Acoustics
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Module 05
Directivity
Lecture 04
Directivity

Hello again, in last over last several lectures we have been discussing the notion of directivity and in that context we have explored the directivity pattern associated with the simple source you also explored the directivity pattern associated with sound source which is having two individual simple sources separated by some distance let us say that distance is D when with the condition that the value of this D is extremely small compared to the wavelength and also the radius or the distance between the point of observation and the source.

So the ratio of this distance and D is extremely large and once we did that then we moved further and then we explored the directivity pattern associated with more than two sources we started with four sources, six sources and then generalized it for situation where we have n sources. So we explored that if we have a very large number of sources and all this sources are lined up along one single straight line and also if these sources are evenly spaced and the volume velocities of these sources are identical and the phase difference between all these sources is exactly 0.

Then we can develop sound projection mechanism where sound prefers to travel along the 0 degree direction and the side lobes for the directivity pattern their magnitude tends to get very significantly suppressed. So today we will continue this discussion on directivity further and today what we are going to explore is the directivity pattern associated with the doublet.

Now a doublet is essentially a combination of two simple sound sources, it is a combination of two simple sound sources which are separated by very small distance and also the phase difference between these two sound sources is pi radians or 180 degrees, so that is what a doublet is, so we will see what kind of directivity pattern is associated with doublets.

Good example of a doublet could be simple speaker which is not mounted on a baffle so on the front side this particular speaker is producing a sound wave and let us say at instant T near the diaphragm the pressure is positive and magnitude with respect to reference pressure which is atmospheric pressure then on the back side of the diaphragm because this loudspeaker is unbaffled so the back side of the loudspeaker on the back side of the diaphragm this pressure is negative so there is a phase difference between the front side and the back side of exactly pi radians. So that is a good example of a doublet.

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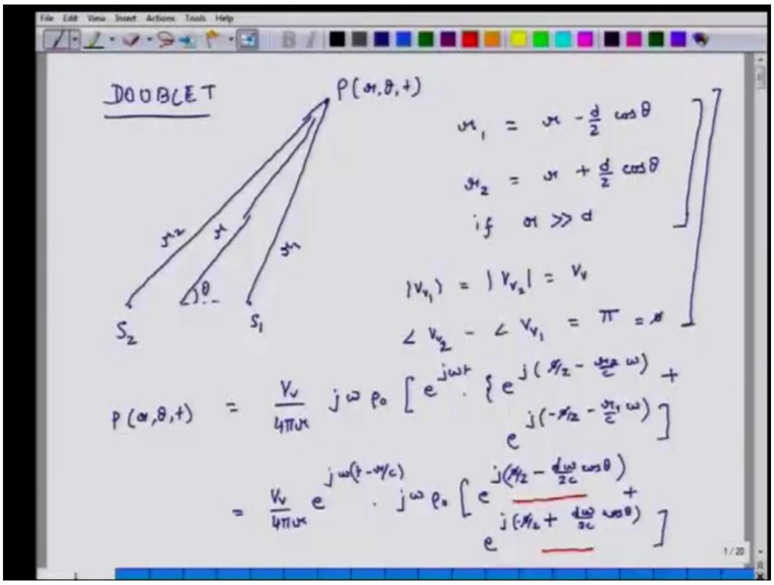

So this is the schematic, so let us say we have two sound sources and let us say this is S1 and this is S2 and I have a far off point and the distance between S1 so let us say this distance, so this is a point of observation and the pressure at point of observation is P which depends on r theta and t, and let us say this is the midpoint so I construct a line connecting the midpoint with the point of observation and let us say that distance is r, the distance of S2 with respect to P is r2 and the distance of S1 with respect to P is r1 and let us say this angle is theta.

So once again with this understanding we can write the relationship between r1 and r2 in terms of r such that r1 equals r minus d over 2 cosine of theta and r2 equals r plus d over 2 cosine of theta, this is true only if r is very large compared to d, so then this is the relationship between r1 and r and r2 and r, these are the relationships.

Further because we are considering a doublet source so we know that volume velocities Vv1 is equal to same as volume velocity Vv2 and that is Vv and the phase difference between that of Vv2 minus phase of Vv1 is equal to pi radians, so these are my basic assumptions and on the basis of these assumptions now I develop the pressure relationship at point P.

So P is a function of r theta and t equals Vv over 4 pi r j omega rho not e j omega t times e j phi over 2, here phi is the phase difference minus r2 over c omega plus e j minus phi over 2 minus r1 over c omega and that means this is equal to Vv over 4 pi r e j omega t minus r over c times j omega rho not, so I have taken j omega t and I have also substituted in one single step r2 and r1 by these respective relations so in the parentheses what I get is e j phi over 2 minus d omega over 2 c cosine theta plus e j phi over 2, this should be negative minus d omega over 2 c cosine theta and excuse me there has to be a positive sign here.

So the term in brackets this can be simplified as it can be simplified further because e to the power of j times some angle alpha is a cosine component and an imaginary component which has a sine term in it.

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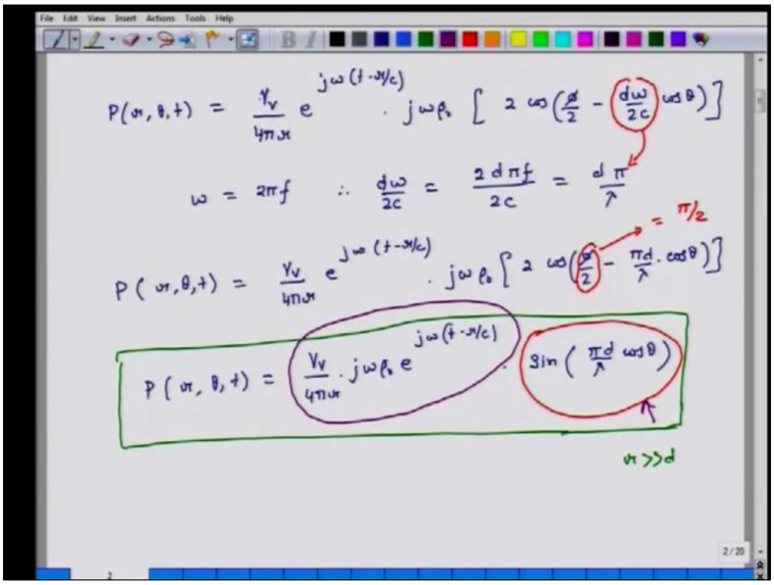

So I can write the expression for pressure, complex pressure as Vv over 4 pi r e j omega t minus r over c times j omega rho not and then in the parentheses if I resolve this two terms in terms of their imaginary and real components and do the mathematics what I get is 2 cosine of phi over 2 minus d omega over 2 c cosine theta.

Now we know that omega equals 2 pi f therefore d omega over 2 c equals 2 d pi f over 2 c is equal to d times pi over lambda so I can replace d omega over 2 c with this so what I get for my expression for pressure in a doublet is equal to Vv over 4 pi r e j omega t minus r over c times j omega rho not 2 cosine phi over 2 minus pi d over lambda times cosine of theta.

Now we know that in case of a doublet the value of phi is pi radians so this term, this equals pi over 2 and if this equals pi over 2 then this cosine of pi over 2 minus some angle is essentially sine of that angle so I can finalize this relationship as P r theta t equals Vv over 4 pi r times j omega rho not e j omega t minus r over c times sine of pi d over lambda cosine of theta.

Now this relation is valid if r is large compared to d now as we saw in earlier cases this entire expression for complex pressure has a term which depends on theta which is circled in red and then there is another term which is circled in purple which does not depend on theta but it changes with r. So my directivity is going to be influenced by this term which depends on theta okay.

Now before we start analysing this equation carefully we have to just revisit this picture where we had located S1, S2 and P, so please bear in mind that S1 and S2 they are located along the X axis while and with respect to the X axis the angle theta is measured for point P, so with this understanding let us look at how the directivity of this system changes once we start changing theta.

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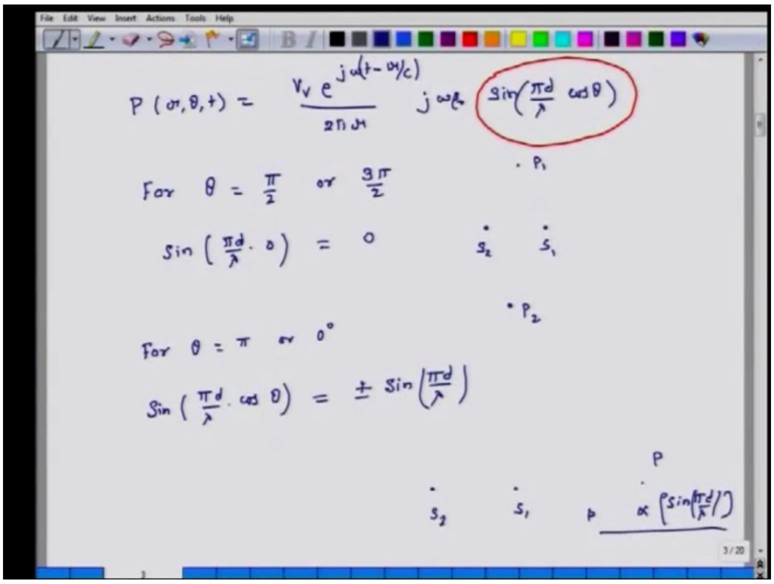

So we will rewrite P r theta t equals e j omega t minus r over c divided by 2 pi r j omega rho not and then sine of pi d over lambda cosine theta and we had said that this term which is circled in red is the component which influences the directivity of the doublet source very strongly okay so for theta equals pi over 2 or 3 pi over 2 this term sine of pi d over lambda times cosine theta is 0, so cosine theta is 0, so this becomes 0.

So what that means is that if I have two sources which are very close to each other and these sources are S1 and S2 then the overall strength of sound at this location because in this direction it is 0 here and also in excuse me, the overall strength of sound at this point P1 and also at P2 it will be 0, because theta equals pi over 2 and at pi over 2 or 3 pi over 2 the contributions of these two sources they tend to cancel each other.

Now for theta equals pi or 0 degrees sine of pi d over lambda equals cosine of either 0 degrees or it is pi degrees, 0 degrees or pi radians so in that case this is equal to if it is 0 degree then cosine of 0 is 1 then it is sine pi d over lambda or if it is pi radians then it is minus sine pi d over lambda then it is plus or minus of sine pi d over lambda.

So what this means is that if I have two sources S1 and S2 then the strength of the signal along the in the 0 degree direction in this, so this is point P, this is one possible location then the strength of this signal at point P will be directly proportional to sine of pi d over lambda. So pressure will be directly proportional to sine of pi d over lambda. So this is how doublets behave that along the vertical axis the pressure field is 0, along the horizontal axis it is directly proportional to sine of pi d over lambda so it all depends on the ratio of d over lambda.

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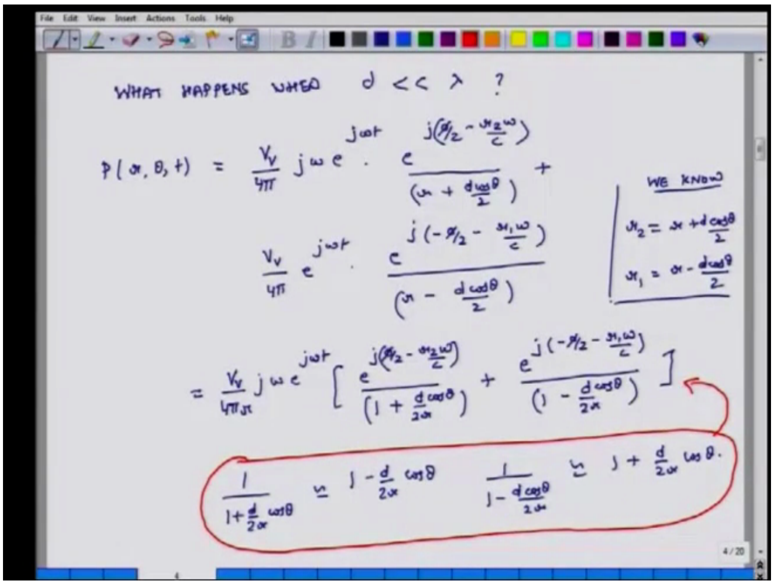

Now let us look at another more interesting case for doublets itself that what happens when d is extremely small compared to lambda, it is extremely small compared to lambda so when d is very small compared to lambda how does a doublet behave what kind of directivity pattern do we see for such doublets so that is what we are going to explore in over next 15 to 20 minutes.

So to understand this we have to revisit our pressure relation and we will rewrite the equation and the equation is for pressure field is going to be contribution of first source plus contribution of second source so contribution of first source is Vv over 4 pi j omega e j omega t times e j phi over 2 minus r2 omega over c divided by r2.

Now earlier please bear in mind that in the denominator we had assumed r because we assumed that for large values of r, r2 and r are almost equal so in that case we had approximated r1 is approximately equal to r and r2 is also approximately equal to r, but we are not going to make that assumption here and we are going to explicitly write term r2, so this is what I have written here is the contribution of the pressure from sound source S2.

Similarly there is a contribution from the first source S1 and that is Vv over 4 pi e j omega t times e j times minus phi over 2 minus r1 omega over c and then again in the denominator I do not put r rather I put explicitly r1 so I am not here going to assume that r1 is same as r and r2 is same as r.

Now we know that if d is small distance between these two sources is extremely small compared to r and that is a valid assumption then we know that r2 equal r plus d cosine theta over 2 and r1 equals r minus d cosine theta over 2 so we are going to put these two expressions for r2 and r1 in my expression for P.

So what I am going to do is I am going to erase this and I am going to replace r2 with r plus d cosine theta over two and I am going to replace r1 as r minus d cosine theta over 2, okay. So I am going to now rework this equation and I am going to organise some terms so Vv over 4 pi j omega e j omega t and from the denominator I am going to take out r as the common number.

So I am going to place r here so what I am left with in the parenthesis is e j phi over 2 minus r2 omega over c divided by 1 plus d cosine theta over 2 r plus contribution from the source s1 and that is e j minus phi over 2 minus r1 omega over c divided by 1 minus d cosine theta divided by 2 r.

Now we know that d over 2 r is very small compared to 1 we know that because d is extremely small compared to r and because of that cosine theta never exceeds 1 so d cosine theta over 2 r is extremely small compared to 1, so if that is the case then 1 over 1 plus d over 2 r cosine of theta is approximately equal to 1 minus d over 2 r cosine of theta and similarly 1 over 1 minus d of cosine theta divided by 2 r is approximately equal to 1 plus d over 2 r cosine of theta. So we put these relationships in this equation in the denominator.

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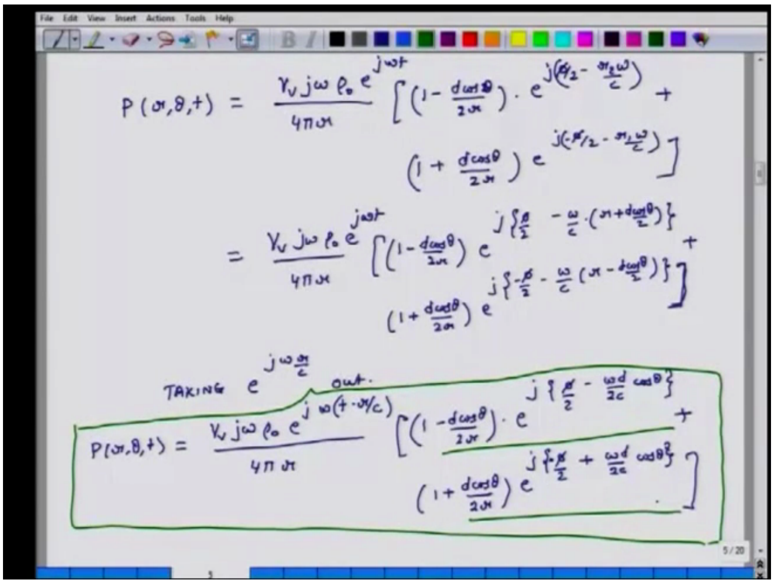

And what we get out is P r theta t equals Vv j omega rho not e j omega t divided by 4 pi r and in the parenthesis what I have is 1 minus d cosine theta divided by 2 r times e j phi over 2 minus r2 omega over c plus 1 plus d cosine theta over 2 r e j phi over 2 minus r1 omega over c okay.

At this stage I in the exponent term, I replace r1 and r2 by these relations, r1 equals r minus d cosine theta over 2 and r2 equals r plus d cosine theta over 2. So what I get is, so I have this Vv over times j omega rho not e j omega t over 4 pi r times 1 minus d cosine theta over 2 r times I have to be careful here e j minus phi over 2 and then I have r2 is r plus d cosine theta over 2 so minus omega over c times r plus d cosine theta over 2 this my first term and this is because of the source S2 and then I add contribution of S1 so 1 plus d cosine theta over 2 r e j here I have excuse me I should not have at, phi should be positive with a positive sign.

So here I have minus phi over 2 minus omega over c and then parenthesis r minus d cosine theta over 2, this is my overall relation. Now we take taking e j omega r over c out of this parenthesis because it is common to both these terms what we get is P r theta t equals Vv j omega rho not e j omega t minus r over c divided by 4 pi r so I have taken out e j omega r and I have combined it with the time term.

So I get omega times t minus r over c and then in parenthesis I am left with 1 minus d cosine theta over 2 r times exponent of j phi minus omega d over 2 c cosine theta plus 1 plus d cosine theta over 2 r exponent of j phi, oh there should be phi over 2 and then this is minus phi over 2 and then I have a positive sign omega d over 2 c cosine theta.

So this is my relation now I am going to simplify this relation further and by decomposing this term and this term and then again clubbing terms which looks similar together.

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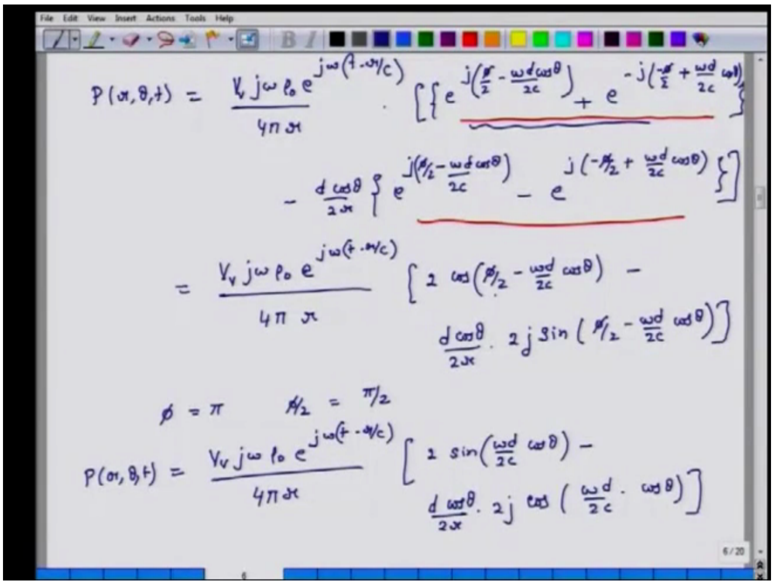

So what I get is P of r theta t equals Vv j omega rho not e j omega t minus r over c divided by 4 pi r times so what I am going to do is club terms which have co-efficient of 1 together and co-efficient d cosine theta over 2 r again together so what I get is e j phi over 2 minus omega d cosine theta over 2 r plus e minus j minus phi over 2 plus omega d over 2 r cosine theta so this is one set of terms plus d cosine theta over 2 r so this is one set of terms and then another set of terms is having a co-efficient of d cosine theta over 2 r and actually there should be a negative sign here and then in parenthesis I get e j phi over 2 minus omega d cosine theta over 2 r, excuse me should be 2 c, omega d over 2 c so it should be 2 c here as well and minus exponent j minus phi over 2 plus omega d over 2 c cosine of theta.

Now if I add these two terms after decomposing them into their real and imaginary parts, these two terms after I decomposing them into their real and imaginary parts then what I get is cosine term and in this case I am left with only the sine term multiplied by j so what I get from such complex algebra manipulations is Vv j omega rho not exponent j omega t minus r over c divided by 4 pi r and in the parenthesis I am left with 2 cosine phi over 2 minus omega d over 2 c cosine theta plus actually it should be a negative sign here minus d cosine theta over 2 r times 2 j sine of phi over 2 minus omega d over 2 c cosine theta.

Now we know that this is a doublet and for a doublet phi equals pi so phi over 2 equals pi over 2 radians so if that is the case then P r theta t equals so once I put pi over 2 in place of phi over 2 this cosine becomes sine term and this phi over 2 term goes away, similarly the sine term becomes cosine terms and phi over 2 term goes away.

So what I get is Vv j omega rho not e j omega t minus r over c divided by 4 pi r and in the parenthesis what I have is 2 sine omega d over 2 c cosine of theta minus d cosine theta over 2 r times, so actually I will write this in the next line minus d cosine theta over 2 r times 2 j and then I have a cosine term omega d over 2 c times cosine of theta, okay.

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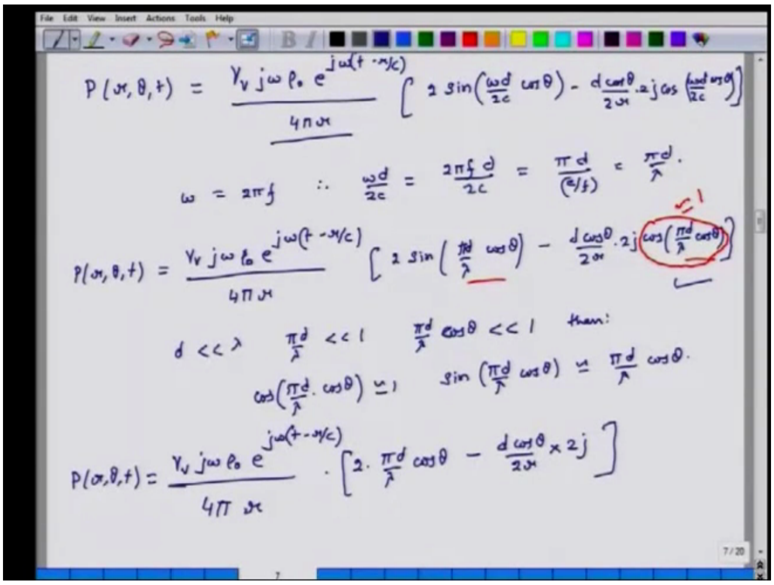

So I will rewrite this equation P r of theta equals Vv j omega rho not e j omega t minus r over c divided by 4 pi r times 2 sine omega d over 2 c cosine of theta minus d cosine theta over 2 r times 2 j and sine of omega d cosine of theta over 2 c. Now at this stage I replace omega by 2 pi f so omega is equal to 2 pi f therefore omega d over 2 c equals 2 pi f times d over 2 c equals pi d over c over f equals pi d over lambda.

So I rewrite this relation as P r theta t equals this whole term, I am not going to rewrite it times 2 sine and omega d over 2 c is nothing but pi d over lambda so pi d over lambda cosine theta minus d cosine theta over 2 r times 2 j times, so I am going to rewrite this relation using this approximation omega d over 2 c equals pi d over lambda and what I get from there is P r theta t equals Vv j omega rho not e j omega t minus r over c divided by 4 pi r and in parenthesis I have 2 sine omega d over 2 c cosine theta minus d cosine theta over 2 r times 2 j and then I have so I put parenthesis here cosine of, so here omega d over 2 c I will replace it by pi d over lambda.

And here it is cosine of pi d over lambda cosine of theta. Now we know that if d is very small compared to lambda, if d is very small compared to lambda then we can also say that pi d over lambda is very small compared to 1 and because cosine theta is at the max only 1 so pi d over lambda times cosine theta which appears here as well as here so this is also very small compared to 1.

If that is the case then this term cosine of pi d over lambda times cosine theta this term this becomes approximately equal to 1 because the term in parenthesis pi d over lambda times cosine theta is very small and approximately equal to 0, and also so I write this approximation so if that is the case then cosine pi d over lambda times cosine theta is approximately equal to 1 and sine of pi d over lambda cosine of theta is approximately equal to whatever value is there within the parenthesis is approximately equal to pi d over lambda cosine of theta.

So now I put these approximations in this equation and what I get is P r theta t equals Vv j omega rho not e j omega t minus r over c divided by 4 pi r times 2 and then sine of this entire term is nothing but same as the term so in parenthesis I have 2 times pi d over lambda cosine theta minus d cosine theta over 2 r times 2 j because cosine theta of pi d over lambda times cosine theta is 1, so I have simplified this even further.

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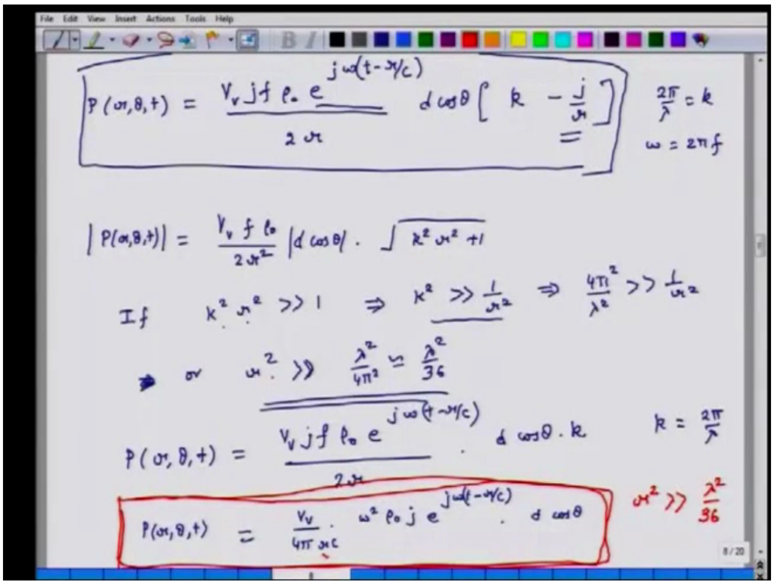

So I will rewrite this equation so P r theta t equals Vv j omega rho not e j omega t minus r over c divided by 4 pi r times 2 pi d over lambda cosine of theta minus d cosine of theta over 2 r times 2 j. Now I can take d cosine theta, I can take this term out of the parenthesis with this is common so what I get is I replace this parenthesis with the following relation so I am taking 3 cosine theta out and what I am left with in the parenthesis is 2 pi over lambda minus j over r and 2 pi over lambda is k which is the wave number so I rewrite this equation further and I write it as k and of course I have also taken this 2 term out so this 2 term cancels with 4 and I am left with, excuse me.

And if I now process this equation further and replace omega by frequency then what I get is, I replace omega by frequency so f omega equals 2 pi f and once omega equals 2 pi f if I put this relation here then pi in the denominator goes away so my denominator becomes 2 r so this is my overall relationship, this is the overall relationship.

Now for a simple source the directionality pattern is symmetric around origin, now we will look at some special cases as to how this directionality pattern looks like for different values of r. So magnitude of P r theta t equals magnitude of this portion, so it is Vv f rho not e, excuse me, so magnitude of this portion is 1, so I put 1 here and then divided by 2 r magnitude of d cosine theta is d cosine theta in vertical brackets times magnitude of k minus j over r and that is nothing but k square plus 1 over r square or if I process this relation further what I can get is k square r square plus 1 and then I take r out so I get 2 r square in the denominator.

Now if k square r square is very large compared to 1 which means k square is very large compared to 1 over r square which means now k is 2 pi over lambda so it is 4 pi square over lambda square is very large compared to 1 over r square and this, so in that case or you can say or if r square is extremely small compared to lambda square over 4 pi square which is approximately equal to lambda square over 36.

So for this condition which means k square r square is extremely large compared to 1 for this kind of a condition I can omit 1 in the magnitude and in that case this imaginary component in the parenthesis it goes away so for this case e r theta t equals Vv j f rho not e j omega t minus r over c divided by 2 r times d cosine theta times k, here k is equal to 2 pi over lambda.

So what we see here is, yeah so this is one relation and I can rework and play with lambda f and c and what I can get is the following relation Vv over 4 pi r c times omega square rho not j e j omega t minus r over c times d cosine theta, so this is one relation and this is good if r square is extremely small compared to lambda square over 36 so for very small values of r such that r square is extremely small to lambda square over 36 my pressure for doublet is given by this relation and what we have to note here is that in this case the dependence on radius is 1 over r okay.

Now for the case r square is extremely large compared to lambda square over 36 if that is the case then in that case the other term goes away. So I have to make a small correction here that we had seen that if k square is extremely large compared to 1 over r square that means that if 4 pi square divided by lambda square is very large compared to r square which means that r square should be extremely large and not extremely small it should be extremely large compared to lambda square over 4 pi square.

So this relation which has been boxed in red is good for the condition when square of radius is extremely large compared to lambda square over 36 what this means is that if I am far away from the doublet source then pressure decays in 1 over r fashion because I have r in the denominator okay. So that is first condition, the second condition is that what happens if r square is extremely small compared to lambda square over 4, so in that case.

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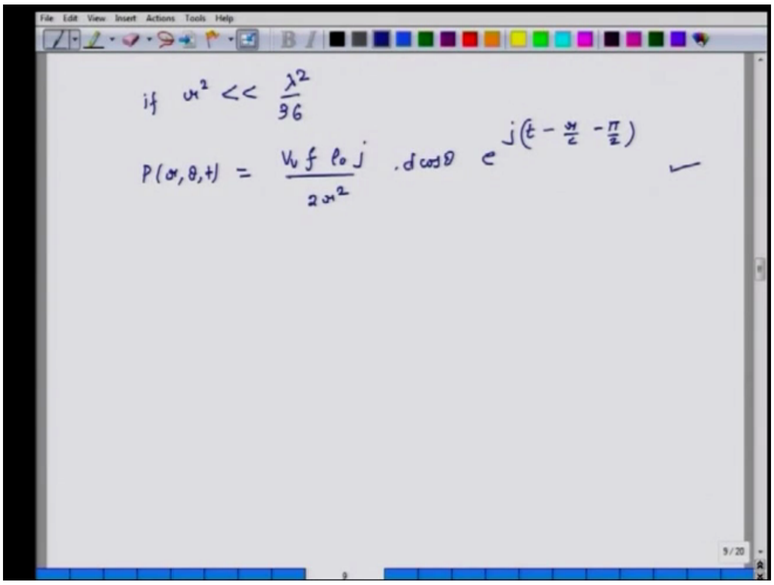

So the second condition is if r square is extremely small compared to lambda square over 36 so in that case using similar approach if r square is extremely small then this term dominates versus this term so I can drop the key term here and essentially what I get is that P r theta t equals Vv f rho not j over 2 r square times d cosine theta times e j t minus r over c minus pi by 2, this is the second relation.

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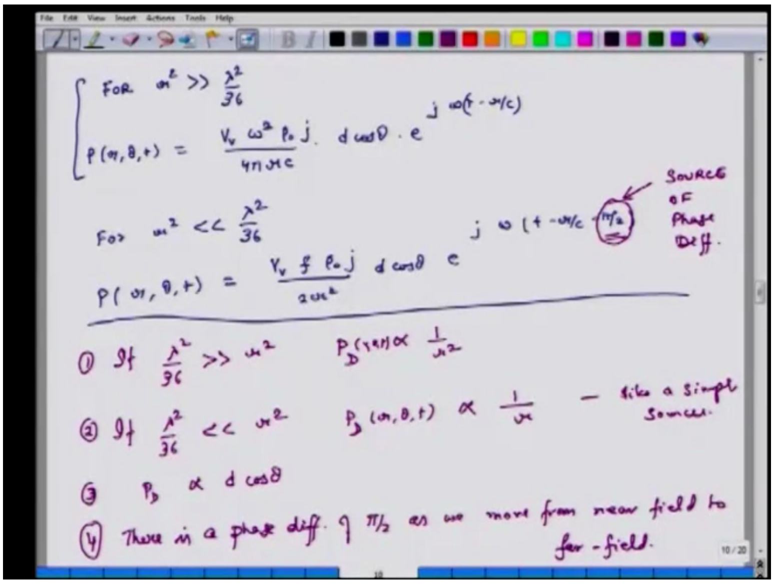

So I will rewrite on a clean slide these two relations for r square very large compared to lambda square over 36, P r theta t equals Vv omega square rho not j over 4 pi r c times d cosine theta times e j omega t minus r over c and if for, so this is one set of relations and if r square is extremely small compared to lambda square over 36, then P r theta t equals Vv f rho not j over 2 r square d cosine theta e j omega t minus r over c minus pi over 2.

So we make couple of observations here, first observation is that if lambda square over 36 is very large compared to r square then pressure of the doublet which depends on r theta t decays in 1 over r square fashion as r grows. Second, if lambda square over 36 is extremely small compared to r square then PD pressure in doublet which again depends on r theta t this also depends on r theta t is directly proportional to 1 over r so this is like a simple source.

So if I am far away from the doublet then the decay of pressure follows the magnitude of that decay follow something similar to that of a spherical source and third thing is PD is directly proportional to d cosine of theta so that put some polarity in the system and the fourth and the last point which I will like to make here is that the phase difference between the pressure when I am near to the system and near to the doublet and far away from the doublet that equals pi over 2 so what that means is that there is a phase difference of pi over 2 as we move from near field to far field and that phase difference comes from this term, this is the source of phase difference, okay.

So this concludes our discussion on directivity and what we have seen over last several lectures is several aspects of directivity as the (())(56:36) into a simple point source which has spherical directivity pattern which is symmetric about the origin and all the directions and also we have explored what kind of directivity patterns are depicted by two sources which are close to each other, multiple sources and finally we have also covered a doublet source.

So this covers directivity in a fairly comprehensive way specially in context of simple and in context of simpler systems of sound sources and we can extend this understanding we have more complex arrays of sound sources we can use this information and this knowledge to predict their directivity patterns as well. Thank you very much.