Acoustics
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Module-4 Monopoles and Dipoles
Lecture 04
Radial propagation of sound, monopoles, and dipoles

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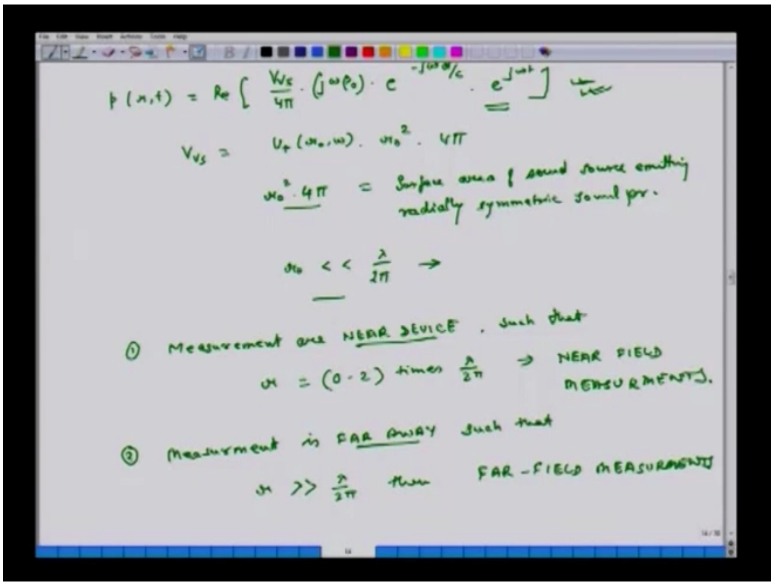

Welcome again so in the previous lecture we had introduced this concept of volume velocity and we had developed a relation for pressure field attributable to monopole and the relation look something like this so pressure field due to a monopole is the real component of volume velocity divided by 4 Pi times j omega Rho not times e minus j omega r over c times e j omega t and this volume velocity was defined as complex amplitude times r not square times 4 Pi there r not square times 4 Pi equal the surface area of sound source emitting radially symmetric sound sound pressure there were sound source and if it is let us say radius is r not and it is generating this sound which is radially symmetric then that is the surface area which is emitting sound I multiply that with velocity and that gives me volume velocity.

The other thing we had done as we were developing this relation was that we had assumed that r not which is the size of the sound source is extremely small compared to lambda over 2 Pi and as long as this assumption held valid this relation would be reasonably accurate. If this particular relation is not valid then we have to go back to the first principles and modify this particular relation so that we also include the effect of size.

At this stage I also wanted to introduce two more terms so one is that if the device is small and I am doing measurements which are very close to the device so such that my measurement are near the device such that r is which is the position where I am taking measurement is 0 to 2 times lambda over 2 Pi then these type of measurements are called near field measurements.

And if measurement is far away such that r is very large compared to lambda over 2 Pi then I have these measurements are known as far field measurements. So I am near field measurements when I am extremely close to device which is emitting sound in the sense that the distance between the device and my observation point is less than you know 0 to 2 times of lambda over 2 Pi then I will be doing near field measurements. And if my location is extremely far such that the radius or the distance between observation point and the device is very large compared to lambda over 2 Pi then what I am doing is they are known as far field measurements. So that is about volume velocity, near field measurements and far field measurements.

Now we move one step further and we start exploring what happens if we have more than one monopoles and then how does the sound field get effected and at this stage we will start talking about the concept of interference, the concept of interference in case of sound waves is not in any fundamental way different from light ways when they interfere from each with each other or for that sake any type of waves when they interfere with each other.

So we will use similar mathematical tools to predict the sound pressure field when there are more than one monopoles in the neighbourhood of each other and essentially what we will do for purposes of predicting the sound field is that we will compute the sound field attributable to one particular sound source the same thing will be done for a sound field attributable to another sound source and then if we have two sound sources we just add these two components and then we get the total final consequential sound pressure field in the place which we are at the place which we are interested in.

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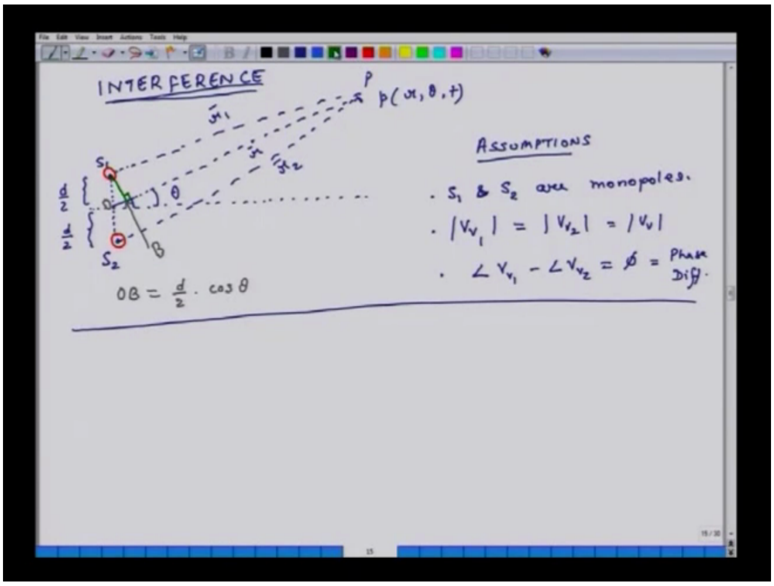

So so we will start talking about interference and specifically in this lecture we will just talk about two sound sources we will just talk about two sound sources to make the discussion simple and relatively easy to understand and then we will see how these two sound sources interact with each other to produce an interference sound pattern at some distance away in the pressure field.

So let us say I have two sound sources and this sound sources S 1, this is another sound source S 2 and this is a geometric line which connects these two. So these two sound sources are separated by distance d such that this is distance d over 2 and this is another distance d over 2. Now I am interested in finding the sound pressure level at this point and let us say this point is P and the pressure here is a lower case P and that is what I am interested in finding.

So this is my origin this point so these are two sound sources and this is my origin, so from with respect to the origin this point P is located and the distance from the origin to point P is r and this angle is theta. So I will just make it a little larger this angle is theta. So the pressure at point P is lower case P and that depends on three variables r which is the distance between P and origin, theta and of course time.

The distance of sound source S 1 is r 1 and similarly the distance of sound source S 2 is r 2. Finally I draw some construction lines, I drop a normal from S 1 on the vector r and if that is the case then this angle is again theta this angle is again theta and if I extend this line so this is 90 degrees and from the origin which is point O to this point let us say this point is B then OB is basically d over 2 times Cosine of theta because the angle between line OS 1 and the normal is again theta and OS 1 is the hypotenuse so the base of this triangle is nothing which is OB is d over 2 times Cosine of theta.

Now as so again I have two sound sources S 1 and S 2, I have a point P which is distance r away from origin I am interested in finding the sound pressure field at point P and see how this pressure varied with respect to r theta and time and that is the basic problem which I am trying to address here. Now for purposes of this discussion we will make a couple of assumptions so assumptions.

So the first assumption is that S 1 and S 2 are monopoles so ideally they are point sources of sound but in reality you cannot generate sound from point sources but if they still have to be like monopoles or approximate to monopoles then the size of these devices sound these sources has to be extremely small compared to the wavelengths they are emitting. So S 1 and S 2 are monopoles.

The second one is that the volume velocity of S 1 or I will call it V V 1 the magnitude of this volume velocity is same as magnitude of V V 2 volume velocity of the second one, so this is another assumption so essentially I have two monopoles, the emitting sound and the strength of these two monopoles is same and the volume velocity is V is equal to V V this is the second assumption.

And the third assumption is that even though they have similar strengths the phase difference between the sounds being emitting by them is not exactly 0 and there is a phase difference between the volume velocities of these two monopoles. So phase of V V 1 minus phase of V V 2 equals Phi and that is phase difference.

So these are the three assumptions, so once again I will formulate the problem or I will restate the problem, I have two sound sources S 1 and S 2 they are in terms of size they approximate monopoles, they generate sound waves and because of these sound waves an interference pattern is getting generated at point P which is located at a distance r from the origin and which is also at an angle r theta with respect to the origin and I am interested in finding the sound pressure field in the neighbourhood of in this entire area the strengths of these two monopoles are same even though they are of in terms of phase by an angle Phi.

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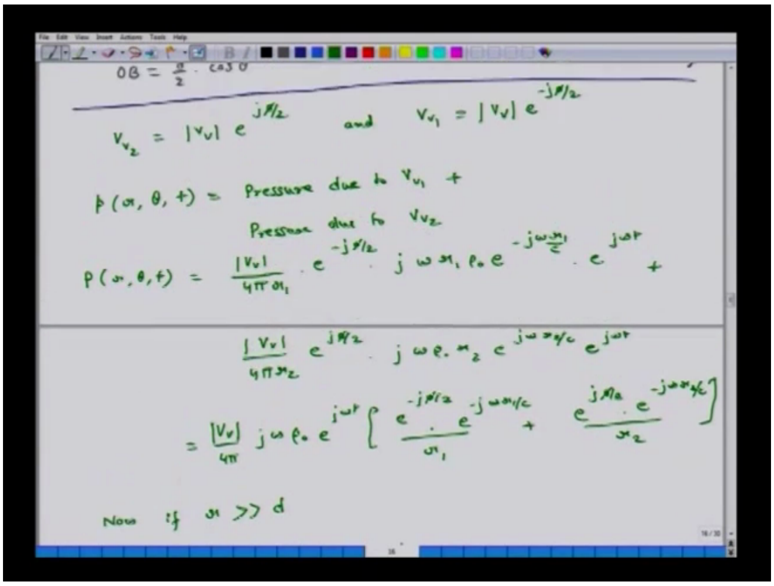

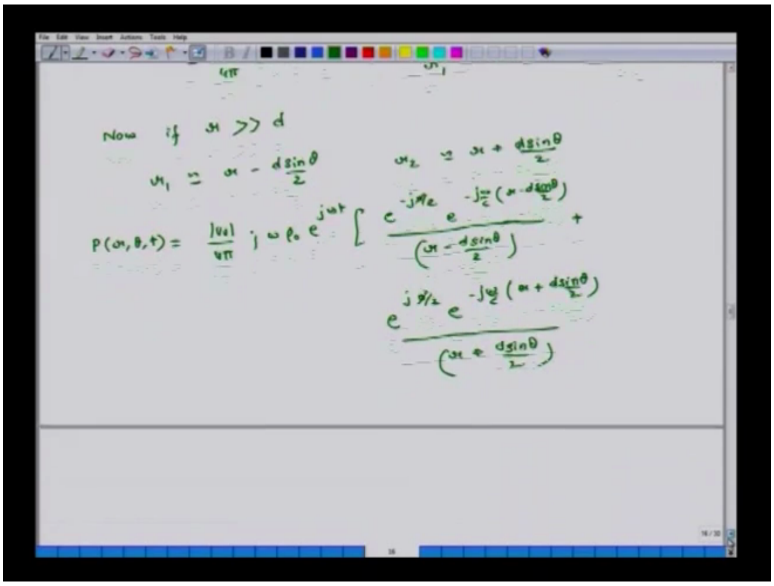

So with that statement I now proceed to start solving the problem, so the first thing is that I will write down expression of V V 2, so V V 2 is essentially it will have some magnitude and that magnitude we know is V V and it will also have some phase e j Phi over 2 and similarly V V 1 is equal to some magnitude component and some phase and we know that because the phase difference between S 1 and S 2 is Phi so its phase will be minus j times Phi divided by 2. Now p of r, theta, t equals pressure due to V 1 plus due to V V 2.

So I will write first complex pressure and complex pressure is complex pressure due to V 1 and plus complex pressure due to V V 2 and then I will add these two up take the real part and then I can get the actual pressure. So this is essentially V V divided by 4 Pi r 1 times e minus j Phi over 2 times j omega r 1, Rho not e minus j omega r 1 over c times e j omega t.

So this is my complex pressure due to first source plus now I have complex pressure due to second source so instead of r 1 now I have r 2.

Now I am going to simplify this whole thing so I have several things common so it is V V over 4 Pi and then I will also make j omega Rho not is common and so is e j omega t and then I have two terms in parentheses minus j Phi over 2 times e j omega r 1 over c and the entire thing is divided by r 1 plus e j Phi over 2 times e minus j omega r 2 over c divided by r 2. Now if r is extremely large compared to d.

So now we are making an assumption here that if r which is this distance between the point of observation and origin if this distance is extremely large compared to the distance between these two sources then if r is extremely large compared to d then r 1 is approximately equal to r minus d Sin theta over 2 and r 2 is approximately equal to r plus d Sin theta over 2. So now I introduce these approximate definitions of r 1 and r 2 in my expression for complex pressure and from that I get complex pressure which depends on r theta and t is V V over 4 Pi j omega Rho not e j omega t e minus j Phi over 2 e minus j omega over c and then I am replacing r 1 by r minus and then again from the denominator I am replacing r 1 by r minus d Sin theta over 2 and then I have another term e j Phi over 2 e minus j omega over c and r plus d Sin theta over 2 divided by r plus d Sin theta divided by 2.

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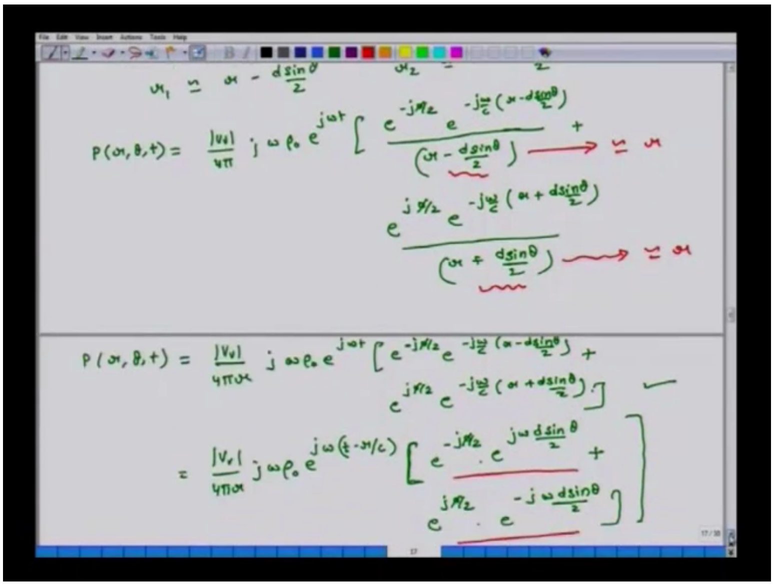

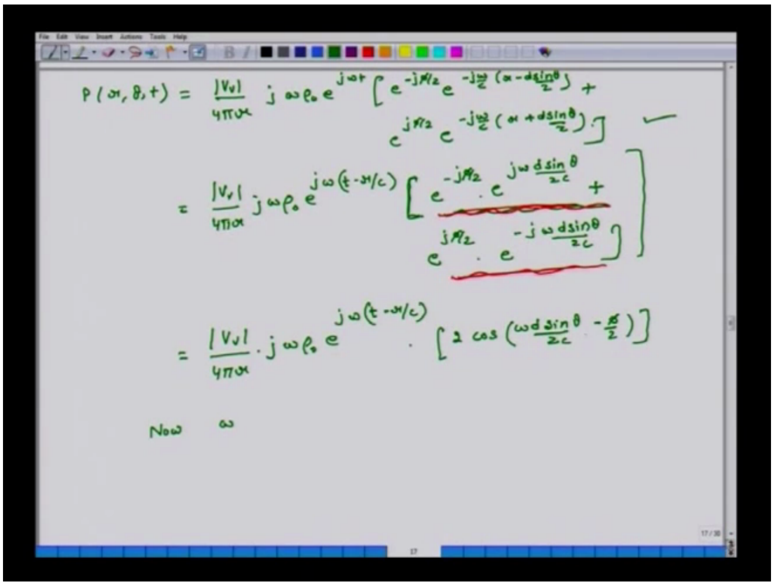

So I have this somewhat complex expression and now I will at this stage I will start simplifying it. So one simplification is that the denominator this is approximately equal to r, similarly this denominator is approximately equal to r. So what I get is P r, theta, t equals V V over 4 Pi r j omega Rho not e j omega t times e minus j Phi over 2 e minus j omega over c r minus d Sin theta over 2 plus e j Phi over 2 e minus j omega over c times r plus d Sin theta over 2. So this is a somewhat simplified expression of the original version.

Now I can make this version further simple by taking e j omega r out of the bracket so what I get is so I have taken r from outside from here so in the parentheses what I am left with is e minus j Phi over 2 times e j omega d Sin theta over 2 plus e j Phi over 2 times e minus j omega d Sin theta over 2, okay. So this is my further simplified expression where I have taken r outside the parentheses and I have clubbed it with e j omega t.

So now what I see in the parentheses which is this term, so the first term is e minus j Phi over 2 times e j omega d Sin theta divided by 2 and the second term is just the inverse of it e j Phi over 2 times e minus j (d) omega d Sin theta over 2. So this is essentially excuse me so we know from complex algebra rules that this these two terms they add up if I expand them in terms of Cosines and Sins then the Sin terms will get cancelled because the Sin terms will get cancelled and we will be left with only Cosine terms.

So that is what I do so I make it further simple so I get V V over 4 Pi r times j omega Rho not e j omega t minus r over c times twice of Cosine of omega d Sin theta over 2 minus Phi over 2, okay. So this is the simplified expression here. Now we know that omega okay I think I I am missing a c here so there should have been a c here and a c here ya so there is a c here as well.

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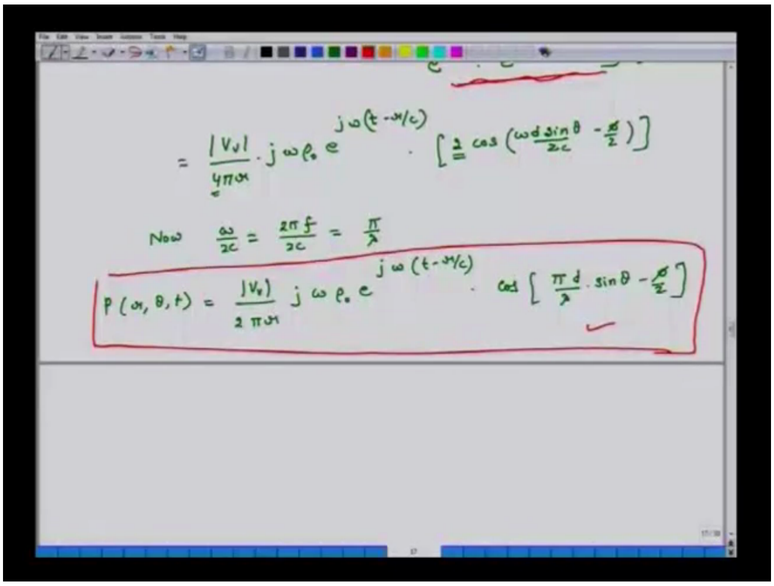

So now omega over 2c equals 2 Pi f over 2c equal Pi over lambda so I can replace omega over 2c by Pi over lambda. So my complex pressure relation is further simplified and also I cancel this 2 by 4 so I am left with only 2 Pi r here so V V over 2 Pi r times j omega Rho not e j omega t minus r over c times what I am left with is Cosine of Pi d over lambda times Sin theta minus Phi over 2, okay. So this is my final expression.

So what this relation shows is that if I know the volume velocity of these two sources and I am aware of the phase difference between the two sources then I can calculate complex pressure using this relation and once I have the complex pressure then I can take its real component and that will give me the actual pressure.

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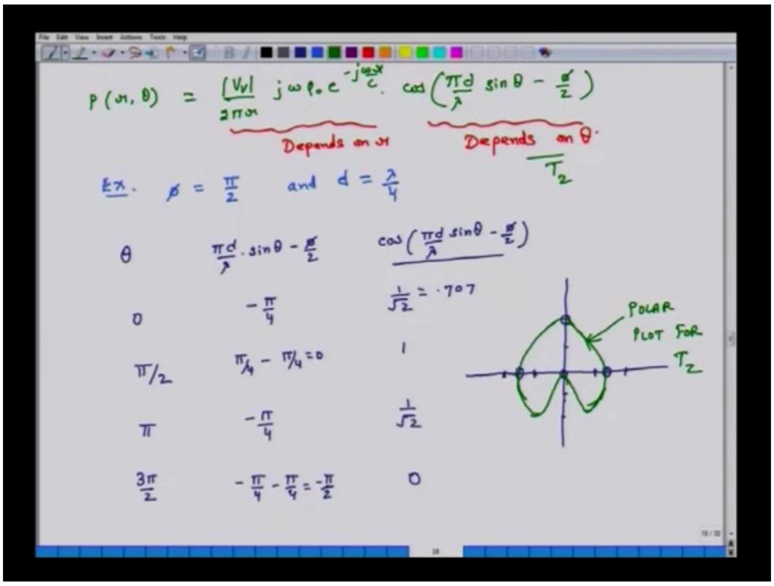

So I will re write this equation again so P and what I will also do is that I will drop the t term because that is just e j omega t so I will just take the drop the dependency you know for purposes of convenience. So P is dependent the complex pressure magnitude is dependent on r and theta and that is V V over 2 Pi r j omega Rho not e minus j omega r over c times Cosine of Pi d over lambda Sin theta minus Phi over 2.

So what we are seeing here is that the first term depends on r and then this term depends on theta the second term depends on theta. So even if the value of r is fixed and if I go around a circle the value of the pressure at different values of theta will be different. So what we are going to do is we will assume some value of lambda and theta and then see some value of lambda and some value of d and some value of Phi and then c how does for a particular set of variables the dependence of theta manifest itself as.

So what we will do is we will do an example and example is such that the phase difference is Pi over 2 and d equals lambda by 4 d equals lambda by 4. So for these two parameters Phi equals Pi over 2 and d equals lambda over 4, we are going to see what kind of dependence on theta is there so we will develop a table. So first column in table of theta, second column in the table will be Pi d over lambda times Sin theta minus Phi over 2 and the third column will be Cosine of Pi d Sin theta over lambda minus Phi over 2.

So when theta is 0 degrees or 0 0 radiance when theta is 0, Sin theta is 0 so first term is 0, Phi is Pi over 2 so this becomes minus Pi over 4 and Cosine of minus Pi over 4 is 1 over root 2, okay when theta is Pi over 2 then what I get here is Pi over 4 minus Pi over 4 equals 0 and here I get is 1, when theta is Pi then again Sin theta is 0, so the first term is 0 and what I get is minus Pi over 4 and here once again I get 1 over root 2 and when this theta is 3 Pi over 2 then Sin theta is negative one so I get here minus Pi over 4 minus Pi over 4 is equal to minus Pi over 2 and then I get here is 0.

So if I do a polar plot of Cosine with respect to theta this is how it is going to look like, so at 0 it is I am just making some marks so at 0 degrees the value is going to be 1 over point 1 over root 2 which is 0.707, so this is 1 so it is going to be little less than 1, so this is one point. At Pi over 2 when theta is Pi over 2 this value is 1 so this is another point, when theta is Pi then again I get 1 over root 2, okay so this is so this is half this is 1 so at theta equal 0 it is 0.707 so it is going to be this is 0.75 so it is going to be somewhere here so this is here.

And then similarly at Pi again it is going to be 1 over root 2 so this is 0.725 so again it is going to be somewhere here and then 3 Pi over 2 it is going to be at 0, so these are the four points. So now I develop a curve and I hope I do a an okay job in developing this curve so it is going to going to look something like this so this is the polar plot for term T 2. So what this polar plot shows is that when theta is 90 degrees I have the maximum amplitude, when theta is minus 90 degrees the amplitude is minimized and I have very interesting polar pattern so even if I keep my radius fixed the plot changes the amplitude changes as with respect to theta.

And this is purely because of interference of sound waves and at different places they add up or they interact in a positive constructively or destructively and we get variation of sound pressure levels as I move in the theta direction. So this is the essence of interference of two sound waves similarly we can use similar theory we can extend this theory very easily to have three sound sources interacting with each other, four sound sources so on and so forth.

One they are several significant application areas of when sound waves are interfering and how do we predict the sound waves, constants we use this approach to figure out the performance of (())(35:20) we also use similar theory to understand how sound propagates when it is being broadcasted over a large area, also there are directional microphones and also directional sound sources where sound pressure field is predicated based on this particular theory.

So I hope the approach of using interference helps you in understanding what happens when more than one monopoles start producing sound in radially in a 3D field and how do they interact with each other to produce constructive and destructive interference pattern in the 3D sound field. So that closes this particular lecture and thank you very much, bye.