

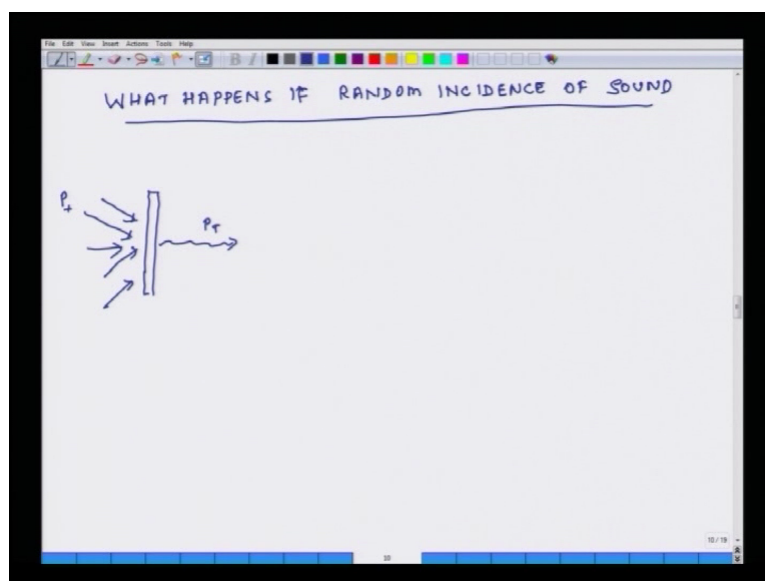
Acoustics
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Lecture 02
Module 3
Sound Transmission through Walls

Okay. So in the last lecture we have developed transmission loss curves for a wall which is impinged upon by normal incidence sound energy and what we have seen is that in the mass controlled region and in the stiffness controlled region we have a positive slope and also a negative slope straight line and the slope of these two straight lines is essentially 6 decibels per decade. Also in theory at a point of resonance there is no transmission loss.

So all the sound which is incident on the wall it passes through the wall but in reality there is some loss through the wall because of damping properties of the system. Now all these discussion was in context of normal incidence sound. So the question is that what happens if we have random incidence of sound?

So once again I have a wall and instead of sound striking normally sound may strike it in all sorts of directions and then the question is what kind of P T do we experience if the incidence sound energy's intensity is related to this term P_i ? Then what will be the transmission ratio and what will be the attenuation and what will be the transmission loss across the wall?

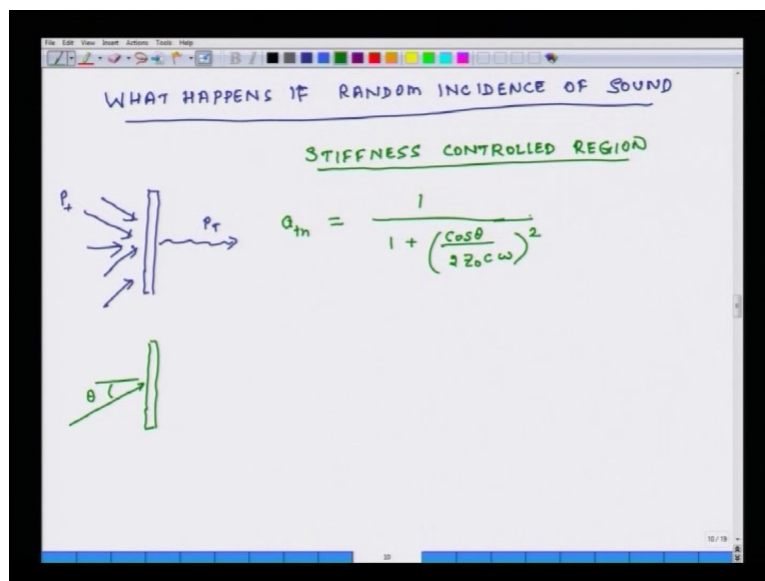
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So once again we are going to break this problem into two components. One corresponds to stiffness controlled region and the other one corresponds to the mass controlled region. So what we are going to start with is this stiffness controlled region. Now the attenuation if I have oblique incidence, so if this is my angle of incidence which is theta then the attenuation is defined and I am not going to do a proof of this term but using existing knowledge I am just going to just replicate the relation.

So attenuation if there is oblique incidence when the angle of incidence is theta then attenuation has been found out to be something like this.

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And what you see here is that this relation is more or less very similar to the relation which we developed for normal incidence except for the fact that I have this cos theta term. All other terms are same. Now if this theta becomes 0, I have normal incidence and this (at) relation for attenuation becomes same as that for normal incidence. Now I know that this is the relation for oblique incidence where angle of incidence is theta but theta can range from 0 to pie over 2 radians. So this is my equation 1.

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WHAT HAPPENS IF RANDOM INCIDENCE OF SOUND

STIFFNESS CONTROLLED REGION

$$a_{tn} = \frac{1}{1 + \left(\frac{\cos\theta}{2Z_0c\omega}\right)^2} \quad \text{--- (1)}$$

But $\theta \rightarrow 0$ to $\frac{\pi}{2}$

And then again using some well proven relations I can write that if this wall is getting impinged upon by all sorts of radiations, by all sorts of sound waves of equal intensity in all directions then my average attenuation is nothing but it is going to be some sort of an integral and this is how this integral looks like. So a t_n attenuation specific to a particular value of angle times cosine theta times sin theta times $d\theta$.

So if I am able to integrate this from theta equals 0 to pie over 2 radians then I get the average attenuation offered by the wall which is being hit by sound coming from all directions.

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WHAT HAPPENS IF RANDOM INCIDENCE OF SOUND

STIFFNESS CONTROLLED REGION

$$a_{tn} = \frac{1}{1 + \left(\frac{\cos\theta}{2Z_0c\omega}\right)^2} \quad \text{--- (1)}$$

But $\theta \rightarrow 0$ to $\frac{\pi}{2}$

$$a_{tn}/A_n = 2 \int_0^{\pi/2} a_{tn}(\theta) \cdot \cos\theta \cdot \sin\theta \cdot d\theta$$

So now what I have to do is I have to integrate this relation with the value of attenuation for angle theta is defined by equation 1. And if I do the math correctly then average attenuation is nothing but this number K_S square times natural log of $1 + 1$ over K_S square where K_S is basically this term, $2\omega Z_0 C$, okay.

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WHAT HAPPENS IF RANDOM INCIDENCE OF SOUND

STIFFNESS CONTROLLED REGION

Diagram: Sound waves with pressure P_i and angle θ incident on a wall. Transmitted pressure is P_t .

$$a_{tm} = \frac{1}{1 + \left(\frac{\cos\theta}{2Z_0 c \omega}\right)^2} \quad \text{--- (1)}$$

But $\theta \rightarrow 0$ to $\frac{\pi}{2}$

$$a_{tm,AV} = 2 \int_0^{\pi/2} a_{tm}(\theta) \cdot \cos\theta \cdot \sin\theta \cdot d\theta$$

$$a_{tm,AV} = K_S^2 \ln \left[1 + \frac{1}{K_S^2} \right] \quad \text{where } K_S = 2\omega Z_0 C$$

And now I know that 1 over K_S square from this relation, this is basically attenuation for (inci) 90 degrees incident angle and this term in parenthesis in the denominator is K_S .

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$$a_{tm} = \left| \frac{1 - \frac{j}{2Z_0 c \omega}}{1 + \left(\frac{j}{2Z_0 c \omega}\right)^2} \right|^2 = \left[\frac{1 + \left(\frac{1}{2Z_0 c \omega}\right)^2}{1 + \left(\frac{1}{2Z_0 c \omega}\right)^2} \right]^2$$

$$a_{tm} = \frac{1}{1 + \left(\frac{1}{2Z_0 c \omega}\right)^2}$$

$$TL = 10 \log_{10} \left(\frac{1}{a_{tm}} \right) = 10 \log_{10} \left[1 + \left(\frac{1}{2Z_0 c \omega}\right)^2 \right]$$

$\hookrightarrow f \ll \frac{1}{2\pi} \sqrt{\frac{f}{cm}}$

(dB)
TL

Graph: A plot of TL (dB) vs frequency f (Hz) showing a slope of 6 dB/oct.

STIFFNESS CONTROLLED REGION

So this is same as this term. So I can rewrite this relation as K_S square times 1 over attenuation.

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WHAT HAPPENS IF RANDOM INCIDENCE OF SOUND

STIFFNESS CONTROLLED REGION

Diagram: Sound waves with pressure P_i and reflection P_r hitting a wall.

$$a_{tn} = \frac{1}{1 + \left(\frac{\cos \theta}{2 Z_0 c \omega} \right)^2} \quad \text{--- (1)}$$

But $\theta \rightarrow 0$ to $\frac{\pi}{2}$

$$a_{tn|_{AV}} = 2 \int_0^{\pi/2} a_{tn}(\theta) \cdot \cos \theta \cdot \sin \theta \cdot d\theta$$

$$a_{tn|_{AV}} = K_S^2 \ln \left[1 + \frac{1}{K_S^2} \right] \quad \text{where } K_S = 2 \omega Z_0 c$$

$$= K_S^2 \ln \left[\frac{1}{a_m} \right]$$

So moving on what I can further write then the transmission loss average implying that energy is hitting the wall from all directions. So if I take some sort of an average of that then average transmission loss is equal to 10 log of 10 times 1 over attenuation average and now in this parenthesis I plug in this number so what I get is.

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$$TL|_{AVG} = 10 \log_{10} \left[\frac{1}{a_{tn|_{AV}}} \right] = 10 \log_{10} \left[K_S^2 \ln \left(\frac{1}{a_m} \right) \right]^{-1}$$

Now what I am going to do is I am going to resolve this. So what I get is because this is to the power of negative 1. So K_S^2 comes in the denominator and then the second component of this log is 10 log 10 of natural log and because once again the power is negative 1, so instead of plus I have put a negative sign here. So that takes care of the

negative 1 on the power thing. So this is 1 over 1 plus K S square. So let us call this relation A.

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The image shows a whiteboard with the following handwritten equations:

$$TL|_{AVG} = 10 \log_{10} \left[\frac{1}{a_{in}|_{AVG}} \right] = 10 \log_{10} \left[k_s^2 \rho_n \left(\frac{1}{a_n} \right) \right]^{-1}$$

$$= 10 \log_{10} \left(\frac{1}{k_s^2} \right) - 10 \log_{10} \left[\ln \frac{1}{1+k_s^2} \right] \quad \text{Ⓐ}$$

Now so remember we are talking about the stiffness controlled region only. Now for the stiffness controlled region we know that if angle of incidence is you know 0 degree then it is normal incidence. So T L normal and we had calculated the relation for this earlier and this is nothing but 10 log of 10 over 1 over attenuation and this is 10 log of 10 1 over 1 plus K S square. So I can rewrite this as 10 log of 10 and what I am going to do is I am going to rewrite this term, I will take in an exponential format so I have an exponential of natural log of K S square, okay.

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The image shows a whiteboard with the following handwritten equations:

$$TL|_{AVG} = 10 \log_{10} \left[\frac{1}{a_{in}|_{AVG}} \right] = 10 \log_{10} \left[k_s^2 \rho_n \left(\frac{1}{a_n} \right) \right]^{-1}$$

$$= 10 \log_{10} \left(\frac{1}{k_s^2} \right) - 10 \log_{10} \left[\ln \frac{1}{1+k_s^2} \right] \quad \text{Ⓐ}$$

$$TL|_{NORMAL} = 10 \log_{10} \left(\frac{1}{a_n} \right) = 10 \log_{10} \left[\frac{1}{1+k_s^2} \right]$$

$$= 10 \log_{10} \left[e^{-\ln \{1+k_s^2\}} \right] =$$

And this term exactly same term but I have just taken a natural log of it and then I have raised it to an exponent so it is the same thing. So now I rewrite it further. So what I get is 10 log of 10 of e times natural log of 1 plus 1 over K S square and this number comes out to be 4 point 343 and then from here I get natural log of 1 plus 1 over K S square, okay. So this is T L normal.

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The image shows a whiteboard with the following handwritten equations:

$$\begin{aligned}
 TL_{AVG} &= 10 \log_{10} \left[\frac{1}{a_{m|AVG}} \right] = 10 \log_{10} \left[k_s^2 \ln \left(\frac{1}{a_m} \right) \right]^{-1} \\
 &= 10 \log_{10} \left(\frac{1}{k_s^2} \right) - 10 \log_{10} \left[\ln \frac{1}{1+k_s^2} \right] \quad \text{Ⓐ} \\
 TL_{NORMAL} &= 10 \log_{10} \left(\frac{1}{a_m} \right) = 10 \log_{10} \left[\frac{1}{1+k_s^2} \right] \\
 &= 10 \log_{10} \left[e^{\ln \left\{ 1 + \frac{1}{k_s^2} \right\}} \right] = \\
 &= 10 \log_{10}(e) \times \ln \left[1 + \frac{1}{k_s^2} \right] \\
 TL_{NORMAL} &= 4.343 \ln \left[1 + \frac{1}{k_s^2} \right]
 \end{aligned}$$

So now I move this on this side so I get natural log of 1 plus 1 over K S square is nothing but 0 point 23026 times T L normal. So this is my equation B and what I am going to do is put equation B into equation A. So I am going to replace this with this term.

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$$\begin{aligned}
 TL_{AVG} &= 10 \log_{10} \left[\frac{1}{a_{m|AVG}} \right] = 10 \log_{10} \left[k_s^2 \ln \left(\frac{1}{a_m} \right) \right]^{-1} \\
 &= 10 \log_{10} \left(\frac{1}{k_s^2} \right) - 10 \log_{10} \left[\ln \frac{1}{1+k_s^2} \right] \quad \text{Ⓐ} \\
 TL_{NORMAL} &= 10 \log_{10} \left(\frac{1}{a_m} \right) = 10 \log_{10} \left[\frac{1}{1+k_s^2} \right] \\
 &= 10 \log_{10} \left[e^{\ln \left\{ 1 + \frac{1}{k_s^2} \right\}} \right] = \\
 &= 10 \log_{10}(e) \times \ln \left[1 + \frac{1}{k_s^2} \right] \\
 TL_{NORMAL} &= 4.343 \ln \left[1 + \frac{1}{k_s^2} \right] \\
 \ln \left(1 + \frac{1}{k_s^2} \right) &= 0.23026 \cdot TL_{NORMAL} \quad \text{Ⓑ}
 \end{aligned}$$

And so what I am interested in finding out is the average transmission loss. So T L average is nothing but $10 \log_{10} \left(\frac{1}{K S^2} \right) - 10 \log_{10} [0.23026 T L_{NORMAL}]$. And if I want to remove this square term from this thing then what I get is $20 \log_{10} \left(\frac{1}{K S} \right) - 10 \log_{10} [0.23 T L_{NORMAL}]$. So that is my relation. So this is my average transmission loss across a wall as long as I am in the stiffness controlled region.

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The image shows a whiteboard with two equations written in green marker. The first equation is $TL_{AVG} = 10 \log_{10} \left(\frac{1}{K S^2} \right) - 10 \log_{10} [0.23026 T L_{NORMAL}]$. The second equation, which is enclosed in a green rectangular box, is $TL_{AVG} = 20 \log_{10} \left(\frac{1}{K S} \right) - 10 \log_{10} [0.23 T L_{NORMAL}]$. The whiteboard also shows a standard software toolbar at the top and a slide number '12/19' at the bottom right.

So likewise now what I am going to do is going to figure out what is transmission loss if I am in the mass controlled region? So for mass controlled region we know the T L normal, we have developed this relation, is basically $10 \log_{10} \left(\frac{1}{\text{attenuation}} \right)$, okay. And this relationship we had earlier proven same as $10 \log_{10} \left(\frac{1 + \omega}{2 Z_0} \right)^2$.

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Handwritten equations on a whiteboard:

$$TL_{AVG} = 10 \log_{10} \left(\frac{1}{K_s^2} \right) - 10 \log_{10} \left[0.23026 TL_{NORMAL} \right]$$

$$TL_{AVG} = 20 \log_{10} \left(\frac{1}{K_s} \right) - 10 \log_{10} \left[0.23 TL_{NORMAL} \right]$$

FOR MASS CONTROLLED REGION

$$TL_{NORMAL} = 10 \log_{10} \left[\frac{1}{a_m} \right] = 10 \log_{10} \left[1 + \left(\frac{M\omega}{2Z_0} \right)^2 \right]$$

Now I want to convert this into T L average for random incidence. So people have found using numerous experiments that if a wall is being hit by sound waves and the frequencies of these sound waves are such that they are coming from all directions and these frequencies are way in excess of the natural resonance of the system then T L average is approximately equal to T L normal minus 5. So this is based on experimental data.

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Handwritten equations on a whiteboard:

$$TL_{AVG} = 10 \log_{10} \left(\frac{1}{K_s^2} \right) - 10 \log_{10} \left[0.23026 TL_{NORMAL} \right]$$

$$TL_{AVG} = 20 \log_{10} \left(\frac{1}{K_s} \right) - 10 \log_{10} \left[0.23 TL_{NORMAL} \right]$$

FOR MASS CONTROLLED REGION

$$TL_{NORMAL} = 10 \log_{10} \left[\frac{1}{a_m} \right] = 10 \log_{10} \left[1 + \left(\frac{M\omega}{2Z_0} \right)^2 \right]$$

$$TL_{AVG} = TL_{NORMAL} - 5$$

EXPERIMENTAL DATA.

So now I have my two conditions. So for random incidences this is the summary. If in stiffness controlled region then my transmission loss for random incidence equals $20 \log_{10} \left(\frac{1}{K_s} \right) - 10 \log_{10} \left[0.23026 TL_{NORMAL} \right]$, where TL_{NORMAL} is $10 \log_{10} \left[1 + \left(\frac{M\omega}{2Z_0} \right)^2 \right]$. Second

thing is if in mass controlled region then T L average equal to T L normal minus 5 decibels and this is based on experimental data so it is an empirical relation.

And here T L N is nothing but $10 \log_{10} \left[1 + \frac{M \omega}{2 Z_0 C} \right]$. Excuse me there should not be a C here, $2 Z_0$ square, okay. So if in mass controlled region this is what I should use. If I am in stiffness controlled region this is what I should use to understand how much transmission loss is going to happen across the wall.

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FOR RANDOM INCIDENCE

① IF IN STIFFNESS CONTROLLED REGION

$$\checkmark TL_{Avg} = 20 \log_{10} \left(\frac{1}{k_s} \right) - 10 \log_{10} [0.23026 TL_N]$$

$$\text{where } TL_N = 10 \log_{10} \left[1 + \frac{1}{(2 Z_0 C)^2} \right]$$

② IF IN MASS CONTROLLED REGION

$$\checkmark TL_{Avg} = TL_N - 5$$

$$\text{where } TL_N = 10 \log_{10} \left[1 + \left(\frac{M \omega}{2 Z_0} \right)^2 \right]$$

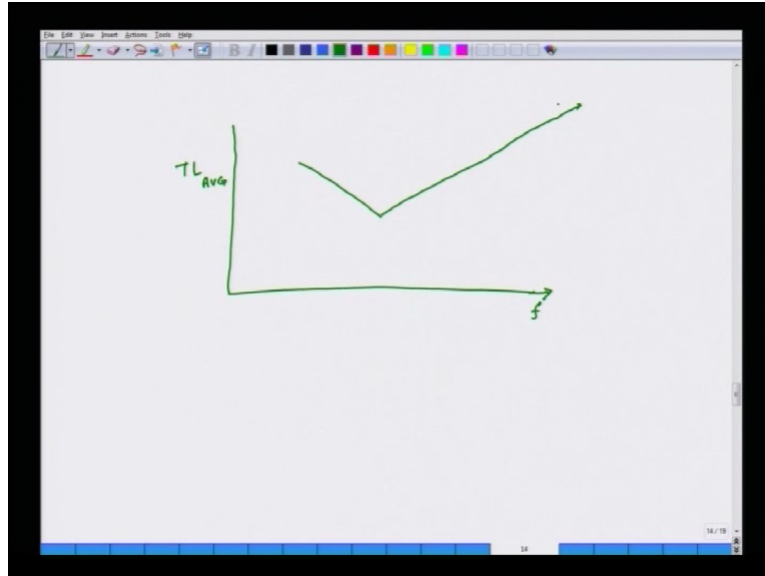
So then a natural question would be how do I know whether I am in mass controlled or stiffness controlled region? So one way is that you can actually figure out the natural resonance of the system by doing some finite element analysis of the wall or the panel which we are talking about or we can use some standard textbooks which have listings of relationships for these natural frequencies of the system.

And we can use that to figure out whether we are in mass controlled region or we are in stiffness controlled region. So now there is one final question. So till so far what we have seen is that even this relationship what it says is that T L average also it increases at something like 6 decibels per octave if I am in stiffness controlled region. So if I am going to reduce my frequency by factor of 2 I will have an improvement in transmission loss even for random incidence by 6 decibels.

Similarly if I am in mass controlled region, if I increase my frequency by factor of 2, I get an improvement in transmission loss again by 6 decibels. So my transmission loss curve will

still look something like this. Now the question is will this line just keep on going forever or after some point of time some other things may start happening.

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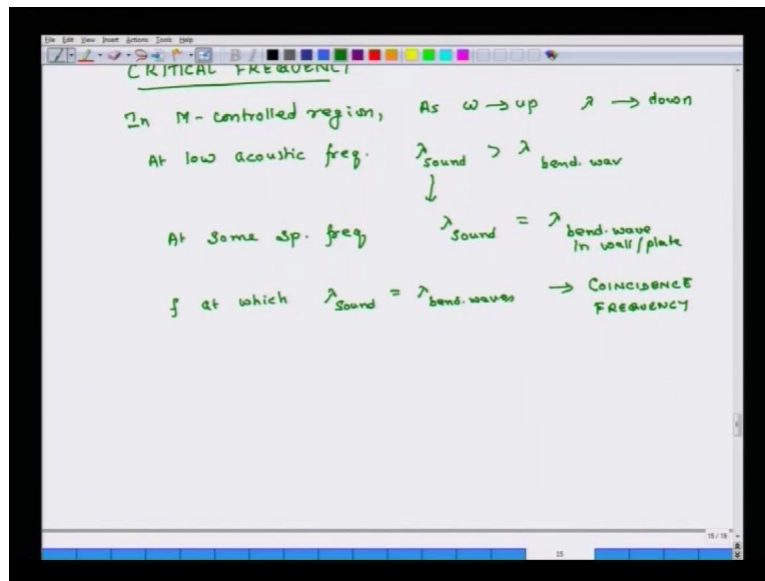


So in that context I wanted to talk to you about a term called critical frequency. So what happens is that in the mass controlled region as my frequency is going up, so in M controlled region as omega goes up wavelength of the sound it goes down. Wavelength starts shrinking. Also these sounds they try to excite some bending waves in the plate or the wall which we are talking about. So, if I have a plate and it is being hit by sound so there may be some bending waves in the plate.

Now these bending waves also have a specific frequencies and wavelengths. And typically at lower frequencies if I hit it let us say at 10 hertz the wavelength of the sound, so at low frequencies lambda of sound is larger than lambda of wave. But as I keep on increasing my frequency this starts coming down and at some specific frequency lambda of sound equals lambda of bending wave.

So this should be bending wave, so these two starts equalling. And when that happens, that particular is called coincidence or critical frequency. So frequency at which lambda of sound equals lambda of bending waves is called coincidence frequency.

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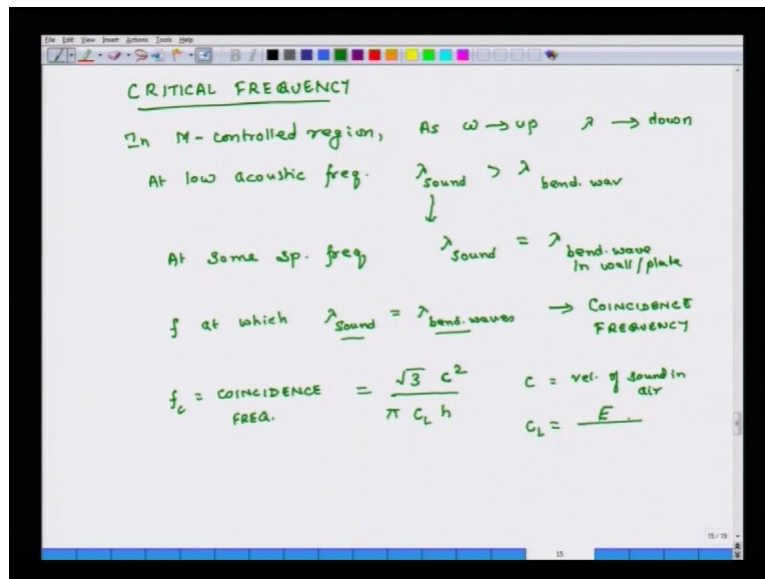
And an interesting thing about this is that mathematics tells us and it is also seen through numerous experiments that at coincidence frequency that is if I am hitting a wall with a particular frequency whose wavelength is identical to that of the bending wave in the wall or the plate then these two waves start reinforcing each other and as a consequence the transmission loss at that particular frequency at coincidence frequency it suddenly drops.

Now if there was absolutely no damping then there would not be any transmission loss in the system. But because there is damping in the system so at this coincidence frequency the transmission loss drops to a certain extent and the extent of that drop is once again governed by damping in the system. And then again after I move beyond the coincidence frequency I start again having more transmission loss.

So what I am trying to emphasize is that at the frequency of coincidence or coincidence frequency the incident sound wave starts reinforcing the bending waves in the plate or the wall which are elastic waves and this causes a significant drop in transmission loss at those specific frequencies. So now what I am going to do is I will just give a relation for this critical frequency.

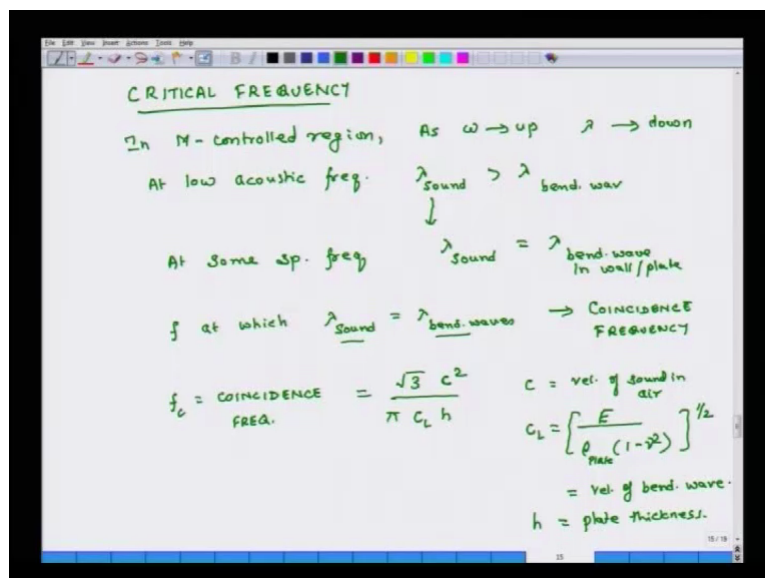
So f_c is equal to coincidence frequency and this is for a plate and there is a mathematical relation for this and this is. So remember this is lowercase c , c is velocity of sound in air. So it is square root of 3 times c square divided by pie times $C L$ constant times h so $C L$ is again a constant and it is the velocity of bending wave.

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So we have two velocities, one is the velocity of sound in air and the other one is velocity of bending wave in the solid or the plate. So C_L is velocity of bending wave in a plate and that is equal to E which is the Young's modulus divided by rho of plate times 1 minus square of Poisson's ratio and then I take the whole thing and I take it square root. So C_L is again velocity of bending wave, okay. And finally h is equal to plate thickness.

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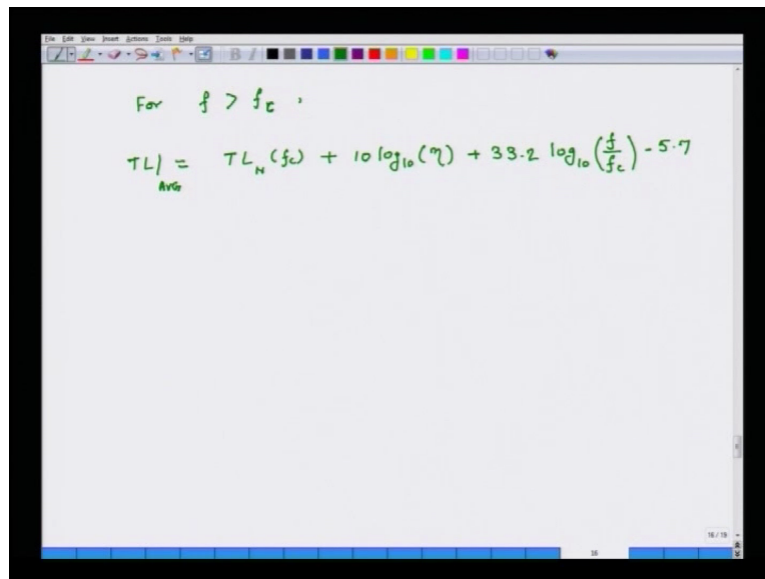


So using this relation you can figure out what will be the coincidence frequency for a plate, okay. So once I know the coincidence frequency then I can calculate the transmission loss of the wall or the plate for a region when I am even exceeding the critical frequency. So for f exceeding f_c where f_c is critical frequency and this is an empirical relation, transmission

loss for random incidence so it is going to be average and it is given by this transmission loss normal at f_c .

So this is not a multiple transmission loss normal for incident waves when it is normal to the plate at critical frequency plus 10 log of 10 of a constant eta and we will define eta later, plus 33 point 2 times log of 10 f over f c minus 5 point 7.

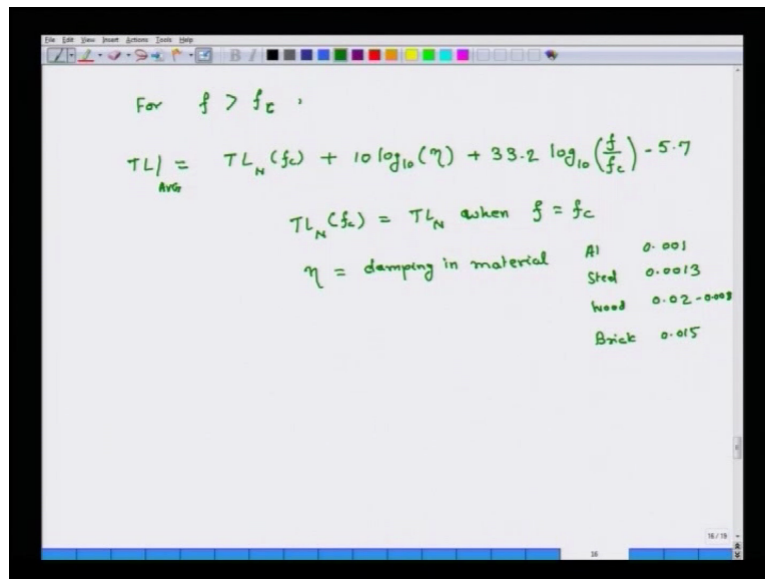
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$$\text{For } f > f_c,$$
$$|TL|_{\text{Avg}} = TL_N(f_c) + 10 \log_{10}(\eta) + 33.2 \log_{10}\left(\frac{f}{f_c}\right) - 5.7$$

So I will just explain $TL_N f_c$ is equal to TL_N when f is equal to f_c . So I will calculate this using the relation which I had shown earlier. Meta or eta equals damping in material and this is a dimensionless number so we can use values like 10 delta for this thing and I will just give you some perspective on this.

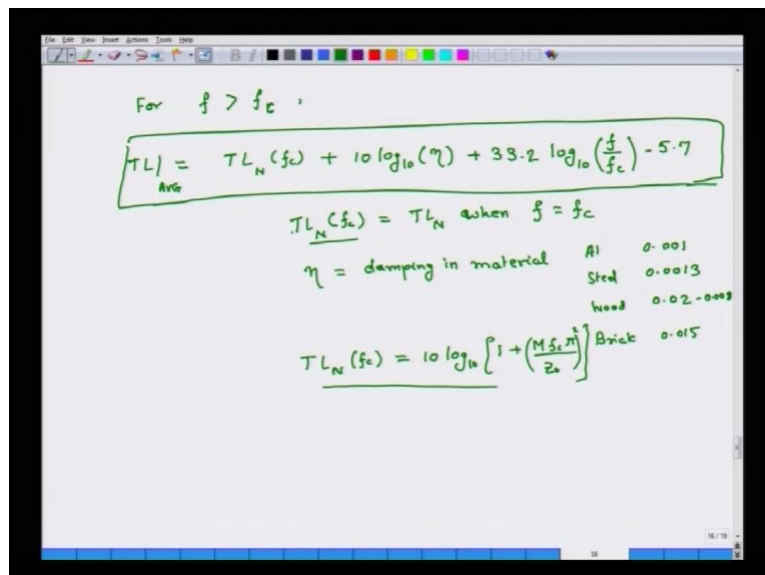
So for aluminium eta is 0 point 001, for regular carbon steel it is a little more but not a whole lot more. If you look at wood it is 0 point 02 to 0 point 008. Brick, it is 0 point 015. So a lot of these metals they have really low damping properties.

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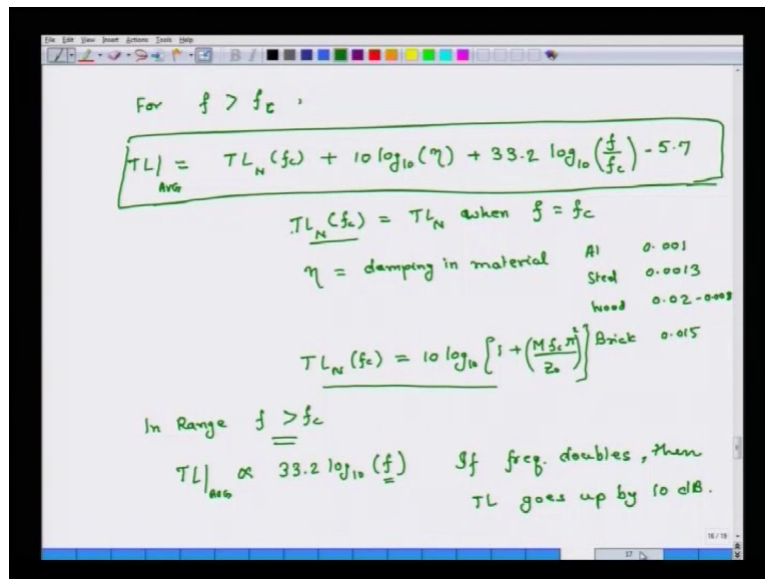
So I will also expand on this term. So TL_N at f_c equals using the relation which we have seen earlier, it is nothing but $10 \log$ of $10 \frac{1}{4} \pi^2 M f_c^2$ divided by Z_0 the whole thing square. So this is TL_N . So using this empirical relation we can figure out what happens after we have crossed the first coincidence frequency.

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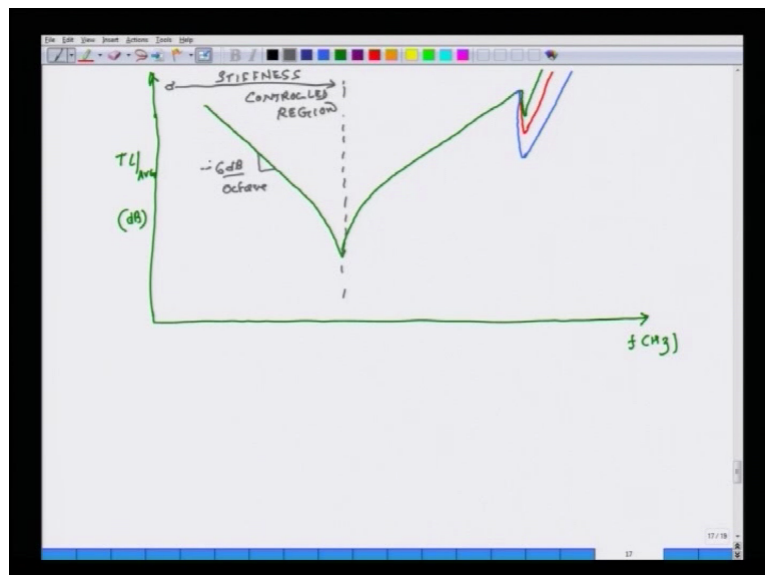
One thing it is important to note here is that if I am exceeding the coincidence frequency then in that range if I double my (frequency) this particular range then TL_{Avg} is proportional to $33.2 \log_{10}$ of f , okay. So if frequency doubles then TL goes up by 10 dB because if I have $2f$ here \log of 2 is 0.3 multiplied by 33.2, I get 10. So if my frequency doubles above and beyond coincidence frequency then I get a 10 dB extra off damping.

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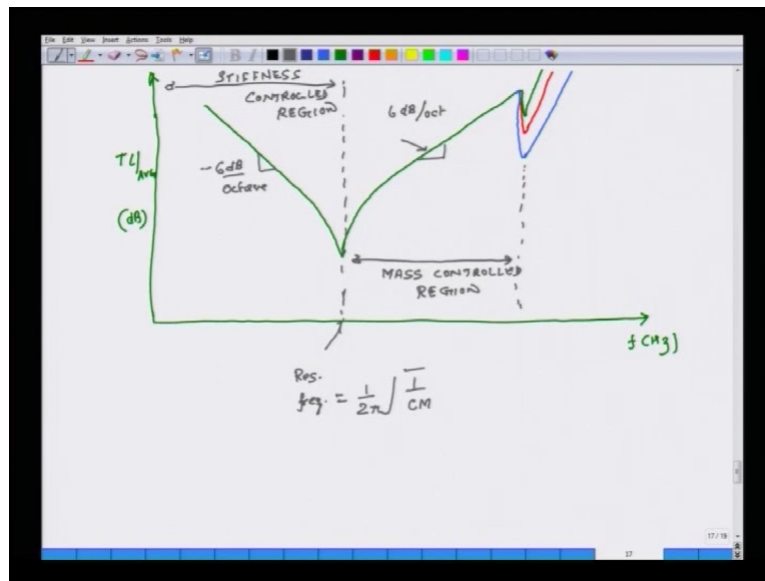
So I will now finally draw this chart once again. So I am plotting TL average and this is in decibels. On the horizontal axis I have frequency in hertz and once again there is a, so I will do a couple of curves here and I will explain what these mean. So this is my stiffness controlled region and the slope here is negative 6 decibels per octave.

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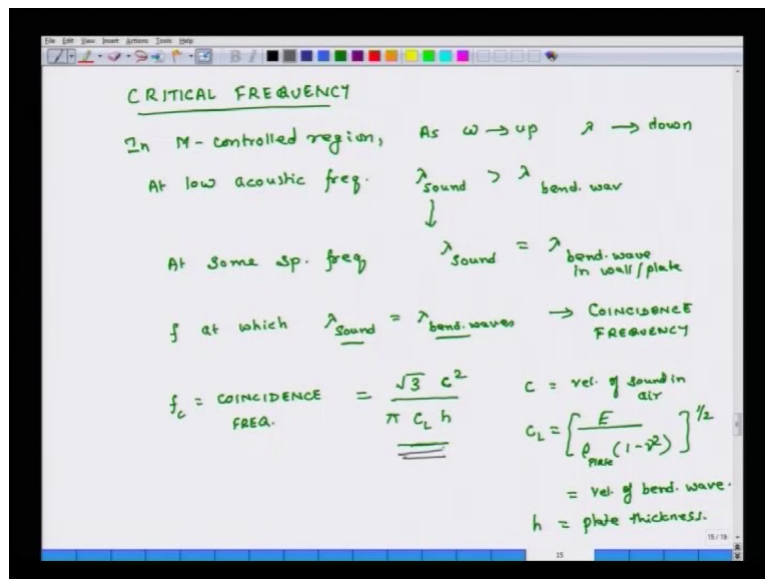
This is resonance frequency and this equals $1 \text{ over } 2 \text{ pie times square root of } 1 \text{ over } C M$ where C is the specific compliance of the system and M is specific mass of the system. Then up to this range this is mass controlled region and here this is decibels per octave. The slope is once again 6 decibels but it is a positive slope.

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Then this is my first coincidence frequency and the value of coincidence frequency has been defined here, okay.

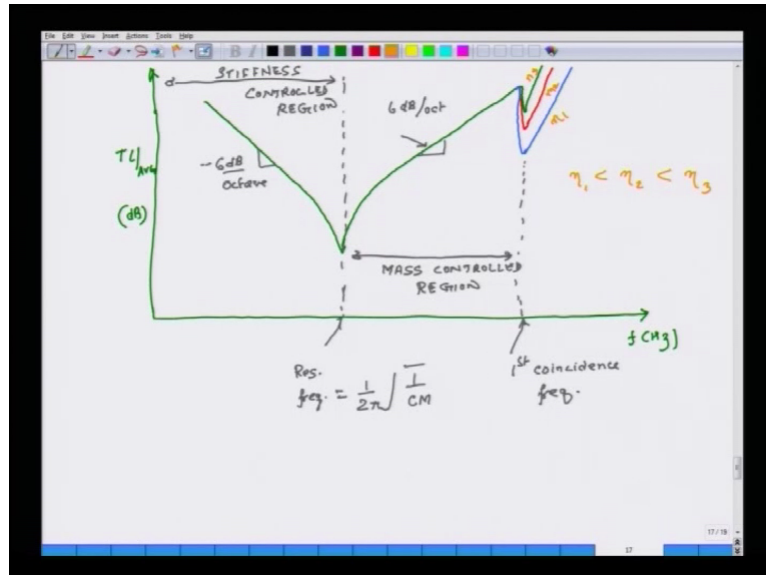
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This is my first coincidence frequency. At first coincidence frequency all of sudden transmission loss drops and the extent of that drop will depend on η which is the damping in the system. So the lower damping, the higher the drop. The more is the damping, less is drop. So these three curves above and beyond red, green and blue they are for different values of damping.

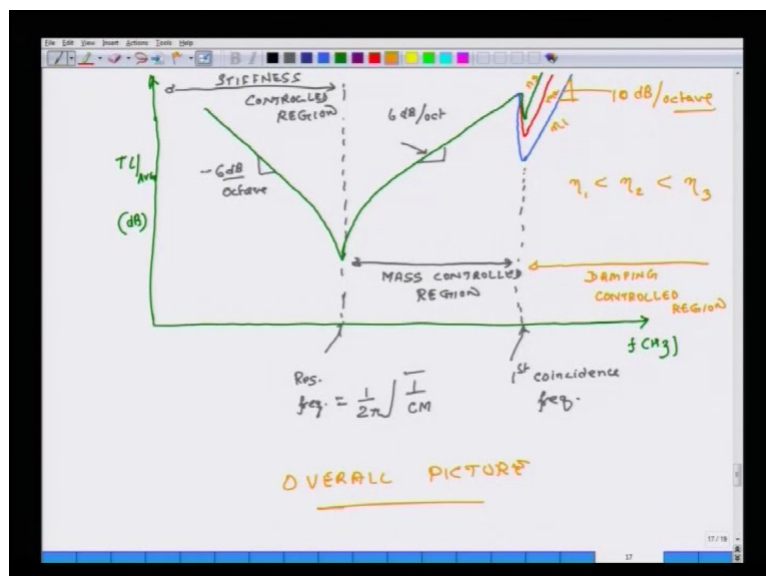
So let us say my damping here is eta 1, eta 2, eta 3. So it is in such a way eta 1 is the least. So I have maximum reduction in transmission loss. Then eta 2 is more than eta 1 and then this is eta 3. So I have lesser drops.

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And then in this region this is called damping controlled region and here as we saw that the slope of this transmission loss curve is 10 decibels per octave. So the slope these three lines, red line, green line and blue line beyond f_c they are all parallel to each other and their slope is 10 dB per octave, okay. So this is the overall picture.

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So what I have explained in last two lectures will hopefully enable you to use this information that suppose tomorrow you have to work on a design where you have to develop wall and you expect certain amount of transmission loss across it because you want to kill sound to a certain extent then the material which has been covered in last two lectures will hopefully give you sufficient information so that you can start developing these kind of structure and you can analyse these walls and panels.

Thank you very much for your patience and we will meet you once again in our next lecture.
Thank you.