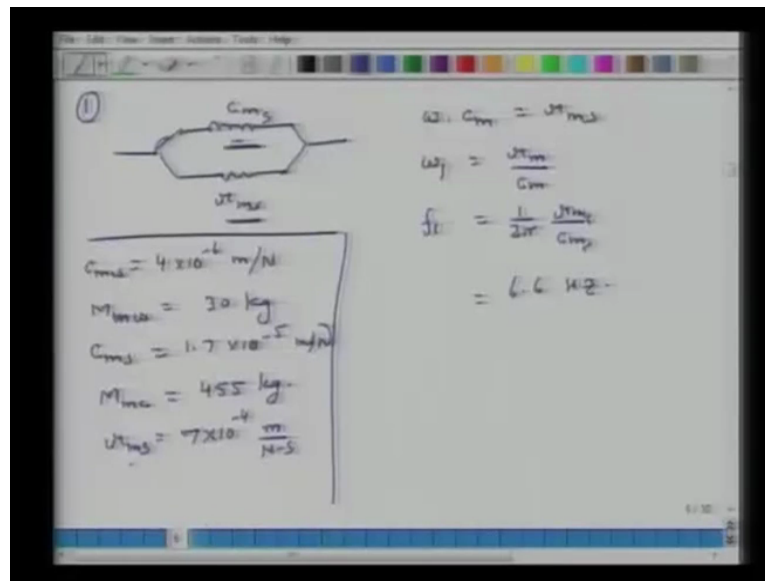
Student: sir in the previous example we added a current source, sir how do I get a mine the direction of the current, what is the direction of the current?

You have to assign the direction the current source in your circuit itself, like in this case you assign a positive and negative based on the physics of the problem. You assign a positive voltage and a negative voltage similarly you have to assign the direction of the current in the circuit itself, so it is your understanding of your physics that will help you understand. We will talk I little bit more about this problem and then will talk about acoustic elements.

So this circuit here the current can go in this loop which we call loop 1 and it can also in go in this loop which we call loop 2 and then within this loop the current can go either through like this through this branch or it can go through this branch. So we will very quickly without doing very elaborate calculations try to figure out what are the parameters at which these breakpoints happen. What is the breakpoint before which current goes through this branch and what is the breakpoint after which current goes through this branch and so on and so forth.

So what are the breakpoints? If we are trying to develop a transfer function which is U_{car} over U_R , suppose we are trying to develop this transfer function and then I develop a board plot for this then the board plot will have some breakpoints, what are those breakpoints going to be? Today will just talk about those breakpoints figure out those breakpoints and later class we will develop an exact board plot without doing solving the whole equation. So my first breakpoint is going to be so my first breakpoint is uhh going to be related to this part, $C M S$ and a mechanical resistance in parallel.

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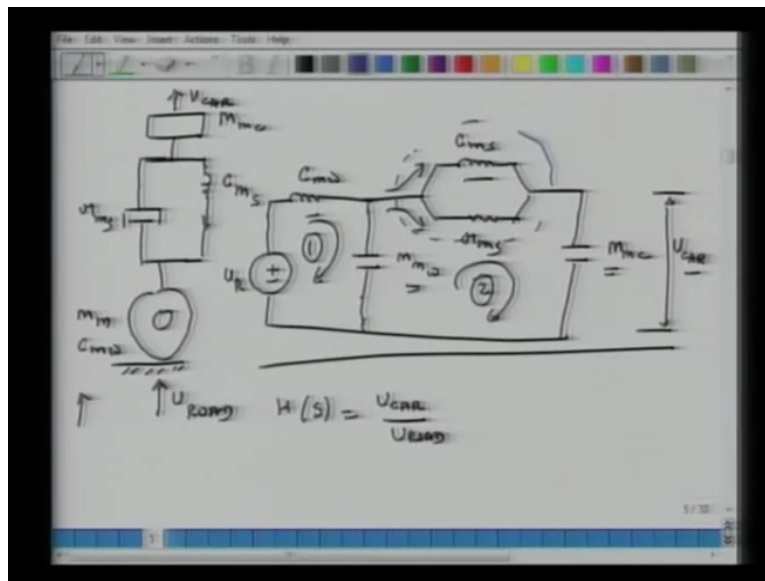


So this my first breakpoint C M S, R M S so for very low frequencies because omega is very low the impedance offered by this inductor will be low they will be a point where this (indu) impedance due to C M will equal R M and (aub) after certain set of that frequency critical frequency R M will be larger than omega times C M right, so that is my first breakpoint. So the criteria for that will be omega times C M equals R M S or omega 1 which is my first breakpoint is R M S over C M or f_1 equals $\frac{1}{2\pi} \frac{R M}{C M}$.

Now will assign some values to these properties, so C M W equals 4 ten to the power of minus 6 meters per Newton, the mass of the wheel is 30 kilograms C M S which is the compliance of the spring is 1.7 times 10 to the power of minus 5 meters per Newton, M M C which is one fourth the mass of the car including the weight of passengers is let's say 455 kg's. These are realistic numbers so that you get a feel, did I miss anything? R M S is 7 times 10 to the power of minus 4 meter over Newton second.

So I plug R M S and C M this is C M S so what I get is f_1 is 6.6 hertz. What does that physically mean? That physically what that means is that the influence of this mechanical damper becomes significant only after 6.6 hertz before that this is playing a bigger role stiffness member is playing a bigger role, so that this is my first breakpoint so this will give you some insight how you design or (devl) you know identify what is the right damping coefficient and the right stiffness for a system which is similar to this circuit.

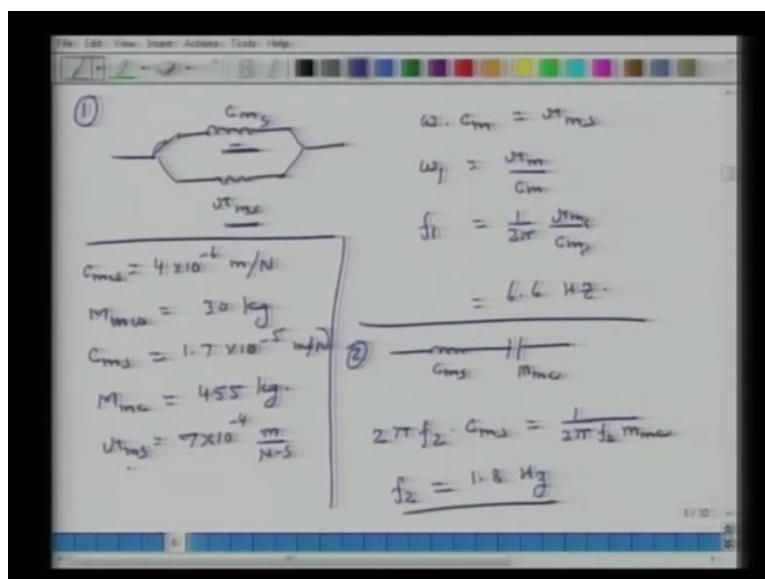
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Now the second breakpoint would be so current is going in this loop which is loop 2 and before a certain threshold that is when C M S is significantly important there will be a resonance point because C M S and M M C the impedance offered by these two will be out of phase, we have the impedance offered by C M S is $J\omega \times C M S$ and the impedance offered by capacitor is $1 / J\omega M M C$ right.

So they are out of phase but as omega develops grows they will be a point when the equivalent cancel each other.

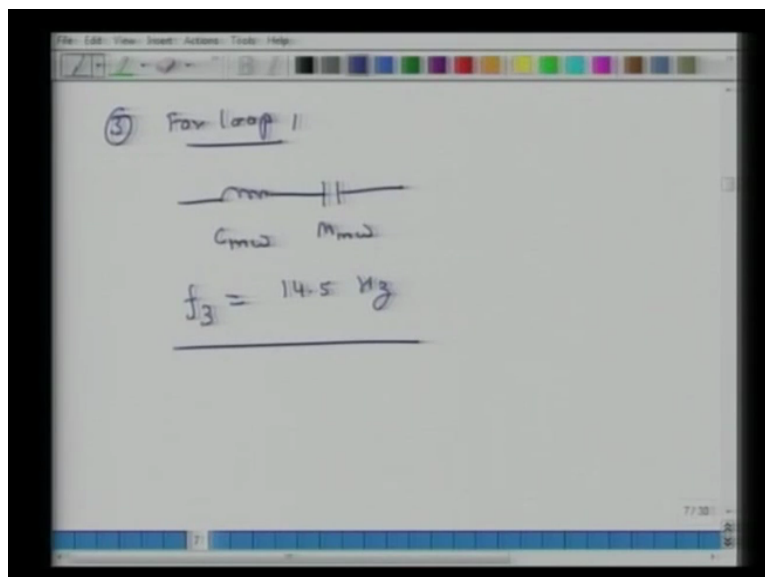
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So my second breakpoint is so before below 6.6 hertz I can neglect this resistance and I have this circuit in an approximate sense they will be some current flowing through the resistance but in a approximate very approximate sense I have C M S and here I have M M C and the condition for resonance would be $2 \pi F^2 \text{ times } C M S \text{ equals } 1 \text{ over } 2 \pi F^2 M M C$ so F^2 when I solve for it is 1.8 hertz.

So in my board plot there will be another break at 1.8 hertz as I am developing a board plot which will be happen later I should be aware that I have to do they will be something special happening at one point and similarly I have an inductor and a capacitor is in series in loop 1 right. So they will be a breakpoint associated with loop 1 also.

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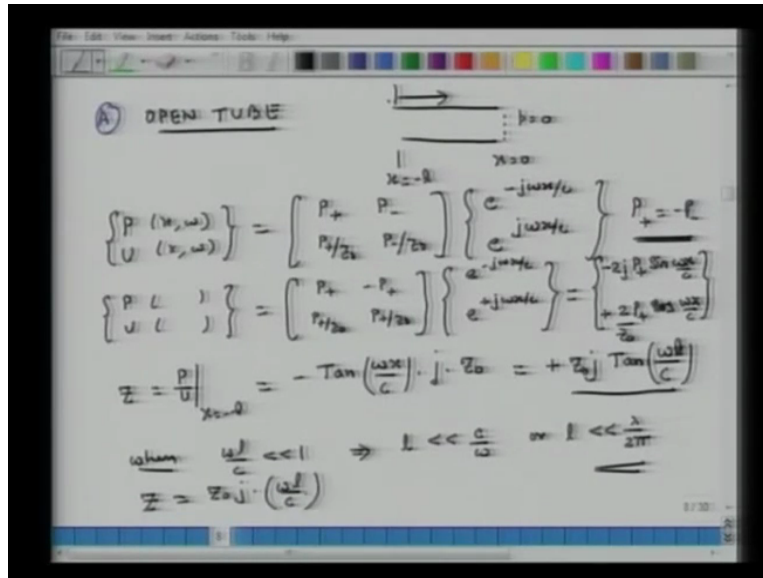


So breakpoint so the third breakpoint is for loop 1 circuit will be something like this M M W, C M W and my breakpoint will be F_3 just as we developed the second breakpoint it comes to 14.45 hertz.

So this is all we will do today in this class here atleast in context of this problem. But we will develop this problem further in some of the subsequent lectures. But we are in mind that what we have done here is we have developed electrically equivalent circuit just by looking at circuit we were able to figure out what are the different transition points based on visual understanding of the circuit and then later we will see what is significant about those transition points and maybe in your once you go back to your home you can try to develop it could be a informal homework.

So now what I will move on to is acoustic circuit, so we have developed electrical networks for mechanical systems now will develop similar network for acoustic systems.

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In an (elec) acoustic circuit is an open tube, so I have a tube which is open at X equal zero and the tube is L long my axis for axis going away from the tube. So my boundary condition at X equals zero as we have talked about earlier also is the pressure at the exit point is zero, the fluctuation pressure.

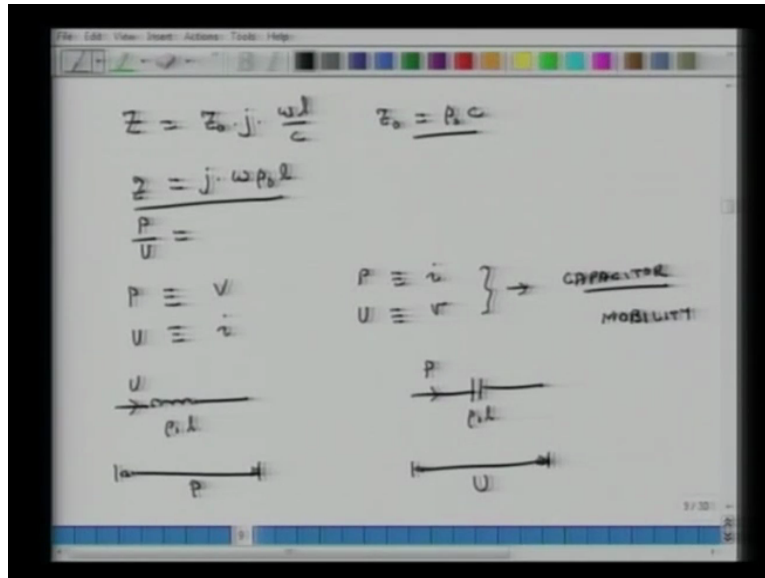
So my relation for P and U which are both functions of X and omega I can write them as P plus P minus P plus over Z knot P minus over Z knot and then I do, E minus J omega X over C, E J omega X over C and once I impose the boundary condition I find that P plus equals minus P minus which we also seen earlier in earlier classes also we have proven. So using equivalence between P plus and P minus what we get is P U again a functions of X and omega I get P plus minus P plus P plus over Z knot then I get P plus over Z knot.

Then I get P minus J omega X over C E plus J omega X over C and if I simplify this then I get minus 2 J P plus sin omega X over C and I get plus 2 P plus cos omega X over C over Z knot. So what we are interested is trying to develop an expression of impedance of the tube at X equals minus L as we are looking into the tube. So the impedance is Z equals P over U X equals minus L and when I put X equals minus L in this entire relation what I get is minus tangent omega X over C times J times Z knot.

And when I put X equals minus L I get plus Z knot J tangent omega L over C, and taking minus sign out then that eliminates the negative sign.

I am putting X equals minus L here so when omega L over C is varying significantly smaller than one basically what that means is L is significantly smaller than C over omega or L is significantly smaller than lambda over 2 Pi. So for this condition when L is significantly smaller than one sixth of lambda this relation becomes Z equals Z knot J times omega L over C.

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So we have seen that Z which is the impedance is Z knot times J times omega L over C. Now we know that Z knot is rho knot C so once I introduced this relation in this I get Z equals J times omega rho knot L, where rho knot is this the density of air and this is basically P over U if I equate P such that is similar to voltage U is similar to current then this relationship looks like the relationship for a for an inductor because there is a J omega here. If I invert this analogies such that P looks like current U looks like voltage which I did in mobility analogy so in such conditions a closed tube looks like capacitor.

So here my thing looks like I have an inductor here of value rho knot L voltage across the inductor is P and the current going through is U and in mobility equivalence I have a small close tube behaving like a capacitor rho knot L voltage across it is Q and the pressure is,

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$$Z_A(x, \omega) = \frac{P(x, \omega)}{V_v(x, \omega)} = \left(\frac{P}{A}\right) \times \frac{1}{V_v} = \frac{Z(x, \omega)}{A}$$

$$Z_A(-l, \omega) = j \cdot \frac{\omega \rho l}{A}$$

ACOUSTIC MASS $l \ll \frac{\lambda}{2\pi}$

$$f = m \cdot \frac{da}{dt}$$

$$p \cdot A = (\rho \cdot l \cdot A) \cdot \frac{da}{dt} \Rightarrow p \cdot A = \rho \cdot l \cdot \frac{dv_v}{dt}$$

$v_v \equiv \text{velocity}$ $p \equiv \text{current}$

$$p = \left(\frac{\rho \cdot l}{A}\right) \cdot \frac{dv_v}{dt}$$

$$v_v = \frac{1}{(\rho \cdot A)} \cdot \int p \cdot dt$$

So now we introduce not new term, term which you have talked about earlier that we will revisit that and that is Z_A or acoustic impedance so again it could be a function of X and ω and that is basically it is a function of pressure which is again dependent on X and ω and volume velocity.

So this is same as P over U both are functions of X and ω times 1 over A , where A is the cross section in this case of the tube is the area cross sectional area of the tube and this is basically $Z_X \omega$ divided by area. So the acoustic impedance for an open tube minus $L \omega$ at X equals minus $L \omega$ is J times $\omega \rho l$ over A and I can use this acoustic impedance in mobility analogy as a capacitor and in impedance analogy as an inductor.

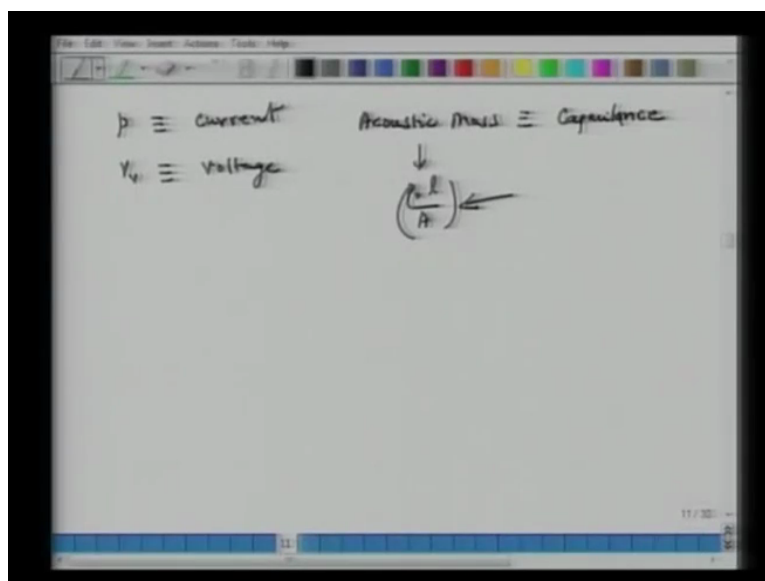
Now will talk something called acoustic mass and will try to develop some sense of will see how acoustic mass behaves like a mass in mechanical area. So here right now is still in doing all this discussions in context of small tube open at both ends and it is small in the sense that its length is significantly less than $\lambda / 2\pi$ again you have a small tube whose length is significantly small in this sense because it is very small the pressure then the air inside that tube (because) based on the some of the discussions which we have talked about earlier that when you look at the velocity profile in such a tube, the velocity profile in such a small tube which is open at both ends is such that it does not change significantly as I move on X axis.

So the air inside such a tube moves like a rigid mass moves back and forth if I have a pressure here and the tube is extending and it is fairly small in length compare to the overall lambda then because of that vibrating piston the air will move more or less like a rigid mass back and forth. So if that is the case then I can use this relation, force equals mass times $\frac{dU}{dt}$. Where this Newton's law is applicable to rigid body that is why I am able to use that.

Now force is pressure times area so P times area where area is the cross section of this tube is mass and mass itself is density times the volume of the tube which is L times A times $\frac{dU}{dt}$. Now I know that area times U is volume velocity so I can say P times A equals ρ times L times $\frac{dV}{dt}$, pressure equals ρ times L over A times $\frac{dV}{dt}$, so what this shows is that if a tube is short in length compare to one sixth of the wave length then because the air is more or less behaving in a very rigid way so it moves back and forth and it is governed by this relation.

Now what you have here is $\frac{dV}{dt}$ term so depending on how you look at it if you move to a mobility equivalence where $\frac{dV}{dt}$ corresponds to voltage and your pressure corresponds to current then a small tube behaves like an acoustic mass in mechanical mass behaves like a capacitor similarly this acoustic mass behaves like a capacitor because then what this tells me is that my $\frac{dV}{dt}$ equals $\frac{1}{\rho \times L \times A} \int P \, dT$. So this is the equation for a capacitor.

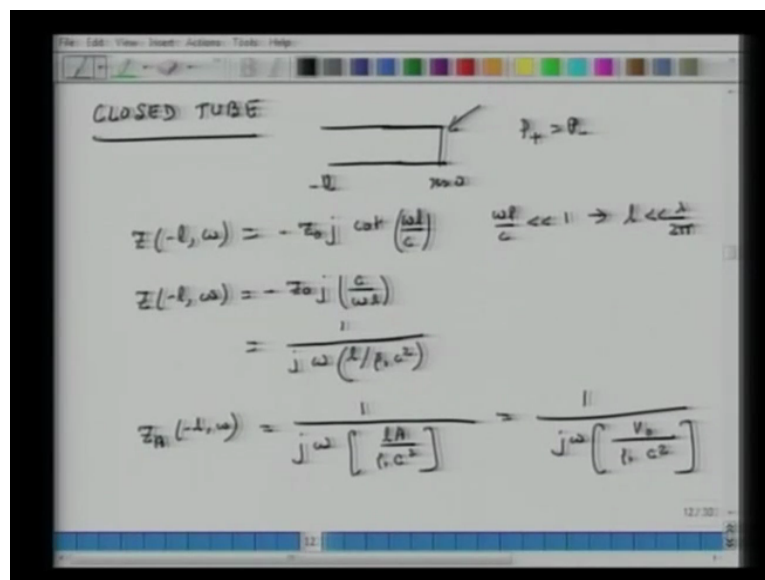
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So in mobility analogy for a short tube P maps to current then V V volume velocity maps to voltage and the capacitance is called acoustic mass and that is the value for that is 1 over the term itself here rho knot L over A so this behaves like capacitance and the value of this is rho knot L over A, so if I have a short tube and if I am using mobility equivalence then the total amount of air all I have to do is it behaves like a mass but its acoustic mass it is not the physical mass and the value of that acoustic mass is rho knot L over A.

So I can construct a circuit and where I short tubes and these tubes physically can be fairly long for instance at 40 hertz the length of a short tube will be anything which is less than lambda over 6 right and the lambda for 40 hertz sound wave will be 340 over 345, 350 over 40 which is significantly large number. So these are not like my micro devices these can be fairly long. We talked about a close tube uuh an open tube now will move to a closed tube.

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So again I have a tube closed at one end where X equals zero it is L long then based on earlier classes we had seen that Z at minus L is minus Z knot J O tangent of omega L over C and the condition here is P plus equals P minus because of deflection. So again for small tubes such that omega L over C is very small that is L is very small compared to lambda over 2 Pie my Z minus L omega is equal to Z knot J times C over omega L and then I then put Z knot equals rho knot C then I get this number as and I bring J under denominator what I get is 1 over J A omega L over rho knot C square.

And my acoustic impedance Z A at minus L omega is basically Z over A as we saw earlier is 1 over J omega times L A over rho knot C square. Now L times A is volume J omega so it is

volume over rho knot C square. When we look at the physics of this tube pressure will be as we saw in earlier lecture pressure will be maximum here and the closed end. Whenever you have a closed wall pressure is maximum here and from here it will start falling as I move away from the tube, but because the tube is not very long in the context of L being significantly less than lambda over 2 Pie that falling pressure will not be significant right.

What that means is that I can approximate the pressure is more or less constant in that tube. It will not go to zero because lambda because the length of the tube is significantly small compared to the wave length one sixth of the wave length. So what that also means is that if I have to understand how gas or the fluid is behaving in this then pressure is not changing significantly then I should be able to use the gas law Adiabatic gas law E V P to the power of gamma equals constant and see what it tells us how the, how the fluid is behaving in this closed tube where pressure is more or less constant so that is what we will do now.

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The image shows a whiteboard with handwritten mathematical derivations. The equations are as follows:

$$P_T V_T^\gamma = C \Rightarrow \frac{dP_T}{dt} V_T^\gamma + \gamma P_T V_T^{\gamma-1} \frac{dV_T}{dt} = 0$$

$$\text{Now } \frac{dV_T}{dt} = A \cdot \frac{dx}{dt} = A u = V_V$$

$$\frac{dP_T}{dt} V_T^\gamma + \gamma P_T V_T^{\gamma-1} V_V = 0 \Rightarrow \frac{dP_T}{dt} = -\frac{\gamma P_T}{V_T} V_V$$

$$\frac{dP_T}{dt} = -\frac{\gamma P_0}{V_0} V_V \quad \left. \begin{array}{l} \frac{dP_T}{dt} = -\frac{\gamma P_0}{V_0} V_V \\ \frac{dP_T}{dt} = -\frac{\gamma P_0}{V_0} V_V \end{array} \right\} \left. \begin{array}{l} c^2 = \gamma P_0 / \rho_0 \Rightarrow P_0 / c^2 = \rho_0 \\ V_V = \left(\frac{V_0}{P_0 c^2} \right) \frac{dP_T}{dt} \end{array} \right\}$$

P T, so P T is total pressure which is P knot atmospheric pressure plus change in pressure P and V T is again total pressure and then I now I differentiate this in time .So differential of P T in time will be same as D P over D T because P knot is not changing with time so I can call it D P over D T times V T gamma plus now I differentiate V T to the power of gamma in time, times V T gamma minus 1 times D V T over D T equals zero.

V T is original volume plus change in volume which was tau and the original volume does not change with time. So I can erase this and I can replace it with D tau, now D tau over D T equals area of the tube cross sectional area times D X over D T equals A U equals volume

velocity. So I can replace this whole term by volume velocity in this relation so we get $\frac{D P}{D T} \times V T^{\gamma + 1} = \frac{D P}{D T} \times V T^{\gamma} \times V$ equals zero. If I solve for $\frac{D P}{D T}$ I get $-\frac{\gamma P}{V T} \times \text{volume velocity}$.

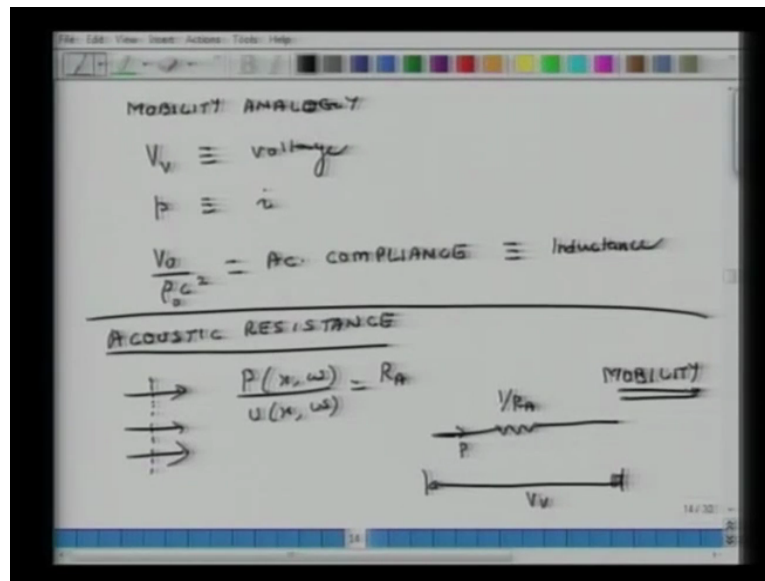
Now we know that in the rim acoustics the fluctuation in pressure are of much lower order of magnitudes than P knot similarly $V T$ is more or less very closely equal to V knot. So I can simplify this further $-\frac{\gamma P \text{ knot}}{V \text{ knot}} \times V$ again we know that C^2 is $\frac{P \text{ knot}}{\rho \text{ knot}}$. So I get $P \text{ knot} \gamma = C^2 \rho \text{ knot}$. So I put this in this relation so I get time rate of pressure equals $-\frac{C^2 \rho \text{ knot}}{V \text{ knot}} \times V$.

The significance of negative sign is as you have it in a spring that when you put a pressure then the volume will reduce and vice versa so that is significance. So base on that I can draw my negative sign and I can say volume velocity equals original volume over $\rho \text{ knot} C^2$ times $\frac{D P}{D T}$, what this is showing me is, that a closed tube behaves like a spring and physically it makes sense if you have a closed volume and you put a pressure on it, it is not behaving like a mass because mass has no way to behaves like a spring and we have done some earlier some problems also.

The stiffness of that is this number it relates to this $\frac{V \text{ knot}}{\rho C^2}$ the acoustic stiffness. The other thing is that as we were developing this relation we did not use the one dimensional wave equation very explicitly we did use in the case of an open tube theory of one dimensional equation. In this case we had made one single assumption that the length of the tube is less than $\frac{\lambda}{2}$. So what that means is that even if I have not necessarily long a tube but it could be any closed volume it could be an arbitrary sack like this.

As long as the longest dimension of this sack is significantly less than $\frac{\lambda}{2}$ the volume velocity at the exit point of that here will be related to change in pressure in such a way and it will behave as a spring as a mechanical spring.

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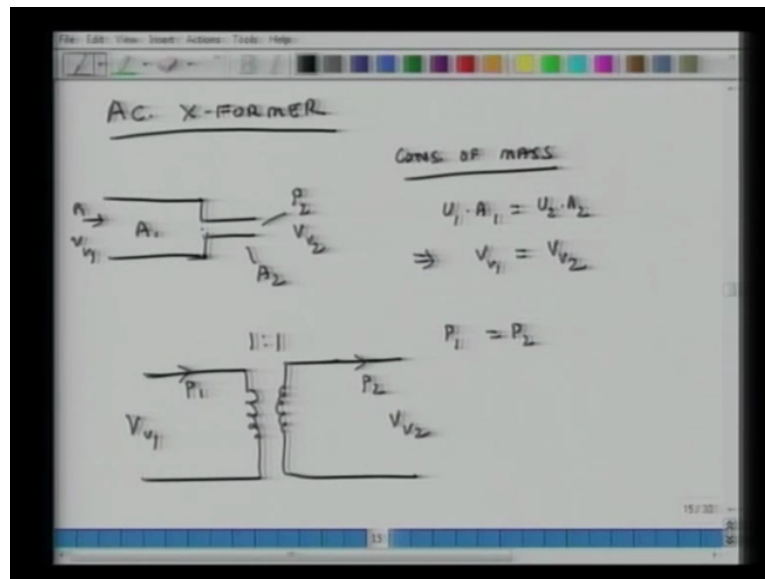


So in mobility analogy, in mobility analogy for a closed sack V V again we saw is similar to voltage pressure corresponds to current and V knot over ρC square is called acoustic compliance and this corresponds to inductance (L)(55:58) relationship exactly.

Volume velocity is something like L times $D P$ over $D T$ this is rate of change of current. So we have developed equivalence for acoustic mass, acoustic compliance and now will go to acoustic resistance and that is fairly straight forward. So you can have a screen of fine (vesh) mesh it could be the screen could be a piece of cloth or it could be criss-cross of wires and as air flows through it because of this viscosity it dissipates energy. So that case P which is function of X and ω over U X and ω is called R_A and in mobility analogy it is 1 over R_A is my acoustic resistance and that is such that my voltage across this thing is V V and the current flowing through this is P , so this is for mobility.

We have done all the three elements resistance, mass acoustic mass, acoustic spring or acoustic compliance and the last one is a transformer an acoustic transformer. What could be an acoustic transformer? Basically you what happens in a transformer, electrical transformer? Current goes up steps up or steps down right, voltage steps up or steps down it has to have a stepped construction because the stepping function is it goes up it is not gradually going up or gradually, say if I have a tube which has two different cross sections then that could work as an acoustic transformer.

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Acoustic transformer, so physical device could be like this. So I have area here A_1 here my area is A_2 my input parameters are P_1 and V_{V1} so again when we are talking about acoustic elements it is not velocity and pressure it is volume velocity and pressure remember that and here my output parameters are P_2 and V_{V2} . So the equivalence relations between two sections of these tube would be what? A conservation of mass or continuity, I should call it $U_1 \cdot A_1 = U_2 \cdot A_2$ is gives (me) volume velocity 1 equals volume velocity 2 that is the first equivalence right.

The second equivalence is that P_1 equals P_2 because at this interface the pressure have to be equal, P_1 has to equal P_2 , so my velocity can go up but the volume velocity will remain same my pressure will also remain same. So this is a little different than regular tubes where water is flowing but when we are talking about sound it is little different. So what will be the turn ratio in such a transformer? 1 is to 1, volume velocity 1 is same as volume velocity 2 and P_1 is same as P_2 . So turn ratio is 1 is to 1.

So what is changing step, going up or down is the velocity but not volume velocity, volume velocity is preserved and finally so we have developed three elements in the area of acoustics, acoustic mass, acoustic compliance, acoustic resistance we have a 4 point transformer and the there are laws of like Kirchhoff's voltage law and Kirchhoff's current law. Similarly here you have similar conservation of that if you have a closed circuit $V_{V1} + V_{V2} + V_{V3}$ equals zero and then also had a node pressures are flowing then $P_1 + P_2 + P_3$ and so on and so forth they also equate to zero.

So what we have done in last couple of lectures is we have developed equivalence between electrical, mechanical and acoustic systems. What is missing is, that do you go jump from acoustic to mechanical, there is a junction interface point. So we have not talked about that, that how do you jump from electrical to mechanical and do you jump from mechanical to electrical? Meaning that if you have current coming in what is the transformation happening that your current transforms to force, what is that relationship? In the next class will talk about that.