Acoustics
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Lecture01
Module3
SoundTransmissionthroughWalls

Helloagain.Intoday'slecturewewillbeusingsomeoftheconceptswhichwehavedevelopedinpreviouslecturesspecificallythetransmissionlinetheoryandusethisapproachtofigureoutwhathappenswhensoundasitpropagatesthroughmediumithitsawall,howdoessoundgetstransmittedacrossawall? Andtosolvethisproblemwewillusethetransmissionlinetheory.Sowhatwewillanalyseisfirsttransmissionofsoundthroughawallwhentheincidentangleofthesoundasithitsthewallis90degrees.

Thatisthefirstproblemwewillsolveandthensubsequentlywhathappensifitishittingatallsortsofrandomanglesandthenusingthisapproachwewillfigureouthowsoundgetstransmittedacrossawall.So thatiswhatwearegoingtodotoday.SoIwillmakeaverysimplepicture.LetussayIhaveawallandthesoundishittingthewallatanincidentanglewhichis90degreessoitisnormalincidence.

Sothisismyincidentwaveandthatismyreflectedwave.Sothecomplexmagnitudeofincidentwaveis Pplus,ofreflectedwaveisPminusandthenthisisthewallinquestion.AndapartofthissounditgetstransmittedtotheothersideandletussaythecomplexamplitudeofthiswaveisPT.

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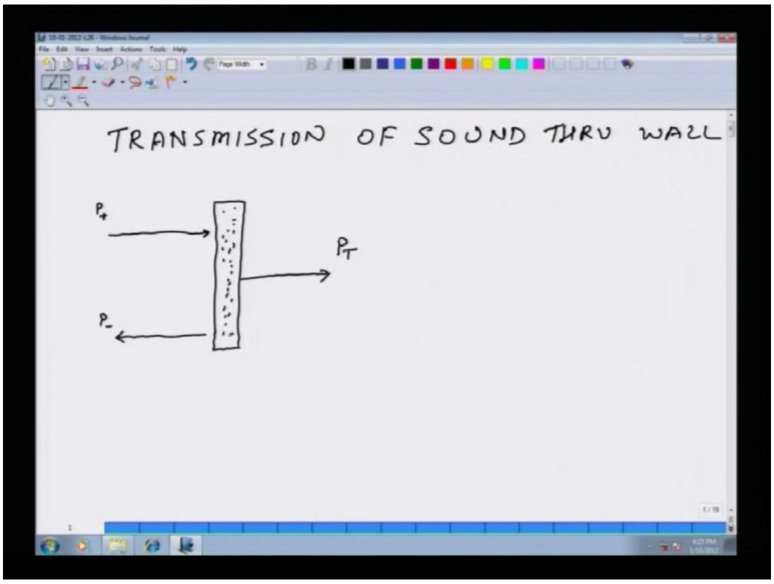
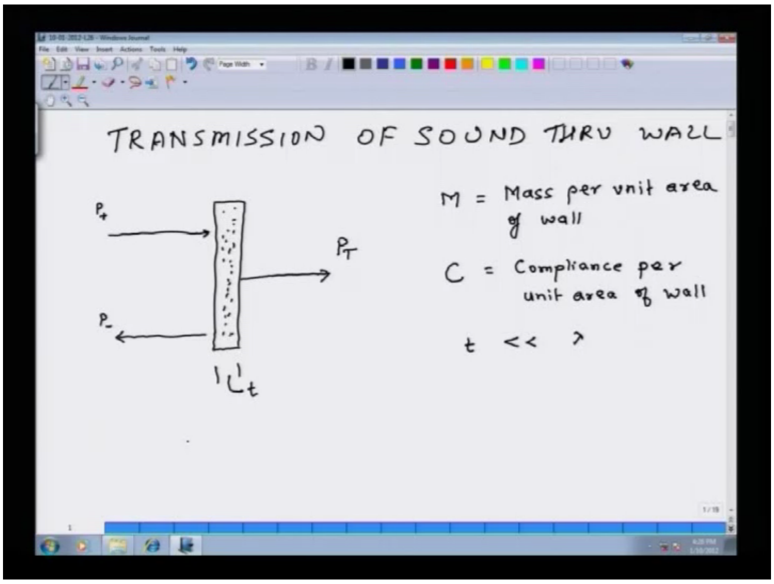


LetusalsoassumethatMisequaltomassperunitareaofwall.So,1squaremetreofthisareagoingtobeM kilograms.Andthiswallcouldalsohavesomecompliancebecausewhensomethinghitsawallthesew allsmaybend.Theyarenotperfectlyrigidsotheymayhavesomecompliance.AndCisthecompliance .Butagainitisperunitareaandthisthicknessistandweassumethatthethicknessofthewallisfairlysma llcomparedtothewavelengthorwavelengthofthesoundwhichishittingthewall.Sotisextremelysma llcomparedtolambda.

(ReferSlideTime:03:55)



Sothisistheproblemformulationandwhatweareinterestedinknowingis,soquestion,ifthequestioni sthatifIknowPplusthenwhatisthevalueofPT?
BecausePplusistheincidentenergygoingintothesystemandPTissomethingwhichiscomingoutbey

ondthewall.SoIwanttofigureoutthevalueofthistransferfunctionPToverPplusandthisiswhatweare interestedinfindingout,okay.

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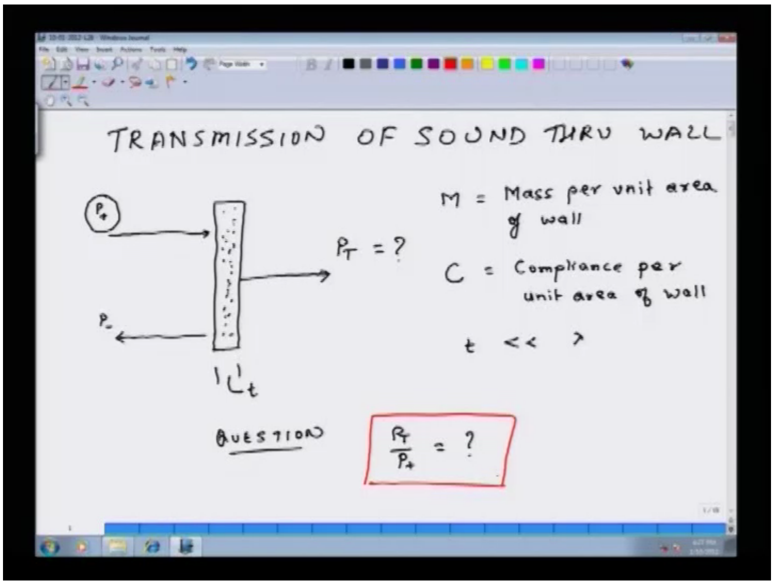
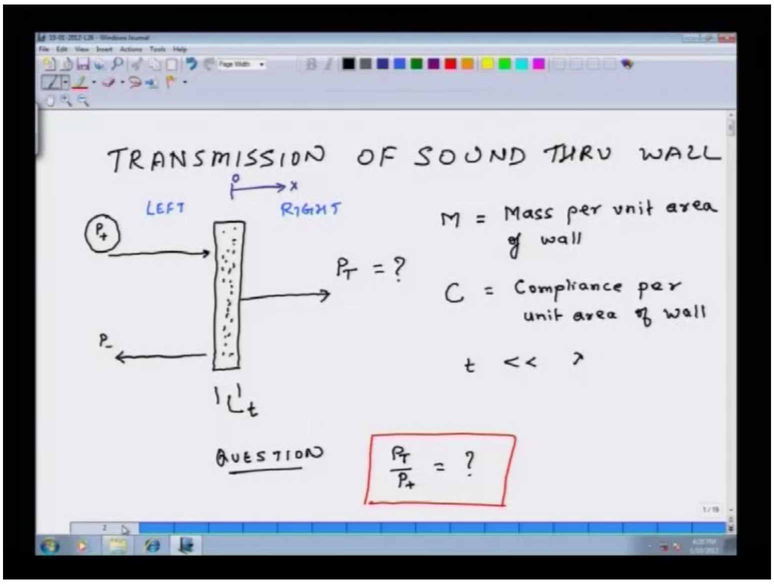


Sooncagainwewillstartwiththetransmissionlineequations.Soforthetransmissionlineequations wewillwritetwosetsoftransmissionlineequations.Sothisistheleftsideofthewallandthisistherights ideofthewall.SoIwillhaveonesetoftransmissionlineequationsfortheleftside,anothersetoftransmi ssionlineequationsfortherightside.AlsowhatIamgoingtodoisIamgoingtoestablishmycoordinate system.Sothisis0.Sowallstandsatthelocationxequals0.SowiththisnowIamgoingtowritemytrans missionlineequations.

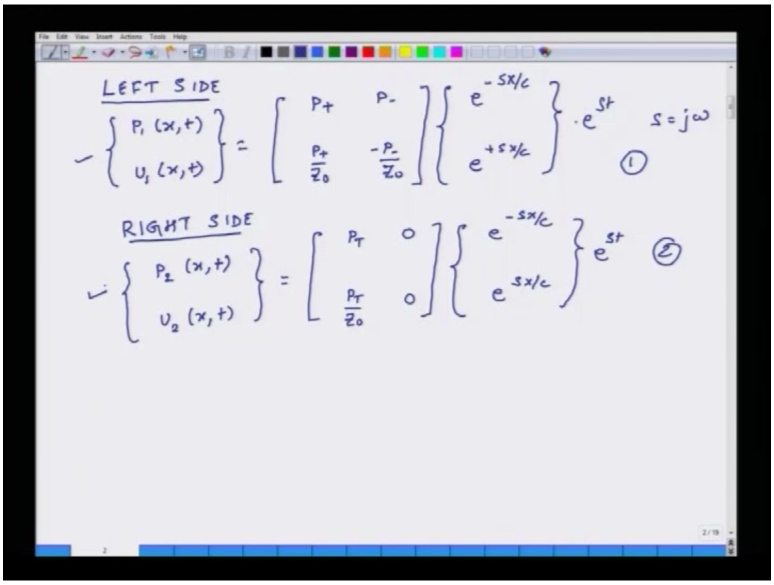
(ReferSlideTime:05:17)



Soforleftside,pressureandvelocityandtheyarefunctionsofspaceandtimeandthisisequaltoPplusP minusPplusoverZ0minusPminusoverZ0.Sothismatrixisgoingtobemultipliedbythisvectoreminu ssxoverC,eplussxoverCtimesestandweknowthatsequalsjtimesomega.Sothisismyfirstquestion. AndthenIhavelikewiseequationsfortherightside.

SoontherightsideletussayIcallpressureasP2andvelocityasU2.SoP2nowwhenweseethisproblemt hereisnoreflectedwavesbecauseIamassumingthatthereisnoreflectingsurfaceonrightsideofthewa ll.Sothereisnoreflectedwaveontherightsideofthewall.SomyPpluswillbesameasPT.Andsincether eisnoreflectedcomponentthiselementofthematrixis0soagainIhavea0.Thisismysecondequation. SoIhavenowtwosetsofequations,onefortheleftsideotherfortherightside.

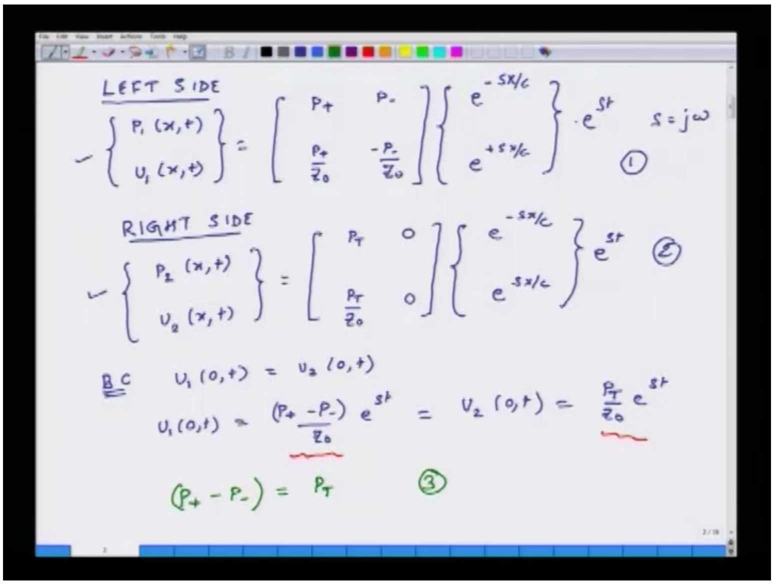
(ReferSlideTime:07:35)



NowatthispointoftimeIstartimposingtheboundaryconditions,okay.Somyfirstboundarycondition isthatwhateveristhevelocityonthissideatxequals0thatvelocityissameonjusttherightsideofthewal lagainatlocationxisequalto0.SoboundaryconditionisthatU1at0tequalsU2at0tandwhatthatgives meifIsubstituteinequations1and2therelationsforvelocityandIputxequals0.

SofromfirstsetofequationsIgetU10tequalsPplusminusPminusoverZ0timesestandthisisequaltoU 2of0tandthatisequaltoPToverZ0etothepowerofstbecausesisidentically0.SothesetermsesxoverC andeminussxoverCtheyvanishandtheybecome1.SonowwhatIdoisIequatethesetwotermsandwha tIgetfinallyisPplusminusPminusisequaltoPT.Sothisisletuscallaconsequenceofthisboundarycon ditionthatatxisequalto0thevelocityonrightsideofthewallissameasvelocityontheleftsideofthewall .

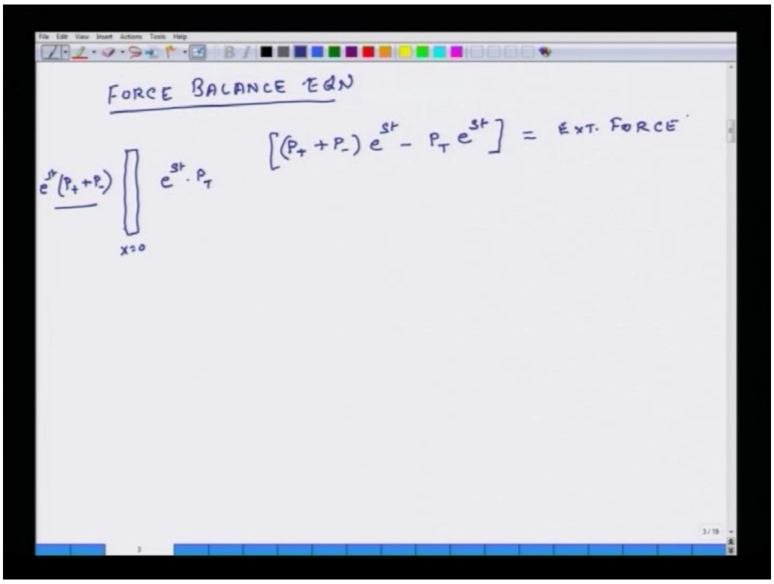
(ReferSlideTime:09:30)



Theotherconditionatthisxequals0locationisthatthewallisexperiencingaforce.Itisexperiencingaforceandthisexternalforceisessentiallythedifferenceofpressuresonleftsideandrightside.Sothisistheoverallexternalforcewhichthewallisexperiencingandthisforcewillbeequaltotheaccelerationofthewalltimesmassplusstiffnessofthesystemtimesthedisplacement.SonowIamgoingtodeveloparelationforforcebalance.

SothisismywallandontheleftsidemytotalpressureisPplusplusPminustimesestofcourseatxequalto0andhowIgetthis?
EssentiallywhatIamdoingisIaminequation1puttingxisequalto0andIamfindingthevalueofpressure.SoontheleftsidethepressureisthisandthenontherightsidepressureisetothepowerofsttimesPT.SomyforcebalanceequationisPplusplusPminusetothepowerofst,thisistheforceonleftsideminusPTetothepowerofst.Sothisistheoverallexternalforce.

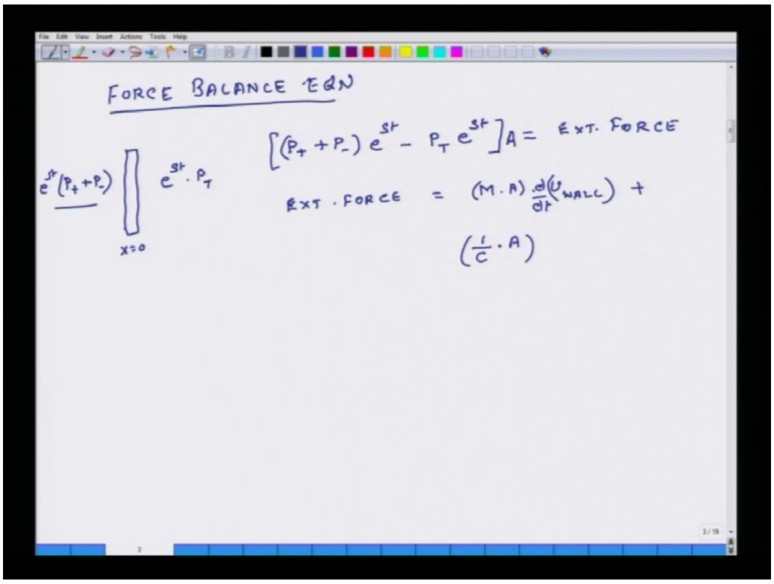
(ReferSlideTime:11:20)



Andthenthisexternalforceweknowifwedrawafreebodydiagramofthissystemisequaltomasstimes acceleration.Sowhatisthemassoftheoverallmass? SoexcusemethisisadifferenceofpressuresandIhavetomultiplythisbytheareaofthewallwhichisA. Sothisistheexternalforceandthisisequaltothemasstimesaccelerationofthewallplusstiffnesstimes displacementofthewall.

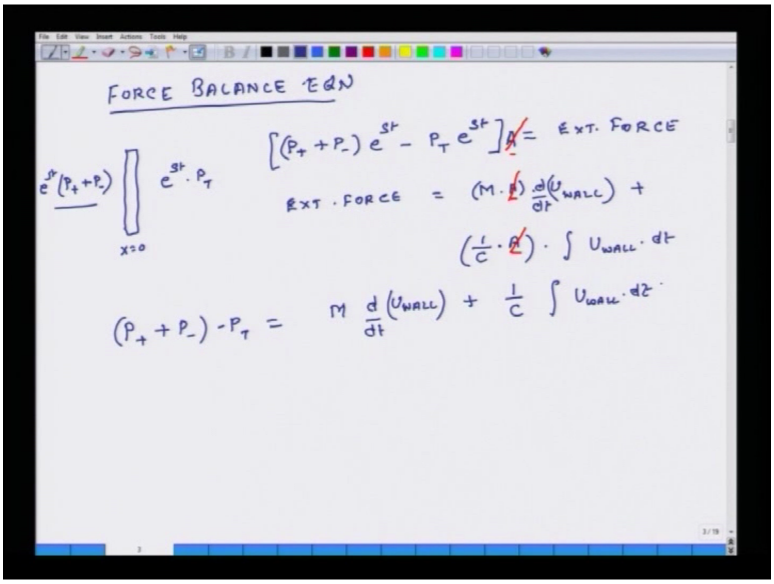
SomassisMtimesarea.ThisismasstimesaccelerationandifIknowthevelocityofthewallwhichisletu ssayUwallthenifItakeadifferentialofthisthenthatistheaccelerationofthewall.Sothisismasstimesa ccelerationplusstiffness.Andstiffnessofthewallisessentially1overcompliancetimesarea.SoCissp ecificcomplianceandthenIamgoingtomultiplythisbyareabecauseitiscomplianceperunitarea.

(ReferSlideTime:12:51)



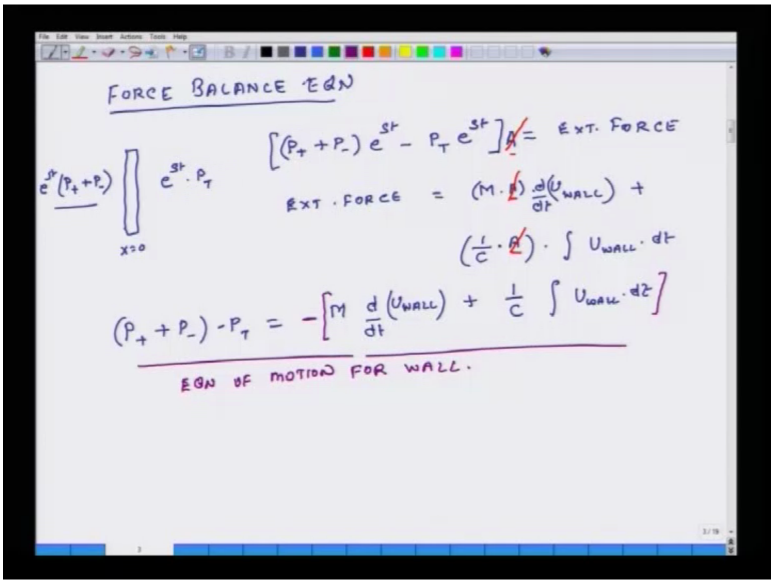
AndthistimesagainifIknowthevelocityofthewallandifIintegratethatvelocitythenthatiswhatIgeta sdisplacement.SoIamgoingtointegratedoverdtUwall.andIseethatareaiscommononleftsideandal sorightsidesoIcaneliminateit.SomyoverallequationbecomessoIhavetomakeacorrectionhere.Thi sshouldhavebeenintegralofUwallwithrespecttotime.SoIwillmakethatcorrectionhereaswell.

(ReferSlideTime:14:15)



NowinthisequationIamalsogoingtoputanegativesignandthereasonIamgoingtoputthisnegativesi gnisbecauseifIhaveapositivepressurethenthepressureisactuallymovinginthenegativedirection.It alwaysactsinverse.Apositivepressureactsinverse.SotoaccountforthatrealityIhavetoputanegativ esignhere.Sothisismyequationofmotion,okay.

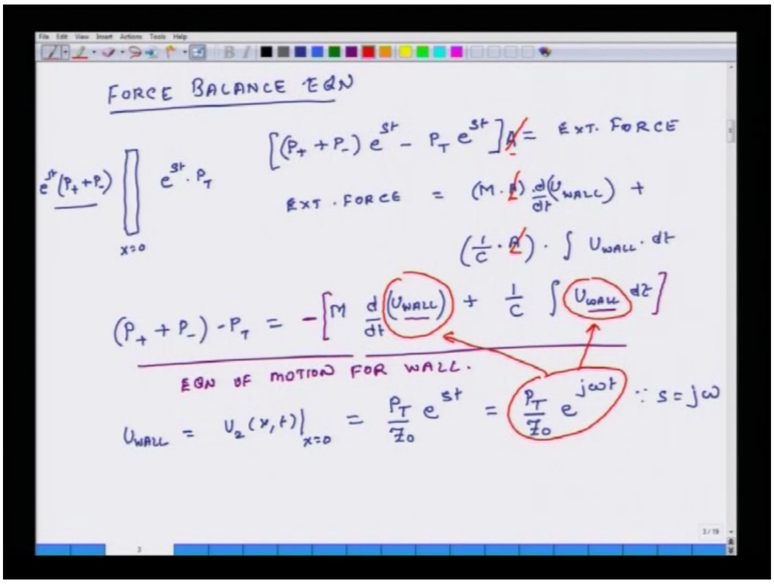
(ReferSlideTime:15:06)



NowwhatwearegoingtodoisontherightsideofthisequationIhaveUwall.SoIwillfindarelationshipforUwallplugintothisandthendosomemoremathematicalmanipulation.SoweknowthatUwalliseqaultoU2atxisequalto0.AndifIseethisrelationforU2thenIknowthatU2isnothingbutU2equalsPToverZ0timesetothepowerofminussxoverCtimesest.

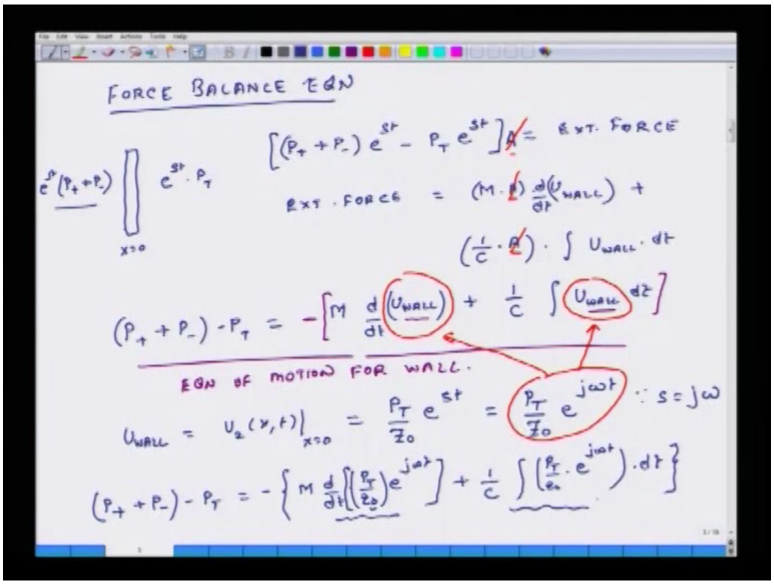
AndifIputxequals0thenIgetU2as,andbecausesisjomegawhatIgetisthisrelationbecausesequalsjomega.SowhatIamgoingtodoisreplacethistermandthistermbythistermbecausethatisthevalueofUwall.

(ReferSlideTime:16:41)



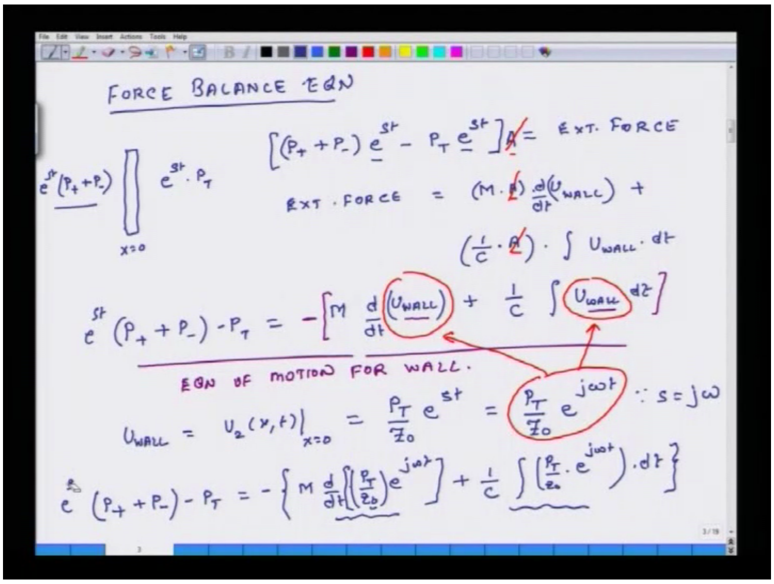
SowhatIgetfinallyisplusthecomplianceterm.SointhisrelationPTandZ0theyareconstants.PTcanbeacomplexentitybutitisstillaconstant.SothenIcanveryeasilydifferentiatethistermandIcanveryeasilyintegratethisterm.AndthatiswhatIdointhenextstep.

(ReferSlideTime:17:47)



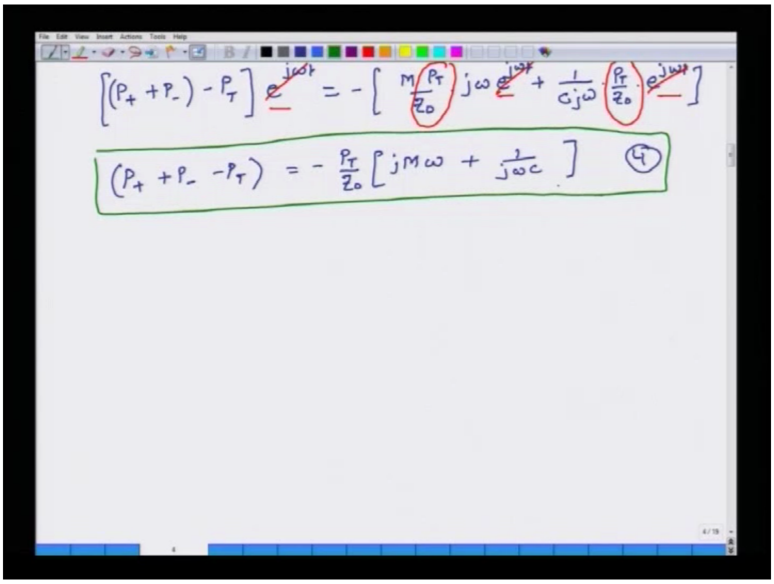
SoafterintegratinganddifferentiatingthesetermswhatIultimatelyendupgettingis.SoIomittedonethingherethatIhadanesthere.SoIshouldhaveesthereandIwillreplacethisestinthisrelationbyejomega.

(ReferSlideTime:18:17)



SomyleftsideisPplusplusPminusminusPTejomegatandontherightsideIgetMPToverZ0timesjomegaejomegatplus1overCjomegatimesPToverZ0timesejomegat.AgainIhaveejomegatcommonandsoIcaneliminatethesetermsandthusmyequationsimplifiesto,thistermandthistermissamesoItakeitoutsidethebrackets.SoIcallthisequation4.

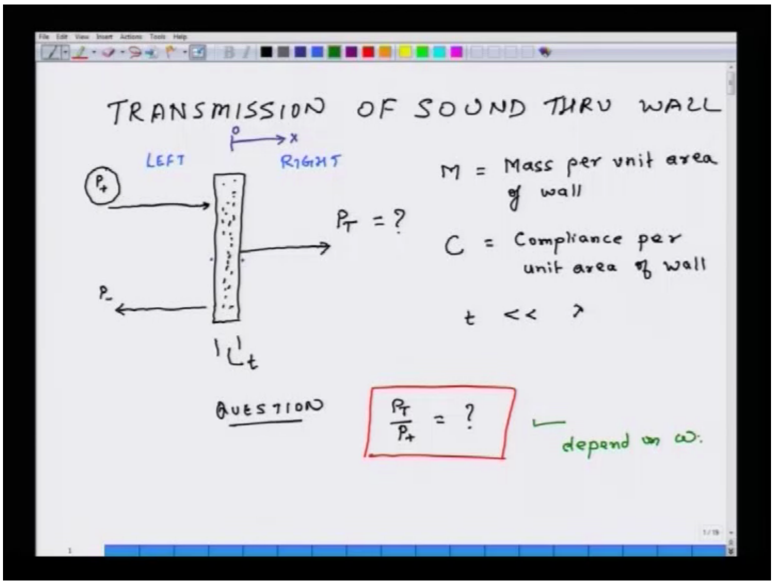
(ReferSlideTime:19:50)



AndwhatthisequationtellsusistheconditionsforforcebalancethatbecausetheforceshavetobalanceoutsoontheleftsideofthetermyouhavetheextraforcePplusplusPminustimesminusPT.Thisissummationofdifferentpressureelementsandontherightsideyouhaveinertialelementandstiffnesselementintheequationandthisequationisequation4.IthasbeendirectlyderivedfromNewton'ssecondlawofmotion.SonowIseequestion4andthenIgobackandIseeequation3.

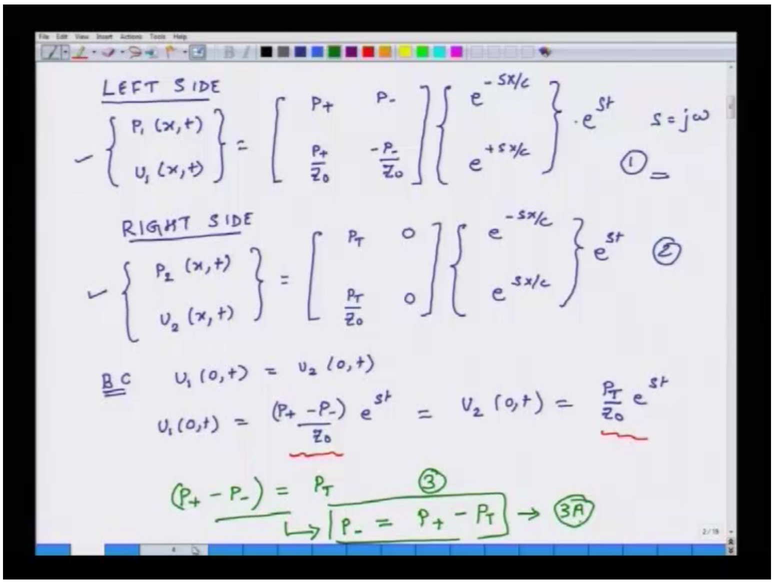
Sobetweenequation3andequation4Ihavethreeunknowns,oneisPplus,theotheroneisPminusandthethirdoneisPT.Theseareconstantsbuttheyareunknowns.IknowwhatisM,IknowwhatisomegaandIknowwhatisCwhichisthespecificcompliance.Sothenusingthesetwoequations,thisequationandthisequation,Iwilliketosolve.Igetmytransferfunctionwhichisthis.Andpleasebearinmindthatthistransferfunctionwilldependonomega.

(ReferSlideTime:21:16)



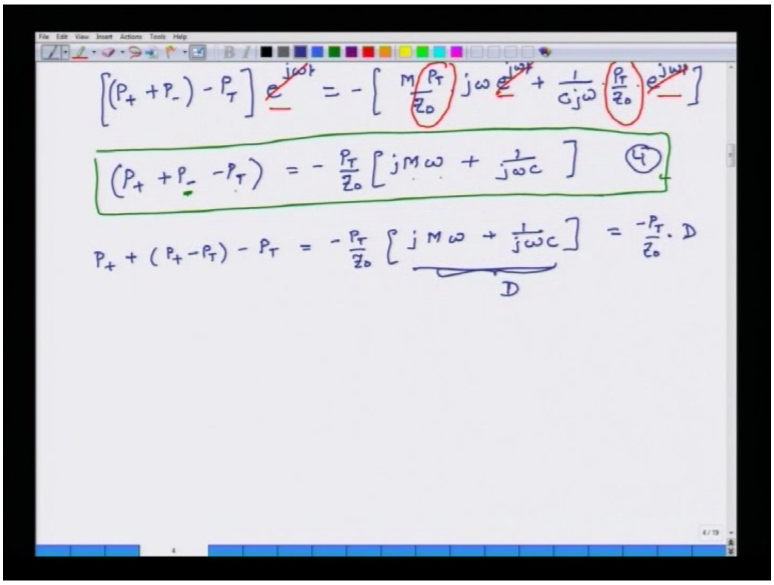
SowiththatintentionwhatIdoisIreframethisequationinsuchawaythatIcanrewritethisequationasPminusequals,soItakePminusontherightsidesameasPplusminusPTandIcallthisequation3A.

(ReferSlideTime:21:46)



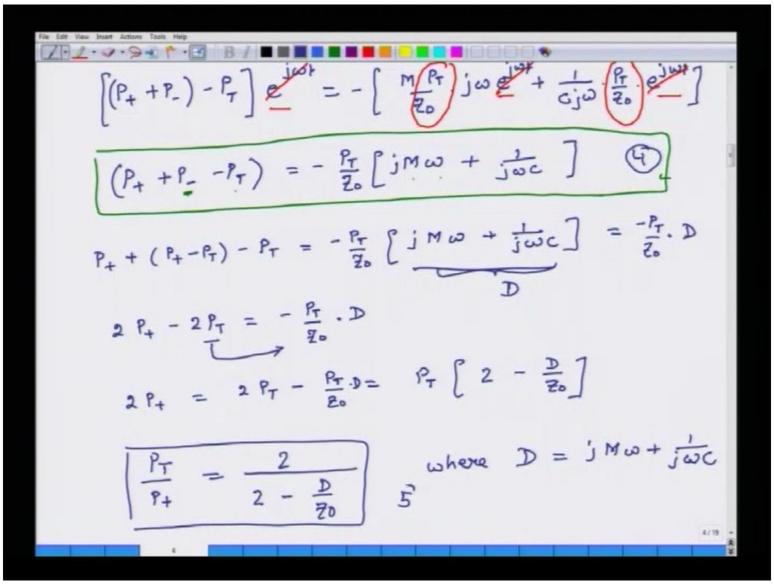
NowIputthisequation3Ainequation4.SoIreplaceherePminusbyPplusminusPTandletusseewhat weget.So,PplusplusPplusminusPT.ThisiswhatPminusequalsminusPTequalsminusPToverZ0jM omegaplus1overjomegaC.AndletuscallthisfunctionForactuallyletuscallthisfunctioncapitalD.S owiththiswhatwegetisthisisequaltominusPToverZ0timesDwhereDisthiswholeterminbrackets,o kay.

(ReferSlideTime:22:51)



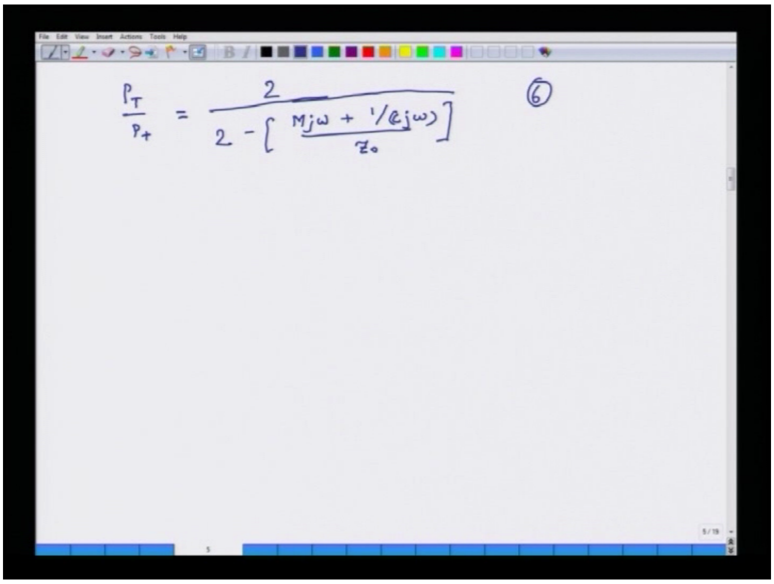
SonowIsimplifythissoIgetontheleftside2Pplusminus2PTisequaltominusPToverZ0timesDorifI movethisonthissideIget2Pplusequals2PTminusPToverZ0andthatissameasPT2minus,soImissed theDhere,DoverZ0.SowiththisIcanwritePToverPplusisnothingbut2over2minusDoverZ0.Thisis myrelationforthisquestionwhereDequalsjMomegaplus1overjomegaCwhereCisspecificcompliance,okay.Soletuscallthisequation5.

(ReferSlideTime:24:13)



SonowIamgoingtopanthisfurther.SoIgetPToverPplusequal2minus,andIamgoingtoreplaceDbythiswholerelation,sowhatIamgoingtogetis,sothisisequation6.Sothisismytransferfunctionandthistransferfunctionitdependsonomega.Itsvaluechangeswithomegaortheangleoffrequencyofthesoundwavewhichisstrikingthewall.

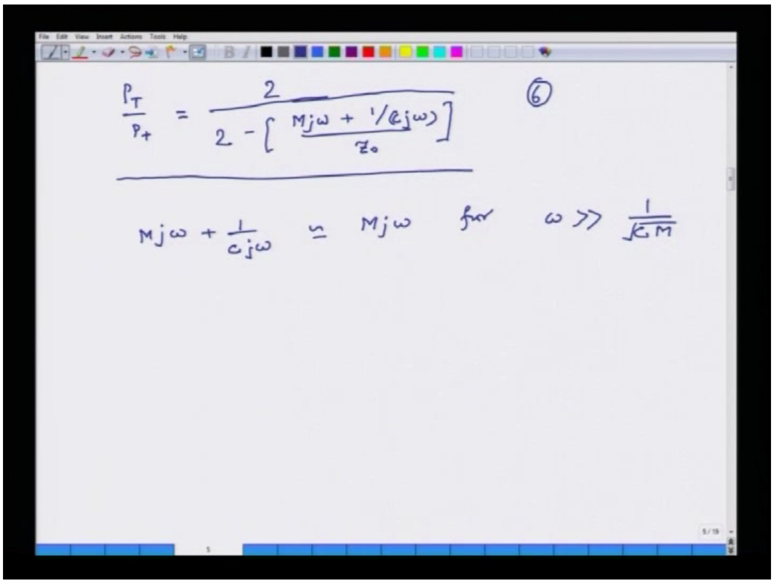
(ReferSlideTime:24:59)



Sousingrelation6IcancalculatewhatwillbethevalueofPTifanincidentwaveofstrengthPplusishittingit?

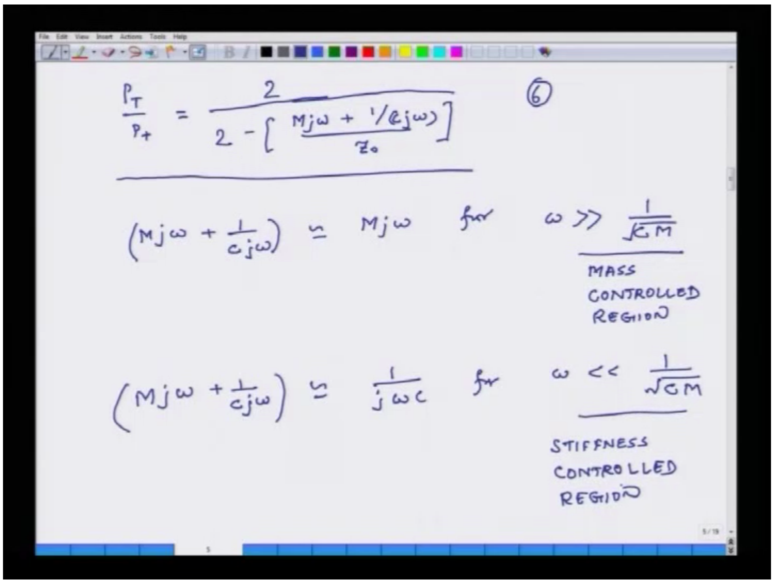
SonowwhatIamgoingtodoisnowIamgoingtodosomefurtherprocessingonthisrelationandbasedonwhatfrequencieswearetalkingabout.SowhatweseefromthisrelationisthatthistermMjomegaplus1overCjomega.SothisIcanapproximateasMjomegaforallomegaorjusttomakeitsimpler,foromegawhichislargecomparedto1overCM.

(ReferSlideTime:26:04)



SoifomegaisextremelylargeandbylargeImeanifitisextremelylargecomparedtothenaturalfrequenciesofthesystemthenthistermapproximatestoMjomega.Thisrangeiscalledmasscontrolledregion. SimilarlyMjomegaplus1overCjomega,itapproximatesto1overjomegaCforomegawhichisextremelysmallcomparedtonaturalfrequencyofthesystemthatis1oversquarerootofCM.Andthisrangeoffrequenciesiscalledstiffnesscontrolledregion.

(ReferSlideTime:27:24)

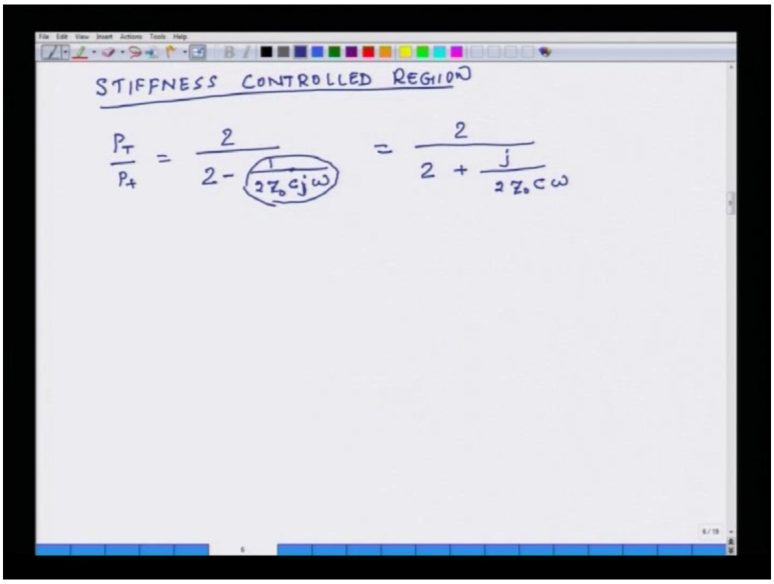


Sowewilltrytounderstandhowthisfunctionworks? Thisfunctionworks,thisisequation6,instiffnesscontrolledregionandalsoinmasscontrolledregion. Nowonceagaintorecap,iffrequencyisextremelysmallcomparedtothenaturalfrequencyofthesystemwhichis1oversquarerootofCMthenthatrangeoffrequencieswhereitisextremelysmalltothenaturalfrequencyitiscalledstiffnesscontrolledregion.

Ifthefrequencyweare(ta)consideringisextremelylargecomparedtonaturalfrequencyofthesystemthenthatrangeoffrequenciesissupposedtolieinmasscontrolledregionbecausethemasstermdominatestheresponseofthesystem.Sowewilllookatstiffnesscontrolled.SoagainIwillwritedowntherelationPToverPplusequals2over2minus.AndweknowthatinthestiffnesscontrolledregionIcanomitMjomegaandIhavetojustinclude1overjomegaC.

SowhatIgetis1over2Z0Cjomega.SoIcansimplifythis.SothereisjinthedenominatorofthistermsoIcansimplifythisbymovingjupwardsbutalsoeliminatingthenegativesign.WhatIamdoingisthatinthistermIammultiplyingthisandalsodividingthistermbyj.SowhatIamgettingis2Z0Comega.

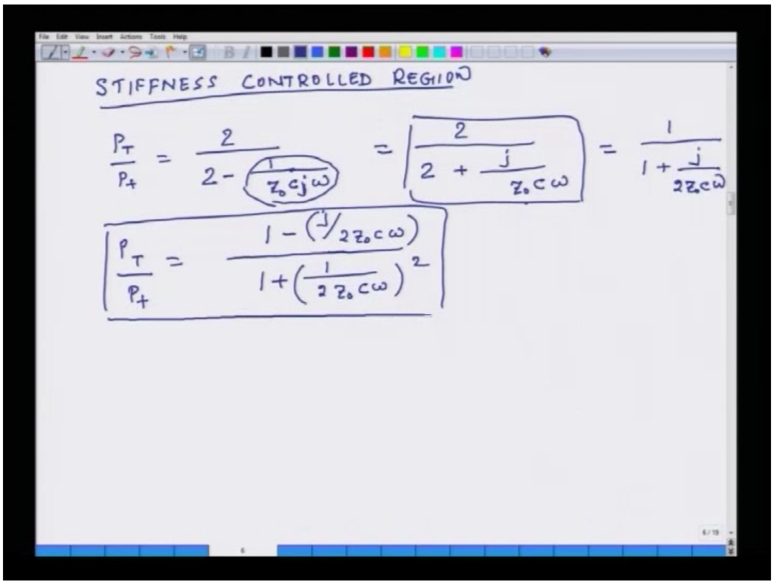
(ReferSlideTime:29:34)



AndnowIamfurthersimplifyingthisandrationalisingthisbytakingthejinthenumerator.SowhatIam goingtodoisIamgoingtomultiplythiswholefunctionandalsodividethiswholefunctionbythecompl exconjugateofthedenominator.SoImultiplynumeratorby2minusjover2Z0ComegaandIdividethe numeratorbythesameterm.

SowhatIgetis,sothis2shouldnotbethere,andherewhatIwilldoisIwillagaindividethenumeratorand thedenominatorby2.SoIgetthisplusjover2Z0Comega.AndnowIamgoingtorationalizeit.SowhatI endupgettingisthisrelation.

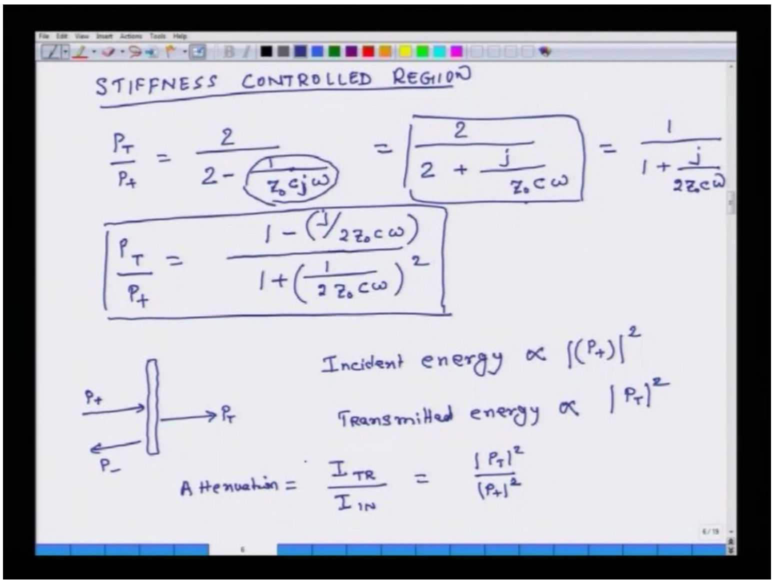
(ReferSlideTime:31:00)



NowweknowthatifIhaveawall,ifthisismyincidentwave,thisisreflectedwaveandthisistransmitted wavethentheincidentenergyisdirectlyproportionaltomagnitudeofPpluswholesquaredandtransmittedenergyisdirectlyproportionaltomagnitudeofP2.Sotheattenuationthatisthelossofsoundassoundtravelsacrossthewallisequaltointensityofthetransmittedwavedividedbyintensityofincidentwaveandthatisnothingbutthisratio.

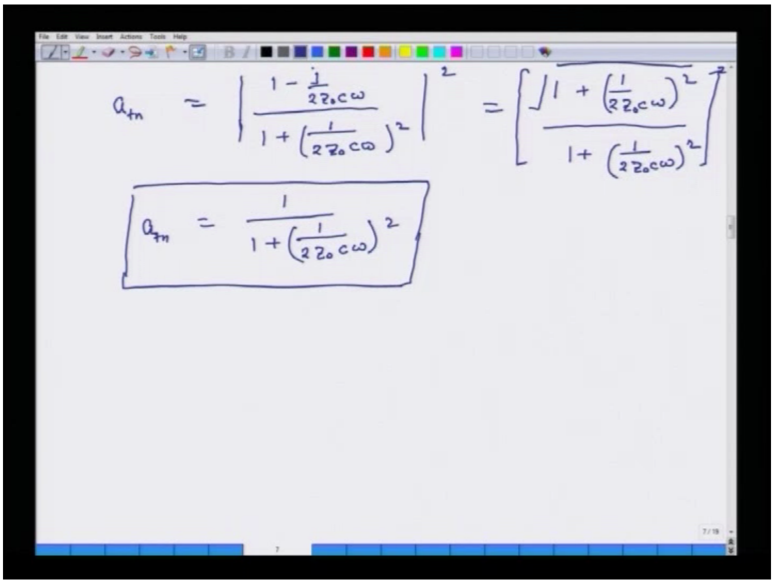
SothisismyattenuationandIcanalsocallthisasthis.SonowIhavethisequationletuscallthisequation A.FromequationAIamgoingtofindwhatisthelevelofattenuation?

(ReferSlideTime:32:27)



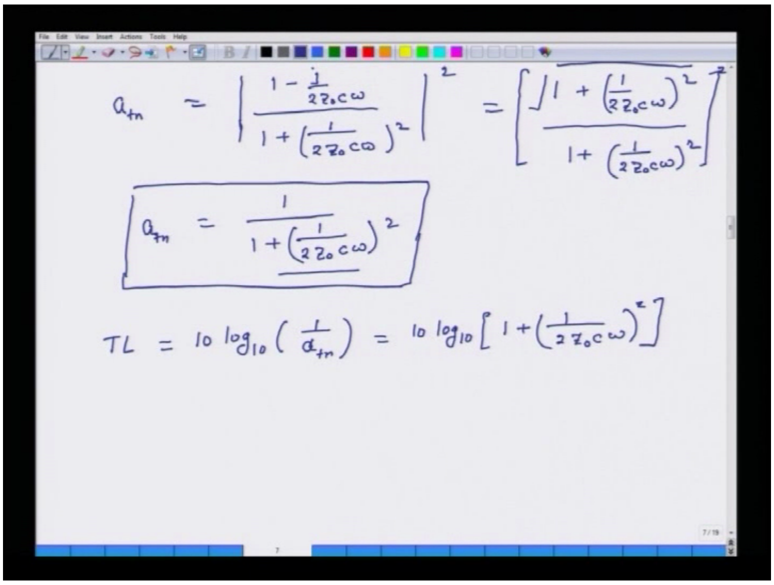
AttenuationIamwritingitasatn,1minusjover2Z0Comegadividedby1plus1over2Z0Comega,thewholethingsquared.AndwhatIdoisItakethemagnitudeofnumeratorwhichis1plus1over2Z0Comega.Thisisthemagnitudeofnumerator.MagnitudeofdenominatorissameasthedenominatorbecauseitisarealnumberandthenIsquarethewholething.Somyattenuationcomesouttobe1over1plus1over2Z0Comegawholesquare.SonowthatIhavedevelopedarelationforattenuationofsoundasitgetstransmittedacrossawall.

(ReferSlideTime:34:20)



NowwhatIwilltrytofindisthetransmissionlossindecibels.SoIcallitTLandthatisdefinedas10logof 10over1overattenuation.Andthatisessentially10log10ofthiswholething.Andbecauseitis1overatt enuationsothis(deno)denominatorcomesinthenumerator.SoIget1over.

(ReferSlideTime:34:57)

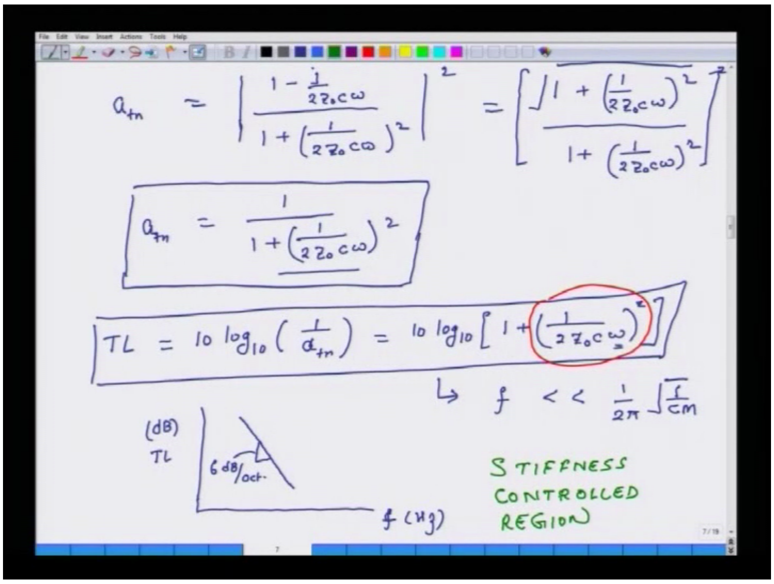


Andpleasebearinmindthatthisrelationisgoodifthefrequencyoftheincidentsoundisextremelysmal lcomparedto1over2pieCM,extremelysmallcomparedtothenaturalfrequencyofthesystem.Theoth erthingweseefromthisisthatifIambelowthenaturalfrequencyofthesystemthenasIkeeponreducing myomega,mytransmissionlosskeepsonincreasing.

SoasIgetcloserandcloserto0hertzIgetanimprovedtransmissionloss.SoifIplotinonalogscalesothisisfrequencyandthisisindecibelsIamplottingtransmissionloss.AndfrequencyIamplottinginhertz. Andbecausethisisalogscalemybodeplotwilllooksomethinglikethis.

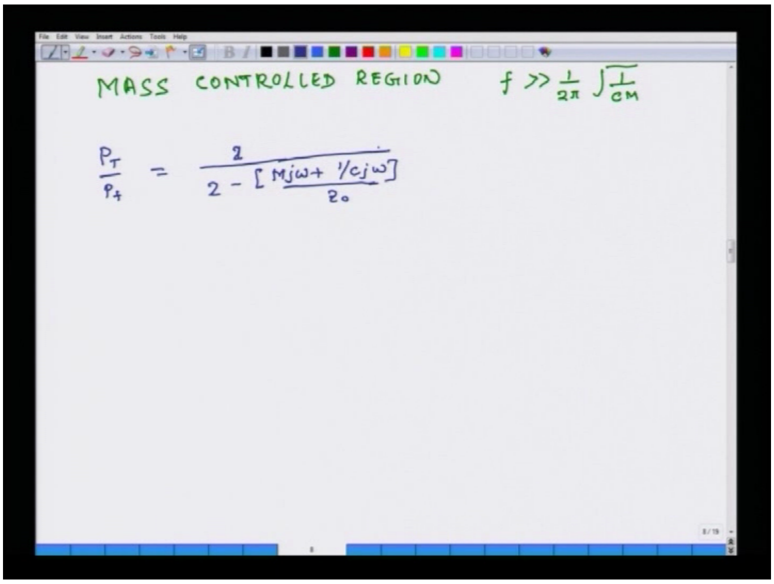
Itwillbeastraightlinebecauseitisanasymptoticresponseandthisslopewillbe6decibelsperoctavebecauseasomegagoesdownIgetthis6decibelsslopebecauseasomegaisextremelysmallthetermintheparenthesis,thistermisextremelylargecomparedto1andwhenItakeitslogIgeta6decibelsperoctaveslopebecausethereisasquaretermthere,okay.Sothisistheresponseofthesystemforstiffnesscontrolledregion.

(ReferSlideTime:37:05)



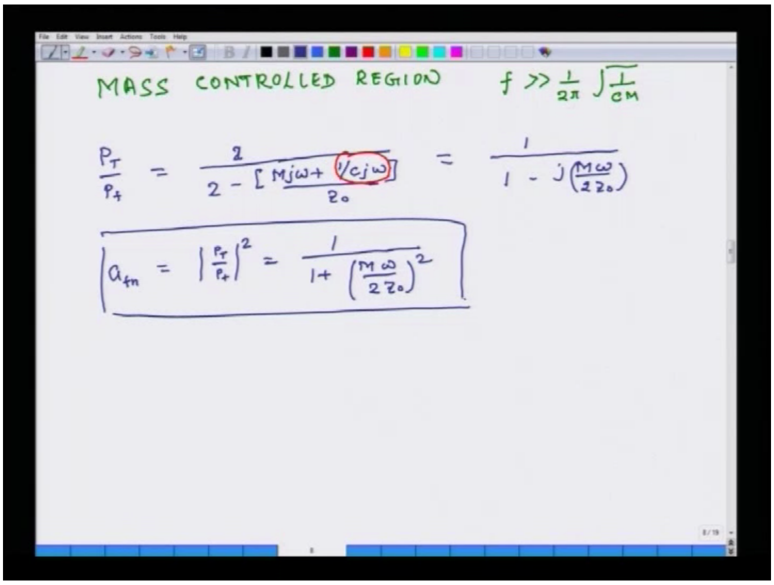
Sonowwemoveonandnowwestartlookingatmasscontrolledregion,okay.AndthedefinitionofmasscontrolledregionisthatmyfrequencyshouldbelargecomparedtoIover2pietimes1overCMsquarerootwhichisessentiallythenaturalfrequencyofthesystem.SoonceagainmyoriginalrelationforPToverPpluswasthisthing.

(ReferSlideTime:38:07)



AndinthemasscontrolledregionIignorethistermbecausetheproductofMandomegaisextremelylargecomparedtothenumber1overCtimesomegabecauseomegaisextreme.SoifthatisthecasethenIcansimplifythisas,andthemathematicsisverysimilarsoIamnotgoingtorepeatthatmathematics,Icansimplifythisas1minusjMomegaover2Z0.Andaswehadcalculatedearlierattenuationinthiscaseisonceagainthesquareofthistermandthiscomestobelikethis.

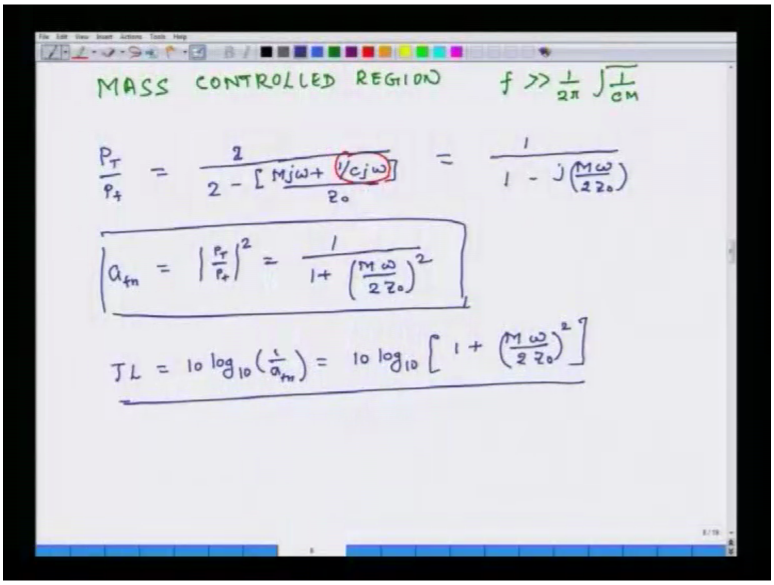
(ReferSlideTime:39:14)



AndfinallywewillcalculatetherelationfortransmissionlossTLandthatisessentially10timeslog10of1overattenuation.Sowhatthatis?

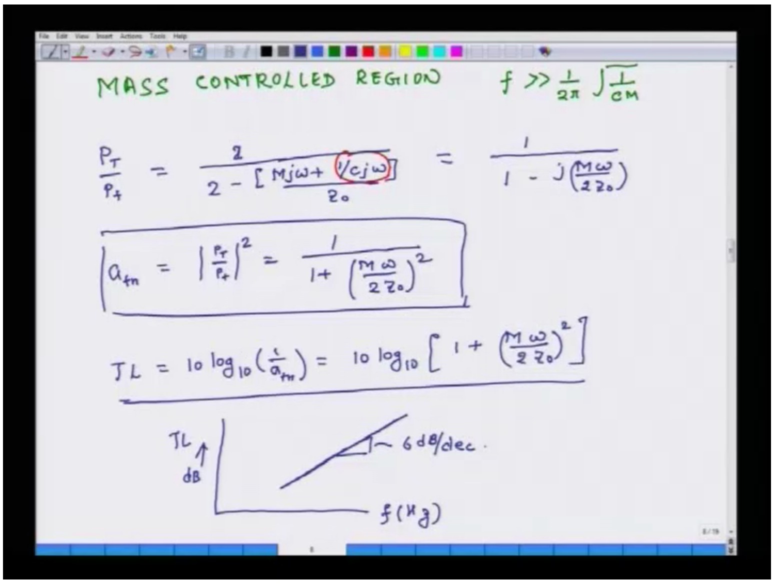
SohereinthisrelationweseethatlikeinstiffnesscontrolledregionifIreducethisstiffnessmyattenuati onwouldgoupandsowouldmytransmissionloss.IncaseofamasscontrolledregionIhavetoincreaset hefrequencytoensurehighertransmissionlosses.

(ReferSlideTime:40:11)



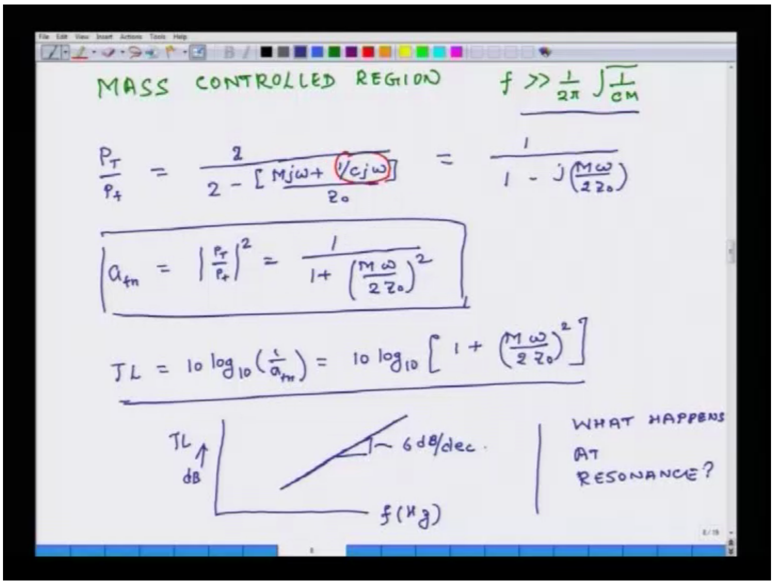
SoifIagainplotthisindecibelsandifIconstructthebodeplotthenitwillbesomethinglikeastraightline .Theasymptoticresponseasomegabecomesverylargewillbestraightlineandthisslopewillbe6decib elsperdecade.Sothisishowthesystemisgoingtorespondasomegagoesup.

(ReferSlideTime:40:48)



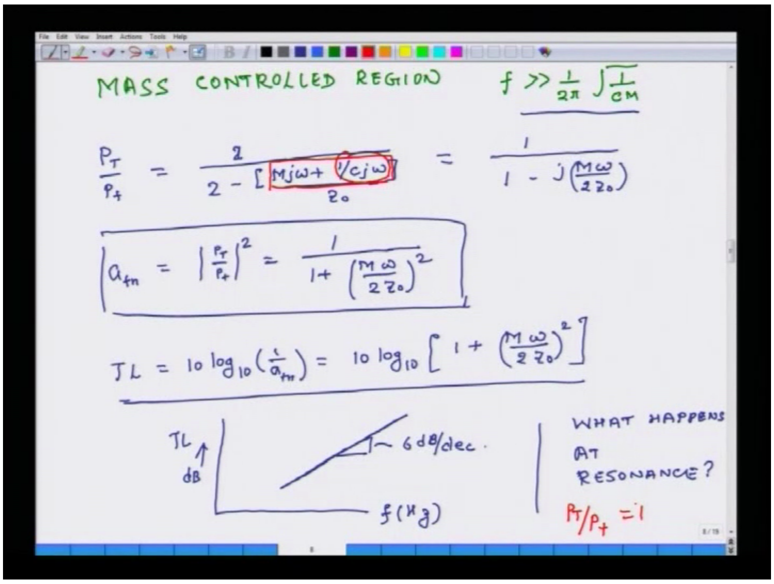
SowhatthatmeansisthatifIhaveawallandifIamstrikingitnormallywithfrequencieswhicharelargec omparedtonaturalfrequencyofthesystemthenasIkeeponincreasingmyfrequencieslessandlesssou ndpassesthroughthatbarrierbecausemytransmissionlossgoesupby6decibelseveryoctave.Everyoctaveitgoesupbyfactorof2thatis6decibels.Nowthequestioniswhathappensattheresonancepoint?

(ReferSlideTime:41:48)



So,whenIhaveaconditionforresonancethenthistermitessentiallyMjomegaplus1overCjomegabecomes0.Sotheinfluenceofmasscounteractsanditcancelstheinfluenceofstiffnessexactlyattheresonancefrequency.Sowhenthistermbecomes0thenmyPToverPplusisessentiallyexactlyequalto1.SoPToverPplusequals1.

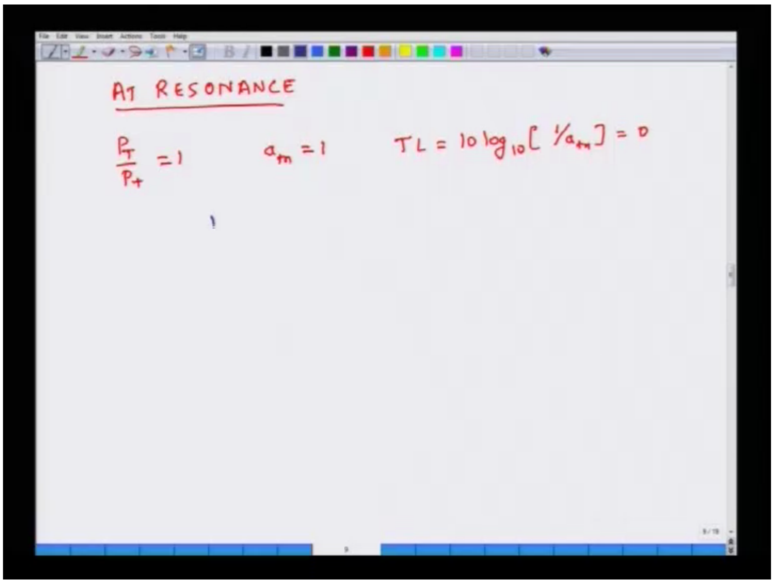
(ReferSlideTime:42:21)



AndthusatresonancewhatmathematicstellsusisthatPToverPplusequals1,attenuationequals1basedonthisrelationandtransmissionlossis0.SoIdonothaveanytransmissionlossacrossthewall.

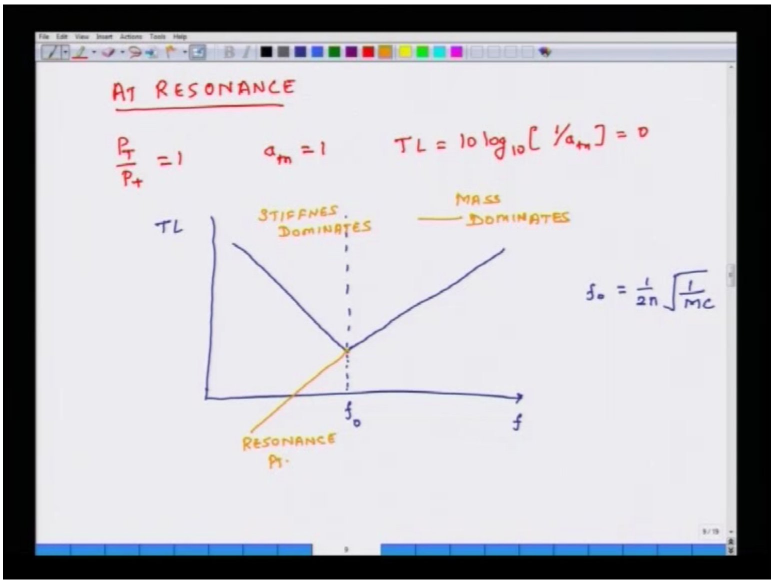
SoifIamhittingawallnormallywithafrequencywhichequalstheresonanceofthesystemofthewallth enmostofthesoundwilljustgoacrossthewallwithoutanydampingout.Butinrealitywhathappensist hatatresonancefrequencythedampingofthesystemstartsplayingarole.

(ReferSlideTime:43:34)



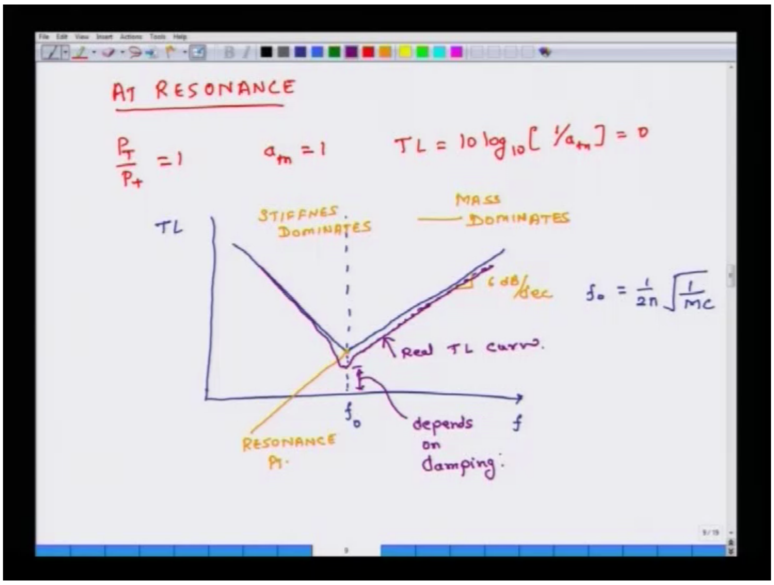
Sotheoveralltransmissionlosscurveitlookssomethinglikethis.Soletussaythisismyresonancepoin twheref0equals1over2pie.Theninthemasscontrolledregionthe(as)asymptoticresponse,thebodep lotwilllooksomethinglikethisandthisishowthebodeplotwilllookinthestiffnesscontrolledregion.S othisismystiffnessdominatesandinthisregionmassdominatesandthisismyresonancepoint.

(ReferSlideTime:44:49)



Nowthesestraightlinestheyhaveaslopeandsameslopeisonthissideinthestiffnesscontrolledregion butitisanegativeslope.Sothisismyidealcurvebutinrealitywehavedampingandalsowehavethisapp roximation,atlowerfrequenciesnotexactlytrue.Sotherealresponse,therealtransmissionlosscurvel ookssomethinglikethis.Soasymptoticallyitgoesandmergersto.Sothisismyrealresponsecurve,rea lTLcurve.Andthisheightdependsondamping.

(ReferSlideTime:46:03)



Thankyouverymuchforyourpatienceandwewillmeetyouonceagaininournextlecture.Thankyou.