Acoustics

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Module-4 Monopoles and Dipoles

Lecture 02

Radial propagation of sound, monopoles, and dipoles

Welcome to this lecture in acoustics, today we will move further till so far what we have been talking about is how sound propagates in an elastic medium and more specifically in fluids how sound propagates in fluids specially in 1-dimension with reference to a Cartesian framework. So essentially what we have covered till so far in detail is propagation of sound through long tubes or ducts.

Now this type of sound propagation has limited applications because most of the sound which we encounter in real applications does not propagate through ducts or tubes, instead it propagates over long distances and most of this propagation is radial in nature. So you may have a sound source and sound gets emitted from it in all directions and it gets radially transmitted over long distances.

And as it travels eventually it may hit some reflecting surfaces and it gets reflected, it also on the path may get attenuated because of absorption characteristics or the medium in which it is travelling or also it may face attenuation because when it hits the reflecting surface part of the sound may be reflected back and part of the sound may get absorbed by the surface which it is striking. But in general propagation of sound in most of the applications which we encounter in real applications occurs in a radial way.

So what we will start talking about today is radial propagation of sound and as we develop a theory or a framework to understand radial propagation of sound we will impose one restriction that is atleast for starters and that will be that they will be we will not account for the reflection of sound. So once you have a point source through which sound is getting emitted and it propagates through air or some other fluid medium then it just keeps on going out forever and it does not hit a reflecting surface.

So that is the type of radial propagation of sound which we will analyse and develop a framework for. Some examples of this type of radial propagation would be for instance you have a bird in the air flying in air and it emits a particular sound and it travels in all the directions as we eventually we listen to that sound another example would be an aircraft flying in air its emitting sound and sound is getting propagated in all directions in a radial way, another example would be an object moving in ocean waters deep into ocean waters and it surrounded on all sides by a fluid medium and once again when this particular object it could be a fish, or submarine, or whatever when it emits sound the propagation of sound is again radial in nature. So that is the context in which we will be developing a framework for radial propagation of sound.

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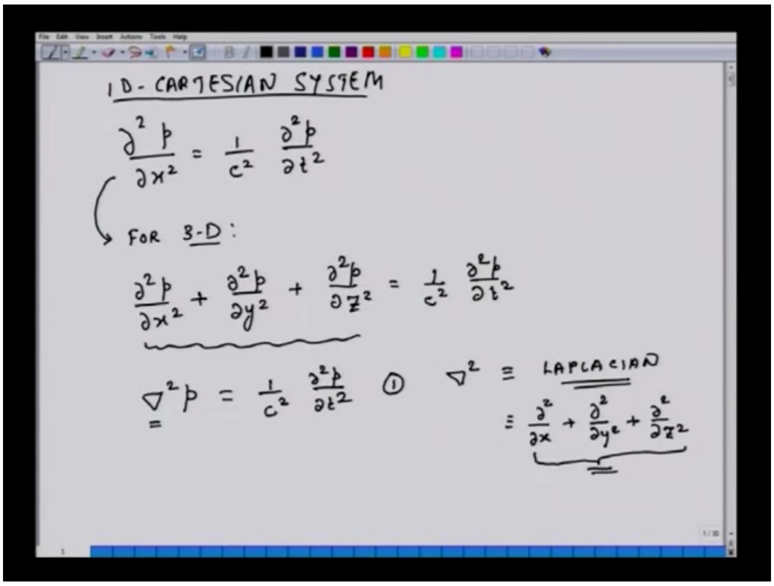

So we will again start with 1-dimensional Cartesian framework and what we had seen that for a 1 D-Cartesian system the relation for pressure which we had developed was something like this so this is the pressure wave equation for propagation of sound as it travels in 1-Dimension specifically if it is moving through wave guides or ducts or some devices of that nature.

Now if we want to extend this for 3-Dimensions then again if we are talking about a Cartesian reference frame then for 3-Dimensional system the wave equation for pressure will look something like this so its second derivative of pressure with respect to x plus second derivative of pressure with respect to y plus second derivative of pressure with respect to z and that the sum of all these three derivatives equals 1 over c square times second derivative of pressure with respect to time.

Now I can re express the left hand side of this particular equation in this format and this particular symbol is known as Laplacian. So essentially this equation for wave propagation in

3-Dimensions has been rewritten as Laplacian of pressure equals 1 over c square times second derivative of pressure with respect to time so we will number this equation as 1 now please note that this Laplacian is essentially an operator it is Laplacian operator and this is essentially this.

So Laplacian operator if I want to write it in a Cartesian framework it will look like this. Another thing you may want to notice that Laplacian operator of if I operate if I perform Laplacian operation on pressure then the resulting entity will be a pure number it will be a scalar number, it will not be a vector quantity and another interesting about this particular operator is that it does not matter if I conduct this operation in a Cartesian framework, or if I move to a cylindrical framework, or I move to a spherical coordinate system if I conduct the Laplacian operation the emergent entity after I have conducted that operation it will remain the same because the right hand side of the equation is not changing it is variation of pressure with respect to time. So if I change my coordinate system from Cartesian to cylindrical to spherical the left hand side also will not change.

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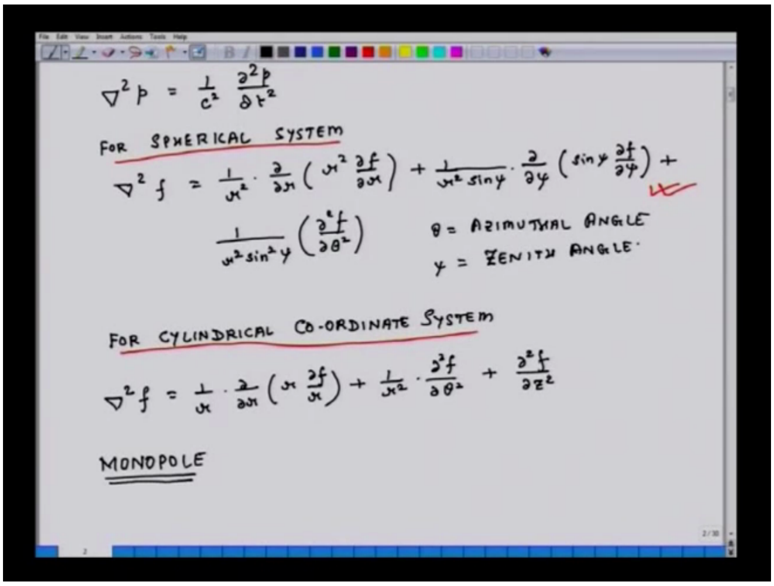

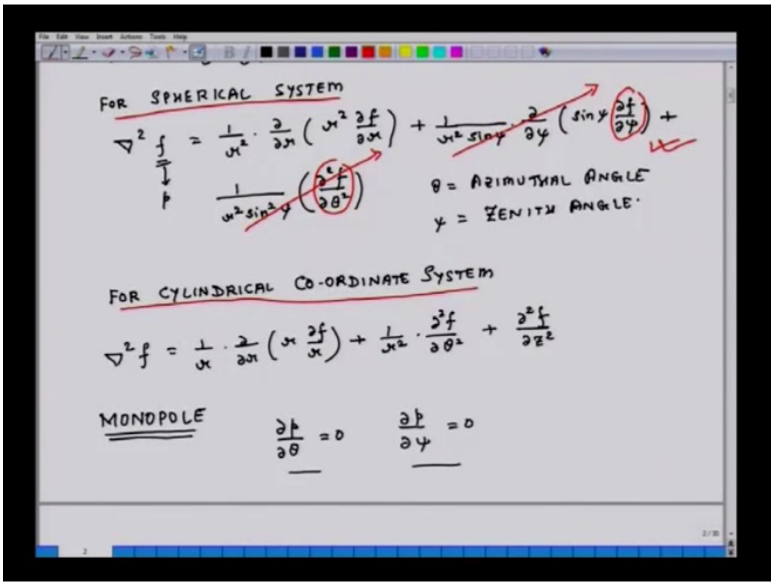

So with that understanding what I will do is I will re express this particular equation which is essentially wave propagation equation in 3-Dimensions and till so far we had seen how Laplacian looks like in a Cartesian reference frame, I will re express the same thing in a spherical system. Now for a for Spherical system Laplacian of a function f is equal to 1 over r times del over del r of partial derivative of f with respect to r plus (excuse I missed a power here and I think I have correct this) plus 1 over r square Sin Psi times del over del Psi Sin Psi times second derivative of this function with respect to theta and in a spherical system r, theta and Psi these are the three independent directions. So theta is known as Azimuthal angle and Psi is known as Zenith angle. So this is the relation for spherical system.

Likewise if I have a Cylindrical coordinate system then Laplacian of a function f scalar function f in a Cylindrical coordinate system I can write it as 1 over r times partial derivative of of this thing with respect to r plus 1 over r square times second derivative of f with respect to theta plus second derivative of f with respect to z and in this case the three coordinate directions are r, theta and z.

So this is how I will write a relationship for Laplacian of a function in Spherical coordinate system and the second relation is for the Laplacian of a function f in a Cylindrical coordinate system. So as I mentioned earlier I am trying to develop a relationship for a Spherical system so I will choose this particular function and then I will see what kind of manipulations I can do and get an equation out of it a wave equation for Spherical coordinate system.

So at this point I will introduce a term called Monopole. So monopole is essentially a point source from where sound is getting emitted. So theoretically its dimension it is a dimensionless entity, it is a point, it is a source of sound and the size of that source is that of a point. So it has virtually zero size and sound is getting emitted from it and then another feature of this Monopole is that its radiating sound in all directions in a Spherical way and the third thing is that the emission of sound from this particular source is equal in all directions.

So essentially what that implies is that the sound field due to a monopole source is spherically symmetric or in other words I can also say is that the variation of sound field in theta direction and also in Psi direction. So in the Azimuthal direction and also in the Zenith direction the variation of sound field is zero.

So for a monopole what I can say is that any variation of pressure field with respect to theta is 0 and any variation of pressure field with respect to zenith angle is 0. So given that understanding if I replace f by p in all these places then and using this understanding that variation of pressure with respect to theta and with respect to Psi they are 0, what I see is that these terms vanish because here I have del p over del Psi and here I have second derivative of pressure with respect to theta so these two parts of the relation they go away.

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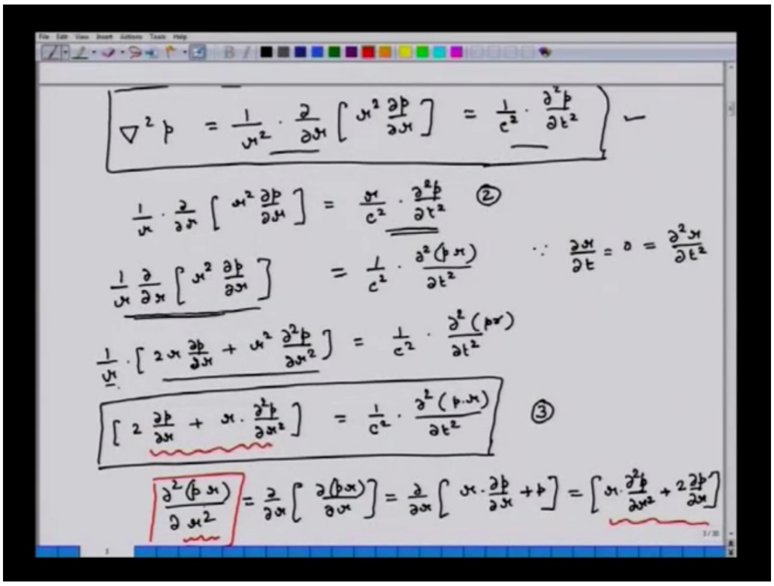

So essentially what that means is that for a monopole the Laplacian of pressure is nothing but 1 over r square times del over del r r square del p over del r and we know that this Laplacian has to equal 1 over c square times second derivative of pressure with respect to time. So this is my pressure wave equation for a spherical coordinate system and that is holds true for a sound source which is a monopole in nature, it has extremely small size, it is emitting sound in all the directions equally and in that kind of a situation the pressure filed is going to behave in such a way that it is consistent with this particular equation.

Now what I will do is I will do some mathematical operations and I will re express this particular equation in another form. So I will re write this equation as 1 over r what I am going to do here is I am going to multiply this side and this side by r so essentially what I get is 1 over r so this is my equation 2.

Now if I look at the left hand side of the equation I see that it is r divided by c square times second derivative of pressure with respect to time and I can re express this side as 1 over c square times second derivative of this function pr over t because partial derivative of r is 0 and also second derivative of r with respect to time in a partial sense is also 0 because this is the case because of this reason I can re express the right side of the equation as 1 over c square times second derivative of p times r with respect to time. So I will now complete this equation, okay.

Now the next thing I am going to do is I am going to expand on the left side I am going to expand on the left side and I think I made a small error here so this should have been r so once I expand on the left side what I get is 1 over r times I am going to differentiate the term in parentheses with respect to r so I get 2r del p over del r plus r square second derivative of pressure with respect to r is equal to 1 over c square times second derivative of p times r divided by over with respect to time and once now what I will do is I will multiply or divide this term in the parentheses with this r.

So essentially what I get is 2 times partial derivative of pressure with respect to r plus r times second derivative with respect to r and there is a partial derivative is equal to 1 over c square times partial derivative of p times r with respect to time, okay so this is my equation 3 this is my equation 3. Now what I do is I do some further manipulation and in right and when I am doing my further manipulation I am going to manipulate another term on this side on again left side of the equation but before I start doing that let us look at this term.

So we will start with the term let us consider this term so this is the term when I am going to p times r and I am going to differentiate partially this term with respect to r and let us see what we get. So if I differentiate it the first time what I I can re write this as and what I get in parentheses is r times partial derivative of pressure with respect to r plus p and now if I differentiate it once again with respect to r this entire thing in parentheses what I get is r times second derivative of pressure with respect to r plus 2 times first derivative of pressure with respect to r.

So what we observe here is that this term and this term they are essentially the same. So I can write I can re write the left side of equation 3 in terms in the same way as this term. So I can replace the left side of equation 3 with second derivative of p times r with respect to r so that is what I am going to do.

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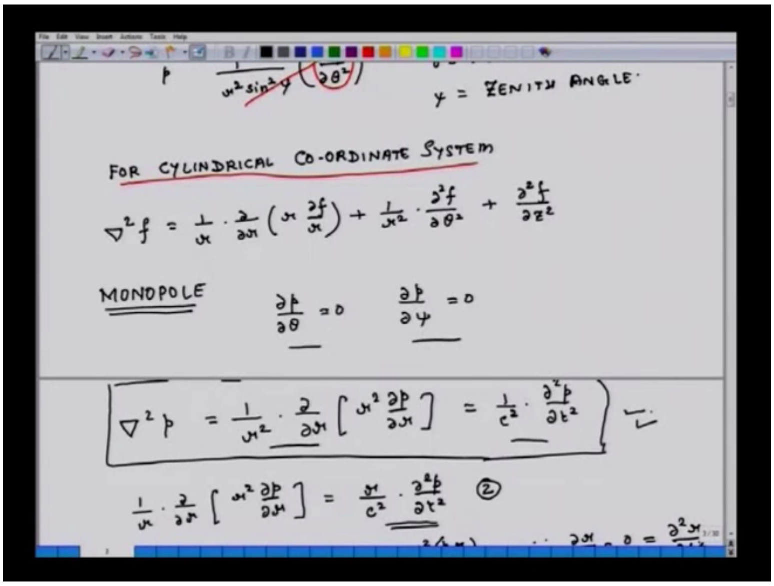

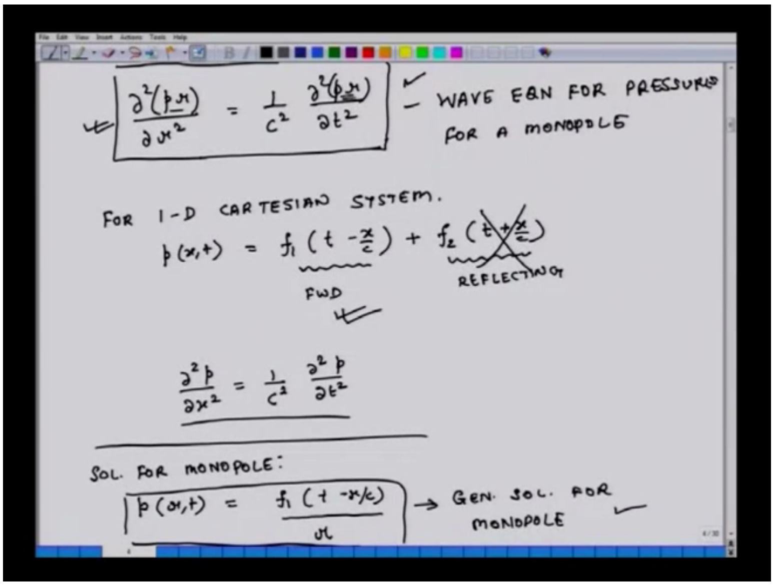

And finally what I get is with that manipulation what I get is so this is the wave equation for a monopole in a spherical coordinate system and the reason I had have developed this particular form of the equation. So earlier this was the wave equation which we had developed but then later we transform that form into this particular form because we will see that this is from mathematical stand point it is easier to handle when we are specifically trying to solve this particular equation. So this is the standard pressure wave equation for a monopole which we have use subsequently.

Now what we are going to do now now that we have developed this particular wave equation is we are going to solve it. So again we will start with the understanding for a 1-D wave equation for a Cartesian framework and see how we solve that particular equation and then we will use similar techniques to solve this particular equation. So for 1-D Cartesian system we had seen that a general form of the solution for a pressure wave equation was something like f 1 times t minus x over c plus f 2 second function of variable t plus x over c this particular term captured the effect of a forward travelling wave and this particular term captured the effect of a reflecting wave.

Now given are basic assumption that we are not accounting for any reflections we will not have a term which is similar to f 2 when we try to solve for a pressure wave equation for a monopole. So this term is going to go away and then we will have a term which is something similar not identical but which is something similar to this particular term when we solve for a pressure wave equation for a spherical coordinate system for a monopole.

So what we see here is that here the term in the parentheses is p times r. Now in a 1-D Cartesian system the pressure wave equation look like. So we immediately see that instead of p we have replaced it in a spherical coordinate system by p times r and x has been replaced by r. So right away we recognize that a potential solution for this particular equation would be. So solution for monopole and this will be a general solution for a monopole would be p of r t is nothing but f 1 t minus x over c but this entire thing will be divided by r because if I now multiply p times r that is same as t minus x over c or rather r over c. So this is my general solution for a monopole.

Now what we will do is we will proof this is a solution which we have assumed and now we will proof whether this is indeed the solution for a pressure wave equation for a monopole.

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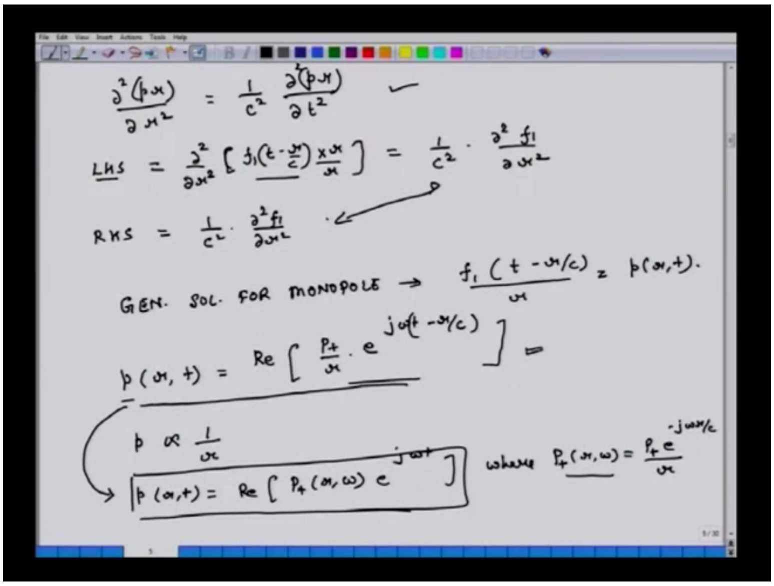

So we will re write the pressure wave equation and then in terms of p and r we will plug in this particular relation. So p times r is nothing but f 1 of t minus r over c, so the LHS or the left hand side of this equation becomes and that is nothing but 1 over c square times partial derivative f 1 with respect to r and then RHS if we do the maths correctly we will again find that this is same as so again because LHS and RHS are same we have verified that this is indeed that so that this particular function f 1 of t minus r over c divided by r is indeed the solution for the equation for a monopole.

So the general solution we will re write general solution for a monopole is and one particular form of this solution could be so we see that this particular form is consistent with the general solution so this is a valid this particular form real part of p plus p plus is a complex number and it relates to the magnitude of the pressure wave so this is real part of p plus divided by r times e j omega t minus r over c and this is a particular type of a solution for a monopole.

One thing we see in this case is that as we increase the radius as we move away from the monopole the value of pressure or the magnitude of the pressure decrease so if we double the radius the pressure goes down the magnitude of pressure goes down by a factor of 2 and so on and so forth. So what we immediately see from this is that pressure is directly proportional to 1 over r or it is inversely proportional to r we will now re express this term as something like this.

So this equation has been re expressed as p of r t equals real of complex amplitude which is dependent on omega and radius it is again very similar to what we had done for 1-D waves moving in a Cartesian reference frame. So it is equal to real of p of r omega times p j omega t and where this complex amplitude p of r omega is defined as p plus which is a complex number times p j omega r over c divided by r.

So what we have covered till so far is a monopole we have developed the equation for a monopole, how pressure varies of a sound source which is close to a monopole and which is emitting sound in all the directions in a radially symmetric way and we have developed a pressure wave equation for such a sound source and then we have also gone ahead and developed a solution for this kind of a sound source and also developed relations for pressure for monopoles. So what we are going to do next is we are going to develop a relation between velocity and pressure for a monopole type of a source.