Acoustics

Professor Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

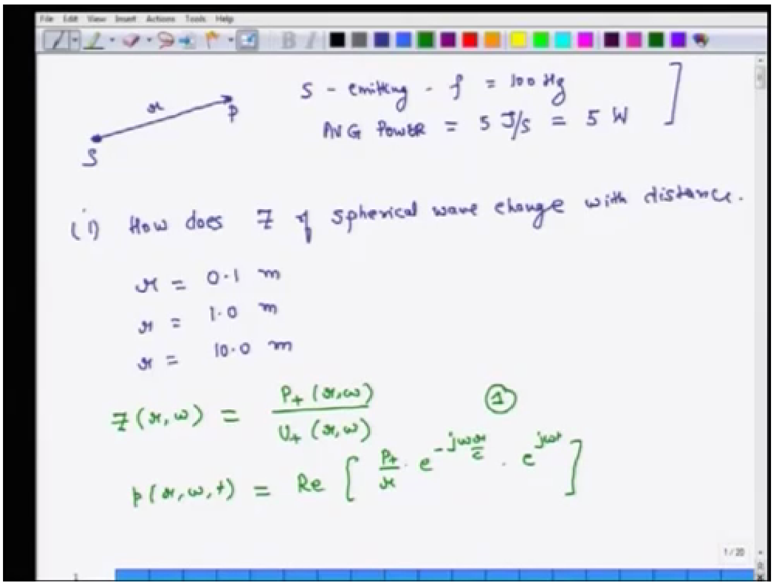
Module-4 Monopoles and Dipoles
Lecture 05
Numerical examples

Hello again over last several lectures we have been talking about spherical sound waves or we have also called this waves which are spherical in propagation and which in particular emanate from a single point as monopoles. So we have dealt with the mathematics of monopoles, we have figured out that these spherical waves or waves emanating from monopole sources they spread out radially and as they spread out radially their intensity decreases with increasing r this is in contrast to plane waves where the intensity does not change as we move out away from the source of the wave.

So what we are going to do is what we are going to do today is essentially solve some problems which relate to a spherical waves and through this problem solving approach what we will try to understand is a how does the complex impedance of these spherical waves change over a period of over distance, second thing we are going to look at is how does intensity change as distance close, another thing we are going to look at is how does particle velocity of these wave fronts it changes with increase in distance, we will also look at volume velocity and also the displacement of particles which essentially can be found by integrating the velocity of a particle over a period of time.

So this is what we are going to do and then in the subsequently we will look at another example and again the purpose of that example would be to understand these spherical waves with more clarity and understand them in a better way.

(Refer Slide Time: 2:41)



So so suppose I have a source and what I am interested in is that as this source is emitting sound waves at some distance r let us say this point is P so point P is the place where I am recording or observing the nature of sound waves S is the place or the point from where these spherical waves are emanating so P is again once again it is the observation point which is at a distance r away and S corresponds to the location of the source or spherical waves.

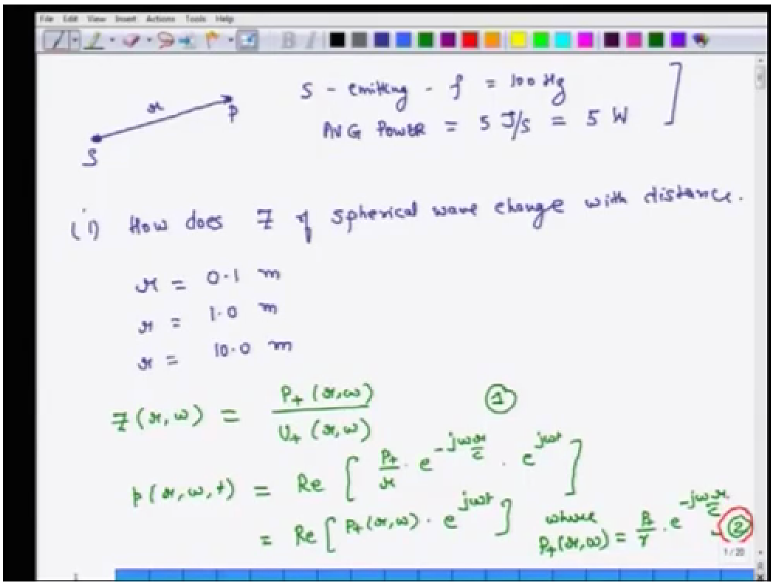
So let us say that S is emitting a spherical wave front such that its frequency equals 100 Hertz frequency equals 100 Hertz and the overall energy released by this spherical source each second is 5 joules so the overall average power average power let us say is 5 joules per second and that is nothing but 5 Watts. So with this definition we will look at different aspects of this particular spherical sound wave front.

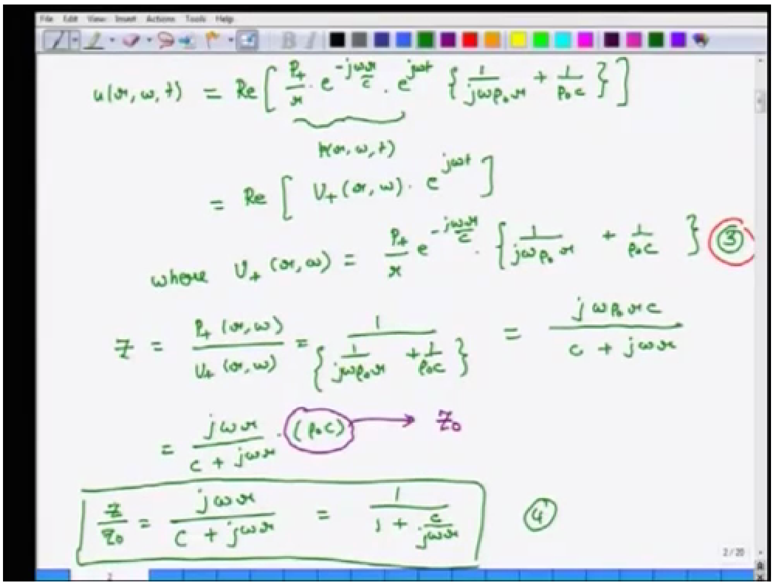
So the first thing we are going to look first thing we are going to look it look at is how does impedance of spherical wave change with distance in particular we will look at three values if r, r equals 0.1 meters, r equals 1.0 meters and r equals 10.0 meters. So at three values of r 0.1 meters, 1 meter and 10 meters what we are interested in finding out is what is the value of impedance and how does this impedance change as I increase the value of r.

Now to do this we know that Z which is impedance is essentially the function of radius and frequency or angular frequency omega and this is nothing but the ratio of P plus which is a function of r and w over U plus which is again a function of r and w so let us call this equation 1. Now we know that P which is pressure in lower case r, omega, t so this pressure changes with respect to radius, it changes with respect to angular frequency and also it changes with respect to time.

So that is nothing but real part of complex amplitude P plus times e minus j omega r over c times exponent of j omega t, okay. So pressure which depends on r, omega and t is nothing but the real component of P plus which is complex amplitude divided by r times e minus j omega r over c times exponent of j times omega times t, so that is the definition of pressure.

(Refer Slide Time: 6:54)





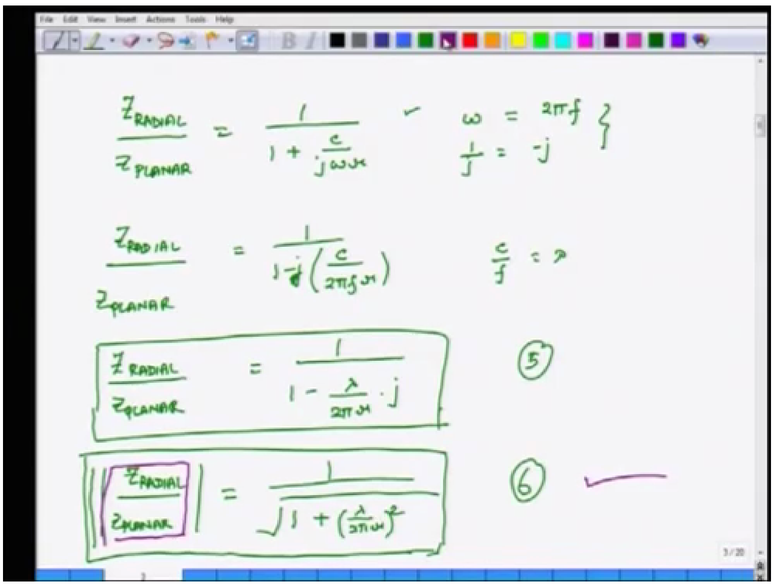
Similarly velocity which again depends on radius, angular frequency and time is nothing but real portion of basically P plus over r times e minus j omega r over c times e j omega t and then I have to so this is nothing but p of r, omega, t so it is a essentially pressure divided by impedance and the value of 1 over z is 1 over j omega Rho not r plus 1 over Rho not c. So and this I can re write it as real of U plus complex basically complex pressure which is a function of r and omega times e j omega t, where U plus of r and omega is equal to P plus over r e minus j omega r over c times this entire thing in parentheses which is 1 over j omega Rho not r plus 1 over Rho not c.

Now we will go back and slightly modify the equation for pressure also so this is the equation for pressure can also be written as real of P plus which is a function of r and omega times e j omega t, where P plus of r and omega equals complex pressure amplitude divided by r times e minus j omega r over c. So I call this equation as 2 and the equation here is 3. Now we know that Z equals P plus r, omega divided by U plus of r and omega and that is essentially.

So if I divide equation 2 equation 2 with equation 3 then I get the value of Z, so essentially what I get is 1 over 1 over j omega Rho not r plus 1 over Rho not c and this if I reorganize all the terms what I get is in the numerator I get j omega Rho not r c and in the denominator I get c plus j omega r and this is equal to j omega r divided by c plus j omega r times Rho not c. Now we know that this term Rho not c equals Z not which is value of (())(11:09) of a plane wave as it propagates through air, so that is Z not.

So I can re write this equation as Z over Z not equals j omega r divided by c plus j omega r and then I can further simplify it as equals 1 over 1 plus c over j omega r, essentially what I have done here is I have divided the numerator and the denominator by j omega r. So this is my expression and I call this expression number 4 and that tells me how Z and Z not are related and what is the dependence of Z on r.

(Refer Slide Time: 12:19)

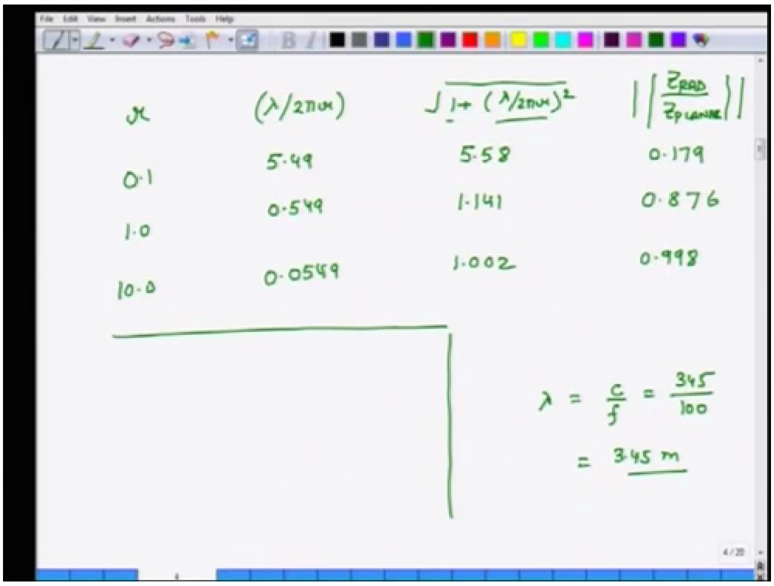


So moving further we know that Z is the impedance for a radial wave and Z not is the impedance for a of a planer wave. So I can re write equation 4 as Z radial divided by Z planer and that equals 1 over 1 plus c over j omega r, next we know that omega equals 2 Pi f and 1 over j equals minus j. So we put these two in this expression and what we get as Z ratio of Z radial and Z planer is 1 over 1 minus j times c over 2 Pi f times r (excuse me) and then we know that c over f equals wavelength.

So I can re write this equation further as 1 minus lambda over 2 Pi r times j and if I have to compute the magnitude of this ratio that is the ratio of impedance of a radial wave and that of a planer wave then the magnitude of this ratio is Z radial divided by Z planer and that is equal to 1 over square root of 1 plus lambda over 2 Pi r times square. So this is my equation 5 and that is my equation 6.

Now what I am going to do is I am going to use equation 6 to compute different values of this ratio or actually this ratios magnitude different values of the magnitude of this ratio and for different values of r this is what I am going to do and I am going to choose three values of r first case r will be 10 centimetres or 0.1 meters, in the second case r will be 1 meter and in the third case r will be 10 meters.

(Refer Slide Time: 15:34)



So I have I am going to construct a table first column will be for r, second column will be for lambda over 2 Pi r, third column will be square root of 1 over lambda over 2 Pi r whole square and the fourth column will be ratio of radial wave impedance and planer wave impedance. Now while we are computing this we have to know what is the value of lambda so lambda equals c over f, c is assumed to be 345 and f is 100 Hertz it is given. So lambda comes out to be 3.45 meters.

So with this value of lambda for different values of r I can calculate lambda over 2 Pi r also I can calculate square root of 1 plus lambda over 2 Pi r whole thing squared and then finally if I take the inverse of this term then I get the ratio of radial impedance and planer impedance the magnitude of that ratio. So first value is 0.1 my lambda over 2 Pi r comes to be 5.49, the square root term comes to be 5.58 and the ratio comes to be 0.179.

Second thing is second case if r equals 1 meters then lambda over 2 Pi r equals 0.549 my square root term comes to be 1.141 and my ratio the magnitude of the ratio of radial and planer impedances it works out to be 0.876. Third case my r is 10 meters which is significantly larger than lambda over 2 Pi so this number is 0.0549, the square root term is 1.002 and the ratio of (planer) radial and planer impedances the magnitude of this ratio it works out to be 0.998 so there you have it.

And essentially you if look at this table what you find is that at small values of r and when I use the term small values of r essentially it is a small value of lambda when lambda over 2 Pi r is significantly small compared to 1 compared to 1 lambda over 2 Pi r is significantly small compared to 1. So in that case when lambda over 2 Pi r is very small compared to 1 or when r is extremely small or when r is extremely large then lambda over 2 Pi r is extremely small and this number my the ratio of impedances works out to be almost 1, which means that if I am very far from my source which is indeed the case when r equals 10 meters.

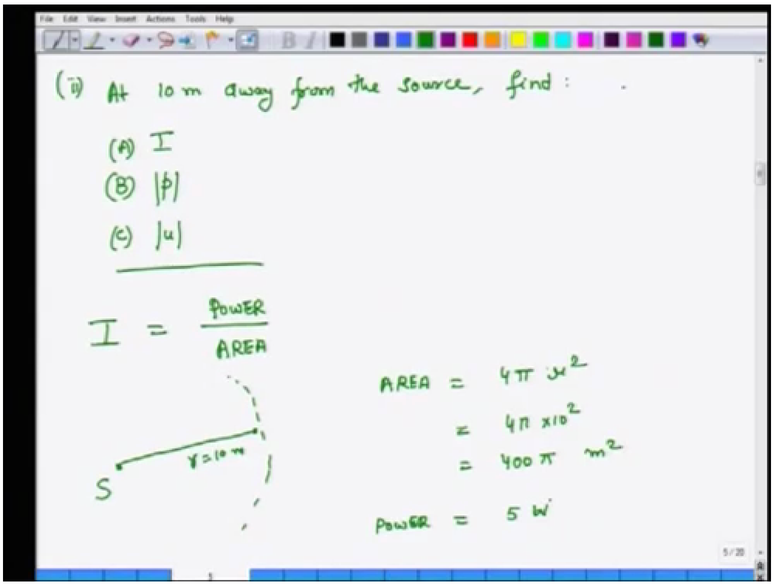
So if I am extremely far from the source then the ratio of planer and radial impedance it comes to be fairly close to 1. So as I am move away far and far away from the source the radial waves start behaving more and more like a planer wave. As I come very close to the source r becomes small lambda over 2 Pi r becomes large compared to 1 and as a consequence of that this term in the square root under the square root sign it becomes large it grows very rapidly and as it grows very rapidly the radial waves impedance becomes significantly small compared to that for a planer wave. So this is the story of impedance.

So once again to recap what we have done till so far is we assumed that there is a spherical source it is emanating radial waves and it is emanating radial waves in such a way that there are no reflected waves so there are no reflecting surface around this particular spherical source, we know how much voltage or how much power this spherical source is releasing on an average and that is actually 5 Watts, we have also said that the frequency of this spherical wave is 100 Hertz.

So for such a source what we have calculated is the impedance of the source at three values of r, r equals 10 centimetres, r equals 1 meter and r equals 10 meters and then we have calculated the ratio of radial wave impedance and planer wave impedance what we have found is that when I am very close to the source then the ratio of radial wave impedance to that of planer wave impedance is small and as I come closer and closer to the source this ratio becomes close to 0.

If I move away from the source and if I am far away from the source what this table shows is that this ratio becomes closer and closer it comes closer and closer to unity. The definition of being far or close to the source is in the context that if lambda over 2 Pi term is very small compared to 1 if it is very small compared to 1 then I am far from the source, if lambda over 2 Pi term is very far (excuse me) is very large compared to 1 then I am to the source. So that is what we learned from this illustration. Next what we are going to do is we are going to compute terms related to intensity and velocity of the particle and pressure generated by this particular wave.

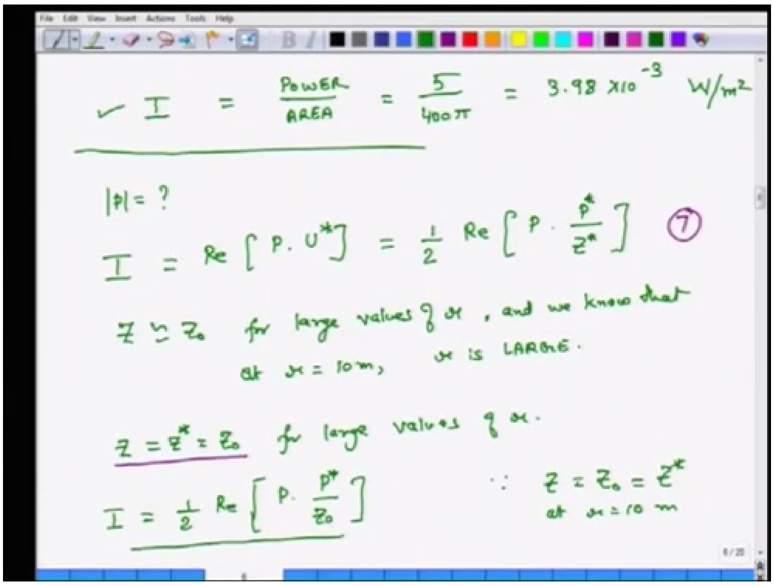
(Refer Slide Time: 22:36)



So my second aim is that at 10 meters away from the source, find I that is the intensity of the sound source. So we have to find excuse me we have to find couple of things we have to find intensity, then we have been asked to find amplitude of pressure and then we have to find amplitude of velocity. So we start by finding intensity of this particular source so we know that intensity is amount of power dissipated or amount of power which is coming out per unit area.

So it is basically power divided by area. Now I have my source here this r equals 10 meters so this is a sphere this is sphere of radius 10 meters. So the area of this particular sphere area is equal to 4 Pi times r square and r is 10 meters so that is 4 Pi times 10 square and that is equal to 400 Pi square meters, also we know from our problem definition that power for this particular source S is 5 Watts.

(Refer Slide Time: 24:47)



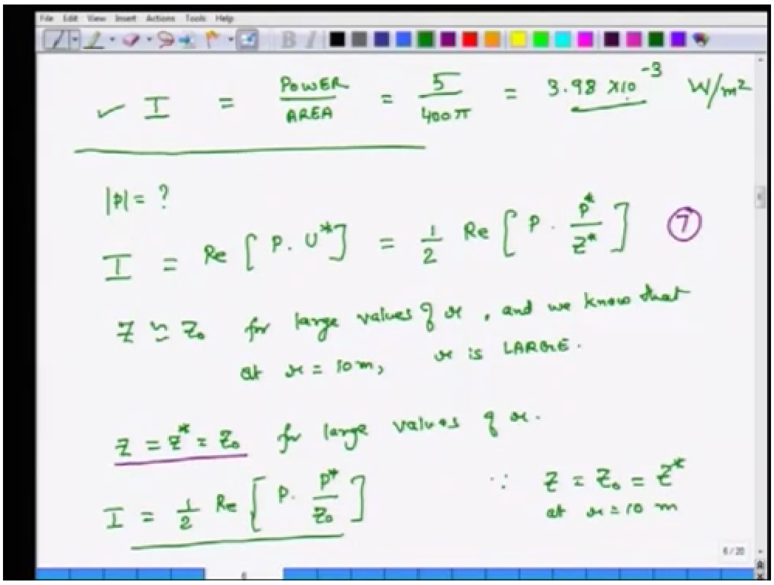
So with this information I can calculate intensity as I equals power over area now this relation power divided by area works only for a spherical source because all the power which is getting dissipated is uniform in all directions. If there was some directivity in the source then we cannot have this relation because in that case we will have to compute value of I on a point by point basis but in case of a spherical source we do not have to worry about it all we have to do is find the total area and then use that area and compute the ratio of power and area to find the intensity.

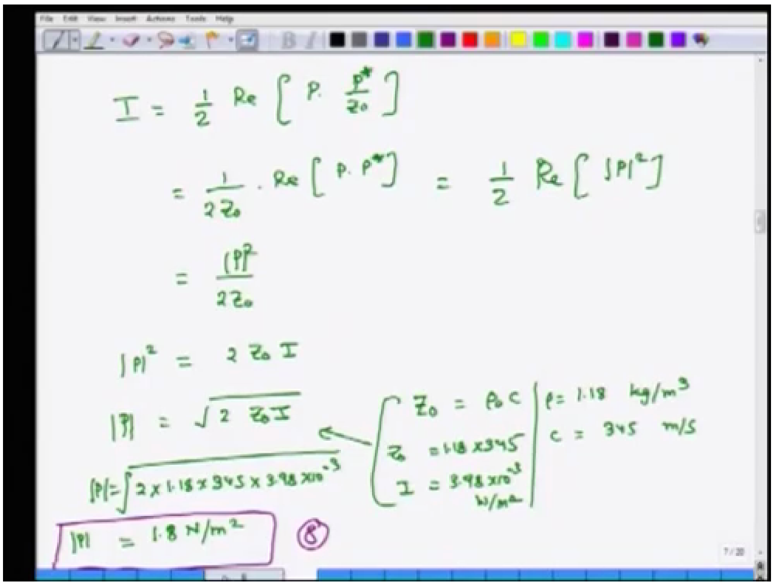
So this is equal to 5 over 400 Pi and if we do the calculations this thing comes to 3.98 times 10 to the power minus 3 Watts per square meter. So we have figured out what is the value of I, our next job is to find the value of amplitude of pressure amplitude of pressure. So to do that so our aim is what is amplitude of pressure? So for that we start using information related to I and we know that intensity is nothing but real part of complex pressure times conjugate of complex conjugate of complex velocity and that is equal to half real component of complex pressure times complex conjugate of pressure divided by complex conjugate of impedance.

Now earlier we had seen you see this in this table that at r equals 10 meters the ratio of radial impedance and the ratio and planer impedance it is virtually 1 it is almost 1 and another way to look at it is that when r is extremely large when r is extremely large this term when r is extremely large this term is virtually 0 in the relationship for Z over Z not. So using this information we can say that Z or Z is approximately equal to Z not and because for large values of r r and and we know that at r equals 10 meters, r is large so r is large and for large values of r Z is approximately equal to Z not which means that Z equals Z not star equals Z not for large values of r.

So with this information we put this in equation so we put this in equation 7 which is this so I get intensity equals half real of P times P star divided by Z not because Z equals Z not equals Z star at r equals 10, actually it is not a strict equality relationship it is a approximate relationship. So at r equals 10 meters, Z is virtually equal to almost equal to Z not is almost equal to Z star so this is what I get and then so I will re write this relation on the next page.

(Refer Slide Time: 29:52)

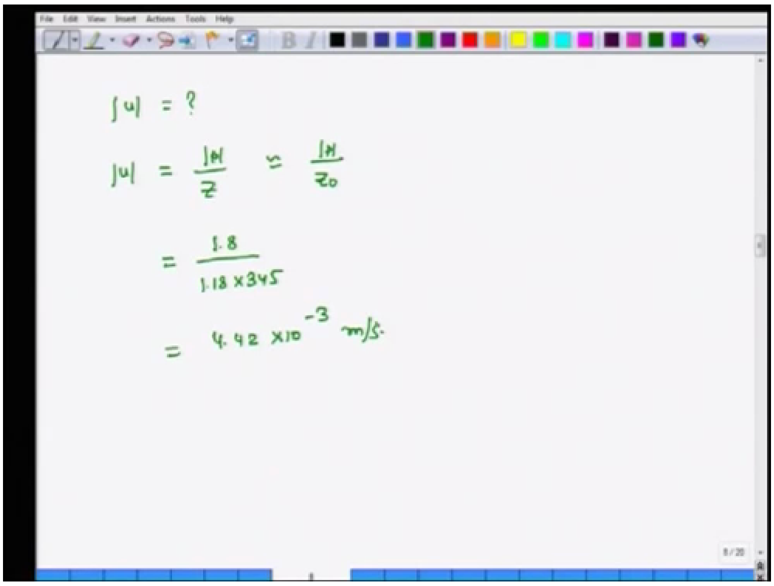




So I equals half real of P times P star divided by Z not equals 1 over 2 Z not times real of P times P star. Now we know that any complex number and its complex conjugate then we multiply we get only the magnitude of that square of magnitude of that term. So this is equal to 1 over 2 real of this thing magnitude of pressure the whole thing squared. So that is equal to so magnitude of pressure square divided by 2 Z not.

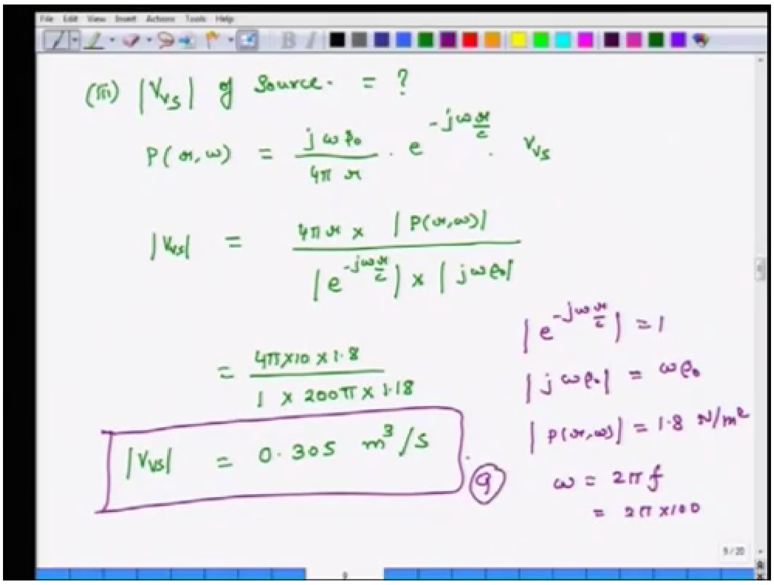
So I can say that magnitude of pressure squared equals 2 Z not I or P equals square root of 2 Z not I and I know that Z not equals Rho not c and Rho not is 1.18 (cubic) kilograms per cubic meters (excuse me) Rho and c equals 345 meters per second. So Z not equals 1.18 times 345, so I plug this and I we have calculated is 3.98 times 10 to the power minus 3 Watts per meter square we calculated it earlier which is this. So I put all this information in this equation I get pressure magnitude of pressure equals 2 times 1.18 times 345 times 3.98 times 10 to the power of minus 3 whole thing we take the square root of and if I do the math magnitude of pressure comes to be 1.8 Newton’s per square meters, so that is my equation so that is the magnitude of pressure 1.8 Newton's per square meter or 1.8 Pascals.

(Refer Slide Time 33:15)



So our next aim is to find (velo) so we have found intensity, we have found pressure magnitude and now we are try we will try to find velocity magnitude. So our aim is to find velocity magnitude of velocity. Now we know that magnitude of velocity is nothing but magnitude of pressure divided by Z and we know that because Z or Z is approximately equal to Z not for large values of r, so I can re write this relation as pressure magnitude by Z not and this I have already found is 1.8 Pascals magnitude of pressure, I divide it by Z not which is a product of 1.18 times 345 and that works out to be 4.42 times 10 to the power of minus 3 meters per second.

(Refer Slide Time: 34:10)



My next aim is to find volume velocity of source which we have been talking about till so far, so what is volume velocity of this source. Now we have seen earlier that the relationship between complex pressure P r, w and volume velocity is this so P r, w is equal to j times omega times intensity divided by 4 Pi r times e minus j omega r over c times V VS. So if I have to compute if I have to compute volume velocity of the source then magnitude of volume velocity, so I am interested in finding the magnitude of volume velocity.

So magnitude of volume velocity equals 4 Pi r times magnitude of pressure divided by magnitude of e minus j omega r over c times magnitude of j omega Rho not. Now magnitude of e minus j omega r over c equals 1 and also magnitude of j omega Rho not equals omega Rho not and we have already calculated that magnitude of pressure is 1.8 Newton's per square meters.

So with that understanding we start calculating magnitude of volume velocity and we find its value, so it is 4 Pi times r which is 10 times 1.8 divided by 1 which is magnitude of e j omega r over c times omega so omega equals 2 Pi f equals 2 Pi into 100 Hertz so that is 200 Pi times density 1.18. So if I do the math what I get is this is equal to 0.305 cubic meters per second, so that is my magnitude of volume velocity and this is my equation 9.