Acoustics
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Lecture 2
Module 1
Review: Linearity, Complex Numbers And Spring Mass System

Good afternoon, so in today's lecture we will cover a couple of review topics as they relate to acoustics engineering, essentially what I will be covering is a bunch of subjects and concepts which you may have got an exposed to during your high school years or even during your first and second or third year while you have been doing your undergraduate program in engineering.

So some of these system concepts we will be mapping directly later in the course in the acoustics course. So it will be fruitful to review some of these and also practice a little bit more before we actually start working deeply into the area of acoustics. So essentially I will be covering 4 key areas, one is that I will and a very brief try to explain what is a linear system because we will be using a lot of linear principles in the area of acoustics.

The second area which I will be covering, again very briefly today is the area of complex variables and then associated with that would be complex time signals and then that from there we will move on to RLC circuits as we use them in electrical engineering because later in the course you will find some of the tools and techniques we use in RLC circuits to be very fruitful and useful in the area of acoustics.

So that is in brief the overview of what we will try to accomplish today. So we will start with linear systems and we have often heard that this system is linear, system A is linear, system B is not linear but specifically whenever we use a term linear there is a context behind it. So suppose one says system A is linear than that person should be unless it is really clear should be explicit in saying that system, this particular system is linear in variables A and B because you can have a complex system and it could be linear in some variables and it could be non-linear in some other variables.

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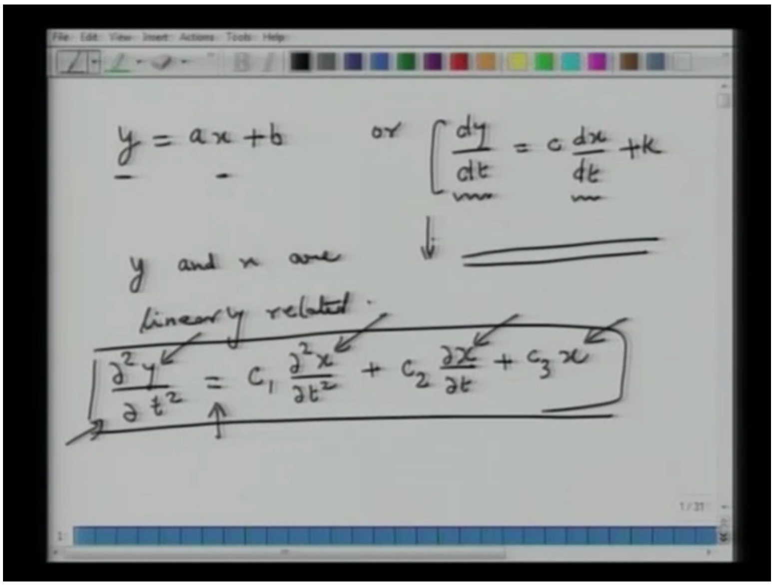

So we will take an example, suppose I have a relationship y equals ax plus b or I have another relationship dy over dt equals c dx over dt plus k. So in this case the system is linear between y and x. In this case the system is linear between dy over dt and dx over dt and of course if I integrate this then I will find that this the same equation is also linear and y and x. So here we can say y and x are linearly related and we can make a similar statement in context of this particular relation also.

So I will give you another example though I have a second derivative of y, so second derivative of y in time let say it equals a constant C1 times second derivative of x in time plus another constant C2 del x over del t plus c3 x. So again y inspecting this whole relation because the power of y is one in this whole relation, power of x a entire relation is one. So you can say that this is a linear relation between x and y because a x and y are on the opposite sides of the equivalent sign and second thing is that both the power of x is 1 and power of y is 1. So even though you are having higher-order differentials the relationship can still be linear with the power of the variables are one and they are on opposite sides of the equivalent sign.

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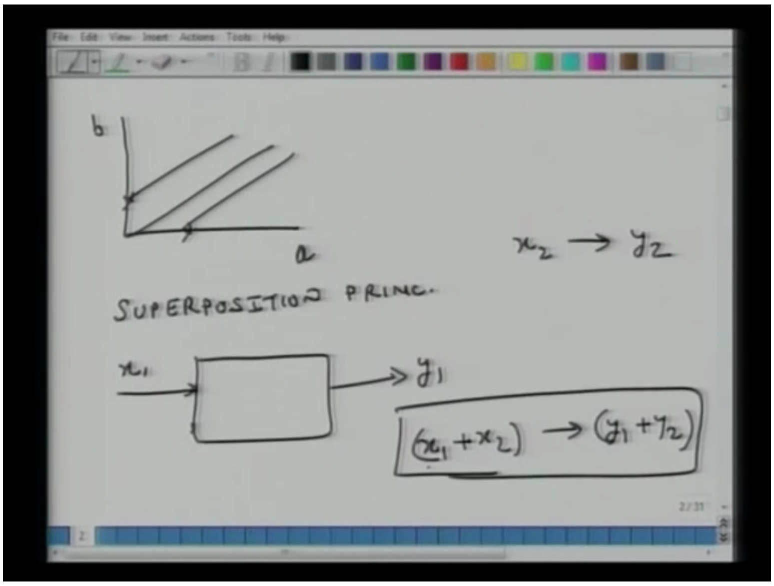

So typically if you have a linear relationship between 2 variables and when you try to plot variable a versus variable b you will get a straight line. The straight line can pass through the origin or it can cut the x axis or it can cut the y axis at non-zero intersect, so that is another feature for a linear system that I have 2 variables and I am plotting these 2 variables against each other everything else being same then I will get a linear response.

The third feature is that I can use super position principal, so in linear system I am able to use a super position principal what does that mean? Suppose I have a system and I have an input x1 and it yields an output y1 and then in another case I provide it an input x2 and it yields an output y2 then if I provide an input x1 plus x2 then if it is a linear system the output will be y1 plus y2.

So this is again an important feature of a linear system which later as we go deeper into the area of acoustics we will be using to understand the response of systems when they are receiving complex time signals. Suppose a system which is linear in nature it is just receiving a frequency input of x hertz x1 hertz at a certain amplitude level and it gives me a certain output response and then it is receiving another input at x2 hertz which is different in frequency and it gives me another response than if I try to sum up x1 plus x2 response that is the output will give summation of the individual outputs.

So this comes in very handy and we also use this idea in the context of transfer functions which we learn later today itself. The third feature of a linear system is that if it is having a particular input shape or a particular type of a signal then the output signal in the steady state situation will be of a similar shape whatever is going in the output is going to be of a similar shape whatever is coming out and this is very important to understand because if we are sure that there is a linear system and it does not matter whether it is acoustics related or any other thing and if we know that if it is a linear system in nature and if it is receiving only sinusoidal inputs and in the output if I am seeing a step function or a non-sinusoidal output then it on my system is not linear or my experiment is not correct or there is something wrong in my understanding or analysis of the whole problem.

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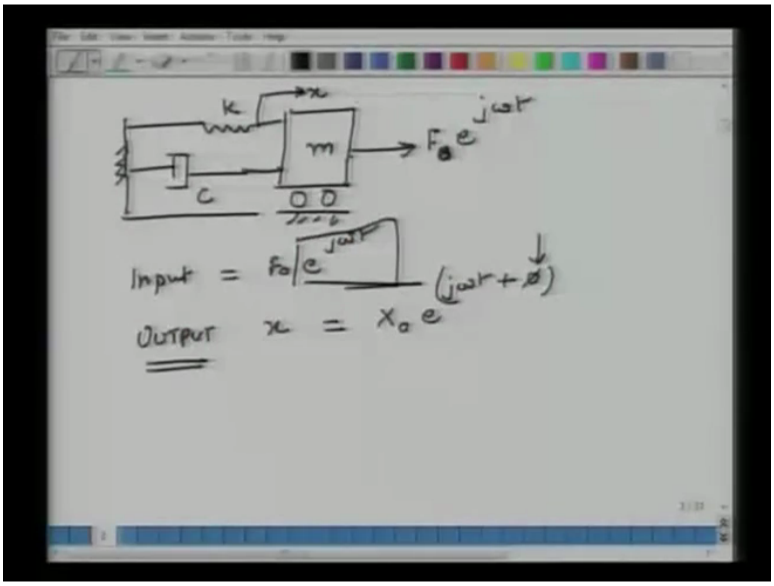

So consider a spring math system which also has a damper and then the spring and the damper are now connected to a (()) (9:20) m, stiffness of the spring is k, the damping constant is c and I am exciting this entire system why F not ej omega t. So my input in this case equals F not ej omega t then my output response. So suppose I am interested in knowing the displacement at this point and suppose I call it x.

So then the output response has, so the shape of the input function is defined by this term, ej omega t, so output response also has to be such that it is some constant times ej omega t plus a phase because the phase this component is not going to alter the shape. All it will do is it will induce a difference in the timing at which the peaks of input and the peaks of output happen.

So phase is 0 then the peaks of input function and peaks of output will coincide and the phase is not 0 then they may not necessarily coincide but the shape of the function for input and out will remain same. Another feature of a linear system is that if I have small changes in the input then the changes in the output response will be proportional to the magnitude of changes which are coming in.

So if I have small changes coming in the changes in the output will be small in a proportional sense because there will be a proportionality factor. If I have large changes coming in the input function then the changes in the output also will be accordingly larger.

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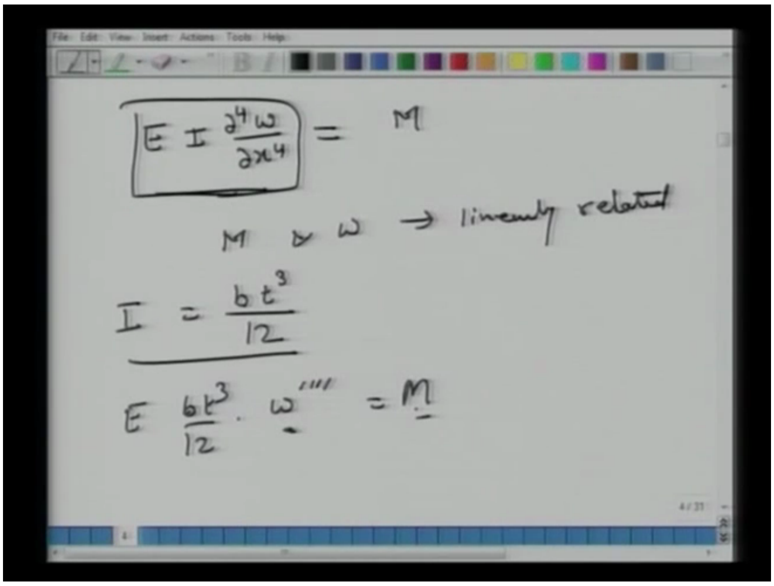

So we will do another example, so I have a beam equation E I del4w over del x4 and where w is the displacement of a beam. So in a beam equation I have this term coming up and this is related to the moment exerted in a beam. Now here M abd w are linearly related. Now we know that I equals the thickness of the beam time cube of it times the width, so bt cube over 12 if it is a rectangular cross-section, so this is I, so if I put this in then I get E bt cube over 12 times 4 derivative of the deflection equals moment.

So again M and w this may appear very trivial but it is important to note because as we do more detailed analysis M and w are linearly related but M and t are not linearly related, so that is all I wanted to capture about what is a linear system, so that once we start solving complex problem in the dream of acoustics and also start doing experiments, we at the back of our minds be sure that the system which we are dealing with is it really linear or is it not so linear? So that we are cognisant of whether our answers or experimental observations are consistent with our assumptions or not.

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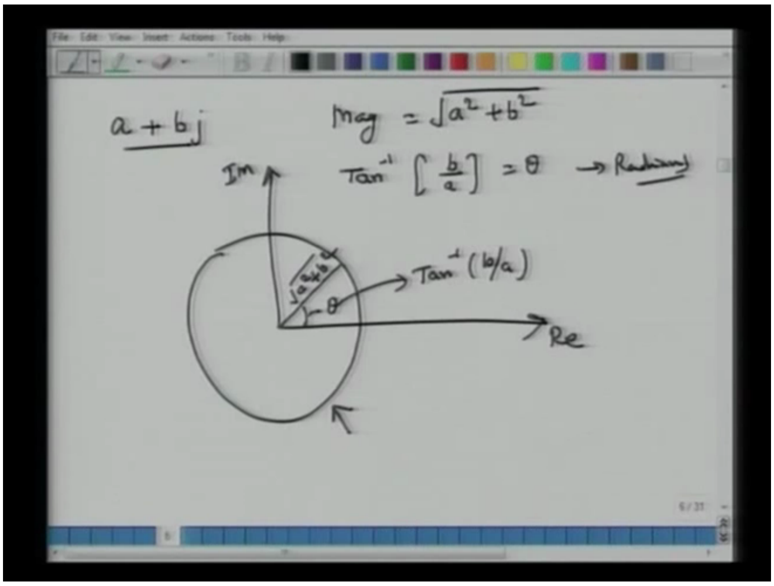

So the second thing I wanted to capture was complex variables. So we know from our high school mathematics and also first and second year of engineering mathematics that exponent of ej theta is cosine of Theta plus j sine theta where j in this course is minus 1, typically we use i as square root of minus 1 but in this course we are using j as square root of minus 1 because we are using i for other variables such as correct.

So if I represent this graphically I have a real axis, I have an imaginary axis; I have a vector as radius r equals one and this angle is theta. So if I draw normal here this value which is along the real axis is cosine theta and the vertical component is sine theta. So this is my function and this is its graphical representation.

So understanding this structure we can then generalize that if I have any complex variable of the firm a plus bj I am able to represent it in such a form, how do we do that? So again let us say there is a complex number a plus bj and if I have to represented graphically than the length of the radius will be the magnitude of this variable. So magnitude is a square plus b square the square root of this.

Similarly the angle or the Valley of Theta will be such that it is Tan inverse of b over a, so this is my theta, so using such an approach again I construct a circle this is my real axis, imaginary axis that is my radius and the magnitude is a square plus b square and that is the Valley of theta and that is Tan inverse b over a and this is in radiance. So if I have any number 3 plus 4 or 1 whatever number I have I should be able to fairly easily represent it in this graphical format.

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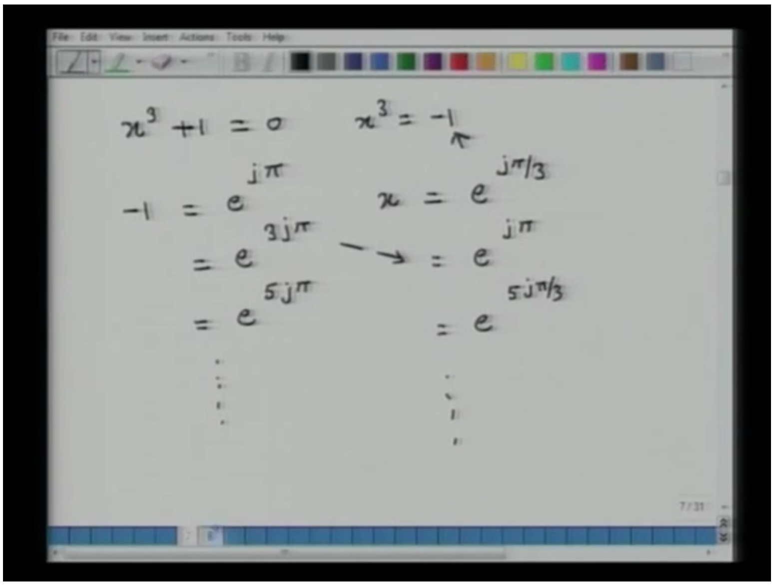

So we can use this complex numbers in a variety of ways, we can also use them to solve equations. So let us say that I have an equation x cube plus 1 equals 0 and if I want to solve for x, so my x cube equals minus 1, okay. Now let us see how we can represent minus 1 in a graphical format. So minus one could be written as ejpi we can also write minus 1 as e3jpi we can also write the same thing as e5j pi and so on and so forth.

So my x is cube root of this thing, so is either it could be ejpi over 3 that could be one solution or it could be ejpi which corresponds to this value of minus 1 or it could be e5jpi over 3 and so on and so forth.

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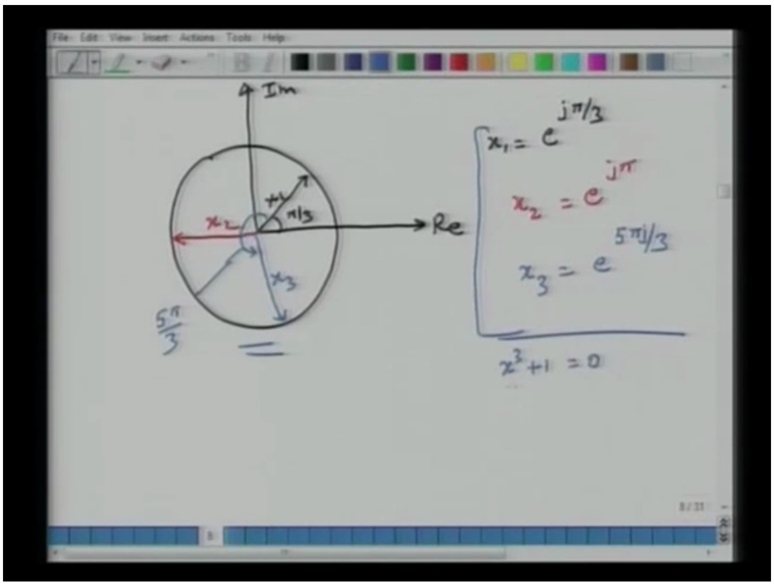

So now let us draw them on a complex plane, so I draw a circle that is my real axis this is my imaginary axis. So my first solution was ejpi over 3, so pi over 3 is 60 degrees, so that is my first solution x1, x1 equals ejpi over 3 and this angle is pi over 3. My second solution x2 equals ejpi, so this is my x2, my third solution is x3 equals e5pi j over 3. So 5pi j over 3 is here this whole angle.

This angle is 5pi over 3, this is my x3 and all other solutions x4 because this will have in finite solutions will be basically repetition, so my x4 will be same as x1, x5 will be same as x2, x6 will be same as x3 and so on and so forth. So I have a graphical representation of 3 unique solutions for the equation x cube plus 1 equals 0. We will very quickly cover some very fundamental basic identities now about complex variables.

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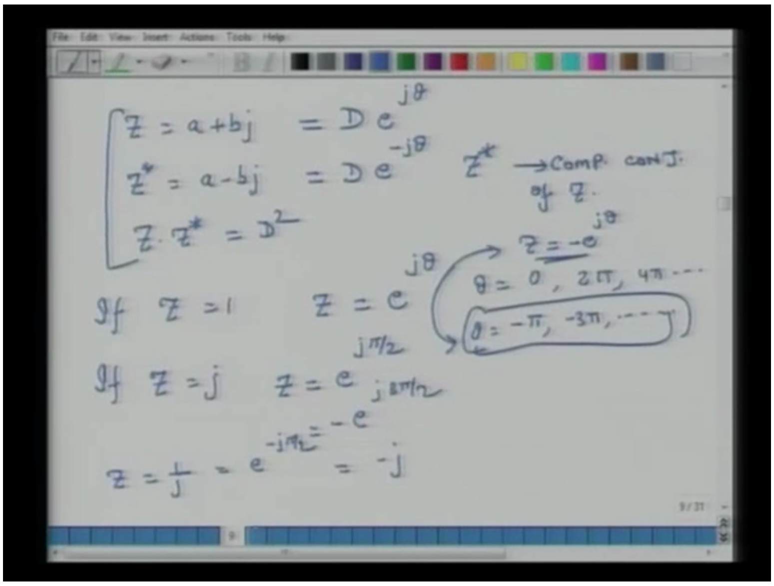

So let us say z equals a plus bj, so I can express it as a real number D times ej theta where theta as we saw earlier is Tan inverse of b over a and D is the magnitude a square plus b square the whole thing square (()) (19:34) and then I have another number z star equals a minus bj, so I can express the same thing as e minus j theta, so z star is complex conjugate of z this is our definition.

So complex conjugate of z which is a plus bj is a minus bj and a property of this is that z time’s z star equals D square which is the magnitude square. Couple of other identities if z equals 1 then in complex plane I can write z equals ej theta where theta could be 0, 2pi, 4pi and so on and so forth or it could be minus pi, minus 3pi and so on and so forth. Similarly if z equals j which is a unique length along the imaginary axis then in complex plane z could be represented as ejpi over 2.

“Professor- Student conversation starts”

Student: If theta is minus 5 then, z is minus 5 because cos pi is minus1 and also cos minus 5.

Professor: Actually you are right, so actually it should have been, yes it should have been 02pi 2 r and alternative solution could be z equals minus ej theta where theta is this thing.

So I have 2 sets of solutions, one is e equals plus ej theta where theta equals 0 2pi, 4pi and so on so forth and the other one is z equals minus j theta where the associated theta value are minus pi, 3pi and so on so forth, okay.

“Professor-Student conversation ends”

So going back z equals j then z could be ej pi over 2 and then we can again or it could be minus ej 3pi over 2. Similarly if z equals 1 over j then I can call it as e minus j pi over equals minus j. So these complex variables can be used in a variety of ways if I have additions of different phases and if I use complex variables I can very easily add the exponential term and then express the exponential terms in terms of cosines and sines and I am able to manipulate complex additions and subtractions of vectors by using the idea of complex variables.

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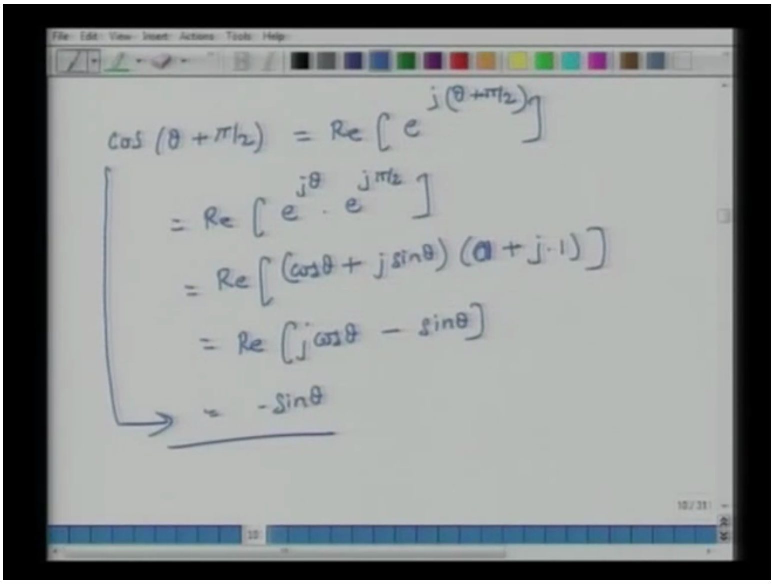

So for instance I know that cosine plus pi over 2 is minus sine theta but if I have to use complex variables it is basically real component of ej theta plus pi over 2 equals real component of ej theta times ej pi over 2 equals real component of cosine Theta plus j sine Theta and then I now expand on ej pi over 2. So cosine o pi over 2 is one plus, no cosine of pi over 2 is 0 plus j time's sine of pi over 2 is 1. So I get real cosine theta times j minus sine theta equals minus sine theta, so I can prove some complex identities also using such a.

This is again a simple illustration but we will use some of these principles and rotations or calculation and that will make our life significantly easy. So we have talked about what is a linear system we have very briefly captured some basic things about complex variables. So now we will talk about complex time signals.

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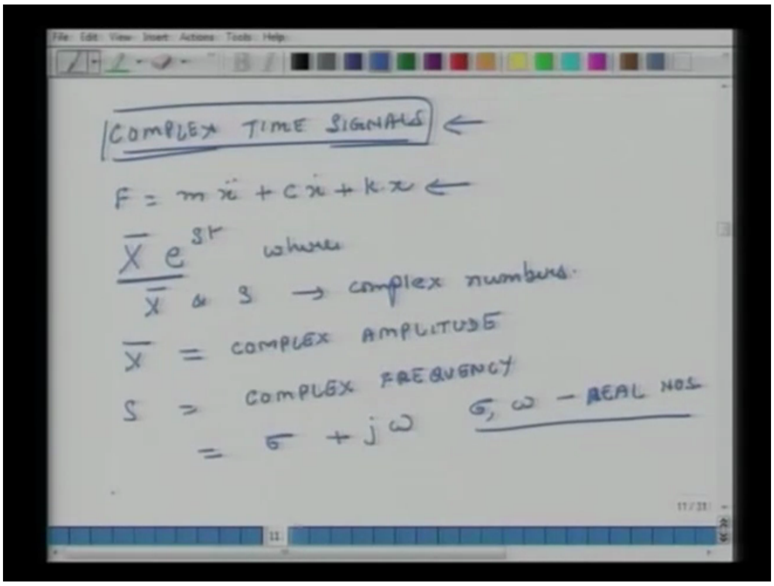

Complex time signals, so it happens that quite often we develop differential equations in time and also our space for instance you have a spring mass damping system such as (()) (24:38), mx double dot plus cx dot plus kx and we have to solve and in such situation this notion of complex time signals comes in very handy. So we will do that, suppose we have F equals mx mass time acceleration plus damping time velocity plus stiffness time displacement.

So solution for this kind of an equation in a very general sense could be of form x bar e st where x bar and s both can be complex numbers. So again a very general solution could be of this form x bar est where x bar is a number and S is a number but they need not be real numbers and x bar is called complex amplitude also S is called complex frequency and this can be decomposed further as Sigma plus j omega where sigma and omega are real.

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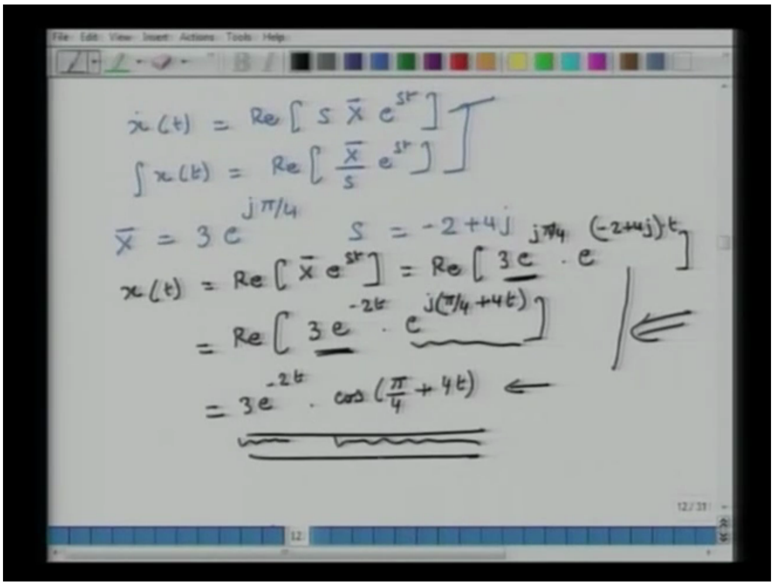

Sigma and omega real numbers but S itself could be complex thing, so if I have this kind of a representation, my velocity will be real component of S x bar est basically what I have done is differentiated the displacement and then I take it real component that is my velocity which I can physically measure. Also know that if I have to integrate this thing, so that gives me x bar over s est. So again differentiation and integration becomes fairly simple process.

"Professor- Student conversation starts"

Student: Over here assuming that x bar does not contain time x bar is a constant.

Professor: It can be imaginary, it can be purely real but it does not have time in (()) (27:26), yes the time is.

Student: Sir basically that is because it is a linear system.

Professor: We have assume the shape of, well I mean I am not using features of linear system to talk about this but I have assumed that x bar est is a complex time signal, it could be anything. I am not saying right now it is a solution of this thing, if there is a complex time signal of this shape than this is by definition called complex amplitude x bar, S by definition we are calling complex frequency which is equal to Sigma plus jw, j omega and x bar is a number, it is not having time embedded in it.

So then if I have to find velocity I know complex displacement, so I just differentiate it and take its real component this is my velocity, if I have to integrate x all I have to do is divide that by S, so that is all I am saying.

So we will do an example, so let us say my x bar equals 3 ejpi over 4 and S equals minus 2 plus 4j, black colour, so my xt equals the way we have defined is real component of x bar est, right? Equals real component of 3ej pi over 4, so again there is no time in it in x bar itself times e minus 2 plus 4j times t. So the time is right now only in the exponential part just to retreat x bar and S are not functions of time themselves.

So now I rearrange, so I get real components of 3e minus 2t. So I am basically breaking up into real and imaginary component and ej pi over 4 plus 4t, did I do the maths right? Minus 2t jpi plus pi over plus 4t, so which tells me, so this is real, so I can take it out times and then the real portion of this guy is cosine pi over 4 plus 4t this is my x, okay.

"Professor- Student conversation ends"

So we have broken it x into 2 components, one as an exponential component and then other one as an oscillatory component and we are multiplying both of them based on mathematical manipulations which we did here.

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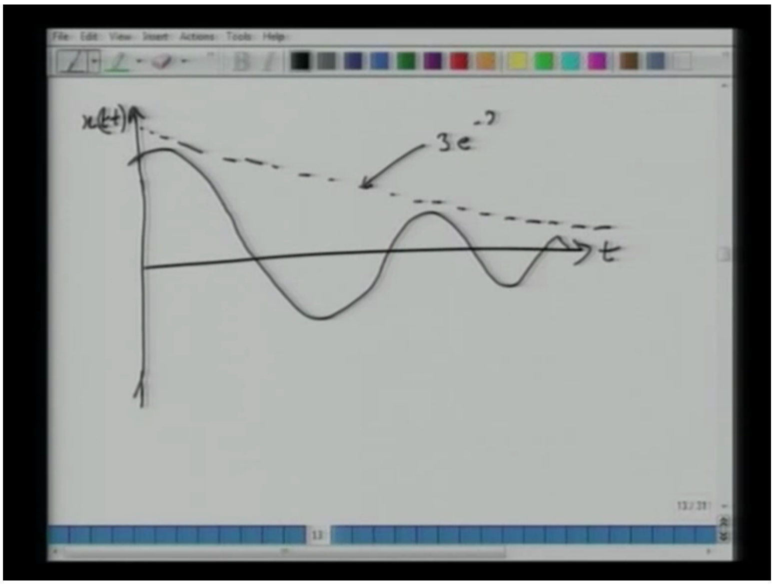

So now we will plot x as a function of time, so we will plot this function over time. So I am plotting x key here this is my times axis and so I get my plot something like this and so on and so forth and the envelope is defined something like this.

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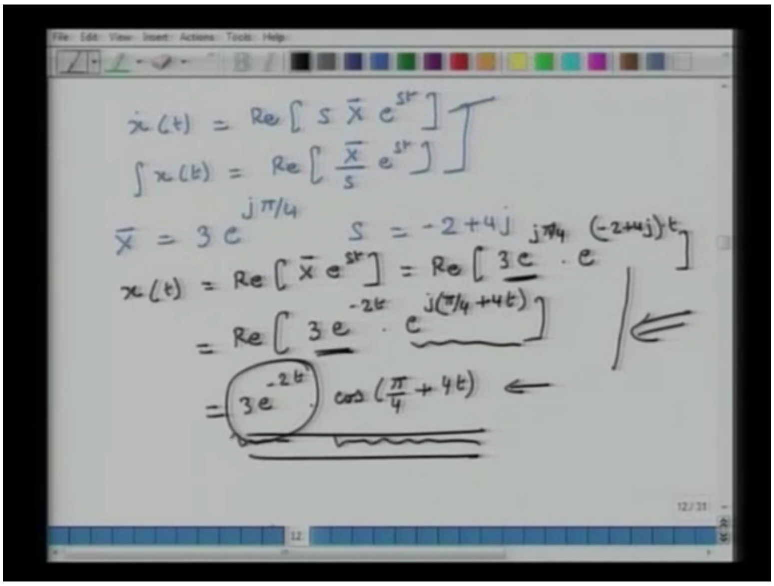

So this is envelope which is of plot of 3e minus 2t which we got from here, so this is defining the envelope.

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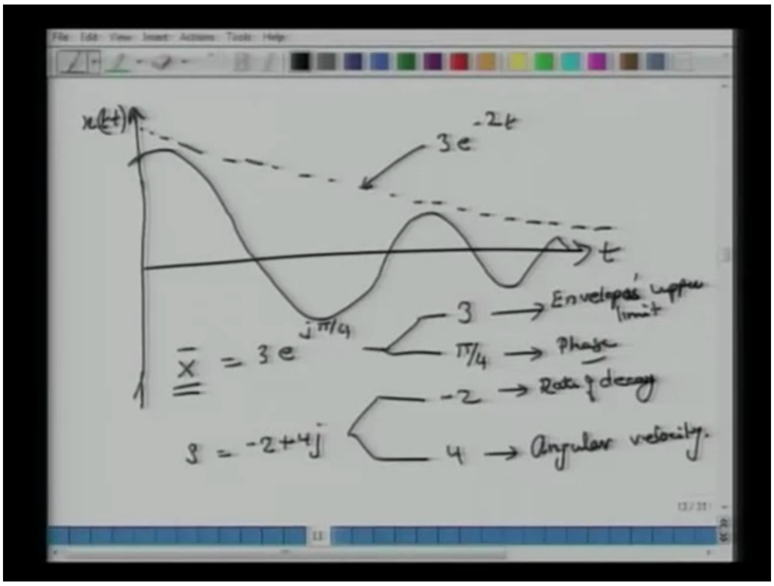

And the oscillating signal is the actual value of the function over a period of time. It happens that because the phase difference here there is a nonzero phase value which is pi over 4. So at t equals 0 this point is not touching the envelope. So based on what we did earlier x bar equals 3ej pi over 4 and this has 2 components, one component is 3 the other component is pi over 4.

So this is like a envelope a limit that is the maximum value a function can get this is your phase and then the complex frequency is minus 2 plus 4j, again it has a real component minus 2 and this is rate of decay and it symptomatic of the damping characteristics of the system and then you have 4 and that is essentially your angular velocity. So you have a complex amplitude which tells you the magnitude which defines the magnitude also and also the phase part of it and then you have complex frequency which helps you understand the damping characteristic of the oscillation or the signal and also the angular velocity of the frequency. So that is all I wanted to talk about complex time signal is at least today and we will talk more about complex time signal is as we move forward.

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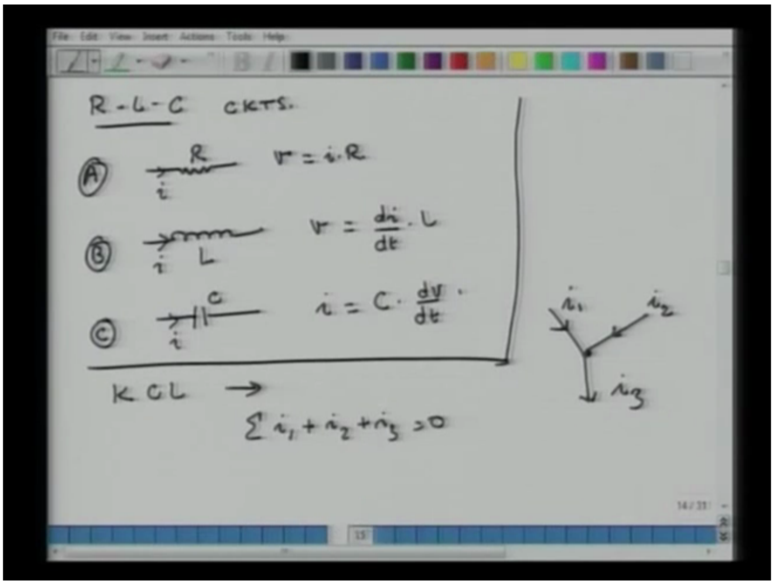

So now we move on to the third vector degree what we will be talking about and that is about RLC circuits. So resistance, inductance and capacitance circuits. So we all know at the first electrical element is very commonly used is a resistance if I have current i and R then voltage across the resistance is i tends R. Similarly if I have an inductor of value L and there is a current going through it i then voltage is date of change of current di over dt times L and then finally if I have the capacitance i is the current going through the capacitance of value C then the current and voltage are related such that current equals capacitance times rate of change of voltage.

So these are the 3 basic elements in electrical circuits which we will be using a lot in acoustics then we have Kirchhoff's current law which essentially says that if I have a note through which the flow of current i1, i2, i3 then Kirchhoff's current law says summation of i1 plus i2 plus i3 equals 0.

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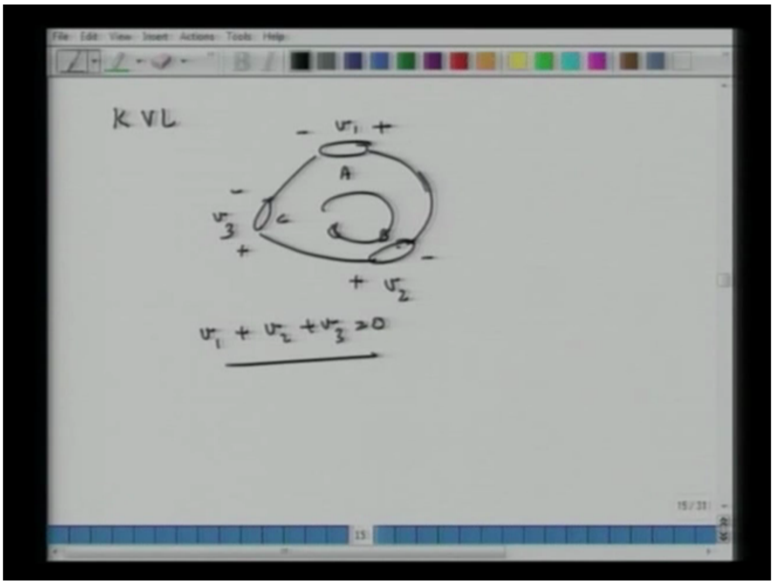

Another law developed by Kirchhoff's is called KVL or Kirchhoff's voltage law. So I have a loop, a close loop of electrical circuit let say around this element I have voltage of V1, here I have voltage of V2 and here I have these are 3 element ABC and here I have a voltage difference of V3 then Kirchhoff's voltage law says that if I go around a loop then it's V1 plus V2 plus with 3 equals 0. So we will be using these notions also in acoustics circuit, so that's why it's good to recapitulate.

So the other thing I wanted to talk about, so let's look at the circuit. So we will do a very simple example and use some of these electrical engineering elements and try to understand them a little better and also recapitulate some of the concept which you have learnt earlier.

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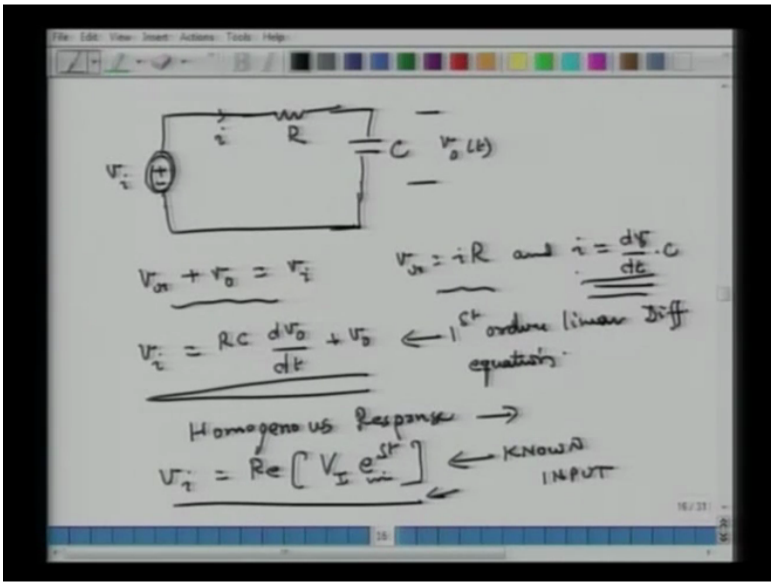

So let’s say I have a circuit with the voltage source Vi which generates current i flowing through a resistance R and also capacitor of value C. So the question is that if I know Vi which could be pulsating then what is the voltage difference across the capacitor as a function of i? So we know that Vr plus Vnot equals Vi from Kirchhoff’s voltage law and we also know that Vr equals iR and we also know that i times i equals d vnot over dt plus, no times capacitance. So current and voltage across the capacitor are related by this thing.

So if I synthesise all these relationships what essentially I get is Vi equals RC d vnot over dt plus Vnot and what you have here is first-order linear differential equations. So this equation where we try to solve it a very general solution will have 2 parts, one will be what we call a homogeneous function and the other will be a particular response. So we will have a homogeneous response and we will have a particular response and the homogeneous response will be highly transient in nature.

So after a certain period of time it will decay, so for instance if I have a system if I have a glass and a moving mass hits it initially the glass will shake but after a certain time it will start moving steadily. So the initial response is called the transient part that dies over period of time. In the lot of acoustic systems we are more interested in study strait response.

So we will focus on little bit more in this course on the study strait response of the system and we will focus not that much on the transient response of the system because a lot of physical devices, suppose you have an engine which is running and once I start an engine there is a perturbation in the system and have hear one particular type of noise coming out from the system but once the engine has started running the sustained noise level is of a different it maybe of a different type and in general people are bothered that over a period of time how does the engine sound rather then what happens right at the point when you are starting an engine.

So we will be focusing a little bit more on the study strait response of our acoustic systems and that we can do in a fairly straightforward way if we use complex variables and also Laplace approaches. So for this particular function let say that Vi is real component of VIest this is known input I know how a power generator is working, so I know this. So if I know this and if the system is linear than my output also has to have a similar shape.

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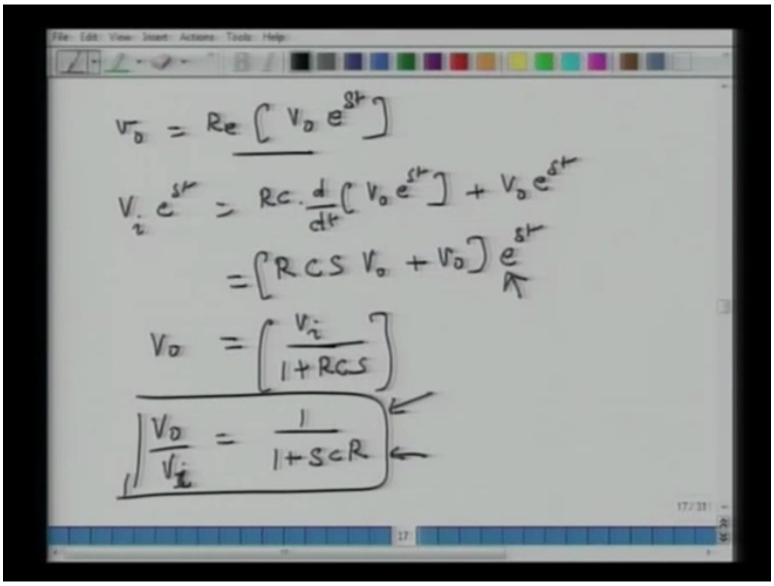

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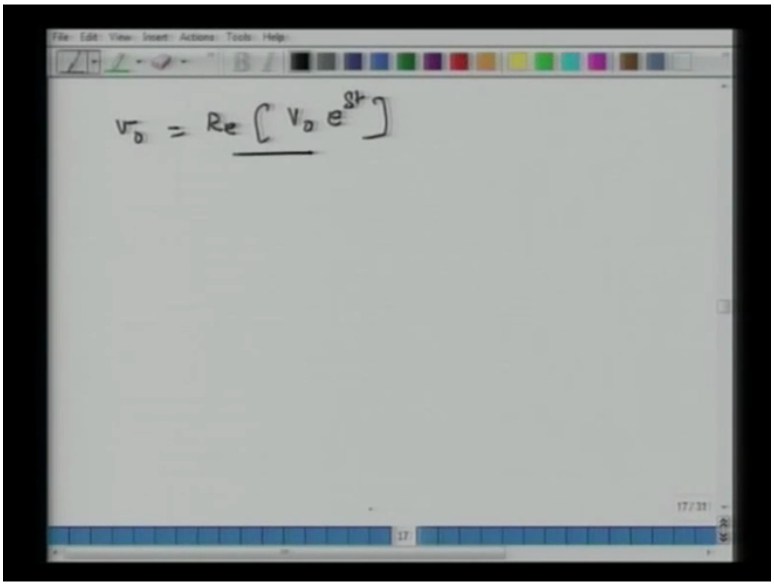

So I can assume that Vnot is a real component of a complex time signal Vnot est.

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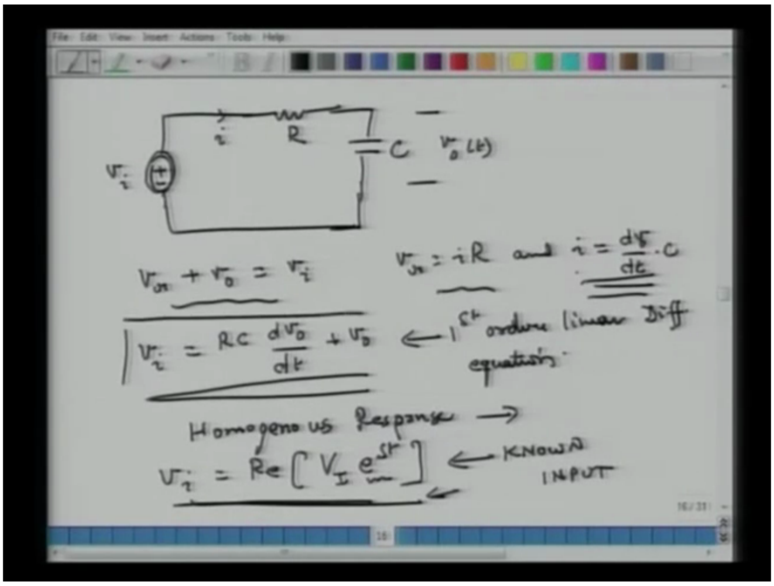

So now I take this relation and I plug it, so I put this relation and the relation for Vnot in this entire equation.

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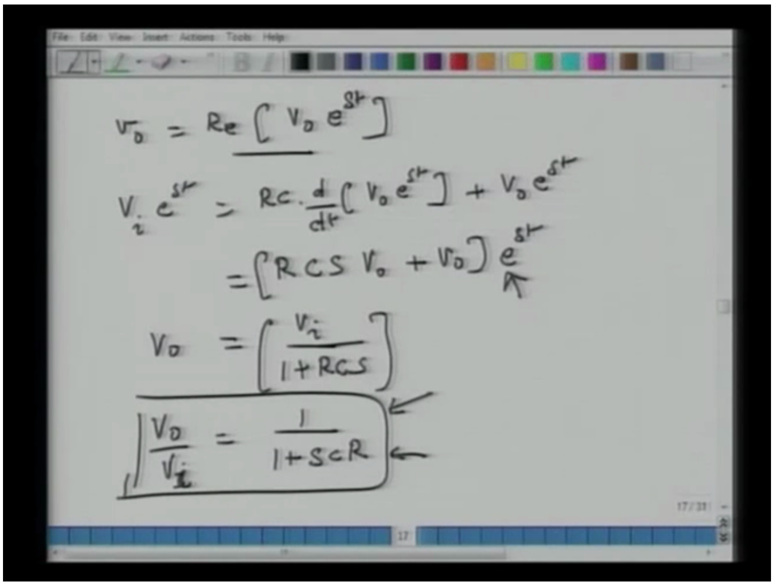

So what I get is Vi est equals RC times d over dt of Vnot est plus not est and now I differentiate and do the maths. So what I get is RCS Vnot plus Vnot est. so I get Vnot equals Vi over 1 plus RCS or Vnot over Vi equals 1 over 1 plus SCR. So this will help me determine the study strait response acoustics.

The time component is embedded in this exponential term I know Vi so I can calculate Vnot and then I take the real component of Vnot est and I figure out what is the actual physical voltage across the capacitance? So again we are seeing that the complex quotation comes in very handy to solve some of these problems especially we are interested in solving steady-state response system.

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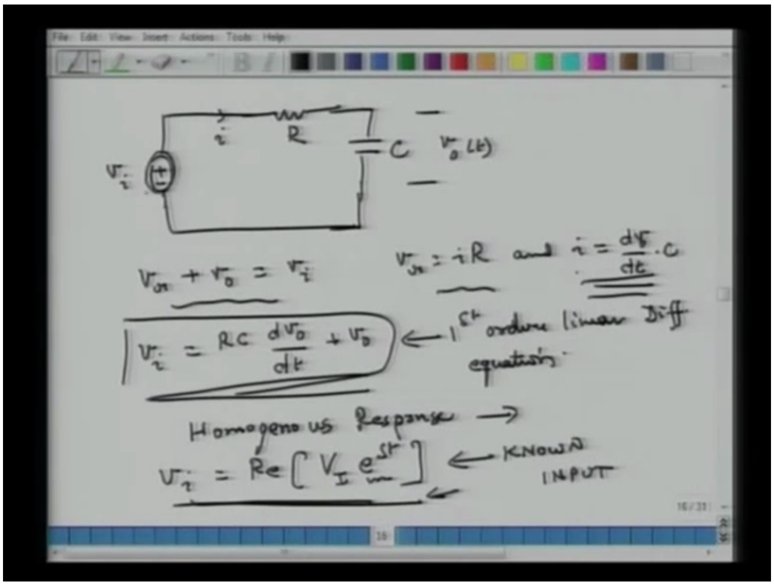

So we have drawn this circuit Vi and Vnot, so input was Vi and output which we were trying to find out is Vnot and we developed a ratio Vnot over Vi is this much.

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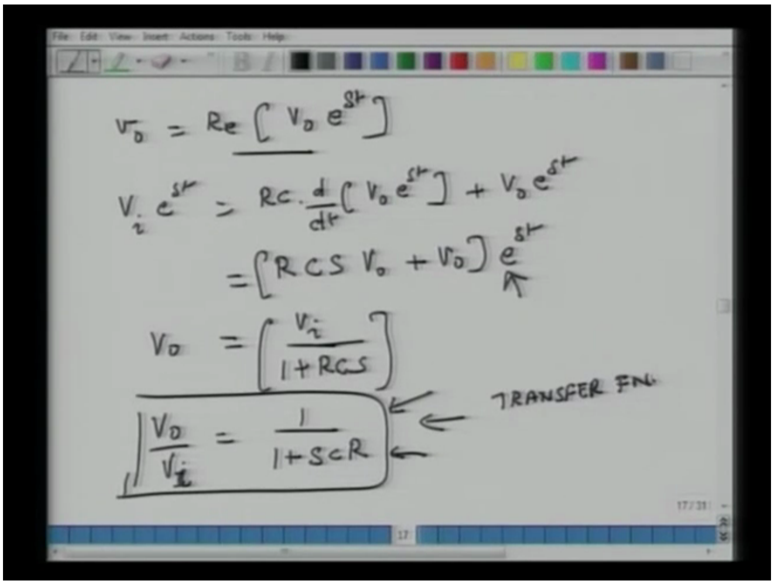

This ratio is called transfer function; in a strict sense here it just happens that Vnot and Vi are the complex magnitudes of the actual voltages. In a very strict sense the transfer function which can be designated as HS is basically response over input but it just happens that est in this strait thing goes away because it is a linear system.

So this is what we call a transfer function and linear systems the time (()) (43:59) component of it will always go away because it's a linear system. So they will cancel out and the idea of transformation works only in a linear framework we don't use a whole lot of transfers in non-linear systems because they depend on time and then for each time instance you could have a different ratio. So then it comes very complicated. So rather there you solve equations in time in real-time itself.

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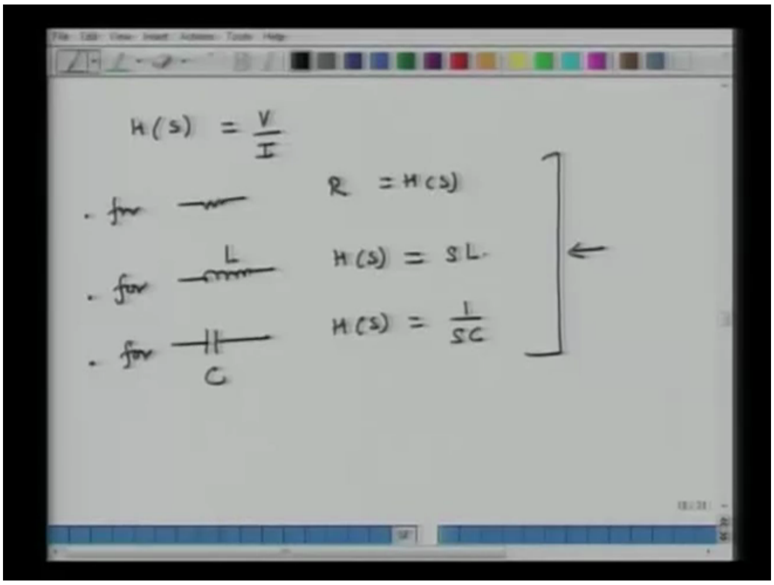

So we will right couple of transfer functions, so HS equals, if my HS or transfer functions V or I then for our resistance my transfer function is R, right? For an inductor my transfer function is, so this is again current going through the resistance and the voltage across resistance the ratio is R.

For an inductor with value L this number would be s times L, for a capacitor of value C the transfer function would be 1 over SC, so again it is handy to remember these ratios, these values and what we will do in another example is that we will construct a circuit, so…

"Professor -Student conversation starts"

Student: Those transfer function what.

Professor: So the transfer function is in these 3 elements, the transfer function is basically the ratio of voltage across the element and the current going through the element not through the whole circuit current going through the element and voltage across the element if I take the ratio of those then I get these values.

Student: So they are the ratio of the complex means complex volts.

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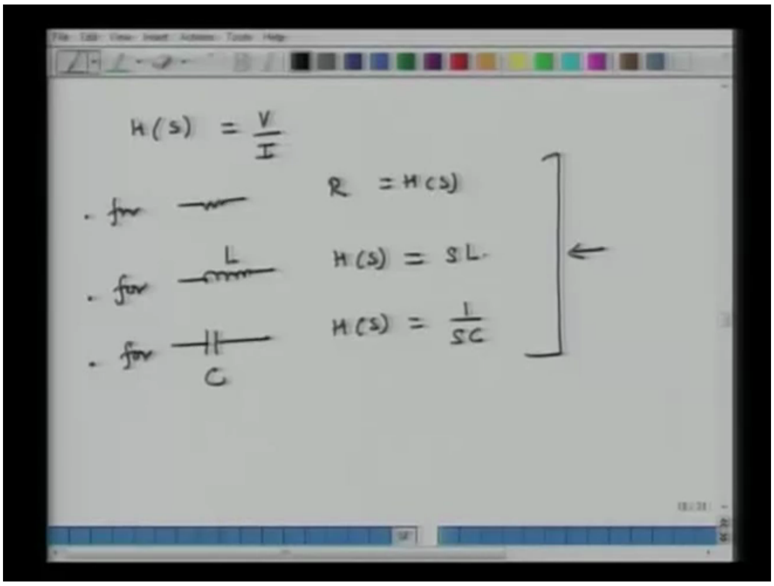

Professor: Transfer function can always be complex because they are ratios of complex entities.

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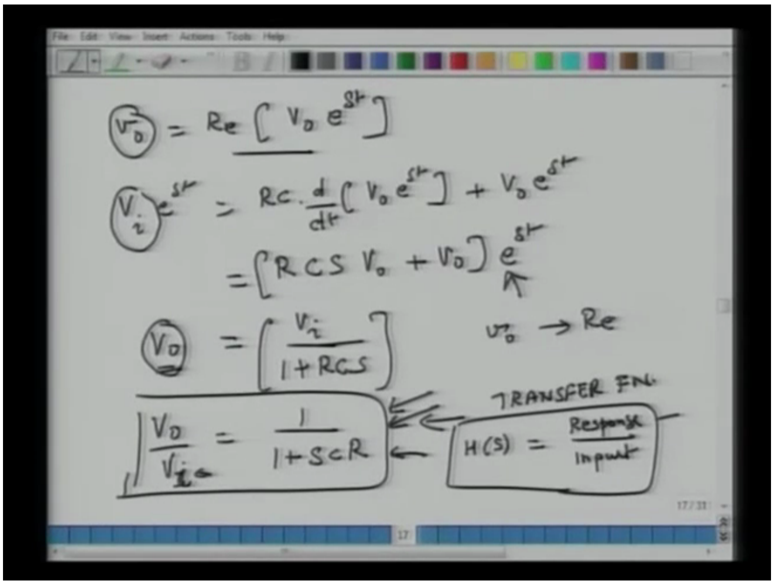

Remember when we did this Vnot is complex and so is VI is complex similarly I can be complex when I want to measure the actual current then I take its real component, the phase is embedded in the (()) (46:30) but the ratios of volt Vnot and VI, so this is complex, this is lowercase that is real, this real number.

Similarly Vnot is complex but lowercase is real when I take real component of Vnot est.

Student: S?

Professor: S is again complex, it can be complex. As we saw in an earlier case if you have vibrations and if they are damping, damping over a period of time it can have a real component and an imaginary component and if it's a pure vibration if it is a pure non-damping vibration then it will be purely imaginary.

"Professor-Student conversation ends"

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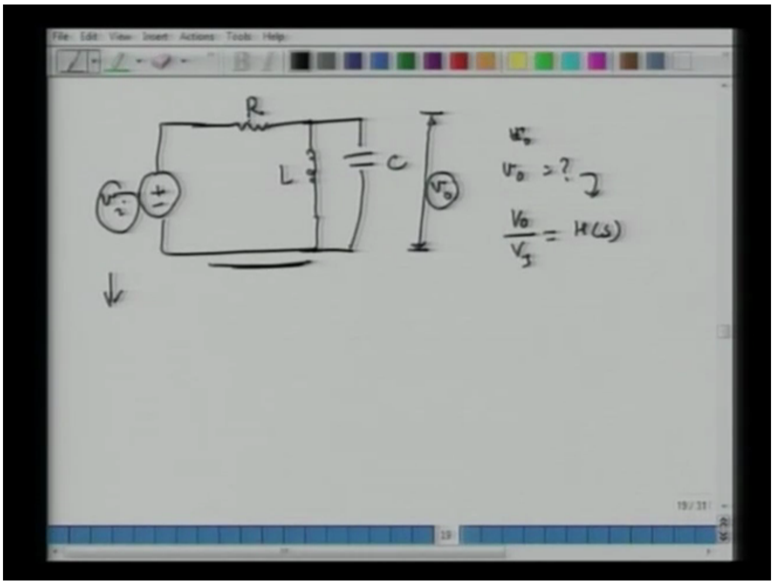

So we will do an example and see how we use some of this transfer functions to simplify circuit analysis. So let's say I have a voltage source and I have a resistance R and inductor L and a capacitor C and I'm interested in finding what is the actual voltage across capacitor? So one approach which we saw earlier was had these developed a differential equation and solve it, is a faster approach, so the question is that what is what is Vo?

So to do this maybe I can be well placed if I can figure out what is Vnot over VI which is complex representation of Vnot and VU, so how do we do that? If I know this ratio that is a transfer function then I can very easily multiply this times this find Vnot on the complex plane take its real value and that gets me Vnot.

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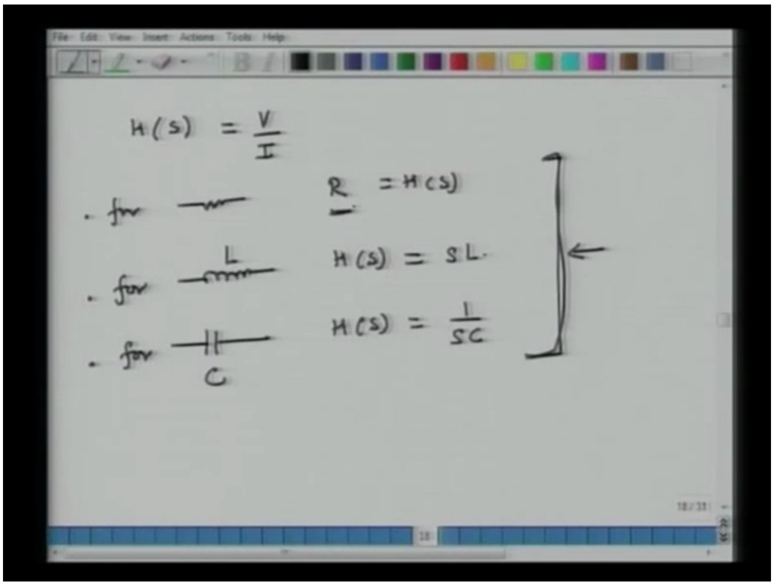

So a relatively straightforward way is that I transform this circuit in frequency space which means that I replace R by its equivalent transfer function which happens to be R, I replace SL by its I replace L by SL and C by SC.

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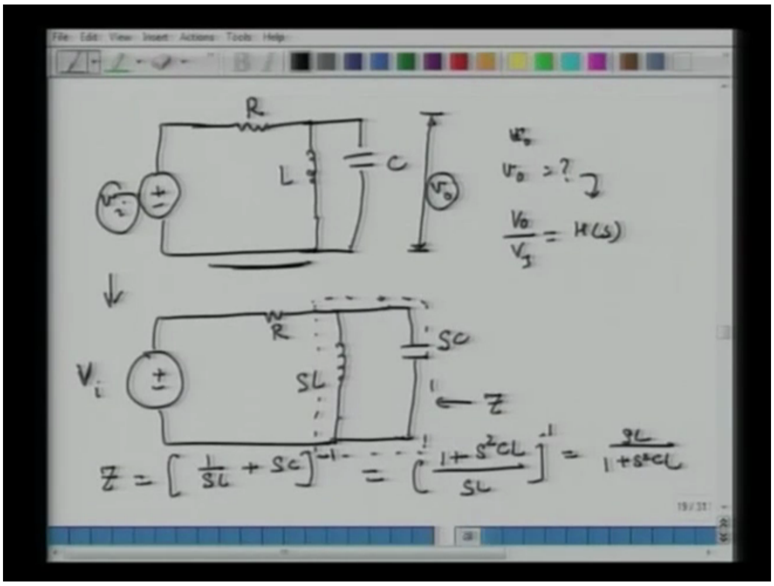

So I am transforming this in frequency space or frequency domain. So VI transforms into a complex entity upper case R remains R, inductor transforms to SL and there is a mathematical proof but we are just capturing whatever is the learning from electrical engineering here directly and this capacitor gets transformed into SC. So now I compute the impedance of this circuit and I don't have to worry about phase and everything.

Because the phases are embedded in these things themselves, so let's just first compute the impedance of this blog. So impedance let us call it as Z, so in that case because SL and SC desire in parallel, it is one over SL we just assume that they behave like resistance even though because there is S embedded here, so 1 over SL plus impedance of the capacitor is this and then I take inwards because they are in parallel and what I get is 1 plus S square CL over SL which is SL over Oh! I have taken inverse of this 1 plus S square C.

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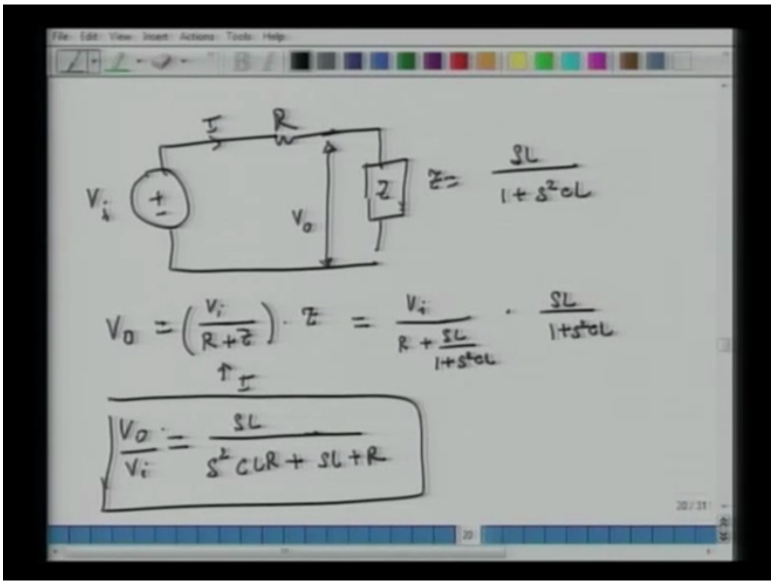

So I make another circuit I have a resistance R, I have this impedance z where z equals we calculate it SL over 1 plus s square cL. So what we have done is just replaced and we have in frequency domain and we are trying to the voltage across this impedance z, so now we can calculate Vnot. So Vnot is what? Is my complex current is I that is basically Vi over R plus z, right?

That is my current complex current I and then I multiply that by z that will be the voltage complex voltage across z, so I do the maths, so I have Vi over R plus SL over 1 plus S square CL that is my complex current times z SL over 1 plus s square cL. So if I do this entire math carefully what I get? My final deviation as Vnot equals Vnot over Vi is SL over S square CLR plus SL plus R.

So I know, now I can take ideal multiply this by Vi and take its real component and I can figure out what is my actual voltage. So again you are using complex variables and also the idea of complex time signals and what we are doing is we are converting circuits which are in time domain into frequency domain by making this transformation, we are replacing S, I mean L by SL and capacitor by C and then you are adding them up appropriately and then calculating complex current, complex voltage and then mapping it back by taking a real (()) (53:07).

“Professor -Student conversation starts”

Student: Even in the frequency domain the Kirchhoff’s law holds?

Professor: It holds.

Student: Is that any special property or it is taken as.

Professor: See the current will still has to hold something conservation laws have to work, similarly energy laws have to obey the same principles rest will hold. If they do not hold when you cannot do a circuit analysis then you need another system they have to hold.

“Professor-Student conversation ends”

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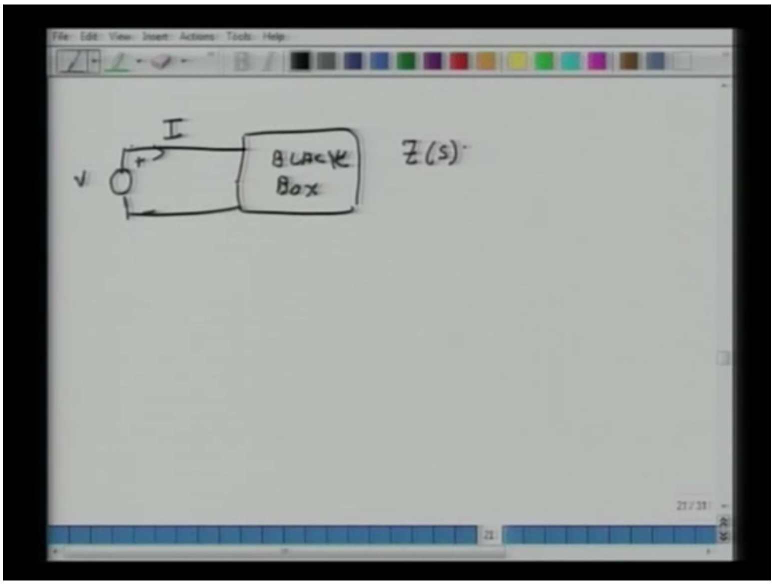

So we will do one final example, so suppose I have of some black box of some embedded, so the complex current is let us say I and complex voltage is V and let us say that transfer function which is a function of s which is complex frequency.

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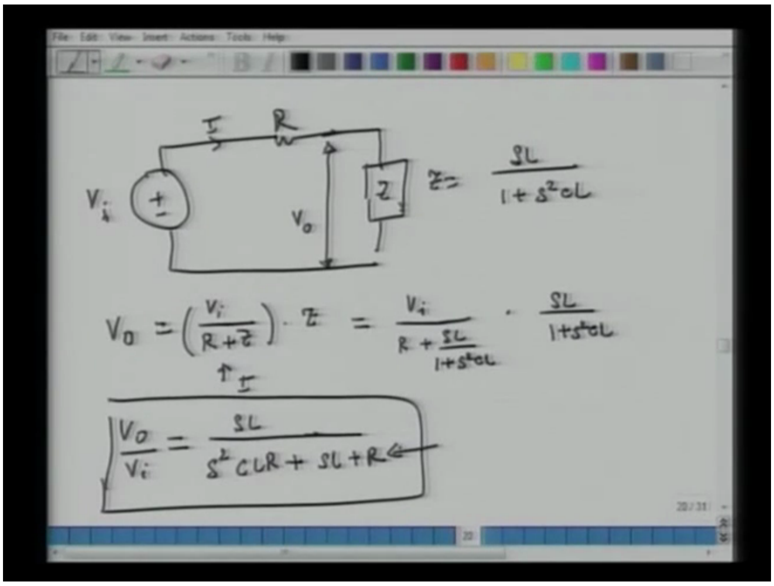

Like in this case complex function is a function of S, it depends on frequency.

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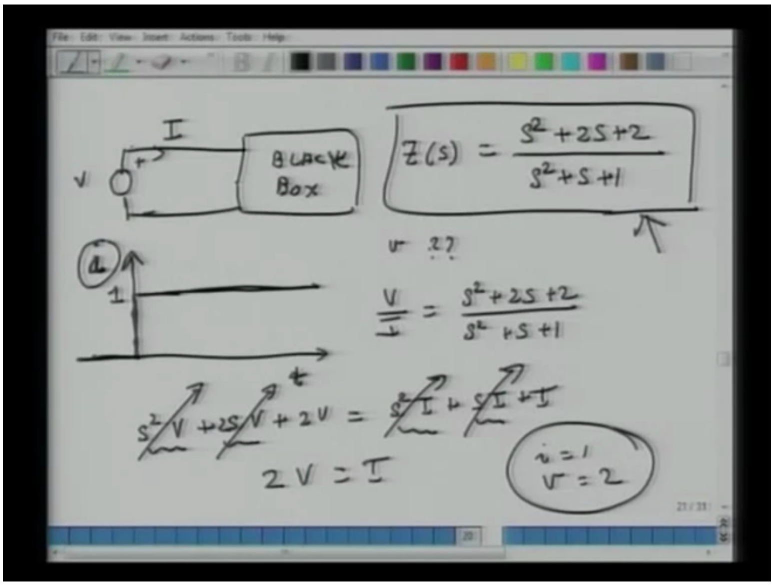

So this is equivalent to S square plus 2S plus 2 over S square plus S plus 1 and the question is that if my current signal, if my current real current i is something like this, so flat line, so it is like a step function at t equals 0 before t equals 0 it is 0, okay. At t equals 0 it goes up and it becomes 1 and this is my time axis. So before t equals 0 it is 0 at t equals 0 it steps up becomes 1and remains constant.

So if this is my current, what will my voltage look like given that this is the transformation? So in this context we should remember that this is a linear system, right? What does that mean? That if my study strait input let say i is constant then my study strait output, if output the unknown is V that will also remain constant because it is a linear system we are using that feature to bring it.

So I go to the complex domain and I say V over I equals S square plus 2S plus 2 over S square plus S plus 1. Now I know, so I expand this I get S square V plus 2S V plus 2V equals S square I plus SI plus I. Now these terms relate to derivatives of current and voltage, second derivatives and first derivatives. So when everything is study strait these terms can be in a study strait situation they will be 0, is that right?

So 2V equals I, so if my current is one and voltage is 2, so transfer functions tell us a lot and they are basically properties of the system they do not change with excitation signals input signals they are properties of the system for instance in an electrical circuit we saw that they are dependent on resistance, inductance and capacity and of course there is a dependency on frequency in that sense they are dependent on the input but they are a property of the system, so if I know my input if I know my study strait input I should be able to figure out my study strait output.

So that is all I wanted to cover for today and we will meet again and we will again review our few more concepts in next one or 2 more lectures and then after that we will move into area of acoustics, thank you very much.