Acoustics
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Module 06
Lumped Parameter Modelling of Transducers
Lecture 01
Generalized Elements

Good Afternoon what we are going to talk about today is primarily about generalized elements and in the previous lectures I had laid out some context as to why we would need these generalized elements since I have not talked and provided much detail about this whole idea of generalized elements we not know much about it but as we go further in the lecture you will see what a generalized element is and why it is needed.

So primarily the logic behind developing this theory or this equivalences that for instance in a sound device such as a loudspeaker you have electrical energy coming in it gets converted into motion or mechanical energy so you have first transformation of energy then you have another transformation where that motion gets converted into sound and to bridge the domain of electrical engineering, the domain of mechanical engineering where you are dealing with motion, stresses and strains and the domain of acoustics you need some mechanism or theory and if we use lumped parameter approaches which work fairly well for devices such as loudspeakers, microphones and a bunch of other acoustic devices.

Then this generalized element approach comes in very handy so we talked about loudspeakers so let us look at microphones. So in general a microphone is a device where you have sound which has been sensed as pressure and then it gets converted into electrical energy, we have five six major categories of microphones.

So the first one is the condenser microphone and what you have there is that you have a membrane which senses pressure and then as that membrane moves back and forth it causes the motion of a plate which is also moving with it, in some cases the membrane itself could be a metallic plate.

So you have one membrane moving back and forth and then the other plate is fixed so this membrane is moving back and forth so the capacitance between the two plates changes and it responds to changes in pressure variation caused by sound disturbances so you sense that change in capacitance and then you convert it into acoustic pressure through again some equivalences then another type of a microphone is a dynamic microphone and there it is essentially an inverse of a regular loudspeaker.

So what you have in a dynamic microphone is a membrane which senses pressure because of that pressure fluctuation it moves back and forth which in turn gets converted into electrical energy, either voltage or current and then that voltage or current is measured and then it gets translated back by some equivalence relation into SPL, sound pressure level.

Then there are piezoelectric microphones where you have the conversion process as sound, sound causes piezoelectric crystal to experience some pressure and when that pressure goes up or it goes down that generates voltage in the piezoelectric crystal, so again you have a transformation of energy, sound to pressure to electricity.

There is another device, a microphone device fiber optic microphone, so what you have in fiber optic microphone is again you have a membrane which is moving back and forth, one side of that membrane could be polished and it could be a reflective surface so light from some other place comes and it gets reflected and as this membrane is moving back and forth the intensity of that light gets modulated.

So then there is another sensing device which senses the change in light intensity and that change in light intensity is then interpreted back into sound pressure level, so in all these measuring equipment, different types of measuring equipment you have transduction happening, so there is a need to have some sort of theory or an approach to which you can bridge the gap between electrical, mechanical sound or electrical optics sound or electrical piezoelectric mechanics sound and so on and so forth.

So what we will cover today is essentially we will develop few types of generalized elements and then we will see how they map into electrical and mechanical domains so that will be the focus.

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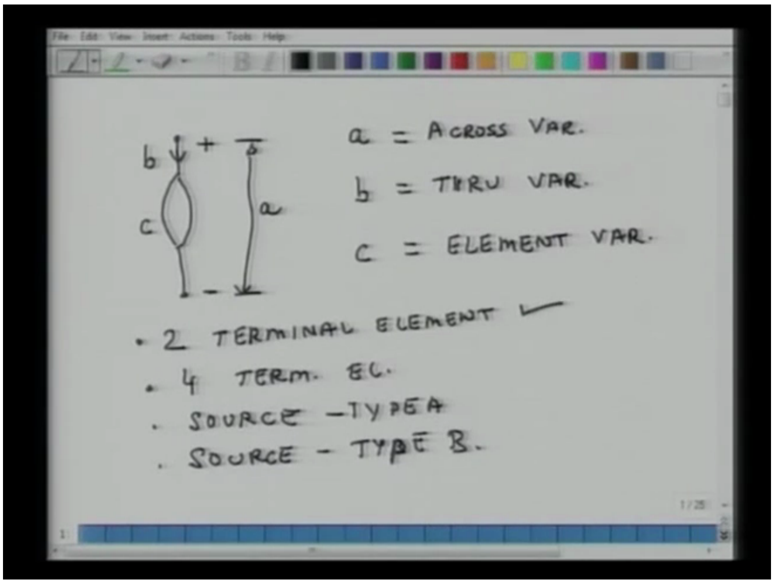

So I can have a generalized element and I can symbolize it something like this it could be positive here, negative here and it could have 3 variables. So the first variable, let us call it a, the second variable let us call it b and the third variable let us call it c and what we say is a is an across variable for sensing a resistor what is across that voltage so voltage could be an across variable, b is a through variable it is flowing through the element and c we call it the element variable and it is essentially a property of that generalized element.

So again if we have a resistor, through variable could be current as current is passing through it and across variable could be voltage and the element variable could be the resistance it is property of this device and this is all for linear theory, these elements as we are developing, we will be using in context of linear systems so what we will develop is we will develop four types of elements the first type will be two terminal element, the second element which we will talk about is a four terminal element, the third element we will talk about will be source and this will be let us say type A.

And the forth element we will talk about is again a source and it will be type B or you can call it type 1 or type 2 so we will start with two terminal element.

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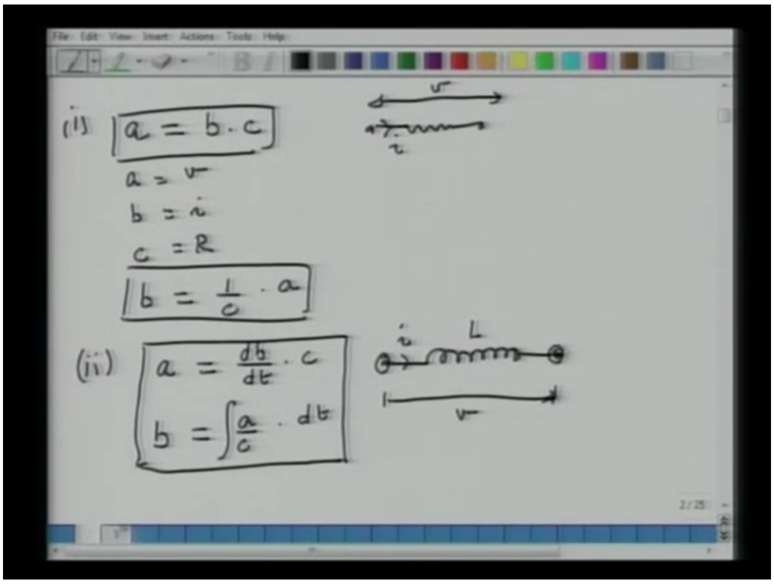

So the first terminal element is as a relationship where a which is the across variable is through variable times element variable (())(08:40) element which has these three variables linked in such a way is a two terminal element of this particular type so for instance you have a resistance there is a voltage across it and there is current going through it so in this case my a is voltage, b is current and c is resistance okay we can also say that b equals 1 over c dot a.

So these are the two forms of a two terminal element and they are of different categories of two terminal so this is first type, let us look at another two terminal element. So in this case the across variable a is equal to time derivative of b times c or b equals a over c if I integrate so again if I map it to electrical domain, this looks like an inductor.

So again my through variable is voltage oh I am sorry current through variable is current, across variable is voltage and element variable is L or the inductance so this is another form, a two terminal element and it is called a two terminal element because it has two physical terminals similarly an inductor has two physical terminals.

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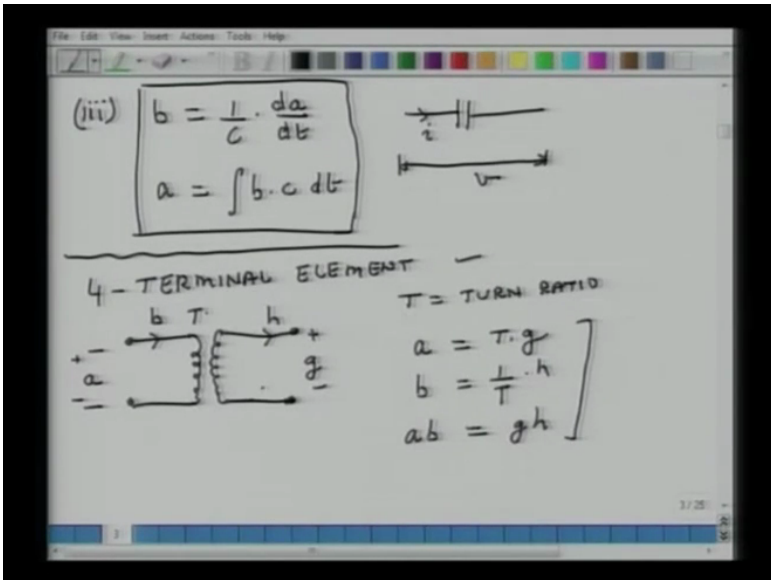

Do the third type of this two terminal element and as you would expect it would look something similar to a capacitor so here the through variable which is b equals 1 over c times time derivative of a, so you have a capacitance capacitor this current going through it and there is voltage and voltage and current are related in such a way as defined by this evaluation and I can also reformulate this relation such that a equals b times c dt when I integrate.

So this is the third category of a two terminal element, if you talk about two terminal element, now let us look at 4 terminal element which is another thing we talked about. So what we are going to 4 terminal element. So can you guys guess a 4 terminal physical device which has four nodes?

Student: Transformer.

Transformer exactly, so what you have is these are two terminals and then the other two terminals are here this is one across variable another across variable is g then there is a through variable d another through variable is h and the element variable is another number we call T.

So T in this case is turn ratio and what we have drawn is essentially an ideal transformer, so in this case the relationship between a and g is a equals T times g, b equals 1 over T times h and then you also know that ab equals gh, okay. So we have right now talking only about electrical side of things but we will see how these ideas also translate into mechanical world. So this is a four terminal element because it has 1, 2, 3 and 4 terminals.

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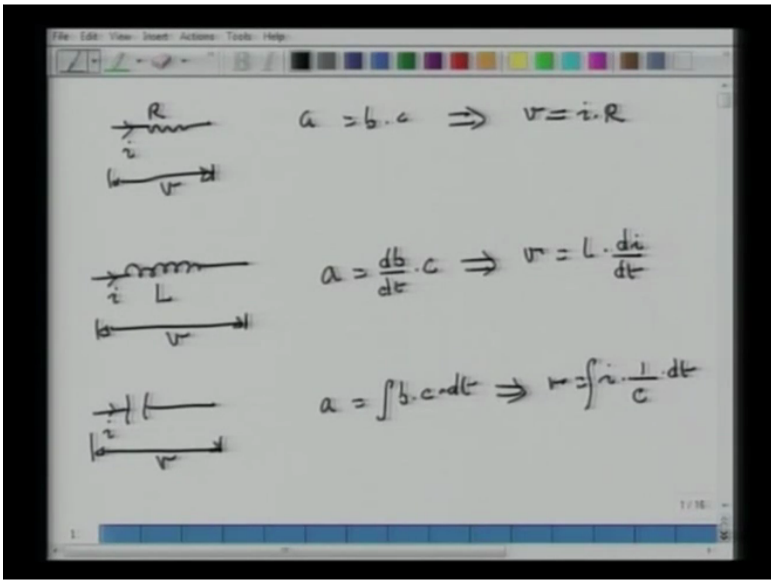

So the third category of terminals was source type A and here what it is, essentially what we are seeing is that the element c is same as a and this is the case for an ideal voltage source as an example that is its internal resistances virtually 0 now I think I wrote it incorrectly because there is an arrow here so this should be b and this should be an ideal current source.

And the fourth one is source type B, it should be B what you have here is a source within element value of c and through variable is b, across variable is a, then in this case you have c equals a and an example would be ideal voltage source.

Student: In the first case it should be b equals c.

b equals c which is what okay, so again we will we talked about different generalized elements and now we will just formally even though we know in electrical engineering which is what we will just formally document so in electrical engineering if I have an resistance, current and then there is voltage across it then a equals b dot c maps to v equals i dot R.

If I have an inductor with an inductance L, current is i, voltage across the inductor is v then a equal db over dt time c this relation maps to voltage equals L times rate of change of current, if I have a capacitor seeing a current i again voltage across it being v then a equals integral of b dot c dot dt this relation maps to voltage equals i times 1 over capacitance times dt then I integrate it.

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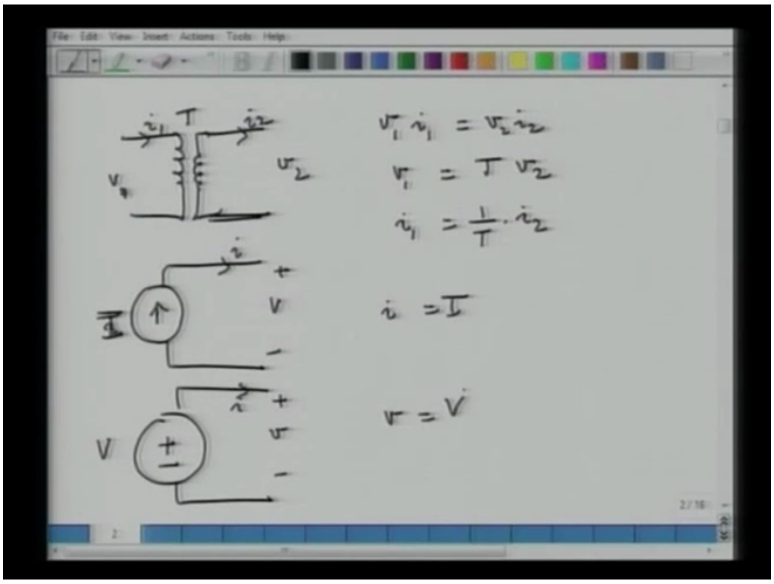

And my four point element my input current is i1, voltage across is v1 and then output voltage is v2, current is i2 and turn ratio is T then the relationships are v1 i1 equals v2 i2, v1 equals this turn ratio times v2 and i1 equals 1 over T times i2 and then finally I will formally list the two sources.

So I have a current source, internal current which is the element variable is I and the current going through the circuit is lower case i voltage across the source is v and here i equals I and similarly I have a voltage source then my relationship will be V. So till so far it is all standard but just to provide a context in, I have a generalized element and I am mapping that generalized element into electrical engineering.

So as the next step what I will do is map it into mechanical area, so what we will find is that if I use a specific type of analogy then we will see that the through variable turns out to be force and we will see it how it so, and the cross variable in mechanical word comes out as velocity, so force is analogous to current it is a through variable and velocity is analogous to voltage, so we will see it.

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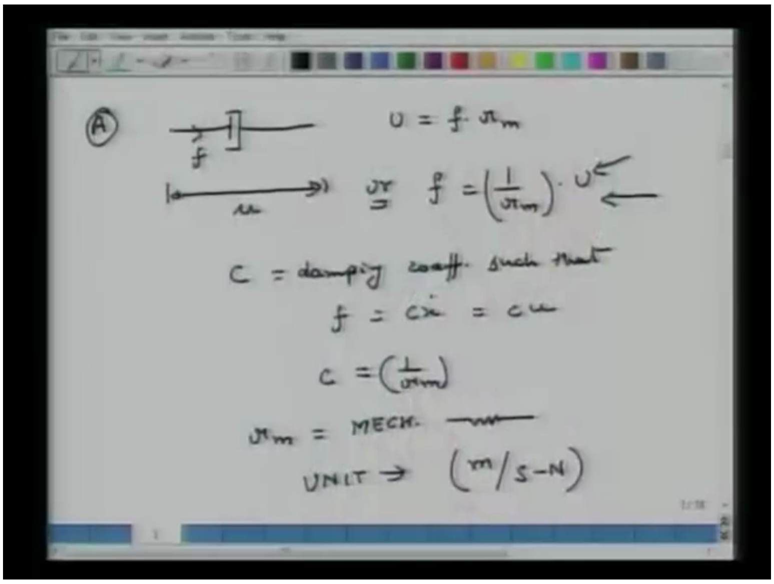

So the first element was a equals b dot c which is a resistance so what is a resistance in mechanical land it is a dash pot, it is dissipating energy so let us look at it. We have a dash pot the across variable is u the through variable is force so u equals force times mechanical resistance or force equals 1 over rm times u okay. So in laws of mechanics you have seen that f equals c x dot this is your x dot, u and c is 1 over rm right. So we have seen this equation.

Student: Sir then how force is through variable?

Because force is felt throughout the flow line, so this equivalence is preserved in spring and in a mass you will see, so it just happens that our traditional damping co-efficient which we use as c we will not use c as damping co-efficient in context of acoustics as it is being thought in this particular course.

So if c is damping co-efficient such that force equals c x dot equals c u then damping co-efficient is essentially 1 over rm and I call this rm as mechanical resistance, what is the unit of rm, it will be inverse of unit of damping co-efficient so that unit is meters per second newtons right, so as you are doing your calculation using this approach you should make sure that you do not equate c with rm it is, there is an inverse relation, that is a mistake that happens very often. So we talked about mechanical resistance or a dash pot.

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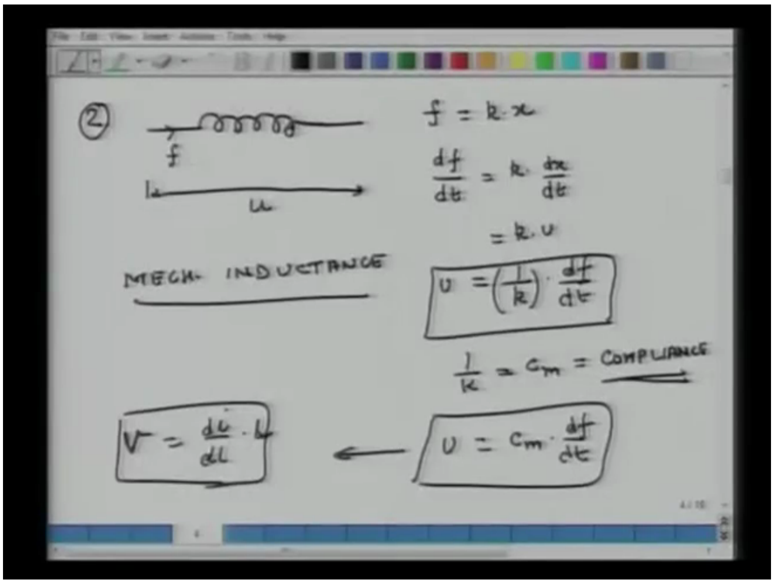

So the second two terminal element is a spring, just happens that the symbol is very similar so my through variable is f, my cross variable is u and we know that force equals stiffness times space value if I differentiate it I get this relation k times u okay.

So if I rearrange I get u equals 1 over k times time derivative of force or I can say if I have to find force is basically, so we will go that close to that later but 1 over k we term it as cm or compliance, so again this relation looks very similar if I write this u equals cm times df over dt this relation looks very similar to voltage equals di over dt times L.

So I am seeing again the same equivalence is being preserved, it is being preserved in the case of mechanical resistance, here i you have a derivative of i you have a derivative of f, you know voltage you have velocity, so that relationship is being preserved. So this is called compliance or it is also called mechanical inductance. Most circuit will use the term compliance but you should know someone uses mechanical inductance we know cm.

So resistance we saw that in case of a resistance the relationships are preserved, u being equal to voltage, equivalent to voltage, current being mapped into force and the element variables.

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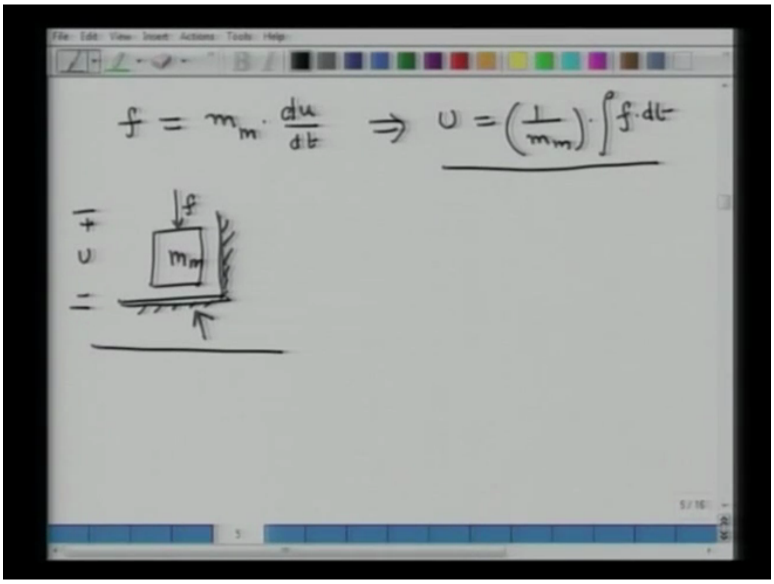

So now let us look at mass that is the third variable. So we know that force for a mass which is accelerating is mm times acceleration which is time derivative of velocity and I can rearrange it so I get u equals 1 over mm times f dt if I integrate it, so again this is again looking similar to the relation as you have in a capacitor so the equivalences are preserved amongst spring is similar to an inductance, mass is similar to a capacitor and a dash pot is similar to a resistor and how do I depict it.

So this is my mass I have through variable f and the cross variable is u which is velocity, this is the inertial frame, this thing is here inertial frame so this is one difference whenever we measure acceleration we measure it with respect to an inertial frame so I have drawn an inertial frame here, so whenever you have a mass in a system when you try to convert it into you know capacitive, something similar to a capacitor you have to make sure that you refer to, you always have a provide with a reference frame in an inertial frame, this is important.

We did what, so is that clear how the analogy between force, velocity and the current and voltage work and also how mass gets translated into a capacitor or spring gets translated into an inductor and dash pot gets translated into a resistance. What could be a four node or a four terminal element in mechanical area? Think about it, you have a transformer in electrical, what could be a four node element.

Student: Gear.

Gear is one, yeah very good. Gears or levers, you have a lever you have f1, v1 you know force and velocity on one side it gets translated into four singular in the other side and it is essentially the same ratio of length, the two lengths so that is what we will talk about and you can just map the same thing for gears also.

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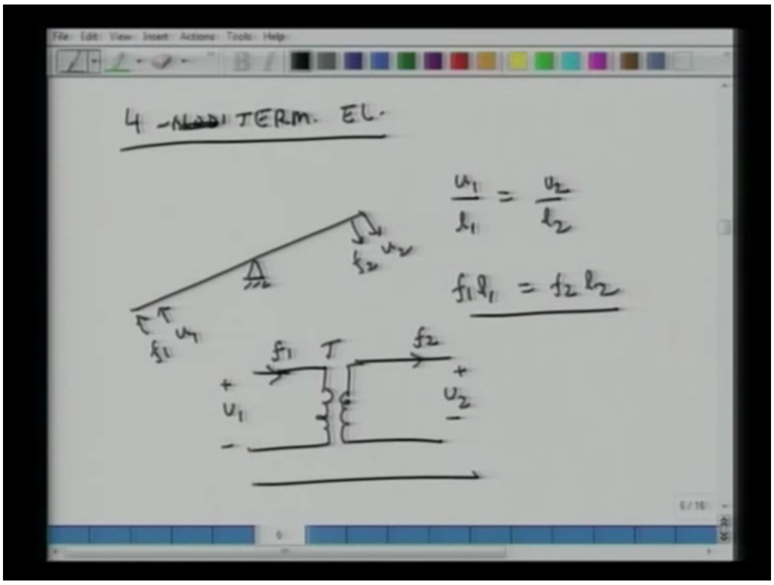

So a four node, four terminal element and the, so I have a lever and I can do the same thing for gear I have a fulcrum, I have two input variables f1 u1, f2 u2 then the relationships are u1 over l1 equals u2 over l2. f1 l1 equals f2 l2 and this is analogous to a transformer, so here it will be u1 plus minus, this is the through variable, this is the second through variable u2 and this is my term ratio, it is the ratio of the lengths and similarly we have a current source and a voltage source in electrical land.

You can similarly have a force source and a velocity source in mechanical land what is a force source if you have there some you know machines, inside testing machines you can either have a force controlled experiment or you can have a displacement velocity controlled experiment so when you are having a velocity controlled experiment essentially you are having a velocity source in the sense if you have a force controlled experiment then you have a force source in the system.

So these equivalences are preserved, a transducer, most of the transducers they control the velocity and the force is as an output of external conditions and the overall impedance outside but the transducer knows how much to move back and forth so if it knows how much to move back and forth then it is a velocity controlled so you have a velocity source.

Now there are you know other types of transducers also so you have to know which type of sources you are placing in the system okay. So we have seen different types of equivalences between mechanical elements and electrical elements so we should also see one more special thing about transformer which could be a gear or an electrical transformer or a lever that it is possible if you have drawn a circle to eliminate the transformer at all by appropriately changing the value of some inductances. So that is what we will see.

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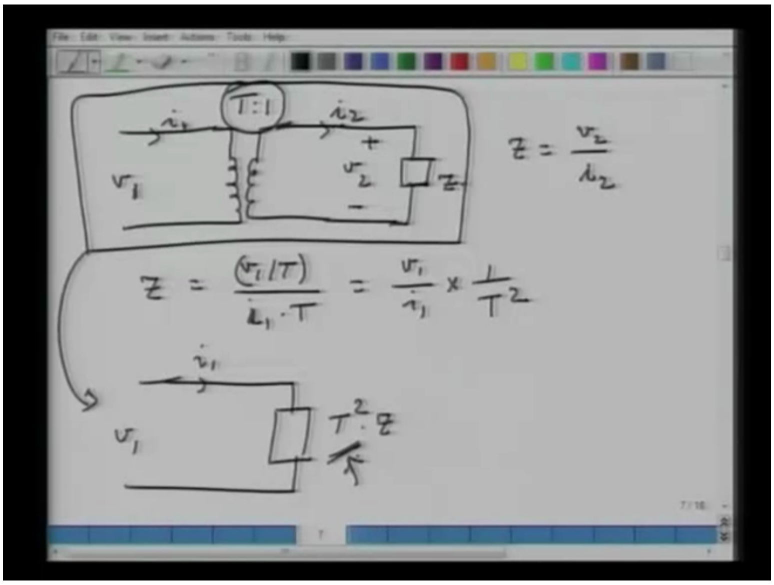

So you have a transformer let us say its turn ratio is 3 is to 1 again these are my 1 2 so and this is connected to an external impedance which is Z okay so we know that Z equals v2 over i2 so Z equals v1 over T over i1 times T right equals v1 over i1 and bear in mind when I am writing v and i actually I should have been a little bit more careful these should be in upper pose because these are vectors because there may be a phase component and Z.

So v1 over i1 times 1 over T square now what this means is that I can replace this entire box by another box which will be something like this right, is that clear, so whenever you have a transformer or whenever there is a transformation happening, this important to understand whenever there is a transformation happening from electrical to mechanical and if you can figure out what is that ratio.

For instance in a coil which moves back and forth force equals BIL right, from laws of electro mechanics so the turn ratio there is BL because force equals BIL so then you can have a mechanical circuit, you can also have an electrical circuit and they are initially broken because of the transformer but you can remove that transformer and combine the electrical and mechanical circuit into one single circuit and you can change the impedance on the mechanical side by using this factor, right. So we will do some couple of examples and how these transformations happen.

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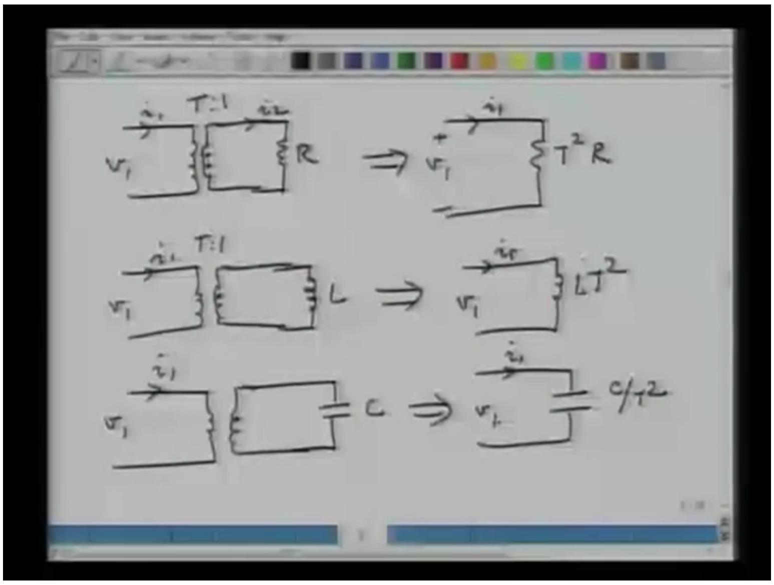

So in case of a resistor and we will do more as we draw complete circuits of loudspeakers or microphones. So you have v1 and I have a resistance r and my turn ratio is 3 is to 1, i1 i2 so this is equivalent to T square R so I can eliminate the transformer and I can just replace it by an increased resistive load or change resistive load, change by factor of T square so the other one is an inductor and in this case I have L as the impedance so this transforms to L times T square and the last one is of the capacitor.

So I have a capacitance C this transforms to C over T square because the impedance is inverse of C. So again some of these concepts are drawn from electrical engineering but you will find them extremely useful as we try to solve acoustics problem and so mechanical problems.

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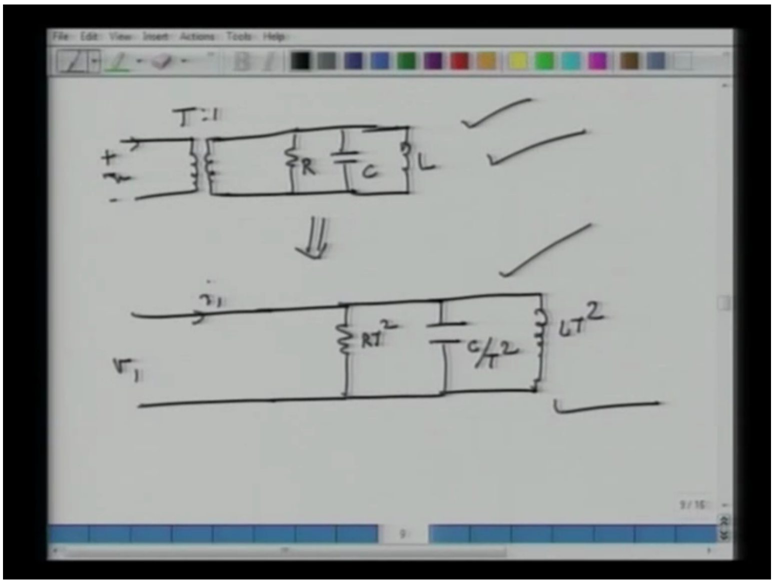

We will do one more example and see how it changes so if my circuit is like this and the turn ratio is T is to 1 let us say it is a RLC circuit R L C then this is changing to, my resistance value changes to RT square L goes to LT square and capacitance changes to C over T square so it does not matter you can solve this circuit or you can solve this circuit the answers will be same.

So depending on the situation you can figure out do I want to solve this circuit or do I want to solve this circuit, whichever appears easier you can use that. So we have talked about how resistances, inductors, capacitors transform into mechanical equivalent and back and forth how transformers can be changed back and forth and also how sources in electrical area map into mechanical area and vice versa.

As we are doing this analysis in circuit analysis for electrical engineering if it is a complex circuit then we use two laws to solve for different currents and voltage, one is the Kirchhoff's current law and the other one is Kirchhoff's voltage law so you can again we will need similar laws for mechanical area, mechanics area to solve electro mechanical problems.

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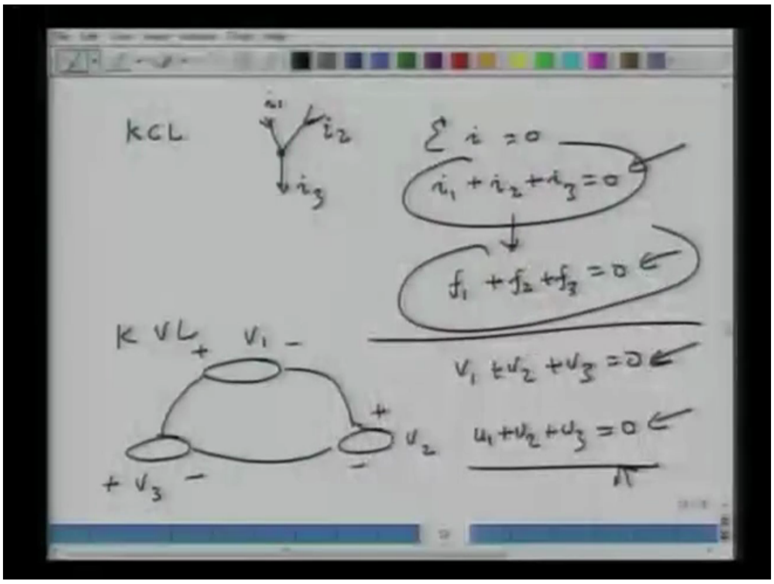

So we have a Kirchhoff's current law where through a node if I have i3 i1 i2 then the Kirchhoff's current law says that through a node i equals 0 that is i1 plus i2 plus i3 equals 0 and this maps, if I have a node a current node for in the mechanical area what this also translates to this f1 plus f2 plus f3 equals 0 and this is essentially your Newton's.

So Kirchhoff's law is similar to Newton's Law, they are not same but they are similar. Similarly you have Kirchhoff's voltage law that if you have three elements and a close circuit you know and the voltage across third element is v3 minus plus here you have v1 plus minus and here you have v2 plus minus then KVL says V1 plus V2 plus V3 equals 0 and this comes from law of conservation of energy, there is an assumption here that there is no external flux but with do not have those conditions then this holds good.

In mechanical land this translates to u1 plus u2 plus u3 equals 0 and that is basically your law of conservation of mass continuity. So these are important rules to remember as you are trying to do an electro mechanical circuit analysis so we will one or two very quick examples and see how we can use some of these techniques.

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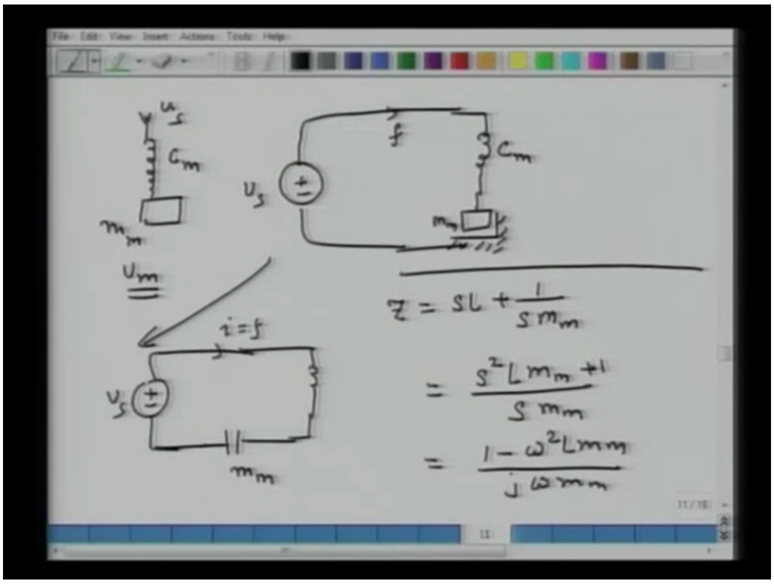

So let us say I have a very simple spring mass system, so the mass is mm the spring has a stiffness of k which is or a compliance of cm and it is being excited by some source velocity so let us say if I am interested to find what is velocity of the mass, we want to know what is the velocity of the mass in other words I want to find if I map into electrical word what is the potential difference of the mass because velocity is mapping to voltage.

So I make a circuit out of it so U s is like a voltage source, what is going through the circuit, force is flowing through the circuit and then I have a spring or a inductor of inductance Cm and then I have a mass which is grounded so I can now convert it into formally electrical circuit so I have a U s my i equals force, U s is velocity source I know this and I can put a capacitance here and again because capacitance is grounded that condition is being satisfied and this value of capacitance is mm okay.

So my total impedance in the circuit Z equals, i equals mm dv over dt, so Z equals SL, S is J omega plus 1 over S mm equals S square L mm plus 1 over S mass so I put S as J omega so what I get is 1 minus omega square L mm over J omega mm is that right, yeah. So now I want to find the velocity of this mass or the voltage difference across this capacitor so I can find the current I multiply by the impedance offered by mm.

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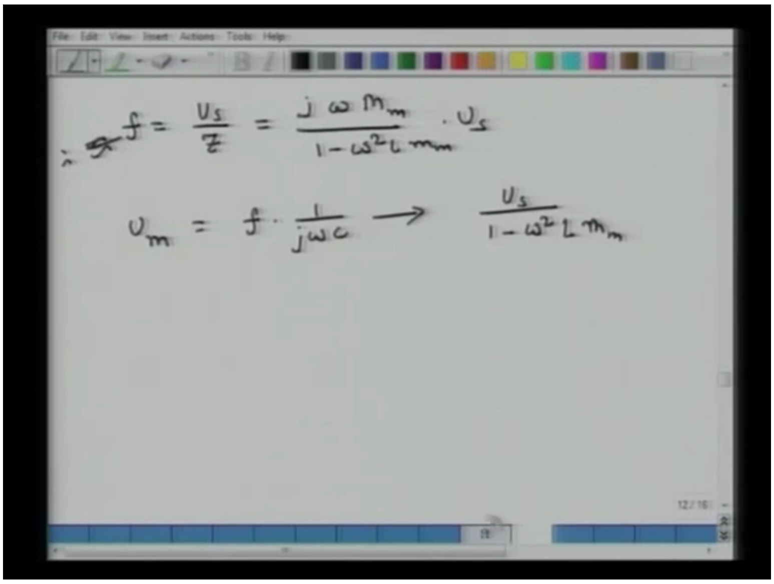

So the force which is same as current when I map it is U s over Z and that comes to be j omega mm over 1 minus omega square L mm times U s. So my velocity across the mass U m is, this is current force right this is equivalent to current so is force times impedance of the capacitor 1 over omega c j so this gives me U s over 1 minus omega square L mm so instead of doing lot of force equilibrium and solving differential equation I am relatively able to solve for the velocity of moving mass by using some concept of circuit analysis, right. So that is the whole point.

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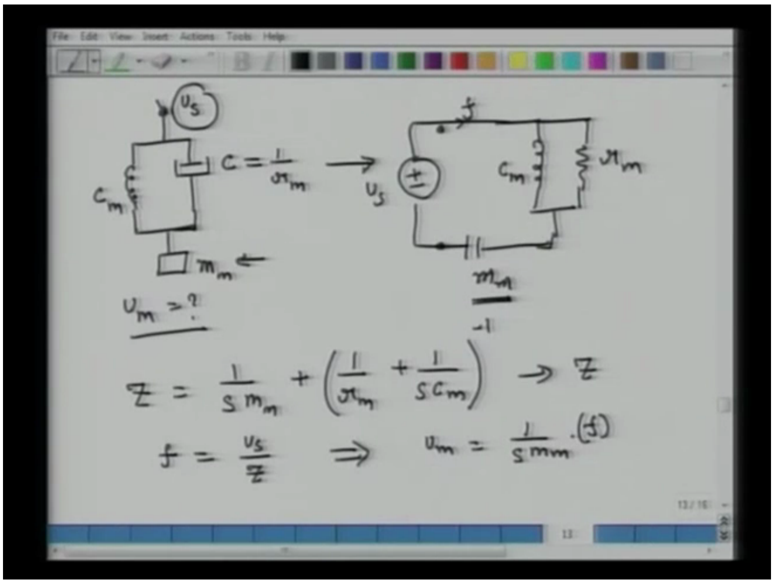

Let us do one more example so now you have a spring of compliance Cm, a dash pot and both these springs and the dash pot are connected to moving mass mm, the damping co-efficient is C which is inverse of mechanical resistance we talked about it.

So if you know the damping co-efficient you should be able to figure out and I am at this point I am exciting it with a known velocity so it is source velocity so then the question is find the velocity of this moving mass so what is this value that is the question, so first I draw circuit and what I get is, so again I have U s here which translates into a voltage source of spring U s, current is f and then I have an inductor, I have a resistance and then I have a capacitance.

So value of capacitance is same as the mass of the object, value of inductance is Cm and this value is 1 over rm. So my overall impedance across these two points is 1 over S times moving mass which is because of the capacitance and then for the inductance and the resistance I get rm plus 1 over s times inductance with the cm and then I take inverse of that.

Student: Resistance should be rm not 1 by rm.

I have take inverse of this resistance and this inductance is

Student: (())(48:34) will be rm that is 1 by c.

Yes I m sorry this should be rm so here I should make it 1 over rm okay so through this I can figure out what is the value of Z using that Z, I can figure out that f equals U s over Z so I know the current flowing through the capacitor or the forcing by the mass and then from here I can evaluate what is the value of velocity.

So which is, so um equals 1 over S mm times force relatively simpler or a straight forward process so in this whole exercise what we have done today is establish couple of equivalents not couple, several equivalents between an equivalent between resistance and a dash pot, a spring and an inductance and between mass and capacitor and then force translates to or becomes equivalent to current and velocity transforms to voltage.

This is one way of establishing the equivalences. Now if you play with the variables in a slightly different way you can establish equivalences which are just the opposite so resistance will still map into a dash pot because it is dissipating energy but a spring can map into, here we saw that spring is mapping into an inductor but you can, you will be able to map a spring into a capacitor.

So either you use those set of principles or you use this set of principle which we spoke today you will get the same answers but you have to be as we are doing this mapping exercise we have to be sure that we are consistence. So there are people who will do the same sort of analysis but in their calculations force will translate into voltage and velocity will translate into current and in that particular scenario spring maps into a capacitor and so on and so forth.

So there are two different, very similar but still different approaches of doing this, conducting this transformations but at the end of the day what we have shown today is that we can bridge the gap between electrical engineering and mechanical engineering and we can create one single large circuit where electrical energy is coming in and as an output what you are having is motion being exhibited by the mass.

So if you have a transducer or a speaker which is moving back and forth and if you want to know that if I am putting in 100 watts of energy at 20 hertz how much this diaphragm is going to move, I should be able to calculate using this approach.

Now I still have not bridged the gap between motion and sound we have not talked about sound at all but then some of the subsequent lectures we will speak about how that motion gets transformed into sound based on some of the transmission line theories we studied earlier 1D wave equation and so on and so forth.

But through this way we have an approach how we can breach the realm of electrical engineering, mechanical and acoustics and in case of most of, not most, lot of electro mechanical acoustics systems like speakers, microphones, sensors this approach works fairly well because most of the elements are very close to lump parameters, so 1D approaches and lump parameters approaches work very well in a lot of applications so it is worthwhile understanding.

The other thing is that once you have created an elaborate circuit you may find that it has seven, five or six resistances, six or seven capacitances, several inductors how do you solve that, so one approach could be that you go through you know do it through the hard way and solve and take a lot of time but there are standard software packages in electrical engineering.

One package is P Splice so you take and it is, it has graphic tools so you pull a resistance, put it on the screen, connect it to a capacitor, connect it to you know a resistance connected to another capacitor or an inductor the way you have constructed the overall circuit excited through a voltage and you will get an output number.

So all that can be automated but we have to know how to construct those circuits so what we are learning is how to construct those circuits and then at some point of time we will see how we can use some of these standard tools, we dump all those circuits into those software tools and then solve for whatever variable we want to. So that is what I wanted to talk to you all today and in the next lecture we will further develop some of this stuff which we have been talking about. Thank you.