Acoustics
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Module 05
Directivity
Lecture 01
Directivity

Hello, in the last lecture we had discussed different aspects of radial propagation of sound waves through some illustrations and through some actual illustration problems. What we plan to do today is continue that journey a little bit further forward and in once we are done with that then we will start talking about a new concept known as directivity and this particular concept of directivity it becomes really important specially in context of radial wave propagation.

So what we plan to do right now is that we will do a problem for illustration purposes and in that context we will consider a sound source which is propagating sound in a radial way and we assume that while this radial propagation is happening there are no reflecting surfaces around this sound source so the only wave which is getting propagated through the sound source is the forward travelling wave and there is no reflection or no backward travelling waves in the picture.

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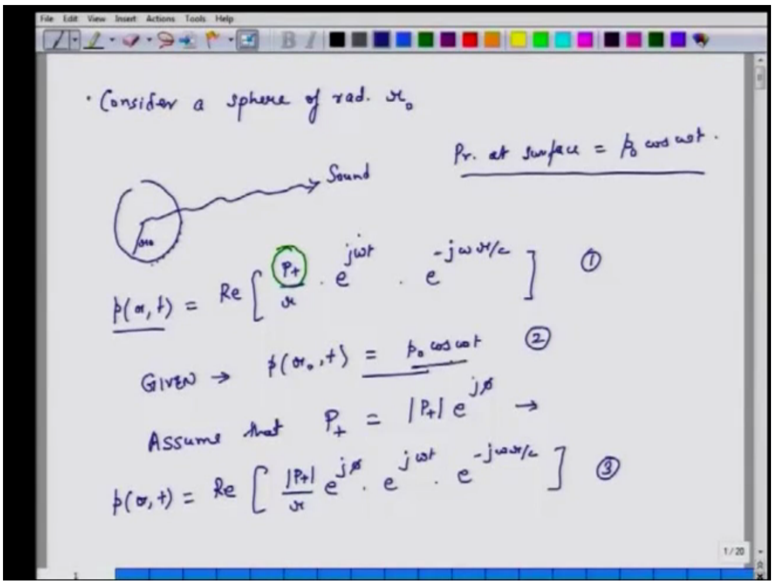

So we consider a sphere and the radius of that sphere is r and that r is very small so sphere of radius r not and this thing this particular sphere is emitting, so this is my r and it is emitting sound waves and what we know is that on the surface of the sphere the pressure is P not cosine omega t, so at this surface my pressure is P not cosine omega t.

So pressure at surface equals P not cosine of omega t and what we are interested in finding out for this particular problem is that what is the value of power flow which is happening from the sphere outwards and this power will be complex in nature so it will be complex power flow it will have a real component and it will also have an imaginary component.

So to do this we start looking at the relation for pressure so we know that pressure for a spherical source depends on radius and it also depends on time, so p is a function of r and t and that is nothing but real of complex amplitude P plus divided by r times e j r omega t times e j, oh excuse me e j, I have to correct this e j omega t times e j omega r over c and because this is a forward travelling wave this negative sign here.

So that is my relation for pressure now we know that at r equals r not this value p is equal to p not cos omega t so we know given that p at r not and at time t equals p not cosine of omega t so I will call this equation 1 this is equation 2, now what we do not know right now is the value of P plus.

So P plus is a constant and it may have an imaginary component as well as a real component and we do not know what this number looks like so let us assume, so we assume that without losing any sense of generality that P plus is having some magnitude and then it also has a phase element to it and that we express as e j times phi.

So now we plug this relation in equation 1, so what we get is p of r t equals real of magnitude of P plus divided by r e j phi times e j omega t times e minus j omega r over c, I call this equation 3 and now I further process this equation so what I get is, so I know that at r equals r not this is my expression for p.

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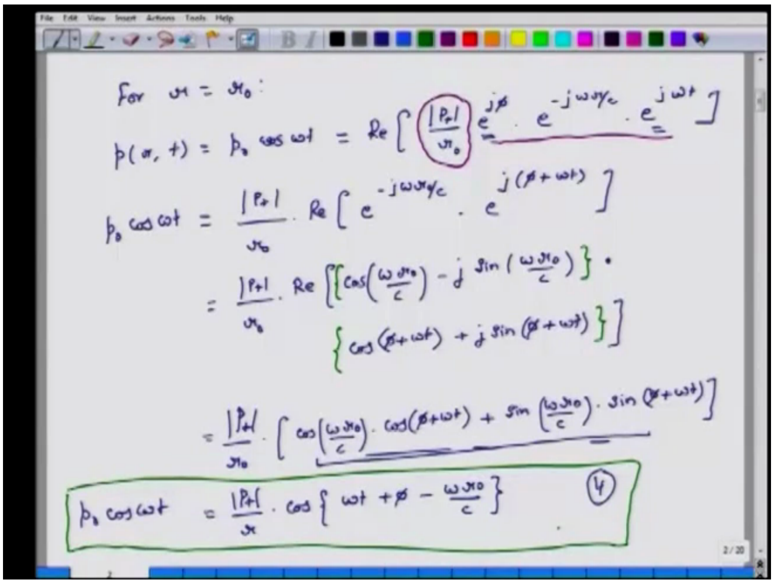

So I plug that value, so I plug the value of r as r not and on the left hand side I put this value so what I get is that for r equals r not, p of r t is equal to p not cosine of omega t and that is nothing but real component of magnitude portion of p plus divided by r not times e j times phi times e j omega r over c times e j omega t.

Now I realize that this term is real so I can take it out of the parenthesis and I then split these complex terms into real portion and imaginary portion so what I get is p not cosine of omega t equals magnitude of p plus which is this divided by r not times real of, so this is equal to, I will rearrange some of these terms here so what I get is e minus j omega r not over c times e to the power of j and in parenthesis I have so I combine this term and I combine this term so I get phi plus omega t.

Moving further what I get is amplitude of p plus divided by r not times real of, now e j e to the power of minus j omega r not c can be expressed in trigonometric terms as a cosine part and it also has a sinusoidal part so what I get is cosine of omega r not over c minus j times sine of omega r not over c and this whole thing is in parenthesis and then I have to add to that excuse me multiply that I have to multiply this with another set of two terms so and that is cosine of phi plus omega t plus j times sine of phi plus omega t.

Now if I take the real portion of this, what I get is amplitude of or magnitude of p plus times and I am now taking only the real components so I get cosine of omega r not over c times cosine of phi plus omega t and then plus sine of omega r not over c times sine of phi plus omega t and that equals so this is basically a function which looks similar to that of cosine of a plus b equals cosine a times cosine b plus sine a times sine b.

So what I get here is P plus divided by r not times this entire thing can be rewritten as cosine of omega t plus phi minus omega r not over c so that is my expression number 4, and on the left side I have p not cosine of omega t. So this is my expression number 4.

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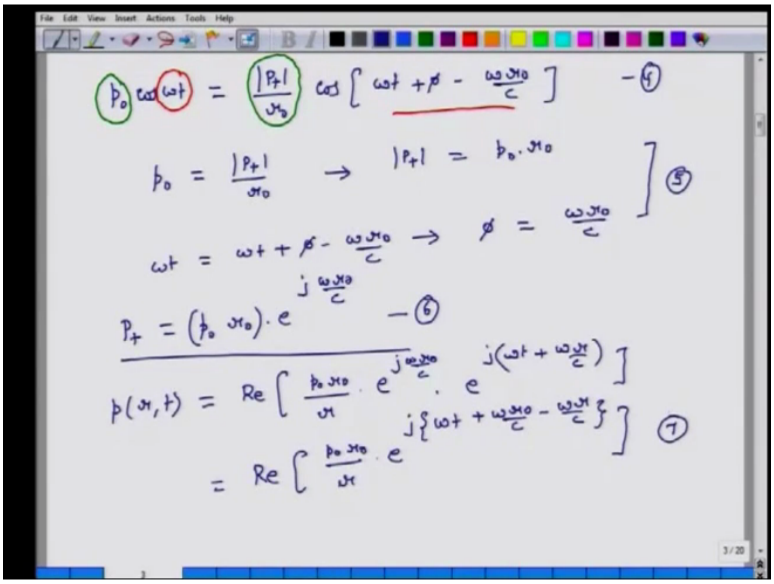

So I will rewrite it here on the next sheet so p not cosine of omega t equals magnitude of p plus divided by r not times cosine of omega t plus phi minus omega r not over c now this expression and this is again expression 4, this expression (())(12:21) hold true in general only if the following conditions are met.

So the first condition is that the amplitude or actually the magnitude of the left hand side should equal magnitude of the left hand side so the first condition for this equation to hold good is that p not should be equal to P plus over r not which implies that magnitude of P plus should equal p not times r not.

The second condition which has to hold true is that omega t, this term should equal this term so omega t should equal omega t minus omega r not over c which implies that phi should equal omega r not over c so these are my equations I will label them as 5a and 5b and once I plug this equations in my expression for P plus what I get is that P plus equals p not r not times e j omega r not over c so that is my equation 6.

Now with this understanding I rewrite my expression for p so we know that p of which is a function of r and t equals real component of magnitude of p plus which is p not r not divided by radius r times its phase component which is j omega r not over c times e j omega t plus omega r over c and I can rewrite this entire thing as real of p not r not over r times e to the power of j times omega t plus omega r not over c minus omega r over c. So this is my expression for pressure

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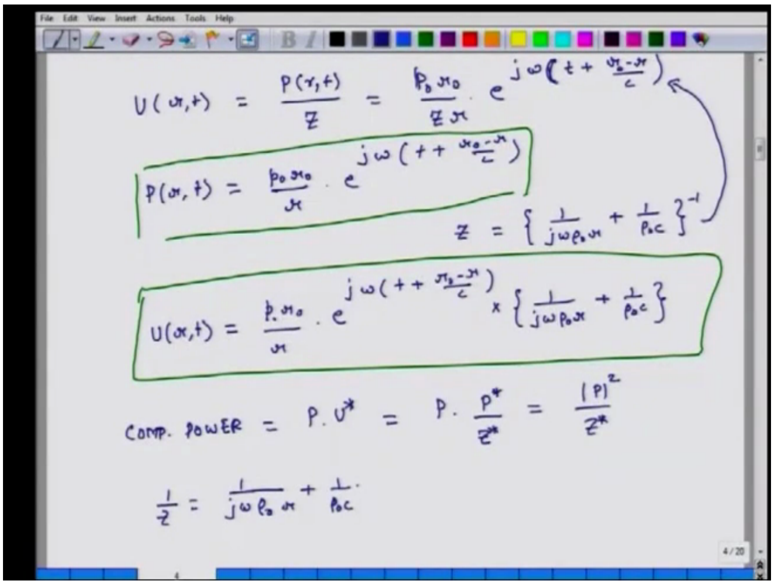

Likewise I can write the relation for velocity, particle velocity is u of r and t and this is equal to p of r and t, excuse me so I will write the expression for complex velocity. So complex velocity is a function of r and t and that is equal to complex pressure divided by impedance so this is p not r not over Z times e j omega and then t plus r not minus r divided by c and the relation for complex pressure is p of r and t equals p not r not over, so there should be an r here and this times e j omega t plus r not minus r divided by c.

Now I can re express I know that in my previous example what is the value of complex impedance for radially propagating waves and we know that value of Z equals 1 over j omega rho not r plus 1 over rho not c the whole thing inverse, so I put this equation this expression in expression for u, so what I get my expression for complex velocity is this, so complex velocity equals p not r not over r times e j omega t plus r not minus r divided by c times 1 over j omega rho not r plus 1 over rho not c, so these are my relations for complex pressure and complex particle velocity.

Now original intent was to find what is the value of complex power, so complex power equals complex pressure times complex velocity and that can be also written as complex pressure times P star over Z star and when I do this and I calculated what I get here is magnitude of complex pressure squared divided by Z star. Now we know that 1 over Z equals 1 over j omega rho not r plus 1 over rho not c.

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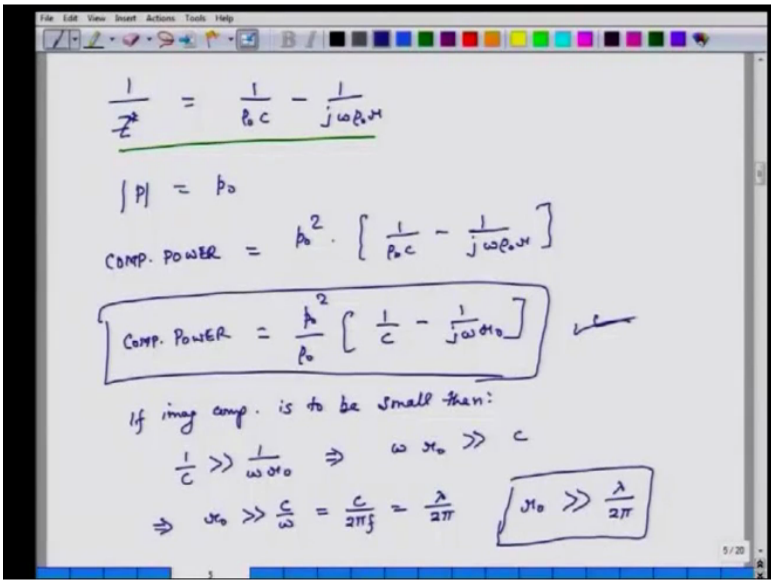

So which means that 1 over Z star can be written as 1 over rho not c minus 1 over j omega rho not r, so now that I know what is Z star and I also know that the magnitude of complex pressure is p not, my complex power equals p not square times 1 over Z star which is 1 over rho not c minus 1 over j omega rho not r and once I simplify this what I get is p not square over rho not times 1 over c minus 1 over j omega r not.

So that is my expression for complex power now there are couple of observations we can make when we look at this relation for complex power, one is, first observation I can make is that this has of course this a real component which is p square divided by rho not times 1 over c and then there is an imaginary component which is p square over rho times minus 1 over j omega r not.

Now if we want to make this imaginary component small. So if imaginary component is to be small then 1 over c should be very large compared to omega r not okay, which means that omega r not should be extremely large compared to c which means r not should be extremely large compared to c over omega and c over omega equals c over 2 pi f equals and c over f is wavelength so lambda over 2 pi.

So the condition for imaginary component to be very small is if r not which is the radius of the sphere which is emitting these sound waves is very large compared to lambda over 2 pi or roughly one sixth of the wavelength of sound waves which are being emitted by the radial source, so that is the essence of this example that if I have a sound source which is emitting sound waves in a radial way.

And if it is a spherical source then it will be emitting complex power and the imaginary component of that complex power will be very small if the wavelength or the one sixth of the wavelength of sound waves which are being emitted are small compared to the radius of the sphere.

So this closes my illustration problem and now I will move on to the next topic which is related to this concept of directivity. Now we have talked about directivity earlier also but not in so much in an explicit sense, so earlier when we talked about directivity it was in context of interference of waves to radially propagating waves which are getting emitted from two point sound sources which are separated by distance and we saw that when these waves at point which is far away meet, they interfere sometimes constructively, sometimes destructively and we have some sort of a polar pattern which is not necessarily symmetric with respect to theta.

So we have what we did see earlier was in not such an explicit sense that when there are two sources, point sources and when they emit sound waves the overall sound pattern is not necessarily radially symmetric and there is some directionality associated with this type of a phenomena.

So what we plan to do in remaining part of today's lecture and maybe also in the subsequent lecture is explore this idea of directivity further and we will start with this discussion on directivity by developing some directivity patterns so that is what we will do and then when we are done with the directivity patterns then we will look at some other ways or metrics of measuring directivity through terms such as directivity index and some other parameters so what the first step what we plan to do is we will start developing directivity pattern for some different set of sources okay.

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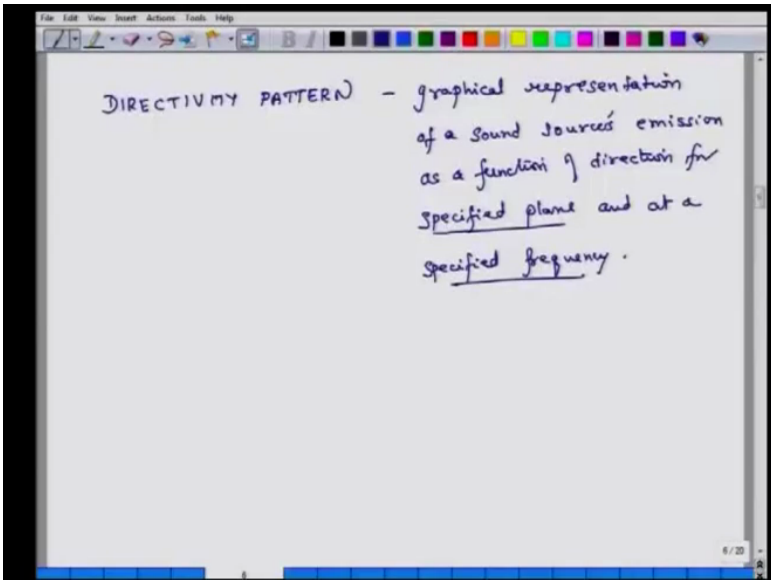

So let us define directivity pattern, so directivity pattern is a graphical representation it is a graphical representation and of sound sources emission as a function of direction for specified plane, this is important for specified plane and at a specified frequency, this is important to understand that these graphical representations are for specific frequencies so you may have one directivity pattern for set of sound sources which may be at a given frequency let us say at 100 hertz and this graphical pattern directivity pattern maybe significantly different if we alter the frequency and we make it 1000 hertz or something like that.

So it is for a specified frequency and in the other thing is that you have these directivity patterns plotted on specific planes so once you change the plane then the pattern may or may not remain necessarily same so we will start by illustrating the directivity pattern of a simple spherical source.

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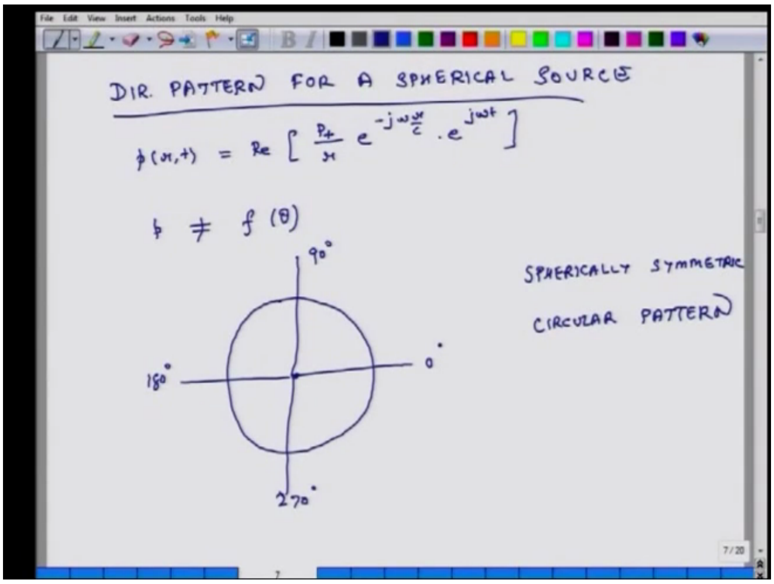

So directivity pattern for a spherical source, okay so we know that the pressure emitted by a sound source, a point sound source and if this is radially propagating wave with no reflections then the pressure function is real of complex pressure constant divided by r times e to the power of minus j omega r over c times e j omega t.

Now we see that in this relation p is not a function of theta, p does not depend on theta, what that directly means is that if I have a sound source let us say it is a point sound source and it is emitting radial sound waves then the directivity pattern for such a sound source will be circular in nature.

So the sound source is located exactly in the center of the circle and that is my directivity pattern so let us say that the SPL so in this case this is my 0 degree, this is 90 degree, 180 degree and that is 270 degrees so here we have spherical and because spherical is symmetric and because we are looking at the pattern on a plane so on that plane this is spherical symmetric translate to circular pattern, okay it translates to a circular pattern.

Now it just happens that for spherical source because everything is circularly symmetric the directivity pattern does not change when we change the frequency and also the directivity pattern does not change when we change the plane of observation so for this particular source, spherical source the directivity pattern does not change with respect to changes in frequency and also with respect to changes in the plane of observation.

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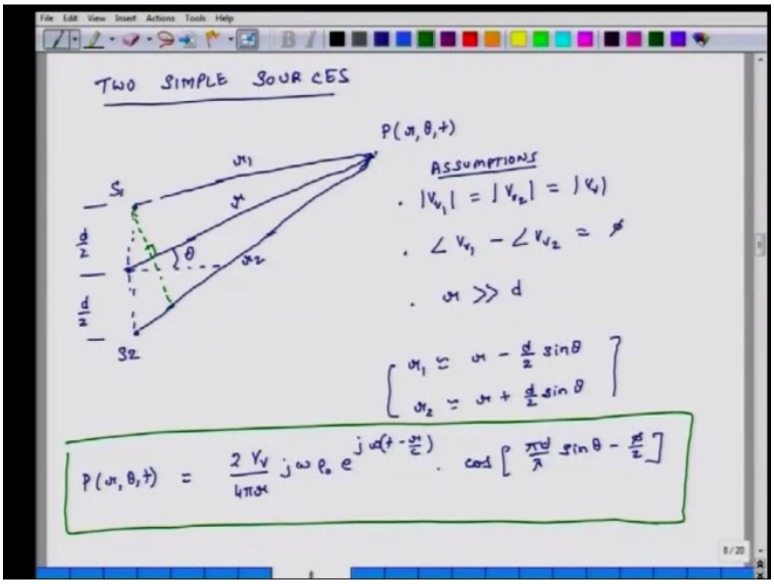

So we now move further and we look at a similar pattern for two simple sources, we are looking at directivity pattern for two simple sources and we are going to refer in this context some of the work which was done earlier in context of interference of two sound waves so let us say we have two sources source 1 and source 2 and they are separated by some distance and the midpoint of these sources is let us say this point.

So this is the midpoint which is not a source it is just a point of reference, the distance between this midpoint and S1 is d over 2 and the distance between midpoint and S2 is again d over 2. Now I have a point far away so at this point the pressure is P and this pressure depends on three parameters r theta and t so I will define T in a moment, so let us say from this midpoint I construct the line which reaches this point and the angle of this, let us say this vector is r.

So the angle of this vector r with respect to the horizontal line, this theta so the pressure at this far point, far away point will depend on how far what is the value of R, what is the value of theta and at what time are we observing so right away we see that whatever the pressure is going to be observed at this point it depends on theta so it is not spherically or circularly symmetric as we saw was the case for simple source which was emitting spherically symmetric or radially symmetric sound waves.

So I construct two more lines in this case one is between S1 and point of observation and let us say that vector is r1 and then the other line is r2 which connects the point of observation and S2 and then I draw perpendicular line from point S1 to this radius r that line and that is that this particular line in green is perpendicular to vector r.

So what we see here is, so what we will do is we will again revisit some of the relations we developed earlier and then we will start talking more about directivity patterns, so earlier what we had assumed was that the volume velocity of sound source 1 was same as volume velocity of sound source 2 and this is Vv, so the magnitude of this volume velocity is same.

Second thing we had assumed earlier was, so these are assumptions. Second thing which we had assumed earlier was that the phase of volume velocity 1 minus phase of volume velocity 2 is some constant phi, and the third thing which we had assumed was radius r is extremely large compared to d, where d is the overall distance between the two sound sources.

So with this understanding we now develop the relationship for pressure, before we do that we would like to know how is r and r1, how are they connected, so what we know is that if r is extremely large compared to d then r1 is approximately equal to r minus d over 2 sine theta and r2 is approximately equal to r plus d over 2 sine theta, okay. So these are the two things.

So with this set of assumptions and these two approximations earlier in our previous class we were able to develop an expression for pressure such that pressure at point, this point P which is r distance away and theta angle away from midpoint so that pressure equals 2 times volume velocity over 4 pi r j omega rho not e j omega t minus r over c times cosine of pi d over lambda sine theta minus phi over 2. So that is the pressure relationship which we had developed earlier.