Acoustics
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Module 05
Directivity
Lecture 02
Directivity

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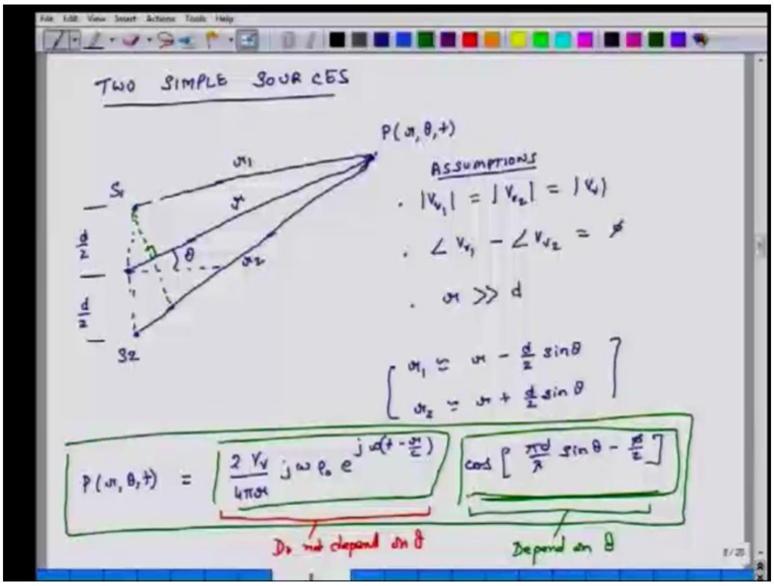

So what we have developed is till so far an expression for pressure at this point P and we have seen that this pressure depends on how far this particular point is away from the midpoint which exist between this two sound sources it also depends on the angle theta and of course it depends on time so that is what this expression tells us.

When we observe this expression further what we find is that there is a set of terms and these terms do not depend on theta, while there is another set of terms and these terms depend on theta, so the terms which do not depend on theta they are not going to materially or in any way influence the polar pattern or the directivity graph, directivity pattern for these two sound sources but the terms which are the second set of terms which are related to this cosine function, they very strongly influence the directivity pattern for sound pressure level measured at point P which is due to sound being emitted by sources S1 and S2.

So if I have to have an understanding of the directivity pattern for these two simple sound sources then all I have to worry about is plot this function which is there highlighted in green and see how or what kind of a directivity pattern image is as I plot this function in polar co-ordinates and that is what I will do as we move forward.

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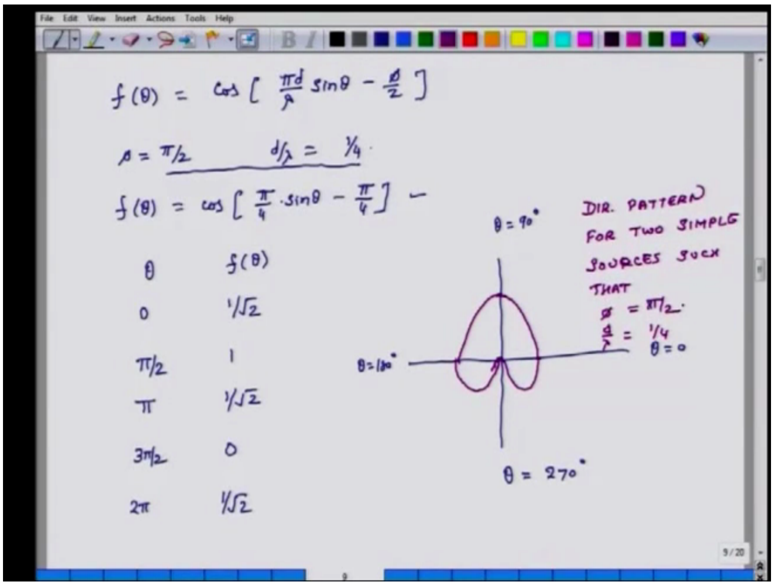

So for directivity pattern I plot this function which depends on theta and what is this function it is cosine of pi d over lambda times sine of theta minus phi over 2, so this equals cosine of pi d over lambda sine theta minus phi over 2. Now to actually construct the directivity pattern I have to know the value of phi which is the difference of phase between two volume velocity sources and also I have to know what is the ratio of d and lambda, and then for different values of theta I can plot this function f.

So we here assume that phi equals pi over 2 and we also assume that d over lambda equals 1 over 4, so with these assumptions my relation or my function becomes cosine of pi over 4 times sine theta minus pi over 4 and this particular function is valid for these two assumptions, hold good. One is the phi which is the phase difference between two volume velocity sources S1 and S2 equals 90 degrees so pi over 2 radians and also if d over lambda equals one forth, now what I am going to do is compute different values of f for different values of theta .

So I have a table theta and f of theta so at 0 degrees, sine of theta is 0 and cosine of, so what I have in parenthesis is minus pi over 4 so f of theta is 1 over root 2 for theta equals pi over 2 sine of theta is 1, I have pi over 4 minus pi over 4 in parenthesis which equals 0 so cosine of 0 is 1, then for theta equals pi and theta equals pi again the first term in the parenthesis becomes 0, so what I get is in the cosine of pi over 4 is 1 over root 2.

And then for theta equals 3 pi over 2 I get my first term is sine theta related to sine theta it is minus 1, minus 1 times pi over 4 is minus pi over 4 so everything in the parenthesis adds up to 0, excuse me, pi over 2, cosine of pi over 2 is 0 and then of course for 2 pi I get again 1 over root 2, so with these points I am going to plot.

So this is theta equals 0 degree direction, theta equals 90 degrees, theta equals 180 degrees and theta equals 270 degrees at theta equals 0 it is 1 over root 2 or 0.707 so let us say this is 0.707, at theta equals pi over 2 it is 1 so it is going to be somewhere here, at theta equals pi again my amplitude of this function f is going to be 1 over root 2 so this is the third point and at theta equals 3 pi over 2 I have the fourth point which is at the centre at origin.

So I am going to construct a graph or a directivity pattern for this source, excuse me, so I think my location of this point at theta equals 0 has to be a little closer which I will put it here somewhere here so now I construct a graph, so it will look something like this so this is my directivity pattern for two sound sources, two simple sound sources such that phase difference between these two sound sources is equal to 0 and d over lambda equals 1 over 4 that is my directivity pattern.

So likewise we can construct similar directivity pattern for variety of combinations of different sound sources, in next few examples we will look at some of, some more directivity patterns and we will see what do they tell us.

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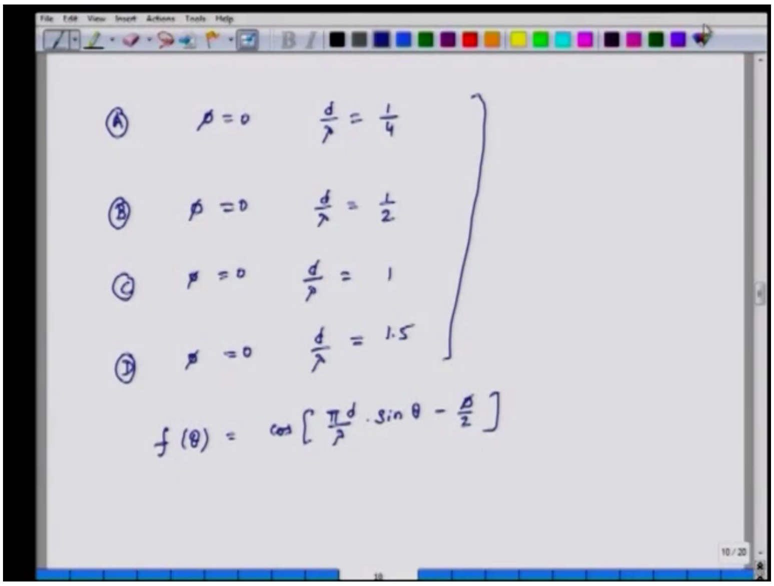

So what we are going to do is we will look at several cases, case A so phi equals 0 that is the phase difference so excuse me here phase difference between the two sound sources was pi over 2.

So what we are going to look at is another set of cases and in all these cases we will have we will assume that phase difference between two volume velocity sources V1 and V2 is 0 and in case A we will have d over lambda we will put it as 1 over 4 then we will look at case B again C is we assumed to be 0 and we will increase d over lambda by a factor of 2 so this becomes half.

Case C, phi again is 0 and we further increase d over lambda to 1 and case D we again assume phi equals 0 and also we assume that d over lambda is 1.5, so in all these cases we again plot the value of f of theta, this particular function f of theta equals cosine of pi d over lambda times sine of theta minus phi over 2 so we plug in different values of phi and d over lambda and then we develop different directivity patterns.

So what I have done is I have already developed these patterns using standard mathematical tools and I am going to show you these patterns so what I am going to show you several figures which I developed using the relation for f theta with varying values of d over lambda and also while phase difference between two volume velocity sources was kept constant and it was kept at 0 degrees.

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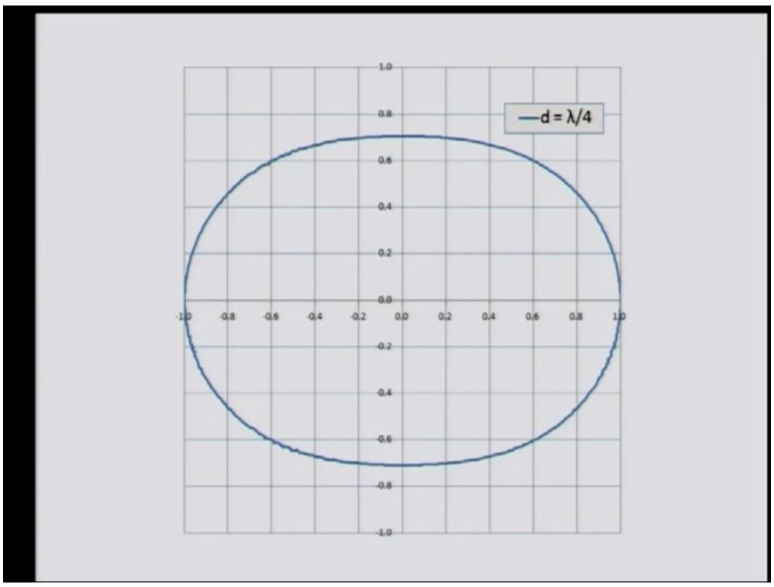

So the first figure is this one and here d over lambda is 1 over 4 or alternatively you can call d equals one fourth of lambda and this particular the x axis, the horizontal axis corresponds to 0 degrees, the vertical axis corresponds to 90 degrees upwards and if I go downwards then it corresponds to 270 degrees so what I see here is that in the x direction or in the 0 direction the intensity of the sound pressure is at its maximum and that is equals to cosine of that entire function at 0 degrees equals 1 however in the vertical direction it goes down little bit and it is somewhere between 0.6 and 0.8, okay and this is a symmetric picture and this is for d equals lambda over 4.

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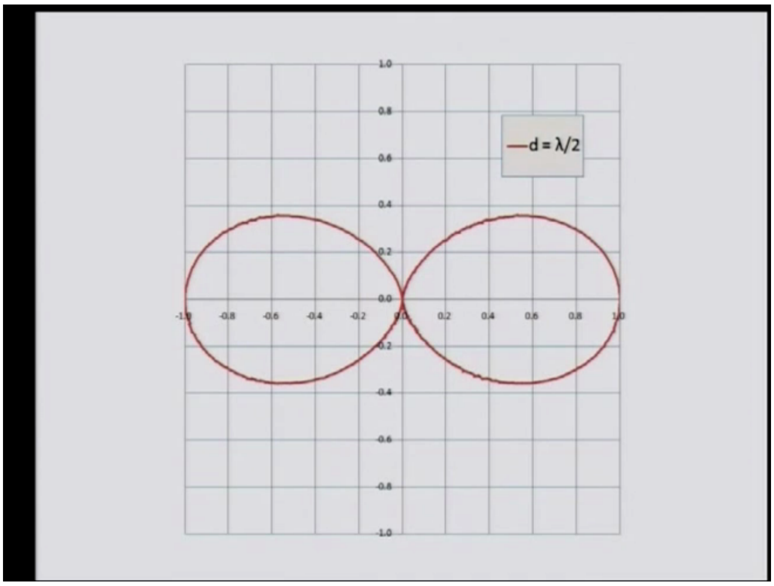

Now let us look at the case when this ratio d over lambda increases and it goes up from 1 over 4 that is 0.25 to half and this is how the picture looks like so here what we see is that once again the strength of the signal is maximum in 0 degree direction and its maintained at 1 in 90 degrees and at 270 degrees the extent of the signal goes down and it comes down to 0 and it is somewhere between 0 and 1 at other values of theta and then unlike the previous example where we had one single big you can call it a bulb, here we have two lobes and these two lobes are, one lobe is in 0 degree direction other lobe points at 180 degree direction.

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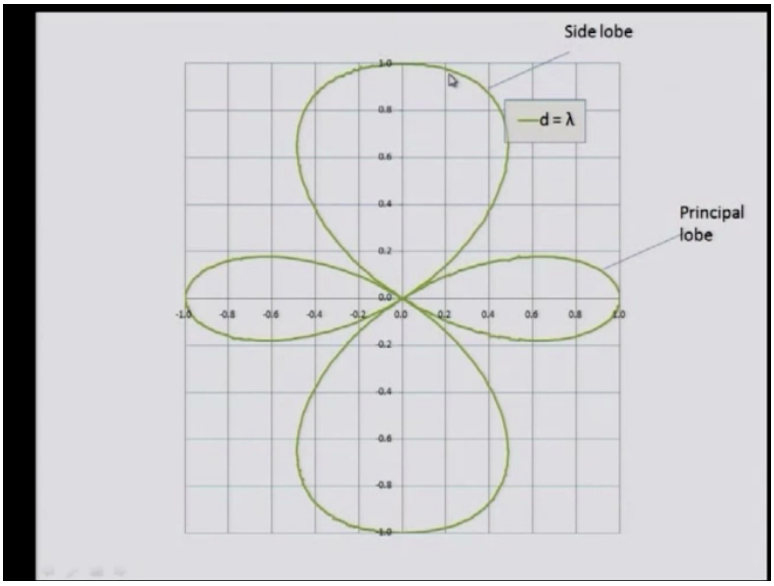

Next is this picture and here d over lambda equals 1 and what we see here is that we have 4 lobes, two lobes correspond to 0 and 180 degrees direction and two other lobes corresponds to 90 and 270 degree direction and then one very interesting feature is that the sharpness of the lobe is much stronger in 0 degree direction compared to the sharpness of the lobe in 90 degree direction.

In 90 degree direction the lobe is fairly wide but in 0 degree it is very sharp and pointed and the 0 degree direction lobe is known as principle one and then this vertical lobes are called as side lobes.

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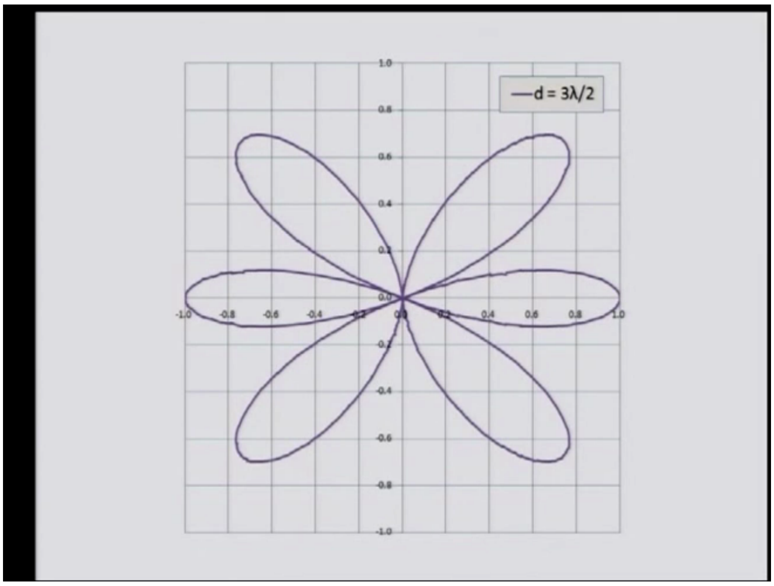

And then this is the picture for when d over lambda equals 1.5 that is when the distance between these two sound sources which we saw earlier that equals to 1.5 times the wavelength of sound wave being emitted and here we have again, again in the 0 degree direction so we see this theme which is very constant that the intensity is maintained at 1 in 0 degree and 180 degrees direction and in other cases it may or may not remain at 1 but in 0 degree direction it is always maintained at 1 and then here we have 6 lobes overall on the directivity pattern graph.

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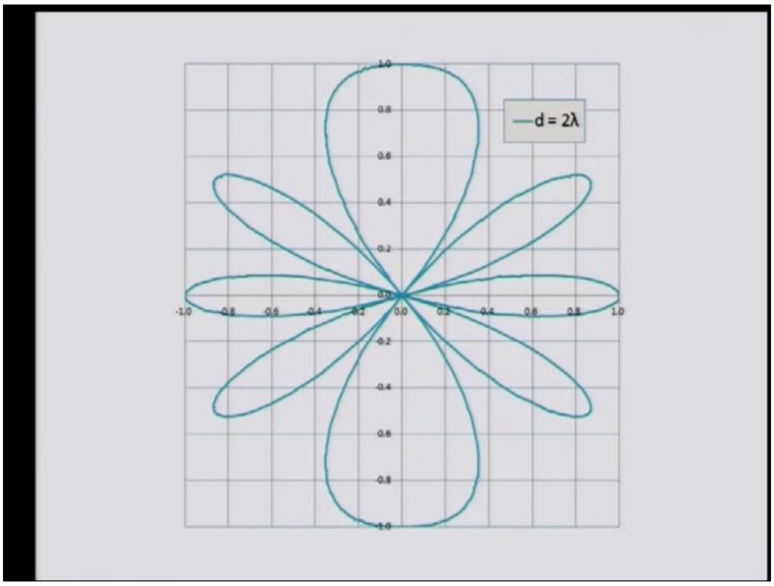

And finally we have d over lambda equals 2 and here once again we have a lobe, very sharp lobe in 0 and 180 degree directions while there are other lobes also and then the lobes in 90 and 270 degree directions, they are fairly wide so you get a feel of what is going on as we keep on increasing the value of d with respect to lambda, what we see is that the larger the value of d with respect to lambda the more the number of lobes we have on the directivity pattern.

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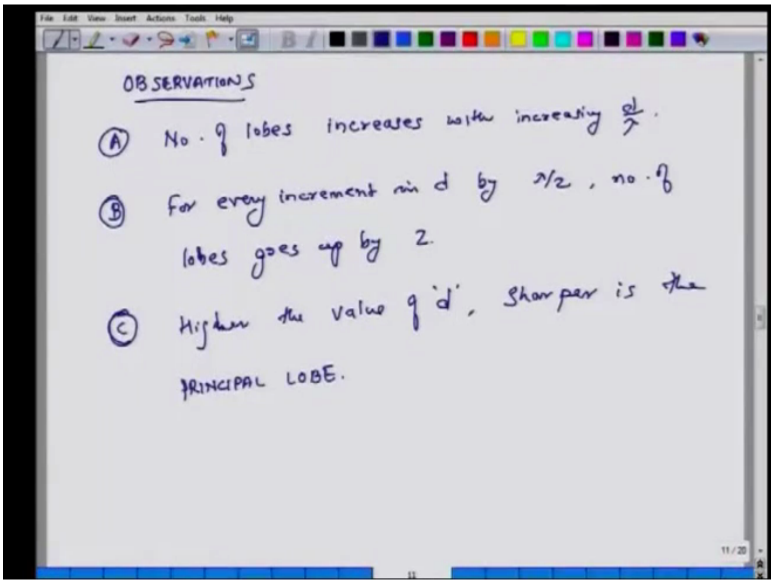

So now we go back and we can make some observations, so some of the observations which we can make after seeing all these directivity pattern graphs are, A that number of lobes increases with increasing d over lambda, B for every increment in d by lambda over 2, number of lobes goes up by 2 so when d equals lambda over 2 then we have 2 lobes when d equals lambda over 4, 1 lambda we have, excuse me, when d equals lambda over 2 we have 2 lobes when d equals lambda then we have 4 lobes when d equals 3 times lambda over 2 we have 6 lobes and when d equals 4 times lambda over 2 that is 2 lambda then we have 8 lobes.

And the third thing is higher the value of d, sharper is the principal lobe, so it is very sharp and focused so at very low values of d you see an overall roughly in the directivity pattern, roughly like a circle but as I keep on increasing the value of d in terms of lambda over 2 then the 0 degree direction lobe and the 180 degree direction lobe they become more focused and sharper in nature.

Now in a lot of cases we have arrays of sound transducers which emit these sounds and we want that these sounds should get focused only in a particular direction so by playing with the arrangement of the number of transducers, the phase difference between these transducers and also the distance between these transducers we can come up with overall directivity patterns which meet our needs in the context of application.

So in some cases we want that all the sound should go only in the 0 degree or in 180 degree direction and not much sound should get emitted maybe in 90 degrees or 270 degree directions, so in those cases, in those types of application we would want that the intensity of the lobes in 90 degree and 270 degree direction it should be somehow suppressed and that can be achieved by playing with these parameters phi, d and also the number of transducers or the number of sound sources which are emitting sound .

So we will explore this area in remaining part of today's lecture and also maybe in next lecture but before that we have to expand or extend our relationship for directivity pattern for multiples sources where we have more than two sources in the field so that is what we are going to do moving forward.

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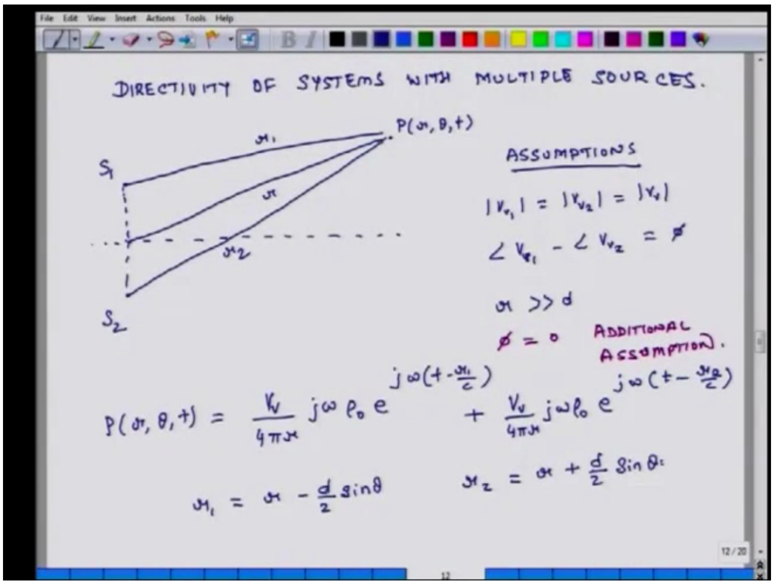

So what we are going to look at is directivity of systems with multiple sources, okay, so earlier we looked at the situation we have two sources next what we will do is we will look at situation where we have four sources, six sources and so on and so forth and then finally we will extend these relations to a generic case where we have n sources in the overall field so but before we start with more than two sources we relook at the results of two sources.

So what we had seen is two sources S1 and S2 and these two sources and we are interested in measuring point, the pressure at point P and this value is P of r theta t and this distance is r, this distance is r1, this distance is r2 and we had assumed so our assumptions were that volume velocity 1 was equal to volume velocity 2 was equal to Vv and then the other assumption was phase of volume velocity 1 minus phase of volume velocity 2 was equal to phi, the third assumption was r is extremely large compared to d.

Now at this stage we make one extra assumption and that is phi equals 0 so this my additional assumption, so if that is the case then my expression for P r theta t can be written as and we are just recapping the relation which we had seen earlier so it is Vv over 4 pi r times j omega rho not e j omega t minus r over c and then plus, oh sorry excuse me it is r1 over c and then and this is t and then the contribution of the second source is V times 4 pi r times j omega rho not e j omega t minus r2 over c and we know that r1 equals r minus d over 2 sine theta and r2 equals r plus d over 2 sine theta.

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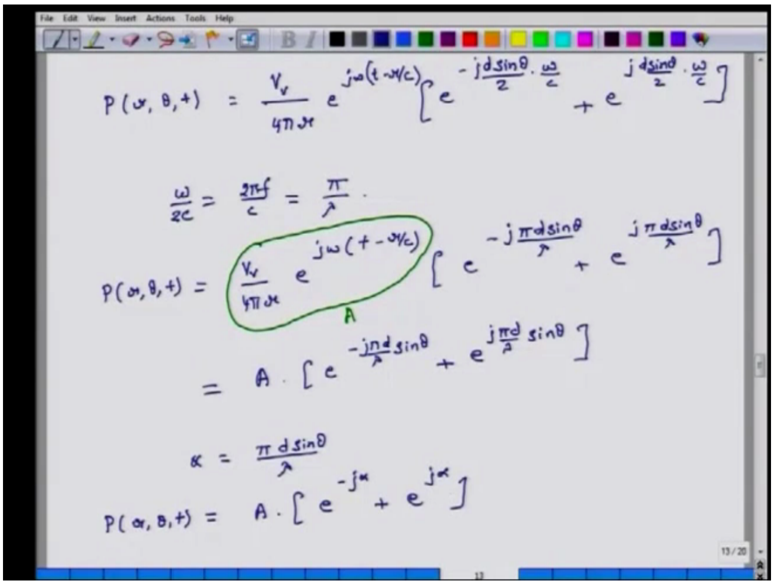

So with that understanding our relation for P of r theta t equals Vv over 4 pi r. So before we do this further we see that in this relation there is no expression or there is no term involving phi because we have involved, we have assumed that phi equals to 0, so this is one difference between the relation between the relation which we developed earlier and this particular relation.

So now we are going to further process this relation and that becomes, so pressure which is a function of r theta t equals Vv over 4 pi r e j omega t minus r over c times e minus j d sine theta divided by 2 times omega over c plus e times j d sine theta divided by 2 times omega over c, okay.

Now we know that omega over 2c equals 2 pi f over c where f is the frequency of the emitted sound and that equals pi over lambda so my this relation for pressure can be rewritten as Vv over 4 pi r e j omega t minus r over c times e minus j pi d sine theta divided by lambda pus e j pi d sine theta divided by lambda and that equals some constant or actually some term so I call this term as A, so this is A so that is my A times e minus j pi d over lambda sine theta plus e j pi d over lambda sine theta. Now I define some term alpha as pi d sine theta over lambda, so my expression for P of which is dependent on r theta and t equals A times e minus j alpha plus e times j alpha.

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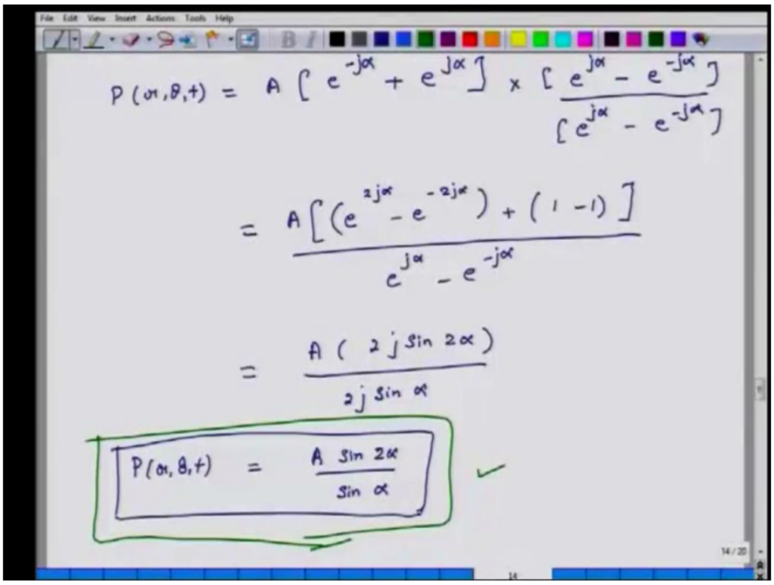

Moving further so I will rewrite here pressure is a function of r theta and t equals A times e minus j alpha plus e times j alpha, and now I can multiply and divide this by this term e j alpha minus e minus j alpha and I can divide it by the same thing and if I do the math I can write this as A times so what I get is e times j alpha times e times j alpha is e times 2 j alpha and then e times minus j alpha minus e times j alpha I get e minus 2 j alpha, so this is one set of terms and then I have cross terms.

And what I get is e times minus j alpha times e times j alpha is 1 and e times j alpha times e times minus j alpha is minus 1 because I have a plus sign and the negative sign on in this two parenthesis and in the denominator I have e j alpha minus e minus j alpha, so this becomes A times 2 j sine 2 alpha using laws of complex variables and this is 2 j sine alpha so this becomes A times sine 2 alpha over sine alpha.

So this is my expression, so what I have done here is I have revisited the case where I have two simple sound sources located at distance d apart and also with and the midpoint of these between these two sound sources is located at distance r apart from the point of observation and in this context we have also assumed that the phase difference between two volume velocity sources equals exactly 0 and with that understanding we have revisited earlier expression for pressure at point P and then we have rewritten essentially the same result but in a different form and this different form is, this form.

So mathematically this form is very similar, it is exactly similar to the form which we had developed earlier where we had this function excuse me, where we had this particular function but in our aim to generalize this theory for n sources we will find that this particular form is very convenient to scale up for n sources and that is what we are going to explore in our next lecture.

So today what we have done in this lecture is we have developed an expression in this particular form for pressure which is equal to A times sine of 2 alpha over sine alpha and this is the pressure for at a point P which is distance r away from two simple sources which are emitting radially I mean radial waves, radial sound waves and the phase difference between these two sound sources is exactly 0 degrees. In our next class we will extend this further and we will generalize it for n sources, thank you very much.