Acoustics
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Module-4 Monopoles and Dipoles
Lecture 03
Radial propagation of sound, monopoles, and dipoles

Hello again welcome to today's lecture on acoustics, in the last class we had started developing mathematical relation for wave propagation or sound propagation in a radially symmetric fashion and specifically what we had developed were two things, one was a wave equation for a monopole a monopole I had explained earlier is ideally a point source or sound which emits sound in all the directions in a radially symmetric way.

So for a monopole sound field generated due to monopole and if there are no reflecting surfaces then the variation or sound field in the theta direction and also in the Psi direction so they are both these variations they are exactly 0 the only variation which happens for the sound field in a spherically symmetric system is in the radial direction.

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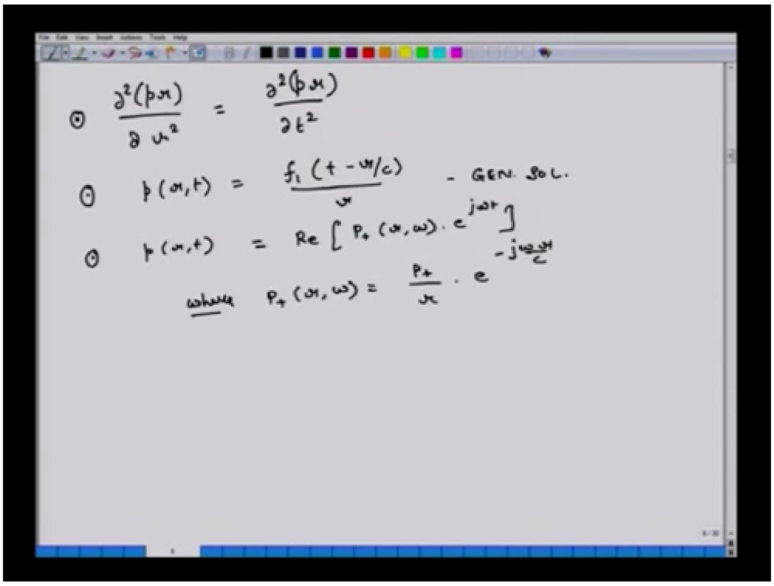

So with this kind of a framework we had developed the pressure wave equation for a monopole which looks something like this and once we had developed this equation we also went ahead and developed a general solution for this pressure wave equation and we found that a general solution for this pressure wave equation looks something like this. Unlike 1-D wave propagation in a Cartesian frame where we had forward travelling wave and also we had accounted for reflections because of the fact that here we are not talking about any reflections till so far.

So because of that we have only a function f 1 which depends on t minus r over c, there is no equivalent function for a reflected wave which would have something like f 2 of t plus r over c because there are no reflections in this system and there is no provision for reflections in this kind of a system and that is the constraint which we have imposed upon ourselves as we are developing the theory for propagation of sound which is getting emitted through a monopole.

Finally we had said that a particular so this is a general solution and then we had stated that a particular solution which is consistent with this general solution is something like this function where P plus of are omega is nothing but a complex number P plus divided by r times e minus j omega r over c. So these are the three things we had talked about in the last class and today what we will do is that we will extend our understanding of a sound field attributable to a monopole further and by addressing two or three important questions.

The first question is how does velocity relate to pressure in this kind of a radially symmetric field, second question would be what is the impedance of what is the impedance in such type of a sound field and third would be we will be introducing a new concept called volume velocity.

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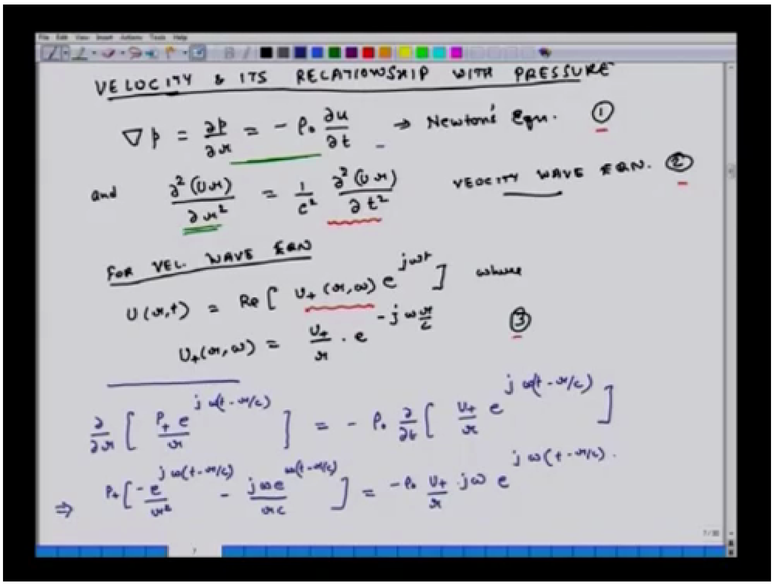

So we will start talking about velocity and its relationship with pressure. To address this question that how does velocity in a spherical spherically symmetric sound field relate to pressure, we will use the Newton's law which we had developed earlier and what we had seen was and we are just drawing analogy from what we had developed earlier for a 1-D wave propagation equation for a Cartesian frame of for a Cartesian frame.

So what we had seen was that p is equal to del p over del r is nothing but minus Rho not del u over del t, so this is essentially directly coming out of Newton's equation or the momentum equation and what this equation says is that the partial derivative of pressure with respect to r equals partial derivative of velocity with respect to time multiplied by density. So this is 1 equation and then we also simultaneously write the wave equation for velocity and the derivation of this wave equation for velocity is very similar to the derivation of pressure wave equation for a monopole.

So I am not going to develop this particular velocity wave equation explicitly but this looks like partial derivative of u times r with respect to r equals 1 over c square 1 over c square times particular derivative of u times r with respect to time. So this is the velocity wave equation. Now from this wave equation just as we had developed a solution for pressure in this form similarly it is very easy to see that for the velocity wave equation for velocity wave equation a possible solution could be of this form where u plus is a function of r omega which is equal to a complex number divided by r times exponent of minus j omega r over c. So this is equation 3, okay.

So the solution for a velocity wave equation we have drawn analogy from the solution for a pressure wave equation which is here and then we have just mapped that understanding while we wrote down the solution for a velocity wave equation through equation 3. So let us look at these three equations, equation 1, equation 2 and equation 3 and if I put this equation 3 in the RHS of the equation 2 so this is my RHS. So if I put in this equation this thing.

And on the LHS side if I put this on the LHS side then what do I get let us look at it, so what we get is so we are combining equation 3, 2 and 1. So what we are going to get is so I am just doing on the pressure side, so I know the expression for pressure I have putted here and I am going to differentiate it with respect to r so this is how I have picked up my equation for the left side of the Newton's equation and then that equals minus Rho not times partial derivative of u with respect to t. So I am going to plug value of u in Newton's equation, so it is minus Rho not and then I am going to partially differentiate u plus over r e j omega t minus r over c.

So once again what I have done here is I looked at the Newton's equation on the left side I have partial derivative of pressure with respect to r and here I replaced p with the expression for p which I had developed earlier and on the right side I have inserted the expression for u and now I am going to do further mathematical operations. So this becomes so we know that P plus now is a pure number and so I will take it out of the parentheses and now I am going to differentiate the entire thing with respect to r.

So what I get is minus e j omega t minus r over c divided by r square minus j omega e omega t minus r over c divided by rc and this equals minus Rho not. Now I am going to differentiate on the right side so what I get is u plus over r times j omega e j omega t minus r over c.

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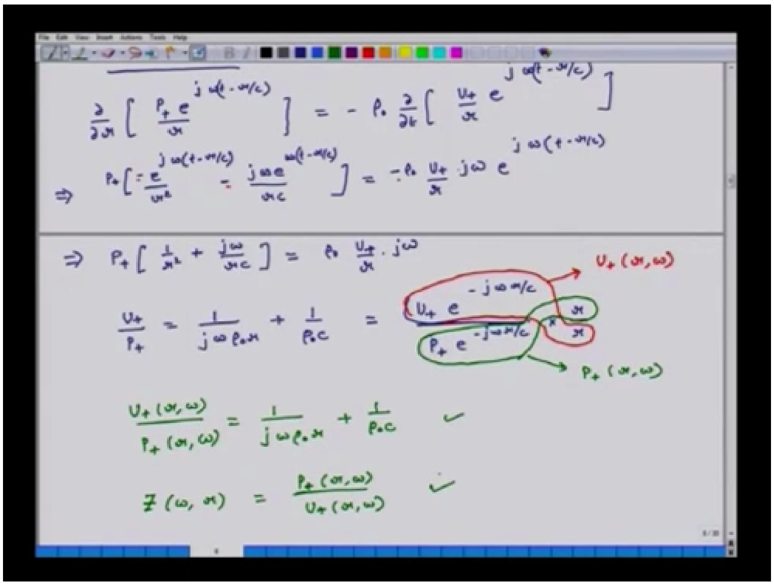

So I will process this now further and what I get here is P plus and I have a negative here I have negative, negative, negative so they all can go away right away. So what I am left with is 1 over r square and also the fact that I have e j omega t minus r over c in every single term so I can also get rid of it. So what I get is P plus times 1 over r square plus j omega over rc equals Rho not u plus over r times j omega. Or if I rearrange this equation I get u plus over P plus equals 1 over j omega Rho not r plus 1 over Rho not c and this is same as I can multiply the numerator and denominator by the same term so this is same as u plus e minus j omega r over c P plus e minus j omega r over c times r over r. So this term is u plus r omega and this term is P plus r omega.

So I can write this relation as u plus of r omega divided by P plus of r omega equals 1 over j omega Rho not r plus 1 over Rho not c. Now I know that Z or the impedance depends on omega and r is essentially P plus or r omega divided by u plus of r omega.

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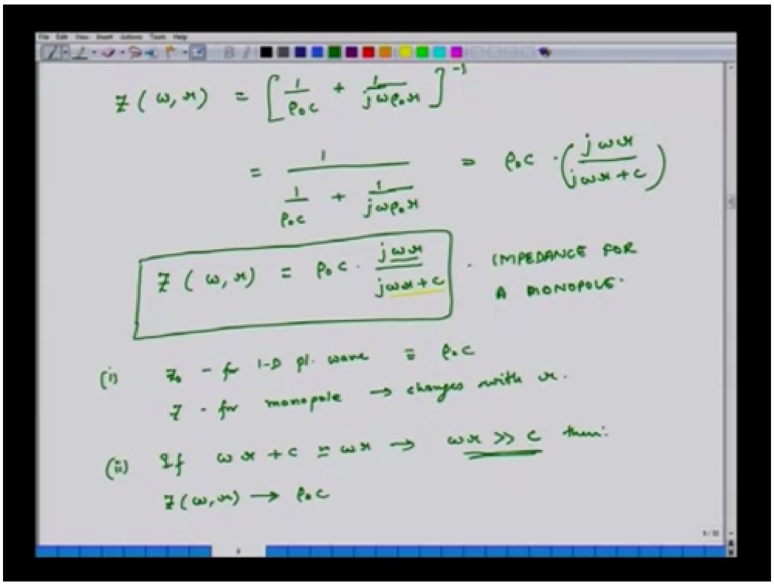

So from these two equations what I get is that Z of r omega Z impedance depends on omega as well as radius is equal to 1 over Rho not c plus 1 over j omega Rho not r and this whole thing is inversed and if I simplify this further what I get is 1 over Rho not c plus 1 over j omega Rho not r and this is nothing but Rho not c times j omega r divided by j omega r plus c. So impedance for radially propagating wave as emitted through a monopole is nothing but so this is the relation for impedance this is the relation for impedance of a monopole and please be aware that this relation is valid to the extent that there are no reflecting surfaces means there no reflections happening in the system.

Now there several observations which we can make as we look at this particular relation, one is that for a 1-D plane wave which would be travelling waveguide or tube for a 1-D wave plane wave the impedance was Z not and this was equal to Rho not c. Here Z is for monopole and it changes with r it changes with radius for a 1-D plane wave the impedance was not changing with radius it was or position it was constant it was constant and its value was Rho not c so this is one very significant thing.

The second thing is if I am far away from a monopole then the numerator which is j omega r and the denominator which is j omega r plus c they gradually approach 1 because the value of c becomes increasingly small with respect to j omega r. So if omega r plus c is approximately equal to omega r which means omega r is extremely large compared to c then Z of omega r approaches Rho not c.

So if I have a monopole and if I am extremely far from the monopole then the impedance which I will observe for the spherical wave which is moving out will be same as that of a planer wave. So essentially what it means is that if I am far away from a monopole then even though the in a strictly mathematical sense the wave propagation is spherical in nature but it approximates to a planer wave travelling out in 1-Dimension so that is (one) that is the second implication. Now this is valid only when omega r is extremely large compared to c and we will qualify that now.

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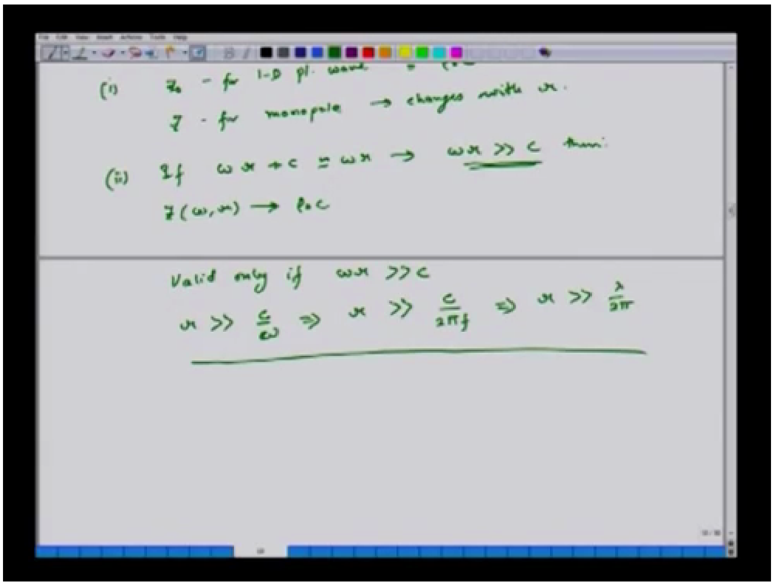

So this is valid only if omega r is extremely large compared to c which means that r is extremely large compared to c over omega which means r is extremely large compared to c divided by 2 Pi f which means r is extremely large compared to lambda over 2 Pi. So whether I am far away from a monopole or not it depends on the wavelength of the wave which is being emitted, if compared to one sixth of the wavelength I am far away from a monopole than my observations of that sound source will be very similar to that of a planer wave 1-D plane wave so that is another important observation which we see here.

So once again in 1-D plane wave the impedance is Z not which is same as Rho not c and does not have any dependency on r or it does not have any dependency on omega. In a spherical wave which is being emitted by a monopole the situation is different and in such a case the impedance depends on omega and it also depends on the position of observation but if I am far away from the monopole in the sense that the distance between the observation point and the monopole is significantly large compared to one sixth of wavelength then in that case the impedance will be approximately same as that of a 1-D plane wave which is Rho not c. So this is how the velocity and pressure are related for spherical waves so this term called impedance.

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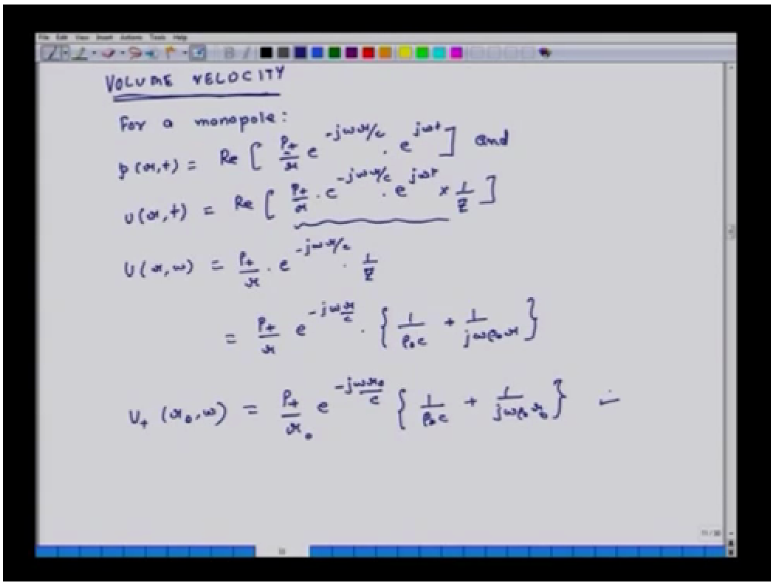

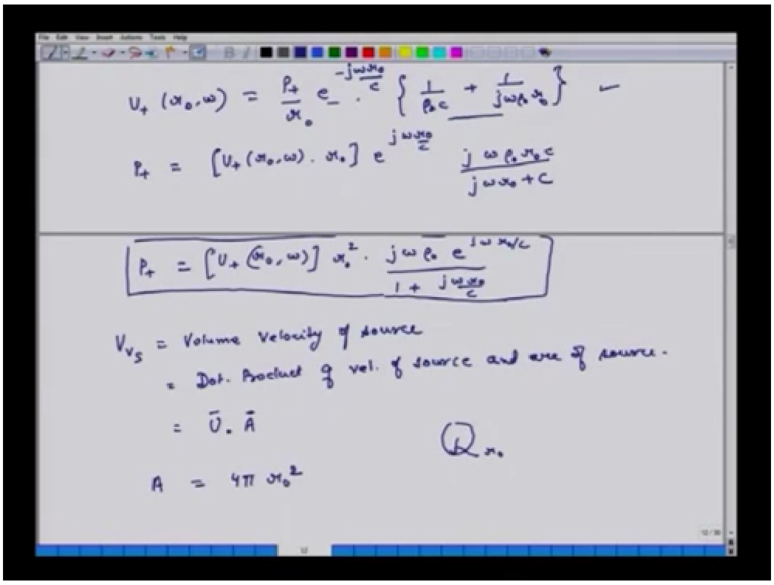

Now we will move on to a new concept called volume velocity, so we know for a monopole we know that pressure is real of and velocity is nothing but real of P plus over r times e minus j omega r over c times e j omega t so I just re written the expression for pressure but now I am going to divide it by impedance and that will give me my velocity. So I have a relation for pressure and I have a relation for velocity and if I know the impedance and if I also know this

P plus then I can find how pressure and velocity of wavelength with respect to r and also with respect to time but in a lot of situations it is difficult to find this value of pressure using physical or actual experimental measurements.

So the way this challenges addressed is by finding a different quantity called volume velocity and which we will develop which we will explain in a while and once we have figured out what the volume velocity is then from that particular measurement then we can calculate the pressure field and velocity field for monopole. So it is for (())(24:18) and for convenience that we have developed the concept of volume velocity and we use this concept to estimate and measure pressures estimate pressure field and velocity field for monopoles.

So we will continue to expand the relation for velocity and what we will do is we know that complex velocity is this entire thing in parentheses and this is nothing but P plus over r times e minus j omega r over c times 1 over Z and this is nothing but p plus over r e times minus j omega r over c times 1 over Rho not c plus 1 over j omega Rho not r.

Now suppose I have a sound source and I am at a distance r 0 away from this, so let us assume that I am r 0 away from this source than u plus r 0 omega equals. So what I have done here I have re wrote written the expression for complex pressure and amplitude complex pressure amplitude and I have replaced r by r not than I am going to re arrange this particular relation and so that I can express it I can develop a relation for P plus.

So P plus is nothing but u plus r not omega times r 0 and then I have to move this exponential term on this side as well. So exponent of j omega r not over c and then I also bring the term Z on the other side and what I get is j omega Rho not r not c divided by j omega r not plus c and this is same as u plus r not omega times r not (excuse me) times r not square times j omega Rho not exponent of j omega r not over c divided by and I am dividing the numerator and multiplying the numerator by c, so what I get is I have eliminated c from the numerator and what I get is 1 plus j omega r not over c. So at this time we introduce this term called volume velocity and we define it.

So suppose I have a diaphragm suppose I have a diaphragm and it is moving back and forth and let us say its velocity is V then the normal area of the diaphragm normal in the sense suppose the velocity is like this and the diaphragm is like this then the normal area of the diaphragm times the velocity will be the total amount of fluid which will be displaced by the diaphragm as it moves back and forth. So that is what is known as volume velocity.

So V VS I call as volume velocity of source and this is nothing but dot product of velocity of source and area of source so it could be a diaphragm, it could be a pulsating sphere if there is a pulsating sphere again it will be total surface area times the velocity at which the sphere is expanding and contracting, expanding and contracting. So I have to take the dot product of the velocity and the area which is moving out back and forth, this is nothing but velocity of source and then there is a dot product times area.

Now in case I have a diaphragm in case I have a diaphragm and it is a circular diaphragm then and if it is a circular diaphragm and let us say its radius happens to be r not or for that matter it could even be a sphere and if its radius is r not then A equals 4 Pi r not square, so I have a sphere this radius is r not and its pulsating sphere so that is the area.

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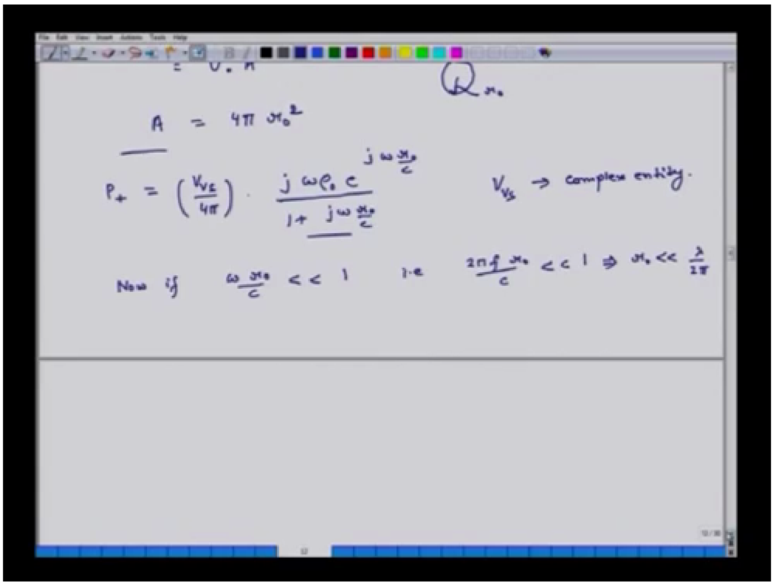

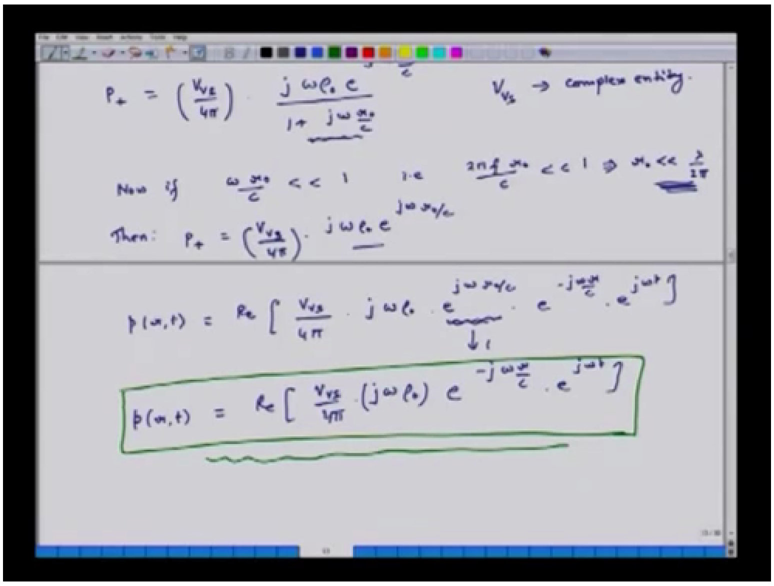

So with that I get I re write this particular equation and what I have here is u plus times r not square is basically V VS divided by 4 Pi because u times r not square times 4 Pi is the volume velocity, so it is V VS divided by 4 Pi times j omega Rho not e j omega r not over c divided by 1 plus j omega r not over c. Now once again because u plus of r not omega could be a complex entity so V VS could be complex entity and it may change with respect to omega and it will also depend on the value of r not. So I have this volume velocity so pressure magnitude complex magnitude pressure is same as V VS divided by 4 times times this complex function.

Now if omega r not over c is extremely small compared to 1 that is 2 Pi f r not over c is extremely small compared to 1 which means r not is extremely small compared to lambda over 2 Pi. Then so if this is the case that the size of this sound source is extremely small compared to one sixth of wavelength if that is the case then the denominator term just collapses to 1 and in this case P plus is equal to V VS divided by 4 Pi times j omega Rho not e j omega r not over c and that is it the denominator is 1 instead of 1 plus j omega r not over c.

So if that is the case then pressure field I can write it as real of V VS times 4 Pi times j omega Rho not times e j omega r not over c times e minus j omega r over c times e j omega t, so I am using my original expression for pressure and I realize that because of this condition r not is extremely small compared to lambda over 2 Pi this term also is approximately equal to 1. So if that is the case then I can simplify this as so this is my simplified expression for pressure.

Essentially what this relation tells us is that if I have a monopole and I know its approximate size and let us say it is emitting sound of a frequency f and monopole is extremely small compared to the wavelength then in that case it is very close an ideal monopole which is of a zero size that is one, second thing is the total area of the this sound source if I multiply that in a dot product since with the velocity of the oscillating or pulsating surface and then I then from that I can get volume velocity and then I can use that volume velocity to compute the pressure field generated by the monopole that is essentially the crux of this entire development which we have talked about till so far.

So we have volume velocity of a system it is a dot product of velocity and area surface area from that I can compute volume velocity and using this volume velocity I can calculate the pressure field using this particular relation with the condition that the size of this source has to be extremely small compared to the wavelength of the sound which is being emitted by the source, thank you very much.