Acoustics
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Module 05
Directivity
Lecture 03
Directivity

Hello in last class we were exploring the notion of directivity of different sound sources or different combinations of simple sound sources and till so far what we have covered is how two sources interact with each other and the consequential directivity patterns which are, which emerge because of the interaction of these two sound sources.

As a follow up to that understanding we want to explore how more than two sound sources interact with each other and what kind of directivity patterns emerge when more than two sound sources interact with each other, in context of that we had developed the relation for directivity associated with two sound sources and one relation which we had developed was this.

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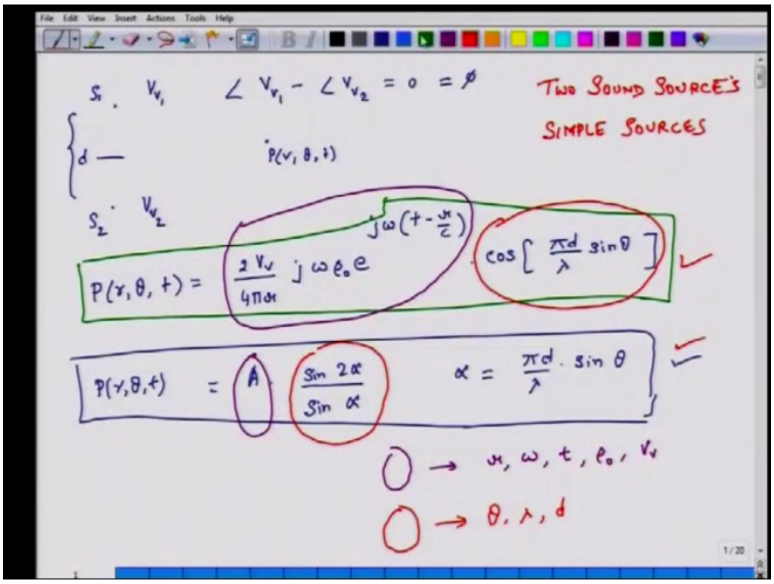

So if there are two sound sources, S1 and S2 and they are separated by distance d and the volume velocity of first sound source is V1, volume velocity of second sound source is V2 and the phase difference of two sound sources is suppose 0 then for that kind of situation we had found that the pressure at a point which is far away the pressure at a point which is far away and which is r distance away from the midpoint of these two sound sources, the expression for that complex pressure can be written as 2 Vv over 4 pi r j omega rho not e j omega t minus r over c times cosine of pi d over lambda times sine theta and minus phi by 2.

But we have this phi is the phase difference between the volume velocity of these two sound sources but because we have assumed that this phi is 0 so I am going to drop this out and close the bracket so this is one relation which we had developed and this relation has been developed several lectures back in time.

In last lecture we developed an equivalent relation and the relation for that is some term which is dependent on radius and omega and time times sine of 2 alpha divided by sine of alpha where alpha equals pi d over lambda times sine of theta so this is another expression for the same pressure, both this expressions give us same exactly the same answer but they are in different forms.

But the reason we have developed the second form is because we will see later that the second form becomes helpful, the approach to developing the second form becomes helpful when we have a very large number of sources and specially if the number of sources becomes infinite.

In both this terms there is a term which depends on the angle and this term is highlighted in red and there is another term which depends on radius, volume velocity of source, omega, density of air and time and I am going to highlight that term in purple, circle that term in purple.

So the term which has been circled by purple colour that depends on radius, omega, time, density and volume velocity while the term which is in red, it depends on theta, lambda and the distance between two sound sources. So our focus as we talk more about this will be on the term which has been circled in red because we are trying to understand the polar pattern or the directivity pattern of the combination of several sound sources.

Now this, these two expressions are for two sound sources which are simple in nature, these are simple sound sources and also the assumption has been that the radius or the point of observation is very far compared to the distance between these two sound sources S1 and S2, now we will go to the next step and we will develop a similar relation when we have four sound sources.

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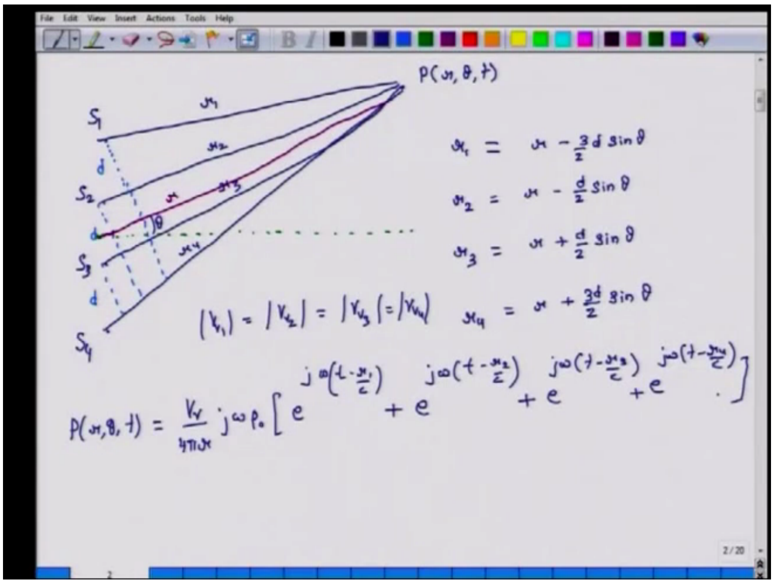

So suppose I have four such sound sources and these are S1, S2, S3, and S4, okay, and the distance between these sources is d, d, and d. So using a similar approach as I did earlier I am going to construct a midline and then there is a midpoint and from this midpoint and then my point of observation which is far away from the sound source is this and I am interested in finding again P r theta and t.

So as I did earlier in case of two sound sources, I am going to construct lines which connect these two sources from S1 to P, S2 to P, S3 to P and S4 to P and I call this lines or vectors as r1, r2, r3 and r4. Just as I did earlier I am going to draw a normal from each of these sources and what I can say is and suppose my midline, the midpoint when I connect to the point of observation is and that is distance r away then I can develop relations between r1, r2, r3 and r4 with respect to r.

So these relations are, so if this is the angle theta then these relations are r1 equals r minus 3 over 2 d sine theta, r2 equals r minus d over 2 sine of theta so I am sorry this theta is this angle, theta is this angle, r3 equals r plus d over 2 sine theta and r4 equals r plus 3 d over 2 times sine of theta.

We again assume that the volume velocity of all these sound sources is same so Vv1 magnitude equals Vv2 equals Vv3 equals Vv4 and we also further resume that the phase angle between each of this volume velocity sources is exactly 0 so the phase is 0. So with these assumptions I can develop an expression for pressure at point P and that depends on radius theta and t and that equals Vv over 4 pi r j omega rho not and then because I have four sources I have to add contribution of each source so the contribution of first source is e j omega t minus r1 over c, contribution of second source is e j omega t minus r2 over c plus e j omega t minus r3 over c plus e j omega t minus r4 over c.

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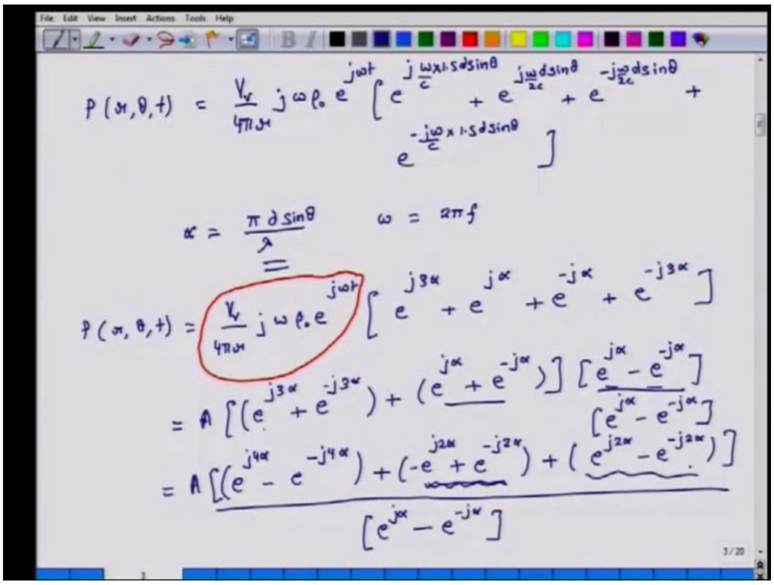

So there are four sources and these are the contributions of each of these sources individually now I can plug in these relations for r1 and r2, r3 and r4 in this equation and because of that what I get is pressure equals Vv over 4 pi r j omega rho not and I can take e j omega t out, so what I get is and in the parenthesis what I get is exponent j times omega times 1.5 d sine theta over C plus e j omega d sine theta over 2 c plus e minus j omega over 2 c d sine theta plus e minus j omega times 1.5 d sine theta over c, now at this stage I put alpha equals pi d sine theta over lambda and I also put omega equals 2 pi f.

So once I do that what I get is P which is a function of r theta and t equals Vv over 4 pi r j omega rho not e j omega t and in the parenthesis I get e j 3 alpha plus e j alpha plus e j minus alpha plus e minus j times 3 alpha where alpha equals pi d over sine theta, pi d sine theta divided by lambda I call this term, so once again this term it does not depend on theta so I call it A.

And essentially what I get is this expression e j 3 alpha plus e j 3 alpha minus plus e j alpha plus e minus j alpha and then I can subtract and divide this entire expression by a common term and this common term is same as that which I used in the expression when I was deriving the expression for the case when we had two sources so I can subtract, multiply and divided by e j alpha minus e minus j alpha divided by e j alpha minus e minus j alpha.

Now if I do the mathematics what I get is outside of parenthesis I have A and then I multiply e j 3 alpha with e j alpha to get e j 4 alpha I multiply e j minus 3 alpha times minus e minus j alpha so I get e minus j 4 alpha so that is 1 set of terms then I multiply e j 3 alpha with e minus j alpha to get minus e j 2 alpha and then I multiply e j e times e exponent of minus j 3 alpha with exponent of j alpha to get e minus j 2 alpha.

And then I multiply this term and the multiplicative term to get e j 2 alpha minus e minus j 2 alpha, all other terms we find they cancel out, so this term and in the numerator I still get e j alpha minus (())(16:43) j alpha. So in the numerator this term and this term they cancel out.

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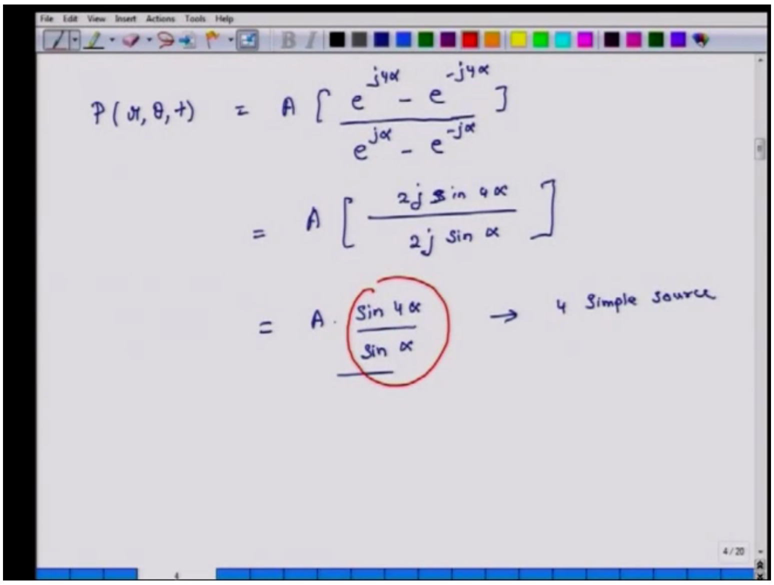

So my expression for p which depends on r theta and t that comes down to A times e j 4 alpha minus e minus j 4 alpha divided by e j alpha minus e minus j alpha and this becomes A and in the numerator I get 2 j sine 4 alpha in the denominator I get 2 j sine alpha so 2 j and 2 j they cancel out and essentially I am left with A times sine 4 alpha divided by sine alpha so this is for the case when I have four simple sources which are evenly spaced they lie in a particular straight line and these four sources have same volume velocity and the phase difference between the volume velocity of each of this sources is 0.

So to compare this with earlier case when we had two sources let us see the theta dependent term was essentially a ratio of sine 2 alpha divided by sine alpha when we had two sources, the theta dependent term when we have four sources is the ratio of sine 4 alpha divided by sine alpha so expectedly we should get when we have six sources we should get something like sine 6 alpha divided by sine alpha so let us explore that.

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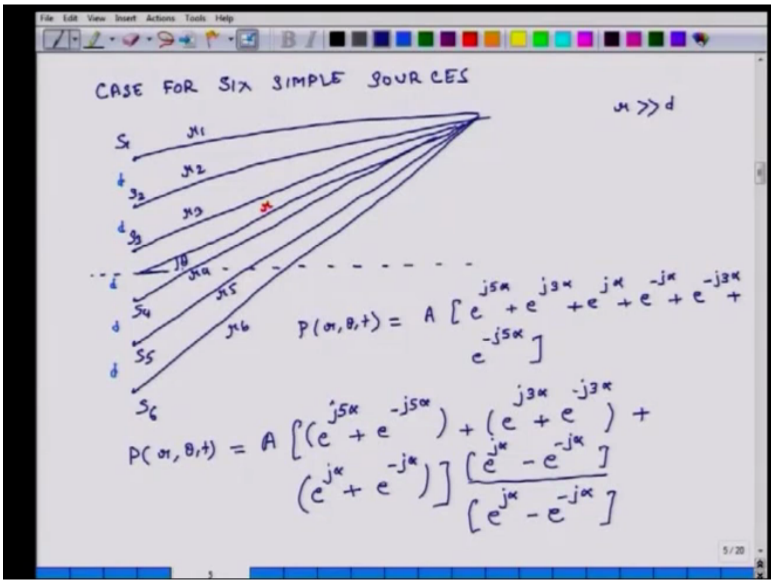

So now we will do case for six simple sources, so we have source 1, source 2, source 3, source 4, source 5 and source 6, this is my midpoint, the distance between each of this source is, adjacent sources is again d I have a far flunk point such that the distance between the midpoint and this far off point is r such that r is very large compared to d.

And then now with this understanding I develop vectors from each of these source points and let us say these vectors are r1, r2, r3, r4, excuse me this is not r4 so that is r3, the midpoint is r distance away so midpoint is r and then I have fourth source this is r that is r4, r5, r6 and these are six sources S1, S2, S3, S4, S5, S6.

So if we use the same logic and if this angle is theta, azimuthal angle is theta then P of r theta t equals using the same logic we can say we can write that this is equal to some variable which does not depend on theta but it only depends on omega r and rho so that is in term A, time is a theta dependent term which is essentially a sum of j times 5 alpha plus exponent j times 3 alpha plus exponent j times alpha plus exponent minus j times alpha plus exponent minus j times 3 alpha plus exponent minus j times 5 alpha.

I can rewrite this expression as P which depends on r theta and t equals A times, so now I am going to organize these terms according to their powers so e to the power of j times 5 alpha plus e to the power of minus j times 5 alpha so that is 1 plus e to the power of j times 3 alpha minus e to the power of j times 3 alpha it should be plus sign here, plus e to the power of j alpha plus e to the power of minus j alpha that is again and then I am going to multiply and divide it by the same term as I did in earlier expressions and this term is e to the power of j alpha minus e to the power of minus j alpha so I am going to multiply it and divide it by the same term.

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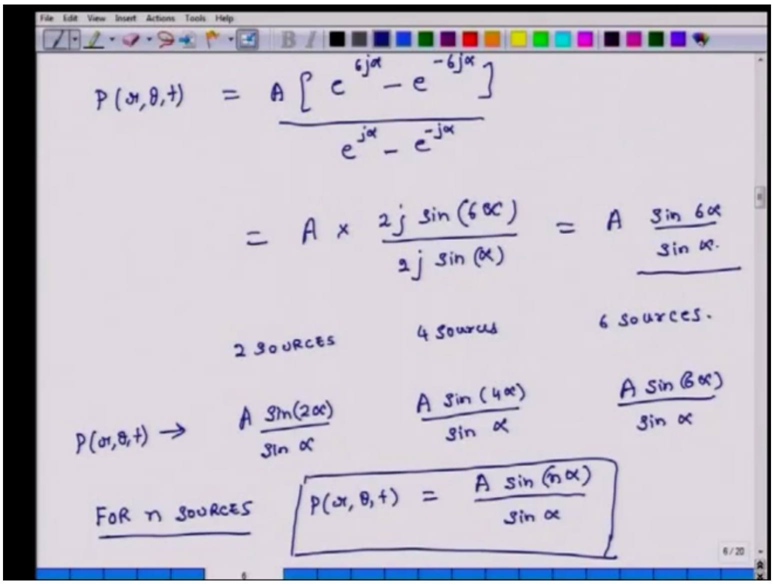

And now when I do mathematical operations on this long equation what I get is P r theta t equals A times e to the power of 6 j alpha minus e to the power of minus 6 j alpha and all other terms cancel out and then in the denominator I have e to the power of j alpha minus e to the power of minus j alpha and this can be written as A times 2 j sine 6 alpha and in the denominator it is 2 j times sine alpha which equals A to the power of sine 6 alpha divided by sine alpha so this is the equation for pressure when I have six sources in a straight line so let us tabulate some of our results so when I had two sources then I had four sources and then I had six sources which I have done 6 cases in case of two sources the overall pressure was expression for pressure, overall expression for pressure was A times sine 2 alpha divided by sine alpha.

When I had four sources the expression for pressure was A times sine 4 alpha divided by sine alpha, and when I had six sources it was A times sine 6 alpha divided by sine alpha. So by process of inductive reasoning we can actually mathematically prove that if there are n sources in an actual case then if there are n sources then for n sources I can actually extend this and I can have a very generic my relation for pressure is going to be P r theta t equals A sine n alpha divided by sine alpha.

This is the relation when I have n sources all in the straight line, all separated by equal distance which is d and all sources are working in such a way that the phase difference between all of them is exactly 0 and their volume velocities are identical in such a case the overall pressure at a point which is far away from the this array of sources is A times sine of n alpha divided by sine of alpha.

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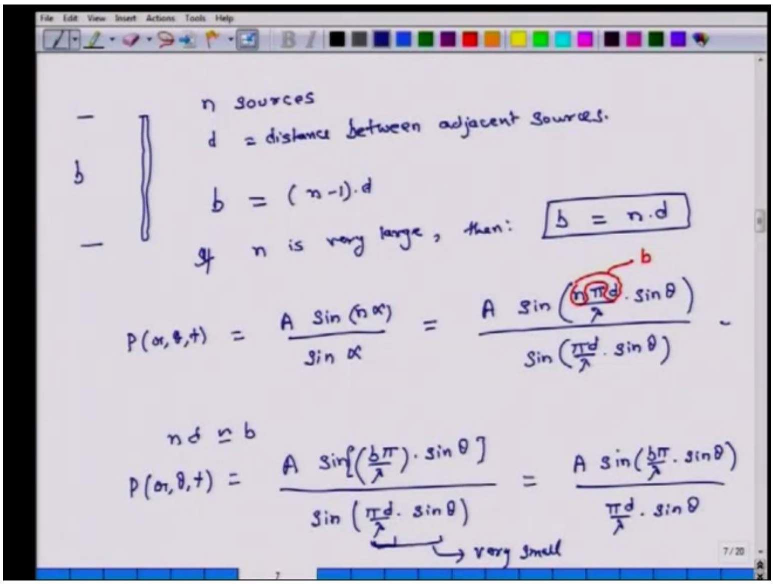

Now let us look at the case when we have infinite number of sources, so we can consider an array and the overall width of this array is b and let us say it has n sources and if these are n sources then the distance and if all these sources are equally spaced then the distance and if the distance between two adjacent sources is d, so in this case if n is very large then the relationship between b and d is nothing but b equals n minus 1 times d.

Now if n is very large then b equals n times d for large value of n if n is extremely large then b equals n times d so in such a case we can write that pressure equals A times sine n alpha divided by sine alpha, this is the relationship which we have developed earlier and now I am going to write the expression for alpha so that is A times sine of n pi d over lambda times sine theta divided by sine pi d over lambda times sine theta.

Now we know that n d approximates b specially if n is very large so then my expression for p r theta becomes A times sine and n times d equals b so I get b pi over lambda times sine theta divided by sine pi d over lambda times sine of theta okay this I can further rewrite this as excuse me there is no, yeah, sine of pi d over lambda times sine of theta.

Now if n is extremely large then this term, if n is extremely large and if b is finite then d will be extremely small and when d is extremely small then pi times d divided by lambda will also be very small and that very small term is being multiplied by sine of theta which never exceeds 1, so in that case again when n is very large then this term is very small and if that is the case then sine of this extremely small term is essentially same as the small term itself so I can again further simplify this relation as sine b pi over lambda times sine theta divided by pi d over lambda times sine theta.

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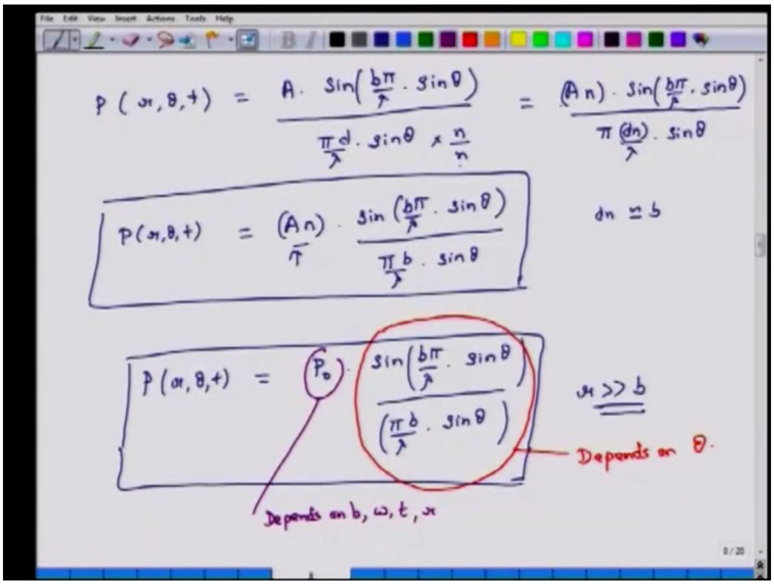

Moving on, so I will just rewrite this equation P of r theta t is equal to A times sine b pi over lambda times sine theta in the numerator and in the denominator I have pi d over lambda sine of, times sin of theta now I can multiply this and divide this by n and what I get is A n times sine b pi over lambda times sine theta divided by so this n comes in the numerator and in the denominator I am left with pi d n divided by lambda times sine of theta, so this is equal to A times n times sine b pi over lambda times sine of theta divided by pi and we know that dn equals or approximately equals b when n is large so this is essentially pi b over lambda times sine of theta.

So this is the expression for the pressure field at a point which is very far away from an array of sound sources which are all lying in a straight line and separated by very small distance and such that the overall length of the array is b then the expression for P is this thing, now A times n does not change with respect to theta okay and that essentially defines that the overall maximum possible pressure in the field will be determined by this term so I can rewrite this equation as that P r theta t equals some constant or some radius dependent number P not times sine b pi over lambda times sine of theta divided by pi b over lambda times sine of theta.

So this is my expression and here I have assumed that r is extremely large compared to b, so this is my expression now again in this expression this term it depends on theta, this term depends on b, angular frequency omega and t and r it depends on the size of the array, it depends on angular frequency, time and radius, how far we are from the array it depends on this. So if I have to know what is the overall polar pattern or directivity diagram of the array then I have to essentially rely on this term which has been circled in red colour and if I am able to plot it that will give me a very good idea of the directivity pattern of the overall array.

So what we will do is that now we have developed this expression for n sources all lined up in a straight line we will look at the directivity patterns of such sources and compare them with some of the sources which we had studied earlier, so that is what we are going to look at.

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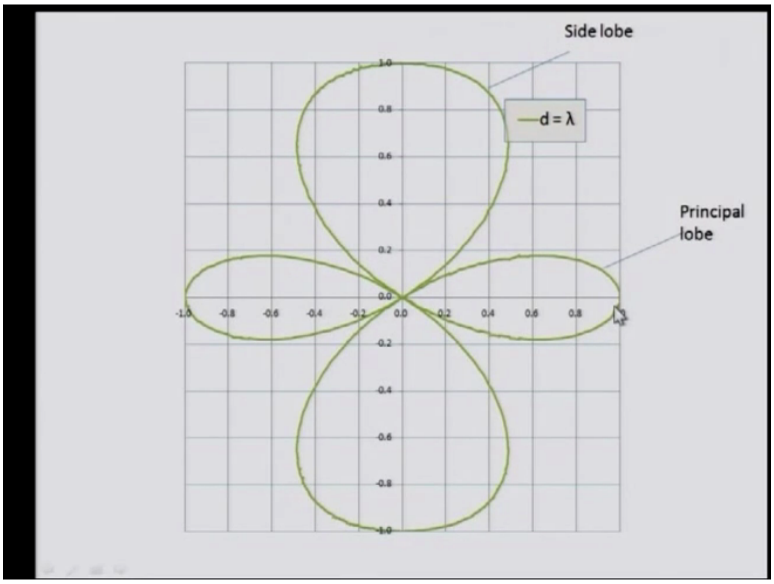

So this first slide is for, is relates to the directivity pattern for two sources, both sources acting in phase, both sources having same volume velocity and these two sources are separated by distance d which equals the wavelength lambda so what we see here is that we have a principal lobe and we have maximization of pressure at theta equals 0 degrees and at theta equals 90 degrees and the width of the lobe in 0 degrees is very sharp, the width of the lobe at 90 degree direction is fairly wide.

Now let us look at the directivity pattern of a source where d equals, of a source or a set of sources where we have a very large number of transducers emitting sound excuse me, the overall length of this array is b and that b equals lambda just as here b equals lambda, in this case the width of the overall array is b and that equals lambda and the number of such sources is extremely large theoretically infinite and we have developed the directivity pattern for this using the relation which we just developed.

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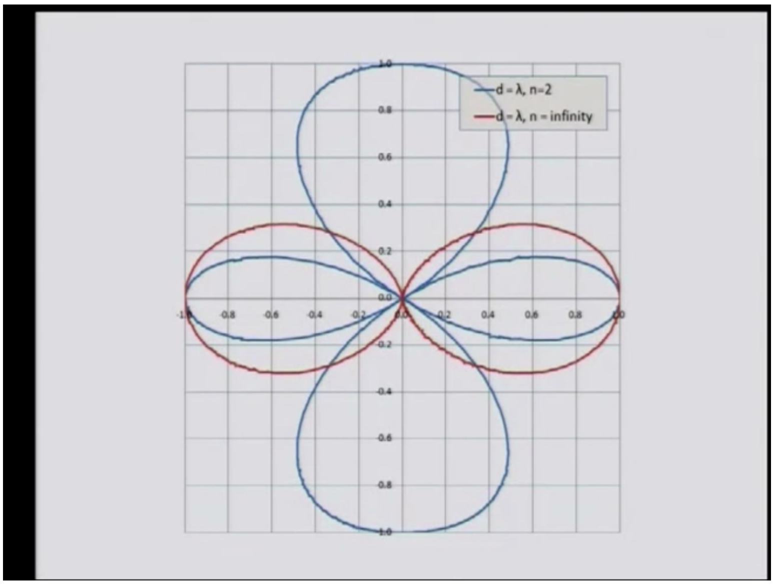

So this is how it looks like okay, so what you have here is there are two curves the curve in blue represents the case when we have just two sources, these two sources are off by distance lambda the curve in red represents a very large number of sources theoretically infinite and the overall width of the array is lambda and we observe several interesting points, one is that the red curve has only two lobes and both this two lobes are in the principal direction while the curve in blue has four lobes two in principal direction, two in 90 degree direction or off axis direction.

So because of this distributed array when and because of the presence of a very large number of sources what we find is that the intensity of the sound which is being propagated in 90 degrees direction and also in 270 degree direction it has gone down to virtually 0 as deflected by the red curve and this is what is known as side lobe suppression.

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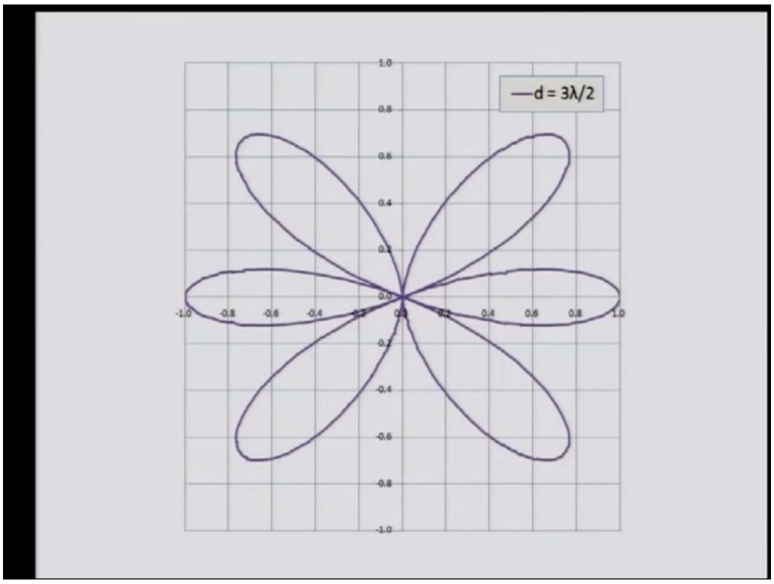

So now let us look at and their curve and this is the curve this is the directivity pattern for two sources emitting sound both sources in phase, same volume velocity for each source and the distance between these two sources is 1.5 times lambda or 3 lambda divided by 2 and what we find is that we have six overall lobes there is a lot of sound emitted in 0 degree direction and also in 180 degree direction but there is also sound getting emitted in 45 degree, 135 degree and so on and so forth.

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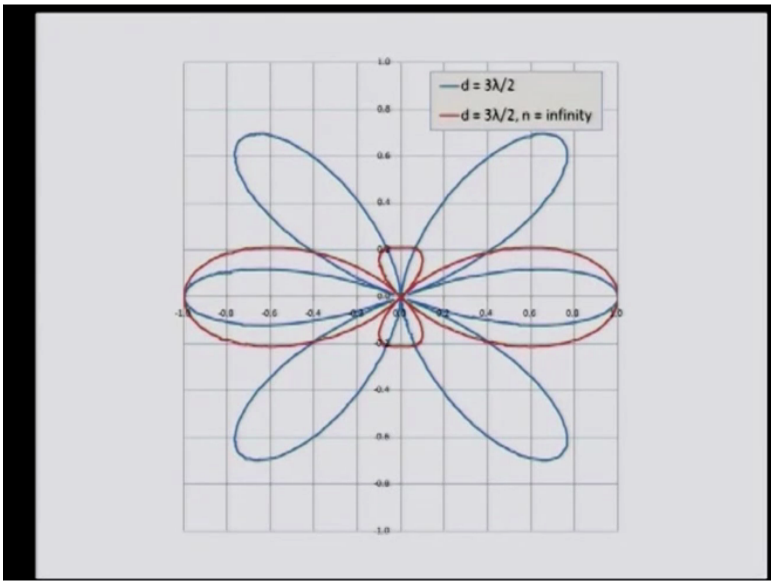

Now let us compare this pattern with the case when we replace these two sources by a distributed source whose overall width is 3 lambda over two so let us look at the comparison of this particular directivity pattern with a pattern which is once again 1.5 times wavelength and let us see how that compares, so again that is the that shown in this slide. So what you see in this slide is basically two curves, the first curve is in blue colour, light blue colour and this corresponds to emission of directivity pattern associated with two sources separated by distance 1.5 lambda and both sources are in phase and they have same volume velocity.

The curve in red the directivity pattern depicted in red corresponds to a distributed source where the number of sources is extremely large theoretically infinite the overall length of the array is 1.5 times lambda same as the overall length of earlier array but what you find here is that instead of six lobes as we saw when we had just two sources we have only four lobes here.

And the second important thing to note is that the overall intensity of sound which is being emitted in 90 degree direction is extremely small compared to the overall intensity of sound when we had only two sources so from when we had only two sources. So once again we find that once if we have the larger, if we have a very large number of sources in all standing, all arranged in a straight line and if they are all in a phase then side lobes tend to get suppressed and the on axis sound propagation does not get affected, it does not get affected.

Now we will close this lecture but before we will close we will just do a very quick calculation of what is the extent of this noise suppression, so to do that lets just observe on this curve that the blue curve and 45 degree direction some has an overall pressure level of something like 0.7 okay while the red curve in the off axis pressure is something like 0.2

So while in the 0 degree direction the overall pressure for blue curve as well as for red curve is identical and it stands at 1, so we will based on this we will do some quick calculation and let us see what is the amount of attenuation we get in off axis direction because of this effect.

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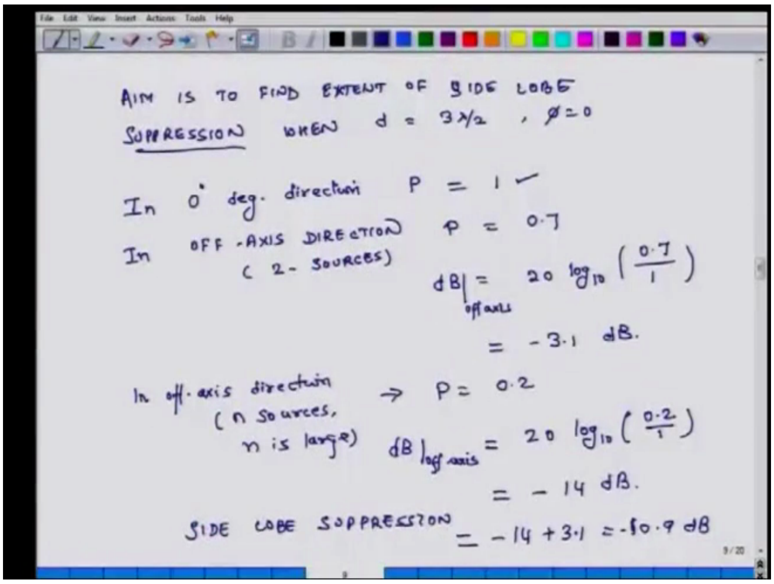

So our aim is to find extent of side lobe suppression when d equals 3 lambda over 2 and phase equals 0 and volume velocities are identical, so in 0 degree direction pressure magnitude is equal to 1 we have normalized it and we call it 1. In off axis direction when we have two lobes, (())(45:16) when we have two sources only then pressure equals 0.7 and if I have to calculate it in decibels then it is dB off axis equals 20 log of 10 0.7 divided by reference pressure which is 1 and that works out to be minus 3.1 decibels.

Next we look at the off axis SPL when we have this whole array because of this we have a very large number of sources, so in off axis direction that is when we have n sources and n is large and that equals pressure, pressure we saw, we measured it was something like approximately 0.2, so in decibels off axis that is equal to 20 log of 10 0.2 over 1 and that works out to be minus 14 dB.

So side lobes suppression that works out to be minus 14 plus 3.1 is equal to something like 10.9 decibels. So just by virtue of having distributed source which implies that we have n sources, very large number of sources over a distance of 1.5 lambda, my side lobe suppression when down to was as high as 10.9 decibels and what essentially this tells me is that I can use this kind of an approach to develop acoustic transducer arrays which can throw sound in a particular direction and this sound moves like a beam because the amount of sound which gets radiated in off axis direction is very little, so with this we close todays lecture and we will further continue our discussion on directivity in the next class. Thank you very much.