Acoustics
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Module-4 Monopoles and Dipoles
Lecture 06
Numerical examples

Hello again, so in my last class we had tried to we tried to understand the nature of a spherical wave front a spherical sound wave front and in that context we tried to understand the nature of complex impedance, how it varies over distance as my as the value of r grows how does this complex impedance for a spherically progressing wave it changes then we also for a particular given wave front or a monopole source which in this case was issuing or emitting 5 Watts of power for this particular source we calculated the magnitude of pressure, calculated magnitude of velocity and also the volume velocity of this particular source.

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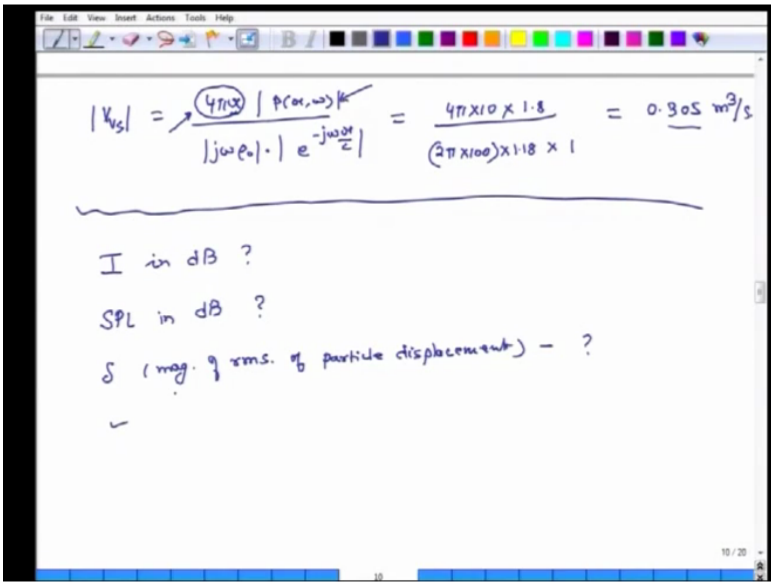

So in context of volume velocity the relation which we had used was this so volume velocity equals 4 Pi r times magnitude of pressure divided by magnitude of j omega Rho not times magnitude of e minus j omega r over c and we then plugged in different values so if r was 10 so this number so it was we calculated it was 4 Pi times 10 and then the magnitude of the pressure is 1.8 so times 1.8 divided by magnitude of j omega Rho not which is 2 Pi times 100 Hertz times density which is 1.18 times magnitude of e minus j omega r over c is 1 and this value came out to be 0.305 cubic meters per second.

So this is the amount of air which the spherical source pushes out each second. Now should be born in mind that this value or this value of volume velocity it depends linearly on radius it depends on radius. So there is a linear term here radius but the other thing so if I increase but it does not mean that if I increase my r by a factor of 2 volume velocity will grow, what will also simultaneously happens is change in pressure because as I move away from the source even though the radius is increasing the pressure or the sphere generated by this particular wave it decreases rapidly.

So even though this term increases, this particular term it decreases as r increases. So the volume velocity is the value of the volume velocity it reflects the position of the point of observation and also the pressure at that particular point of observation. So having said that we will continue understanding this particular source which is emitting 5 Watts of power at 100 Hertz and our next step is that we will try to figure out what is the value of I that is intensity in decibels, what is the value of sound pressure level in decibels and what is the magnitude of velocity displacement.

So what we are going to find out is I in decibels? We are also interested in finding out sound pressure level in decibels? And we are also interested in finding out delta which is the magnitude of RMS amplitude of particle velocity, so this is magnitude of RMS of particle displacement, so what is the value of I? What is the value of SPL? And what is the value of delta?

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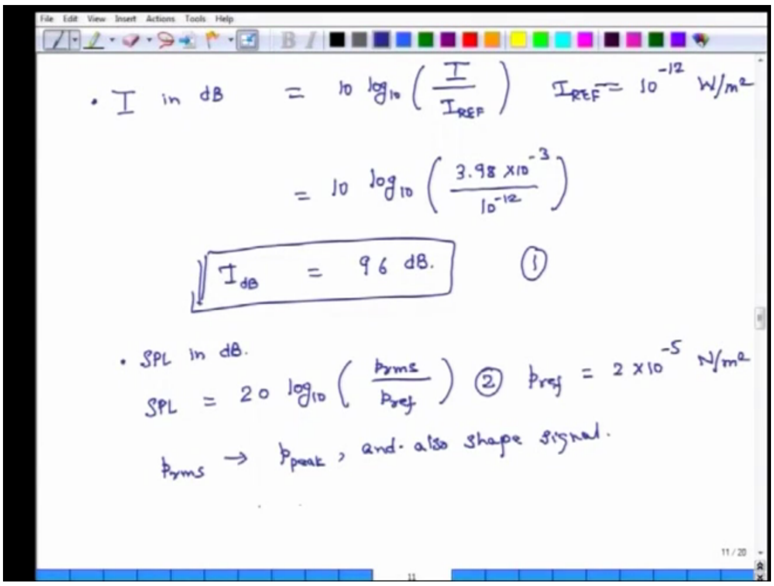

So we start with I so I in decibels it is defined as 10 log of identity in base 10, so I take a log of the ratio of amount of power dissipated per unit area which is I divided by reference value of I and I REF I REF for air is assumed to be 10 to the power of minus 12 Watts per square meters. So we compute further 10 to the power of log base 10 times the value of I which was earlier calculated in last class it is found to be 3.98 into 10 to the power of minus 3 Watts per square meter and reference value of intensity is 10 to the power of minus 12 Watts per square meter. So if I do the math what I get is that the value of I is 96 decibels I equals in decibels is 96.

Our next aim is to compute SPL or sound pressure level in decibel. So SPL in decibels is defined we have seen this definition earlier is 20 logarithm again base 10 to rms pressure divided by reference pressure ya and p ref or reference pressure is 2 times 10 to the power of minus 5 Pascals or Newton's per square meter. Now rms pressure so we call this equation 1 so equation 1, this is equation 2. Now rms pressure it depends on peak pressure and also shape of signal and it also depends on a shape of signal so for instance.

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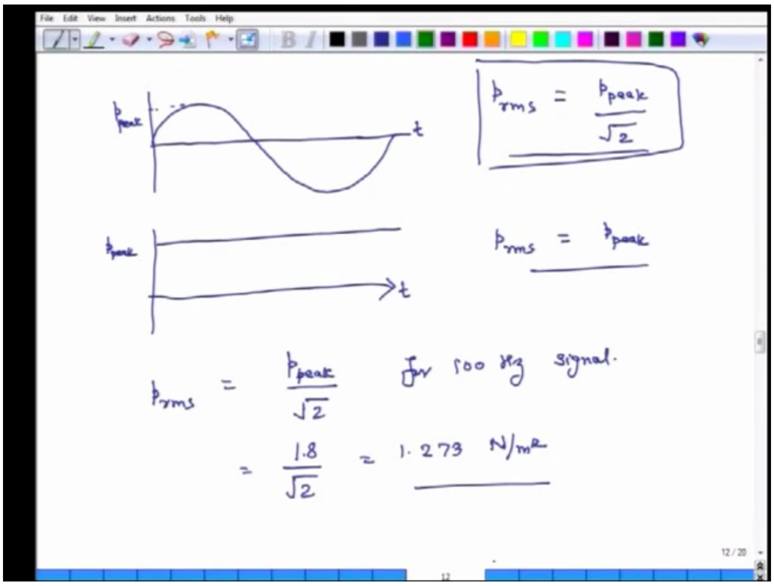

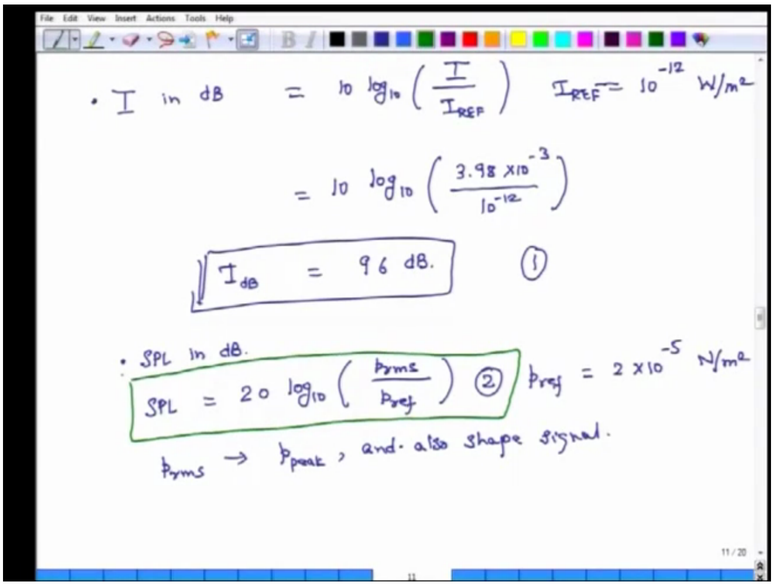

So let us look at this further so if I have a Sin wave, if I have a Sin wave this is my peak value let us say we call it p peak then for a Sin wave p rms equals p peak divided by square root of 2 let us look another Sin signal and let us say this is a flat signal constant. So p peak is p peak and this is my horizontal axis is time so it is a constant signal. So in this case p rms equals p peak.

So to figure out the value of rms of a signal we have to know a value of its peak and b also the shape of this signal. Now in this particular case we have been given that it is a 100 Hertz sinusoidal pulse sinusoidal signal or sinusoidal energy form which is being emitted by the source. So in such a case p rms will be p peak divided by square root of 2, so with this understanding p rms is equal to p peak divided by square root of 2 for 100 Hertz signal and in our last class we had computed peak value of signal to be 1.8 Pascals I divided by square root of 2 and this comes to be 1.273 Newton's per square meters. So I plug this value in my relation for SPL.

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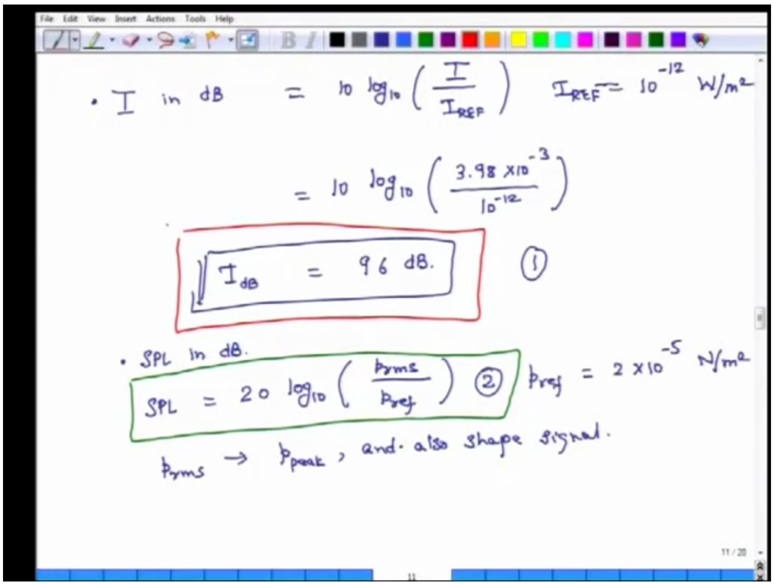

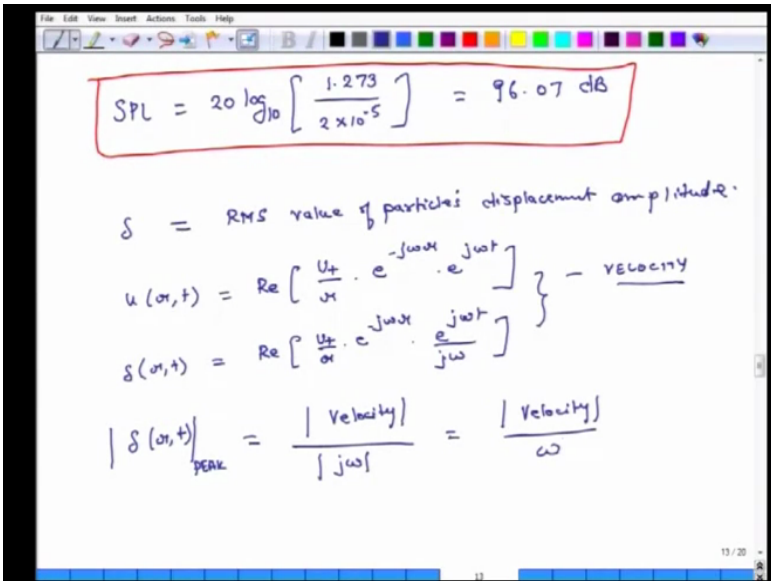

So SPL is equal to 20 times log in base 10 p rms which is 1.273 divided by reference pressure which is 2 times 10 to the power of minus 5 and that works out to be 96.07 decibels which is virtually same as this value so this value and this value will come out to be virtually same or identical. Our third goal is to compute delta, what is delta? It is equal to RMS value of particles displacement amplitude.

So till so far we have calculated pressure, we have calculated velocity and we have calculated volume velocity, what we have not calculated is the displacement how much each particle is moving in terms of millimetres or meters. So to compute displacement we have to know how velocity and displacement they are related. So we know that the relationship for velocity is this so u depends on r and t and actually it also depends on omega.

So this is equal to real of u plus over r times e minus j omega r times e j omega t, so this is my relationship for velocity and my displacement relationship I can get by integrating this velocity relationship over time. So what I get is real of u plus over r times e minus j omega r times e j omega t divided by j omega. So from these two relations what I get is that displacement of r over t and if I have to calculate its peak or magnitude, so peak equals magnitude of velocity divided by magnitude of j omega my velocity was so this is nothing but velocity by omega.

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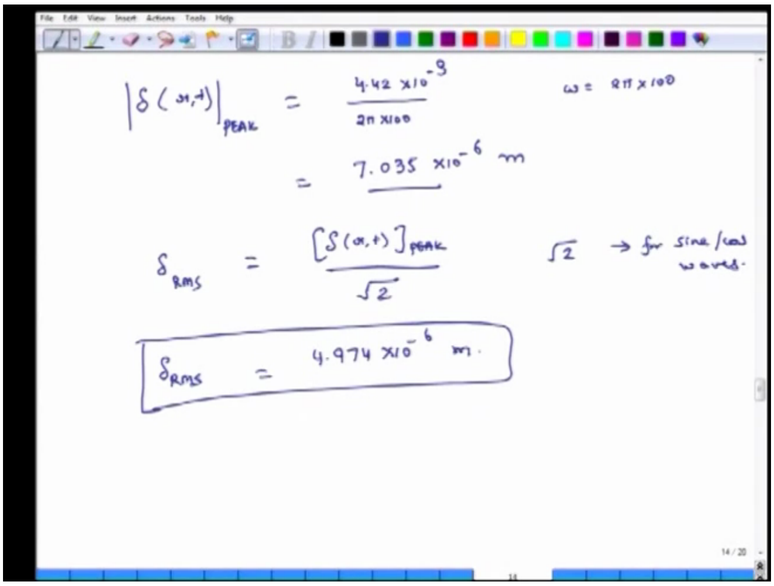

So the peak value of displacement is computed to be velocity peak or the amplitude of that velocity was 4.42 times 10 to the power of minus 3 meters per second and then I divide it by omega which is 2 Pi times 100 Hertz. So this works out to be 7.035 times 10 to the power of minus 6 meters or 7 microns very small distance and thus my RMS value is this number divided by square root of 2 because our signal is of a sinusoidal form. So it is basically peak value of this divided by a factor square root of 2 and this is factor for Sin waves or Cosine waves. So this works out to be 4.974 microns.

Finally we move further and we say okay now that we have also calculated displacement and velocity and all other parameters at r equals 10, one question we can ask ourselves is that how does the sound pressure level in decibels when it changes so at some location at r equals 10 meters it is 96 Pascals. Let us say that if I go down further may be at some location this sound pressure level may go down to 70 decibels. So if I know the (loca) if I know the decibel level at a given point can I figure out d distance between that point and the source so can I figure out the value of r.

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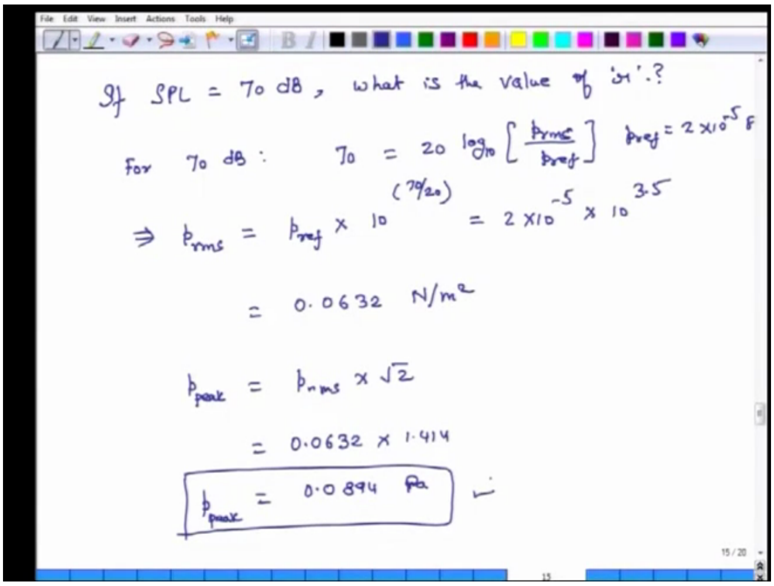

So here the question is if SPL equals 70 decibels what is the value of r? So we start figuring out the value of r and the first thing we do is we have to find what is the pressure when decibel level is 70. So for 70 decibels 70 equals 20 times logarithm p rms divided by p ref and p ref equals 2 times 10 to the power of minus 5 Pascals. So from this equation we can say that p rms equals p ref times 10 to the power of 70 divided by 20 and that is equal to 2 times 10 to the power of minus 5 times 10 to the power of 3.5 and this equals 0.0632 Newton's per square meter, now this is rms pressure.

So our next thing is that what is the value of p pressure, so then peak is equal to p rms times square root of 2 now this factor of square root of 2 once again I have to say that it is valid for a sinusoidal or a Cosine type of a signal. So it is p rms times square root of 2 and that means it is 0.0632 times 1.414 and this equals 0.0894 Pascals or Newton's per square meter, okay. Now we use this relation in the relation for pressure and then from that we can figure out the value of p.

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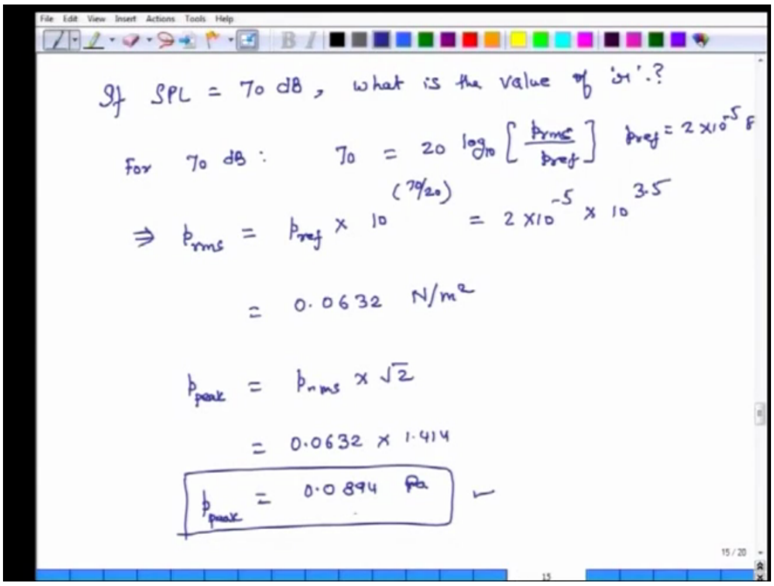

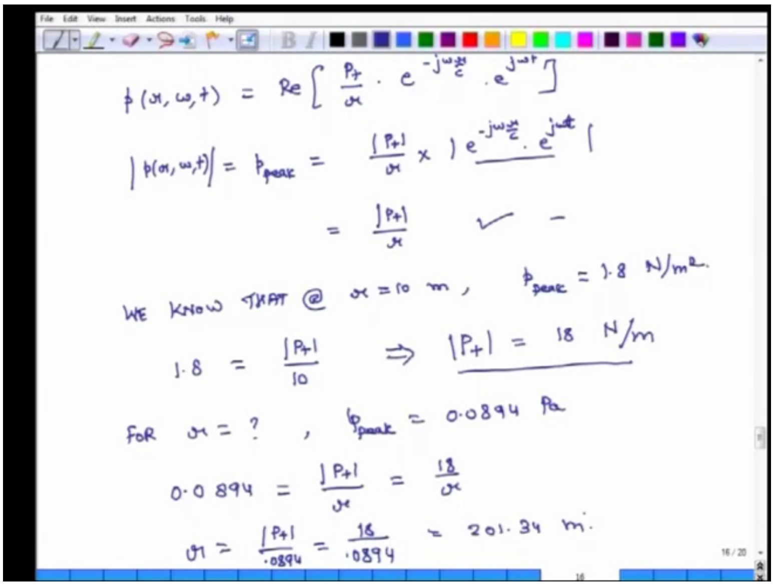

So we know that p r, omega, t equals real of P plus divided by r times e minus j omega r over c times e j omega t. So if I take if I want to know the magnitude of p r, omega, t then what I get is p peak equals magnitude of P plus divided by r because r is real times magnitude of e minus j omega r over c times e j omega t. So magnitude of this entire term is 1 so this is nothing but P plus over r but I do not know the value of P plus, so first I have to find what is the value of P plus.

So we know that at r equals 10 meters, p peak is equal to 1.8 Newton's per square meters. So 1.8 equals P plus times divided by 10, which means that P plus magnitude is 18 it is 18 Newton's per meter, wrote that the unit of P plus is Newton's per meter and once I divide that

P plus divided by r I get the value of p peak. So this is the value of P plus. So with this understanding I put this in this equation.

So for r equals unknown P peak equals we have calculated corresponding to 70 decibels 0.0894 Pascals. So now I have to find what is the value of r? So 0.0894 equals P plus divided by divided by r and we know that magnitude of P plus is 18 divided by r. So from this equation r comes out to be P plus divided by 0.0894 equals 18 divided by 0.0894 and that equals 201.34 meters.

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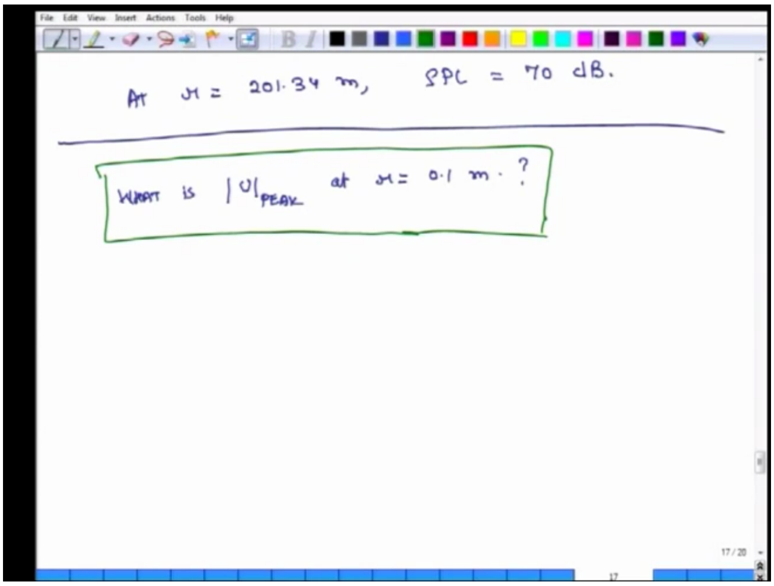

So what we mean we say that at r is equal to 201.34 meters, SPL equals 70 decibels SPL equals 70 decibels. Now finally we look at the value of u, so the last part of this question is that what is u peak at r equals 0.01 meters 0.01 meters or 10 centimetres. So is it that at 10 meters if u peak is x, then at 0.1 meters is it just 100 times that or is it something different so that is what we are going to compute is the last part of this exercise. So this is our goal, what is the value of u peak at r equals 0.1 meters.

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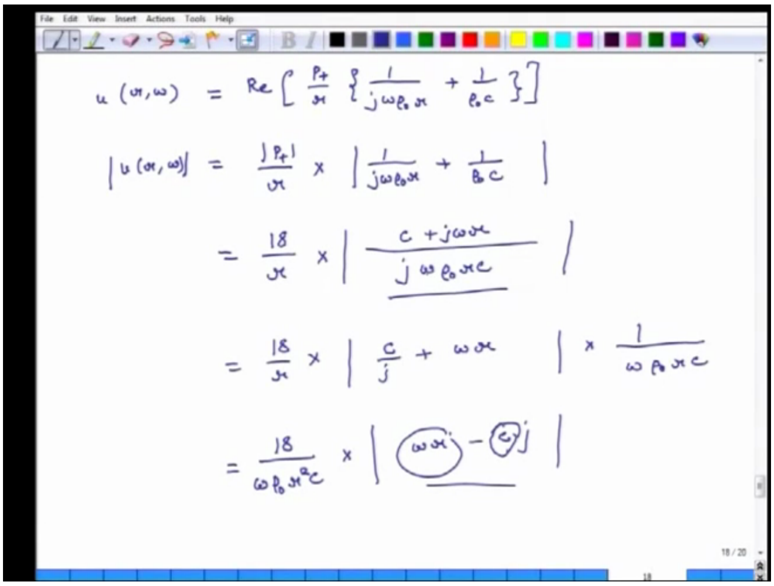

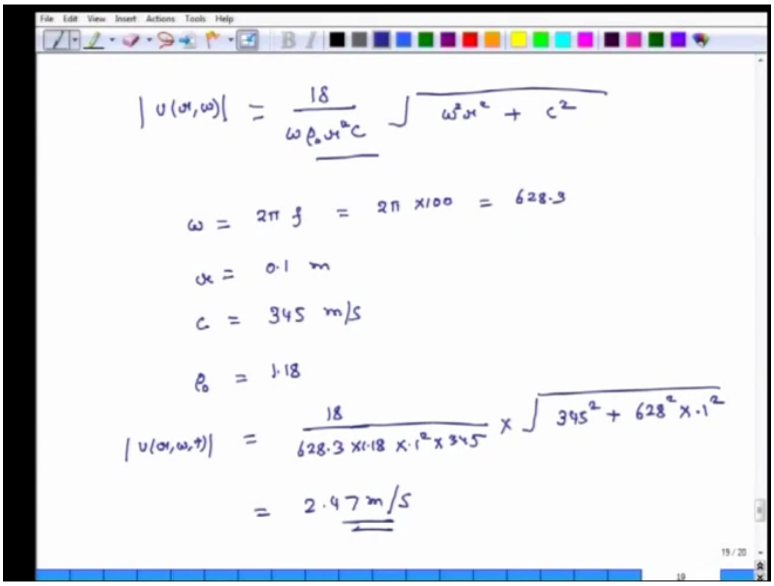

And we start figuring out the answer to this question by first writing the expression for u complex amplitude of velocity which is u r, omega and that is equal to excuse me the relationship or velocity is ya real of P plus over r that is related to pressure times the impedance divided by the impedance which is 1 over j omega Rho not r plus 1 over Rho not c and if I take the magnitude of this complex amplitude of velocity then I have to take magnitude of magnitude of P plus times magnitude of this entire thing 1 over Z which is 1 over j omega Rho not r plus 1 over Rho not c.

Now amplitude of P plus is 18 and then this is r and then I have to take the amplitude of this thing 1 over Z. So first I will simplify this, so what I get on the numerator is c plus j omega r and on the denominator I get j omega Rho not rc and this becomes 18 divided by r times, so what I am doing here is I am simplifying this term.

So I take omega I this divide excuse me I take omega Rho not rc out of the bracket out of this these vertical brackets so I get here is omega Rho not rc and then in the (parent) in the inside these vertical brackets I get c over j plus omega r. So if I simplify this reorganize this information further what I get here is 18 divided by omega Rho not r square c times omega r minus c over j. And then finally what I get is this equals 18 divided by omega Rho not r square c times and if I have to calculate the magnitude of this it is basically square of this entity and square of oh excuse me so I made a error here this c over j should be replaced by c times j. So I have to take the magnitude of c j also which is c.

So what I get is omega r square whole thing square plus square of c and now I know that omega equals 2 Pi f is equal to 2 Pi times 100 Hertz so that is 628.3, r we have been told that it is 10 centimetres so it is 0.1 meters, c equals 345 meters per second and Rho not equals 1.18. So with this information once I plug in all this information in this relation what I get is magnitude of velocity at r equals 10 centimetres equals 18 divided by 628.3 times 1.18 times 0.1 square times 345 multiplied by square root of this whole thing which is 345 square plus 628 square times 0.1 square, which works out to be 2.47 meters per second, okay. So at 10 centimetres the value of this peak velocity at 10 centimetres is 2.47 meters.

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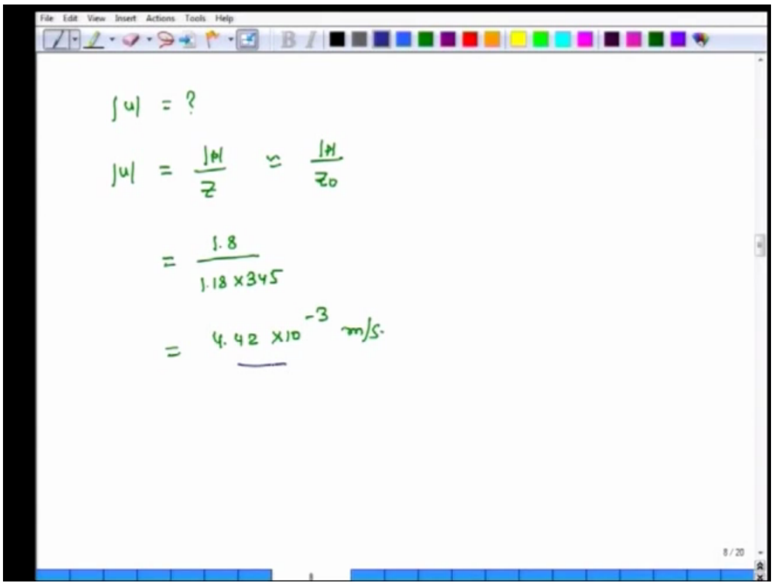

And earlier we had computed that at 10 meters the value of that velocity was computed to be 4.42 times 10 to the power of minus 3 meters per second. So it is not a linear relationship that if I go 10 times further down my velocity is going down by a factor of 10 and so on and so forth because of this complex impedance terms of course once I am very far out in the far field then complex impedance or excuse me the value of impedance offered by a radial wave is essentially more or less same as that of a planer wave so then things start becoming more like planer waves but in case of in case or in the range when we are fairly closed to the source the impedance offered by a spherical wave is significantly different than that offered by a planer wave.

So with these two examples I hope I have been able to illustrate the overall behaviour of radial waves when there are no reflecting surfaces around the radial wave and what we have found in this entire discussion which has extend over today's lecture and also the previous lecture is that A if I am very close to the source than the behaviour of a spherical wave and the behaviour of a planer wave they are significantly different, B if I am far away from the spherical source than the planer wave and a spherical wave they behave more or less very similarly.

The reason for this similarity when I am far away from the source is because the impedance in of a spherical wave and that of a planer wave when I am far away from the source they are approximately the same. Finally the criteria for being near the source or far away from the source depends on the relationship between radius and wavelength so if my radius or the distance from the source is very large compared to lambda over 2 Pi which is approximately one sixth of wavelength of the wave.

So if that radius is very large compared to one sixth of wavelength then I can say that I am far away from the source and in that case the planer wave and the behaviour of a spherical wave will be very similar. If I am close to the source which means in mathematical terms that if the radius is very less compared to that one sixth of wavelength then I am in proximity of the source and in that case the behaviour of a spherical wave and that of a planer wave will be markedly different. So with this I think I have explained to you the sufficient clarity how radial waves propagate in free space in absence of reflecting surfaces, thank you very much.