Acoustics
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Lecture 07
Numerical Examples

So now I am interested in finding out how good my assumption was in terms of whether this is an adiabatic process or not. So what we are going to just verify is, how good was my assumption? So this is what I am going to figure out.

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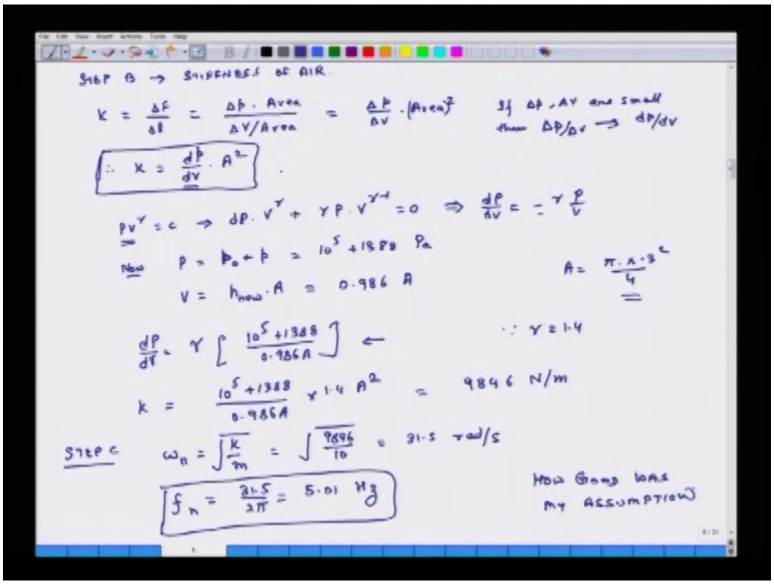

So we know based on literature survey that for atmospheric conditions the thermal diffusion waves, it moves at a speed called C t and that is equal to omega over 160 where omega is the frequency of oscillation. So if I have an air column and there are pulsations going on in this air column, there are pressure fluctuations happening in this air column, the heat generated in such an air column is going to propagate at this speed C t.

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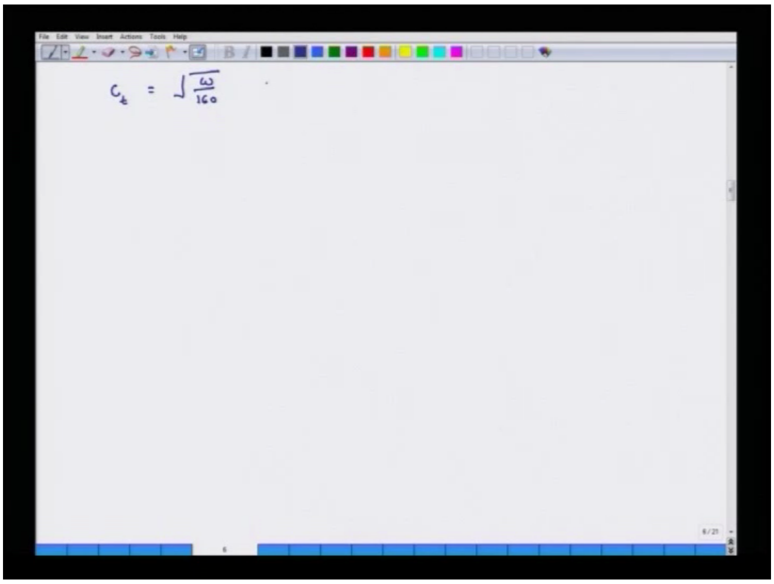

And what this means is now omega is 31 point 5 so the value of C t is and that comes to 0 point 035 metres per second. So this is how fast heat is going to move out or move into the system. Now just consider this air cylinder. This distance is 0 point 3 metres. So from the centre this distance is r and r equals 0 point 3 divided by 2, point 15 metres per second. So suppose at the centre there is a small amount of air which is getting compressed because of this vibration in the system.

Now when it is getting compressed what does that mean? Heat will get generated and this heat is going to try to move out. It is going to radially propagate outwards and it will try to escape the system.

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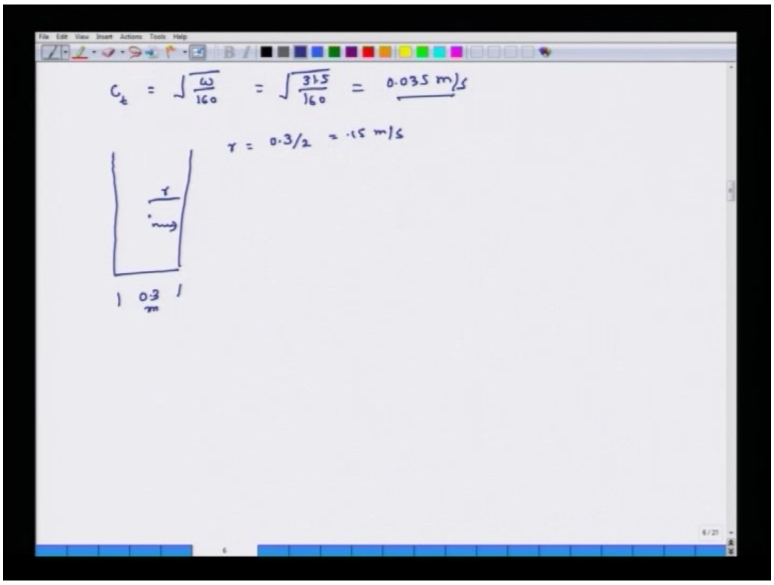

Now let us try to figure out how much time it is going to take for this heat to move out. Now I know that time delta t is equal to this velocity of heat wave, distance it has to travel divided by C t and this is going to be 4 point 3 seconds. So this heat will take 4 point 3 seconds to move from this point to the outward of the cylinder, to the outside boundary of the cylinder.

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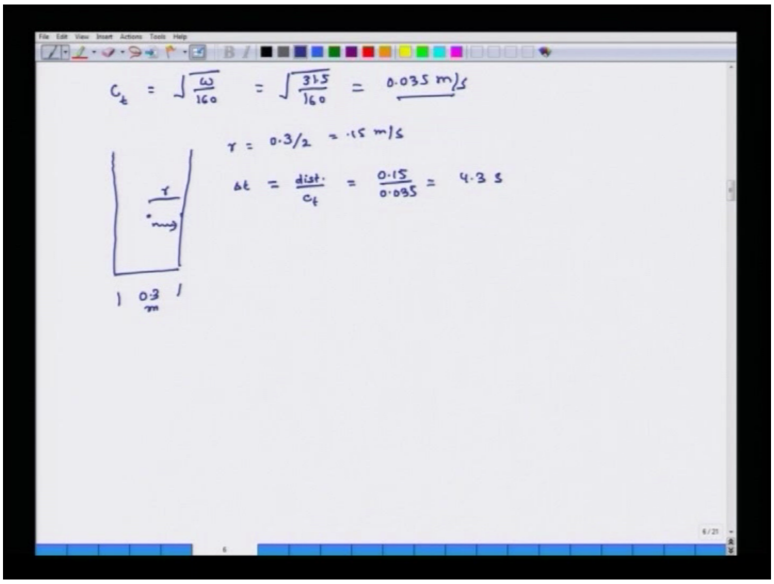

Now in 4 point 3 seconds the piston and consequently the gas will go through a series of oscillations. It is not that once the thing has gotten compressed it will stay in that situation for all the time. It will again you know expand and compress and expand and compress. So in 4 point 3 seconds we would have 4 point 3 times 5 because 5 is the natural resonance of the system, natural frequency of the system, and that equals 21 point 5 oscillations.

So what does that mean that by the time heat moves a little bit out in the radial direction you start having again expansion and as a result cooling starts happening in the centre so heat again comes back in. Then once again heat radiate comes back in. Then in the next compression cycle again heat gets generated, moves a little bit out and it again moves little bit in. So as a consequence heat is kind of trapped in the cylinder primarily because of the fact that C t is extremely small.

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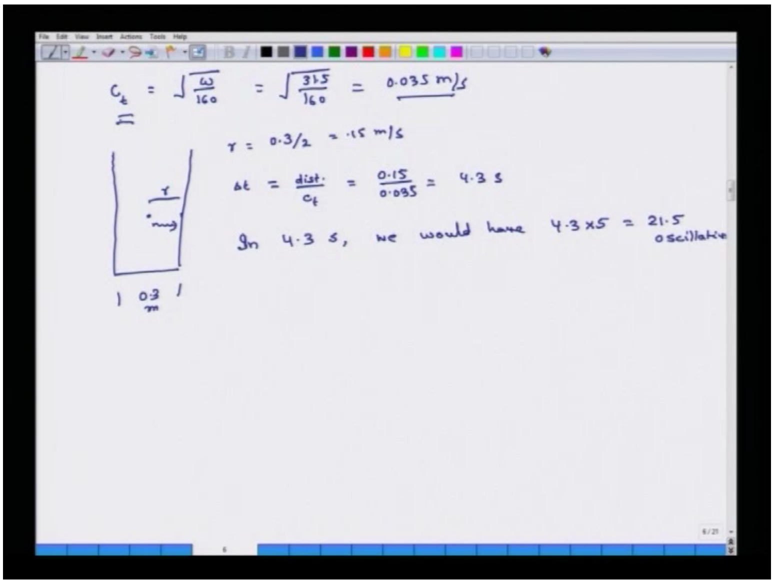

Now if the C t was extremely high compared to the natural frequency of the system then it would have been different. But because the thermal diffusion, the waves speed is extremely small, heat is unable to escape the system. And what that means in the sense is that the system is more or less adiabatic. Now of course there will be some heat escaping from the edges. The bulk of the heat which is contained inside the cylinder, deep in the cylinder, it remains there.

So our assumption that the cylinder is going to behave in an adiabatic way, the air column is going to behave in an adiabatic way is a reasonable one and our answers are correct. So that was the second problem. The first problem was about developing a pressure wave equation if we had an isothermal system instead of an adiabatic system. In the second problem which we just concluded was about developing a relation for natural frequency in air column.

Now we will do two more problems and these will be in context of transmission lines and transmission line equations. So this is the third problem. So what we have here is let us say we have a 1 D tube and this 1 D tube is terminating in some material and this value is R. The impedance of this end of the tube is R. And here I have a loud speaker and it is generating sound which propagates in this tube.

Let us say the length of this tube is L and we have to figure out the value of this L based on some conditions we will talk about. And what we are interested in finding is before we go there we have a velocity mic here. So this is a velocity mic and what we are interested in finding out is how much time delay for particle velocity between speaker on the left and microphone on the right? So the time delay that is what we are interested in.

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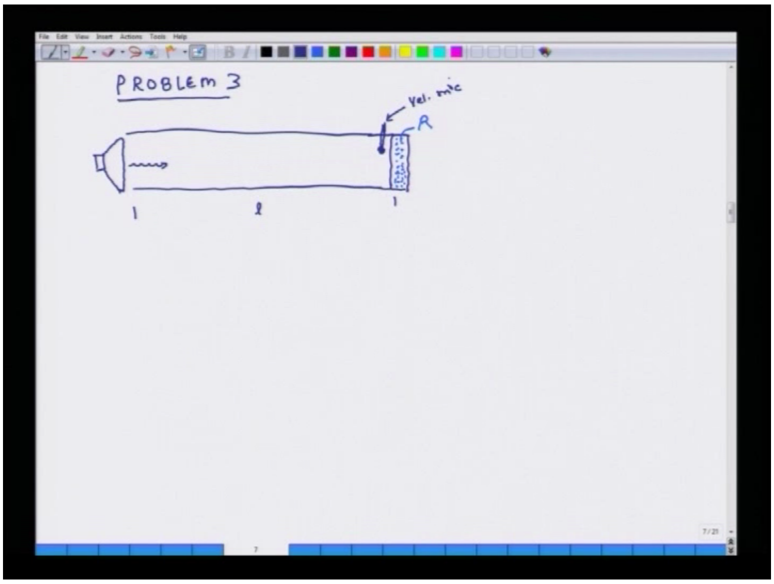

And in this context what we are trying to find out is that what is the value of this terminating resistance? So this is first question. What is the value of this terminating resistance such that there are no reflections? What does that mean? That as sound comes from open end of the tube to the closed end sound will come and get reflected back and we want to prevent these reflections from happening.

So what kind of value we should have at the terminal point? So that is one thing we are interested in. I will also put my coordinates system x is equal to 0 here and x is growing in this way, okay.

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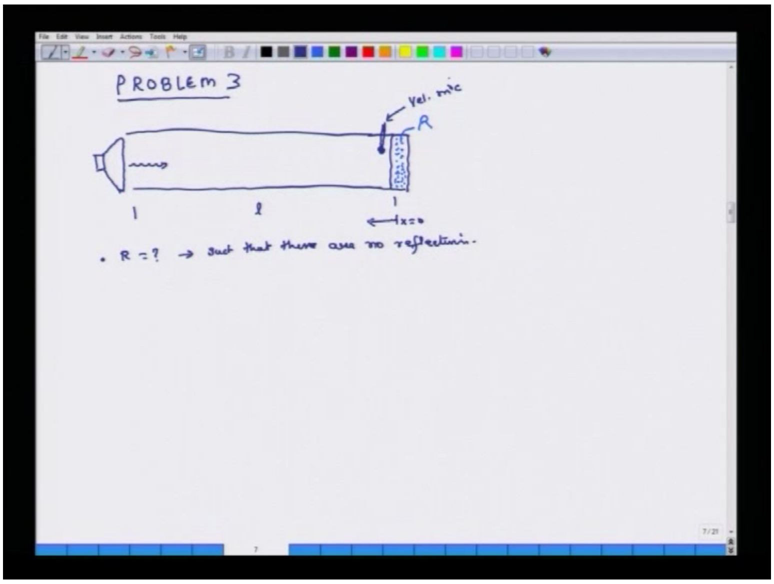

The second question we are interested in knowing is that if there has to be a delay, so our first goal is to determine the value of R such that there are no reflections at extreme end of the tube, at closed end of the tube. The second goal or second thing which we are interested in finding out is what is the value of transfer function for velocity? So this microphone is going to measure some velocity.

So what is the transfer function U mic, velocity measured microphone divided by velocity measured at source. So you have U source here and U mic is at this point. So what is this value?

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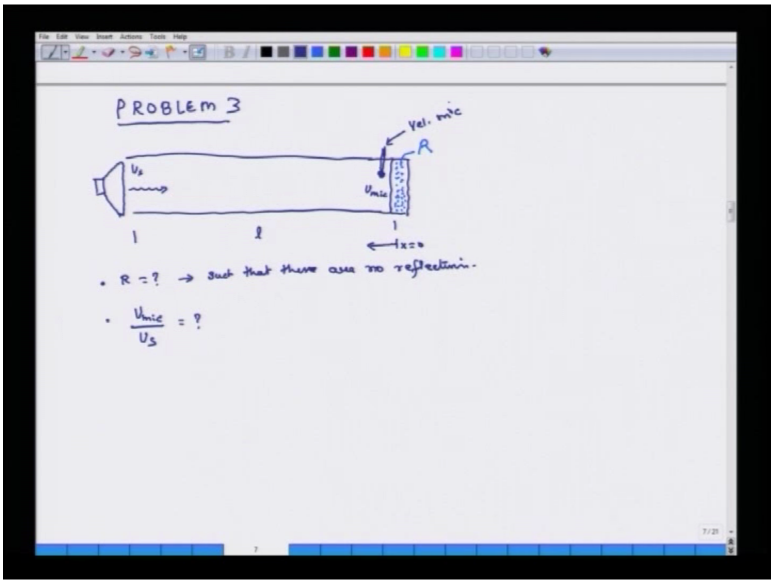

Specifically what is this value such that the system is providing a 0 point 3 second delay? And the third thing is what is the value of L if I have to have a 0 point 3 second delay? Okay. So these are the three questions. So we will start with our transmission line equations. So pressure and velocity we know are related to P plus and P minus using these equations. Now we know that at x is equal to 0 there is no reflection going to happen. So this is my boundary condition. There is no reflection.

We do not want the reflection there. So if that is the case then at x is equal to 0 P x t or actually it should be 0 t and U of 0 t is equal to, so this has to be positive times e s t. And from these two I can develop a relation for transfer function. So P over u at x is equal to 0 equals R. And when I take the ratios of these two entities I get Z 0 because this Z 0 is embedded here. So when I divide P plus e minus s x over c times e s t by this entire number then I get this relation.

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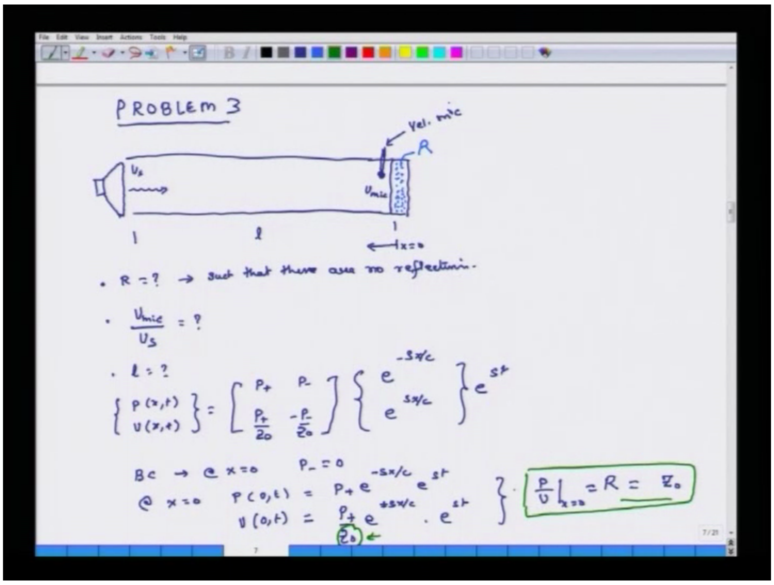

So we know that R has to be same as Z 0 and we know that Z 0 is nothing but P 0 C. So that addresses the first part of my question that what is the value of R. What is the value of R if we do not want to have any reflections?

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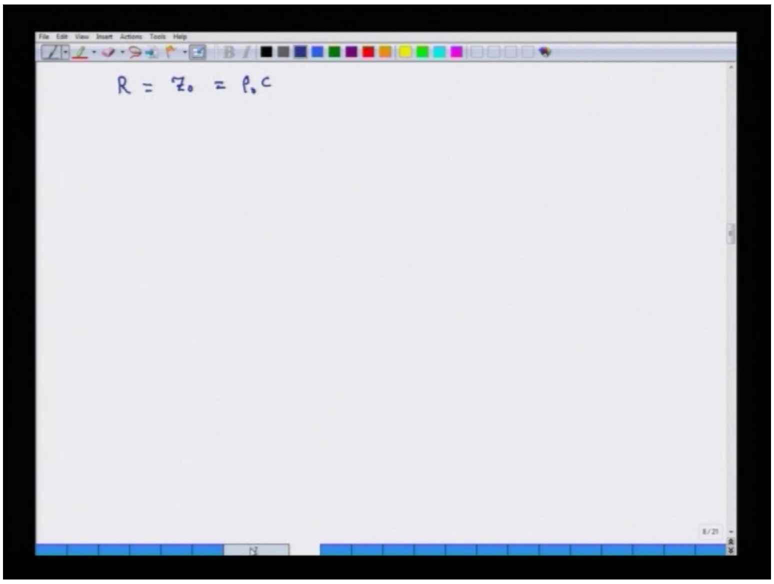

Now my second question was what is the value of this transfer function U microphone with respect to U s that is source velocity? So U mic over U s. What is this value? So now we know that as P negative you know as this term is 0 because there are no reflections, so P of x t is equal to P plus e minus s x over c times e s t and U of x t equal to P plus e minus s x over c e s t over Z 0. Now from this relation I can find the value of U mic and also I can find the value of U at source. So U mic is equal to U x t at x is equal to 0.

And this is I have to put x equals 0 in this relation. So this is equal to P plus e to the power minus s x over c when x is equal to 0 is 1 times e s t over Z 0. And U s is equal to U L t, excuse me, x is equal to L because my source is located at x equals L and that is equal to P plus e minus s L over c e s t over Z 0. So U mic over U source, if I take the ratio of these two terms what I get is e.

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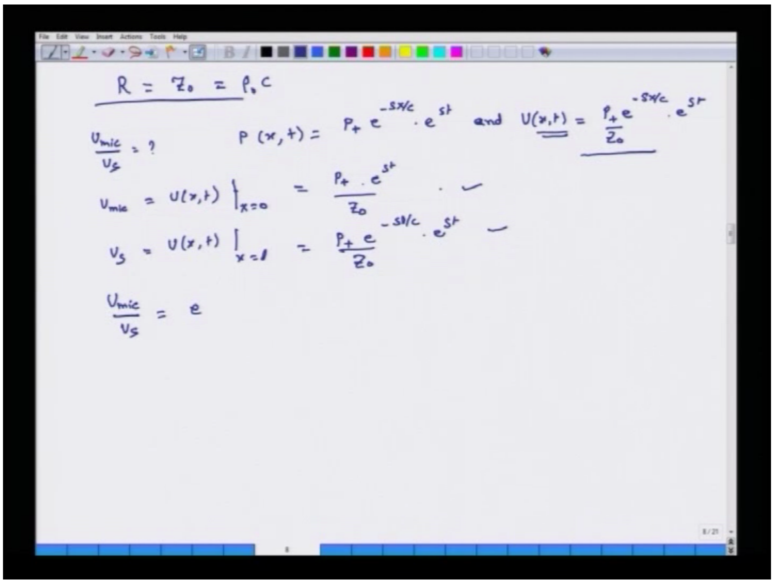

So I will calculate U mic over U s and before I start doing that I have to make a small minor modification in this picture. I had indicated that x is growing in this direction which is not right. Actually x is positive in this direction because that is my direction of P plus.

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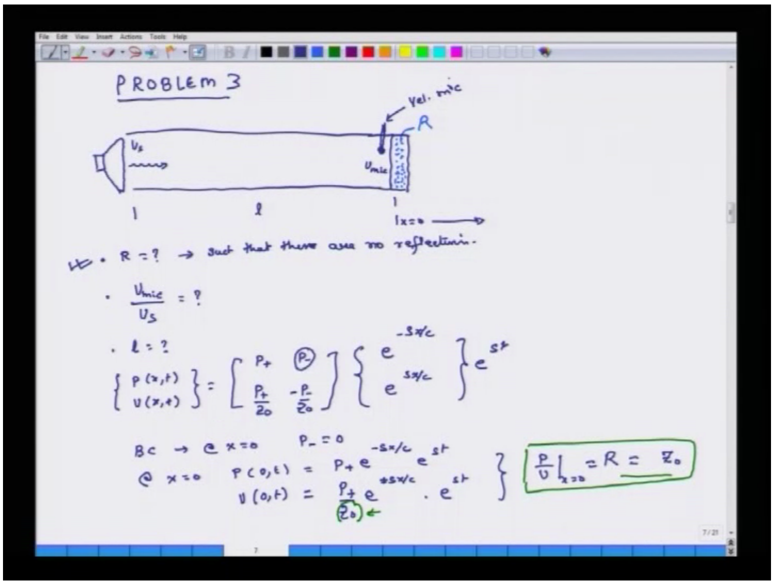

So if that is the case then x is equal to 0 at closed end and x is equal to minus L at open end. So with this modification I go back and here in this relation I make another small modification and x is equal to minus L. And once I put x equals minus L, this negative sign becomes positive. So what I get here is e to the power of minus s L over c. This is the value of the transfer function. Now we know that s is equal to j omega so U mic over U s equals e minus j omega L over c. So this is my transfer function for U mic over U s.

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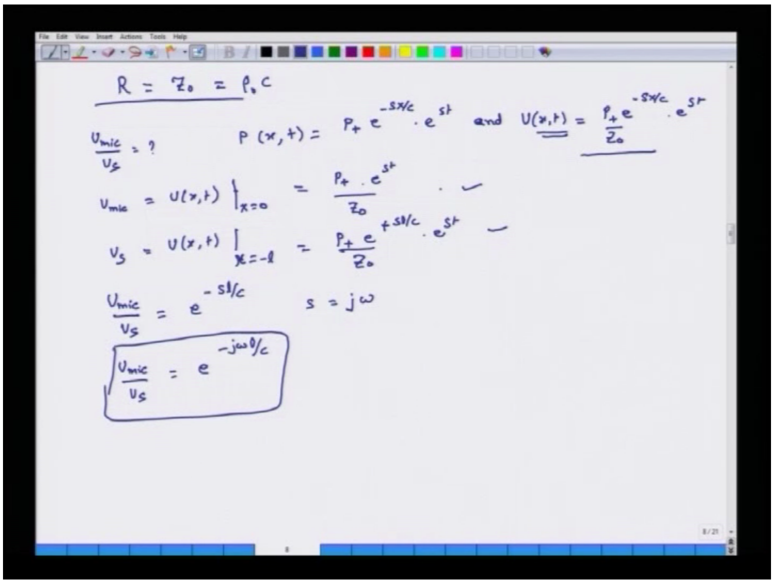

Now the question was that what is the value of L when I have a time delay of 0 point 3 seconds? So if there is a time delay of 0 point 3 seconds what it physically means is that it will take 0 point 3 seconds for sound to travel the distance of L and in that case L is equal to time it takes to reach the other end which is 0 point 3 seconds times velocity of sound. So it is 0 point 3 c.

Now if I put this value L in this relation then I get U mic over U s is equal to e minus j omega, and I am replacing L by 0 point 3 c, times 0 point 3 c over c and what I get is e minus 0 point 3 omega j. So this is the transfer function U mic over U s which is consistent with device which provides 0 point 3 second time delay and also ensures that there is no reflection happening at the close end.

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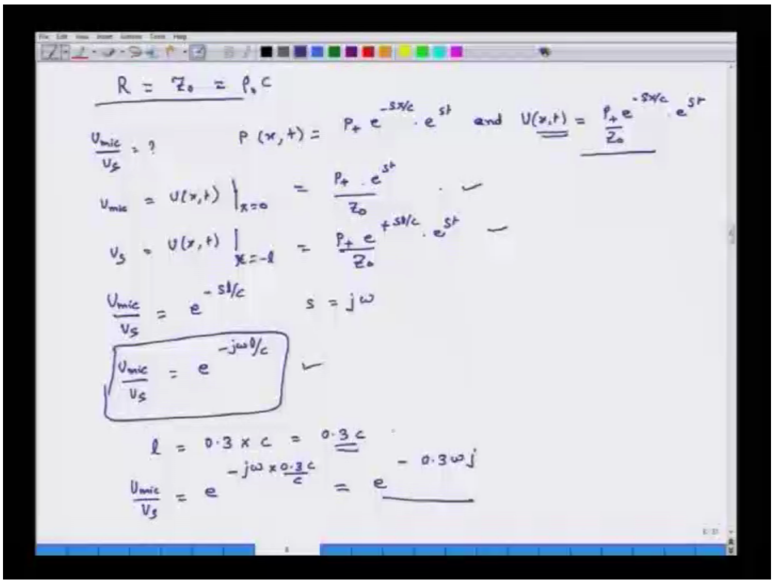

And the final question was what is the length of this tube? So we have already calculated this. The length of the tube is 0 point 3 c and if I want to find out the actual number then this times 345 equals 103 point 5 metres. So that is the third problem which we have covered today and the final problem which is the fourth problem which we will do today is again related to this transmission line theory. So I will frame the problem for first and then we will start discussing its detail.

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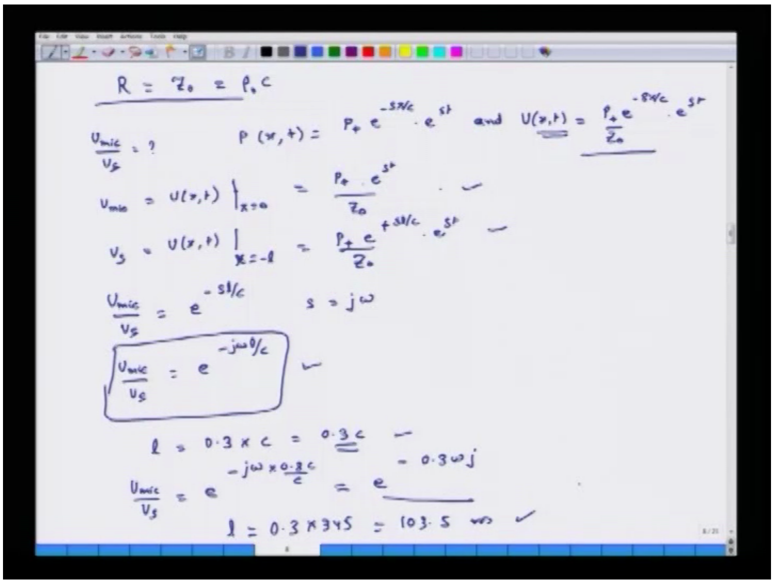

And in this case the problem is not specifically in the area of acoustics but rather in the area of electromagnetism and electrical engineering. But we will see that whatever concepts which we have developed related to transmission line theory apply in area of electromagnetics as well. So that is one reason because later we will use a lot of mappings from mechanical to electrical to acoustics and develop a lump parameter model for complex acoustical systems and in that context we have to become familiar with these equivalences.

So what we have here is this kind of a system. So let us say I have a plate. So this is one plate and then I have another plate. These are two parallel long plates and they are metallic plates, uniform thickness across the length and let us call this L and let us give a number let us say L equals 30 centimetres.

So these are two parallel plates and once again x equals 0 here and x is growing in this direction. And then at this end of the plate I am putting a voltage. And this voltage is V and the value of this voltage can change with respect to, so it is a function of omega. So it can change with respect to omega, input voltage.

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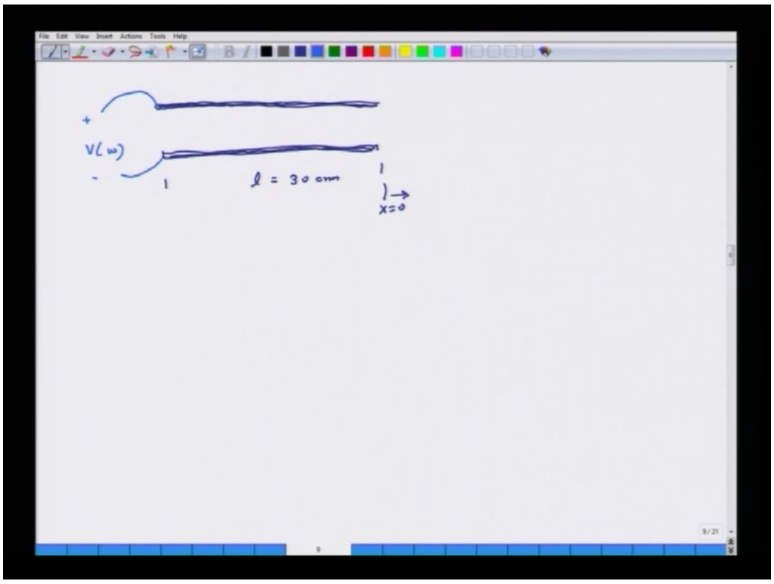

What we are interested in finding out in this system is what will be the nature of this lumped element with the same impedance Z equals V over I. So what does that mean? Now I am applying some voltage on the system. As a consequence there will be some current going into the plate and I am interested in finding out Z and how this Z going to behave and the way I define Z is its ratio of V and I. And this value of Z it depends on x. So I will rewrite this relation.

So Z will change with respect to x and with respect to omega because V is changing with respect to x and omega and I is also changing with respect to x and omega. So what I am interested in finding out is value of Z at x equals minus L. So this is x equals minus L. Here x is equal to 0. So this is the first thing I am interested in finding out.

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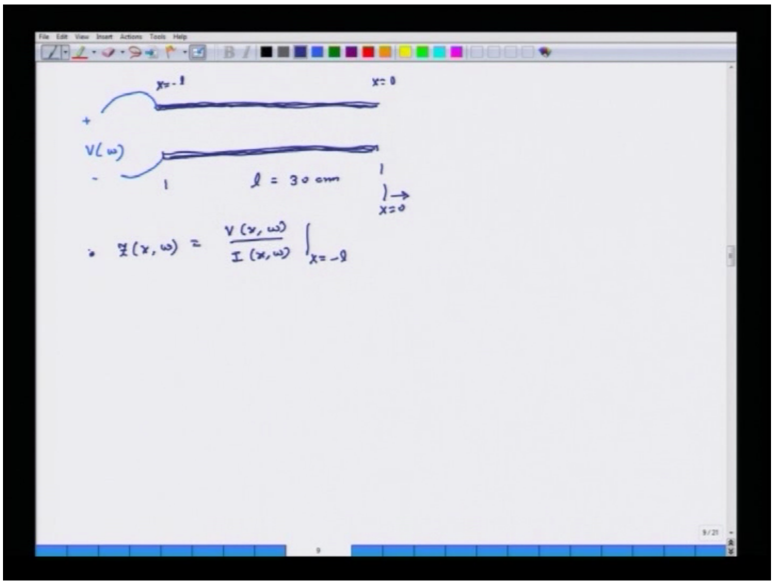

And then special case of that expectation is that I will find the relation Z x omega at x equals minus L and then also what is Z as omega becomes extremely small? So this is a special case. So I will find the exact relationship and then I will find an approximate relationship and then we will see how good is our approximation within 5 percent. So that is the second thing. And then the third thing is we are going to find something related to the capacitance of the system.

So we know from principles of electrical engineering that capacitance between two parallel plates C is equal to A epsilon 0 over d where A is the area of the plate, epsilon is a material property, its permittivity of the medium between these two plates in this case it is air or vacuum and d is the distance between these two plates.

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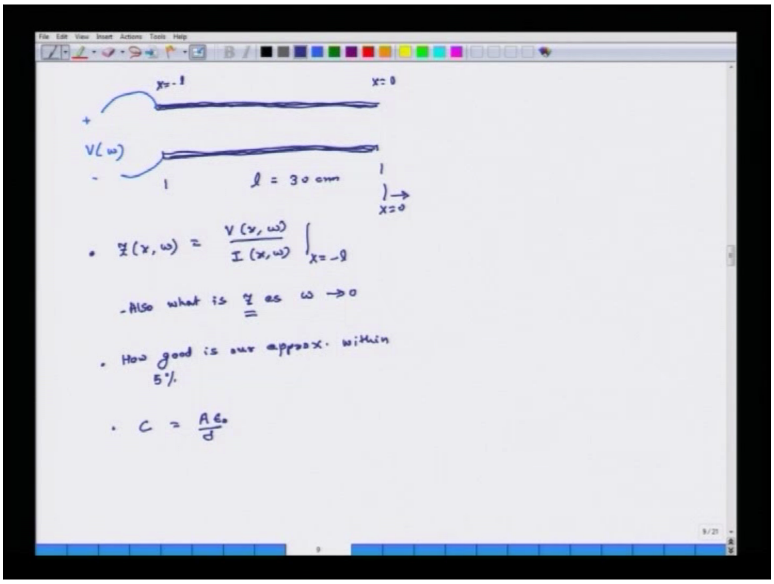

So if I know the capacitance of the system I can take a metre and find the capacitance of the systems. If I know this then what is the value of Z 0? Now this is how the problem has been formulated. To solve this problem I have to know how V propagates along the length of the plate and how I propagates along the length of the plate. So given, V and I both they have a behaviour which is very similar to what we saw in acoustics.

So voltage and current, there is a V plus component so there is a voltage and then at this end there may be some reflection. So this is given. Similarly these are the transmission line equations for voltage and current.

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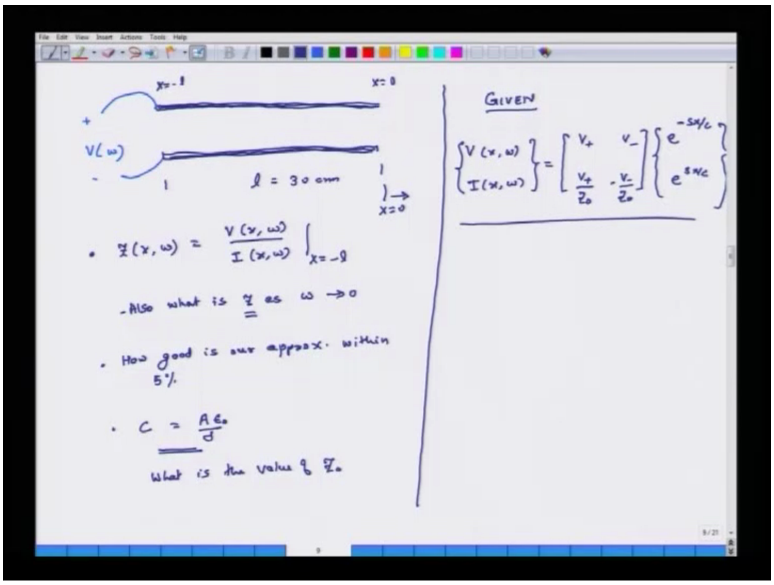

So we will use these transmission line equations to come up with answers on these three points, okay. So C in this case is velocity of wave propagation and that equals 3 times 10 to the power 8 metres per second which is basically the velocity of any electromagnetic wave. So this is what we know. Z 0 as we saw in acoustics here also we call it characteristic impedance. And this is again a property of the material and it does not change from one system to other system. So with this background we will start solving this problem.

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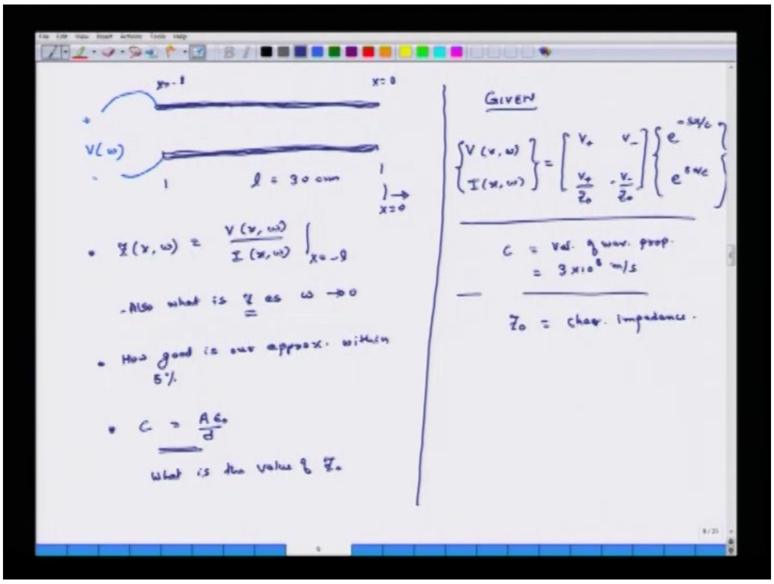

So we will write down the relations for transmission line equations and then we will impose boundary conditions at x equals L and at x equals 0 and then we start solving for V plus V minus and so on and so forth. So one thing we know that at x is equal to 0 there is no current flowing at x is equal to 0 because if at x is equal to 0 there was a current then it will (vialite) violate Kirchhoff's current law because no current can pass from this point to this point.

So there is no current at x equal to 0. So we will apply a boundary condition. B C 1, at x equals 0, I is equal to 0. And what that means is my relation for current is I is equal to V plus over Z 0 times e minus s x over c minus V minus over Z 0 e times s x over c. So my current is 0, so 0 equals V plus e minus 0 s over c over Z 0 minus V minus over Z 0 e plus s times 0 over c and because e to the power 0 is 1 so I get V plus equals V minus.

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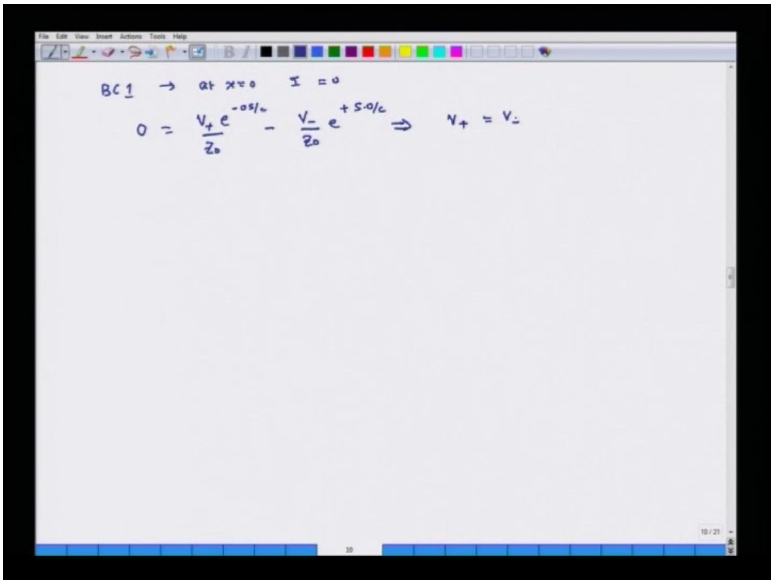

Now this result is very similar to what we saw in the area of acoustics that if I have a closed boundary condition then the velocity at that point is 0 and what that means is that at that particular position P plus is same as P negative. So this is my first equation. Now the second condition is that B C 2 and what it is that voltage at x is equal to minus L. So at x is equal to minus L my equations are, and here I am going to put instead of x, L.

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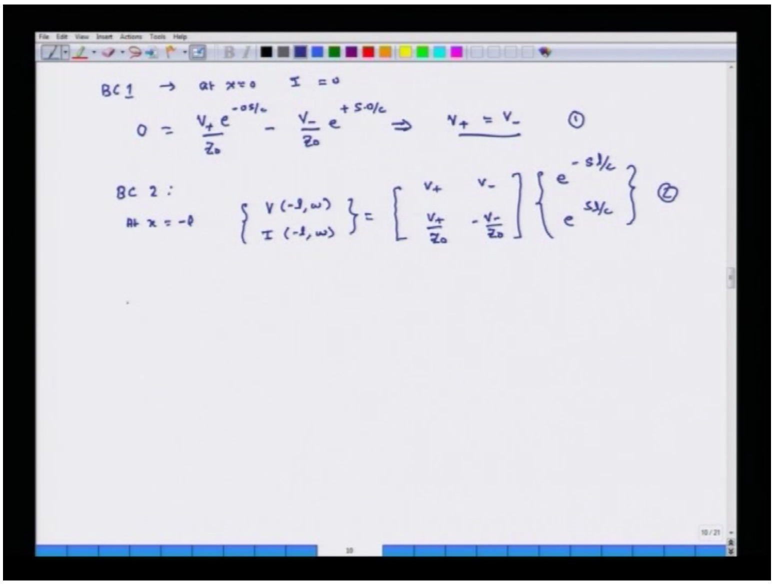

So now if I put 1 in 2 then I get, I am replacing V negative by V positive so what I get is V plus times e to the power of minus s L over c plus e to the power of s L over c. And s is equal to j omega so I get V plus e to the power of minus j omega L over c plus e to the power of j omega L over c and this is same as 2 V plus cosine omega L over c.

Similarly I is at x is equal to negative L is same as once again if I put V plus equals V minus in this relation, the second relation for current then current at x is equal to minus L, I can solve for it and I get this relation, 2 V plus by Z 0 times j sin omega L over c.

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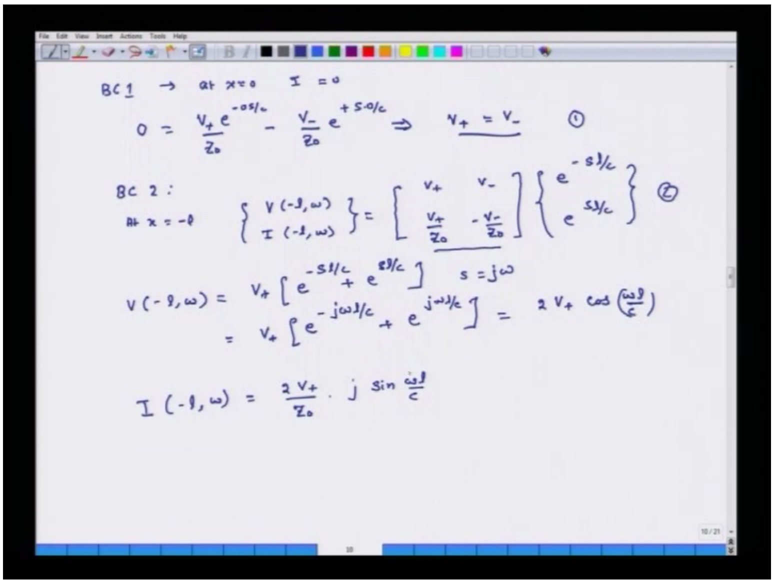

Now what we wanted to know was what is the value of impedance at x is equal to minus L? So I am going to take a ratio of V and I. So therefore Z, then what I get is Z 0 over j times cotangent omega L over c. So this is my relation for Z at x equals minus L.

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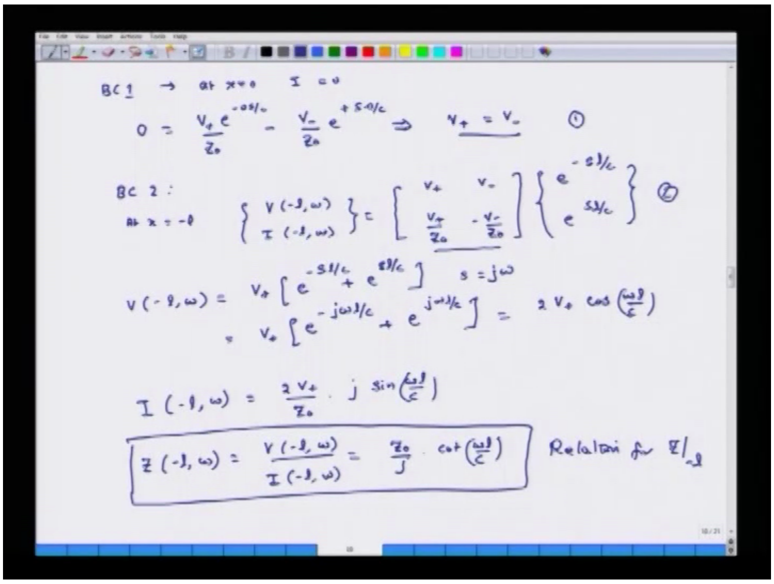

Now my goal was to find what happens at extremely low frequencies? What is the value of Z as omega tends to 0. So as omega tends to 0 this cotangent term it starts approaching as omega tends to 0, cotangent of omega L over c starts approaching 1 over omega L over c. So as omega approaches 0 the cotangent of omega L over c approaches 1 over omega L over c.

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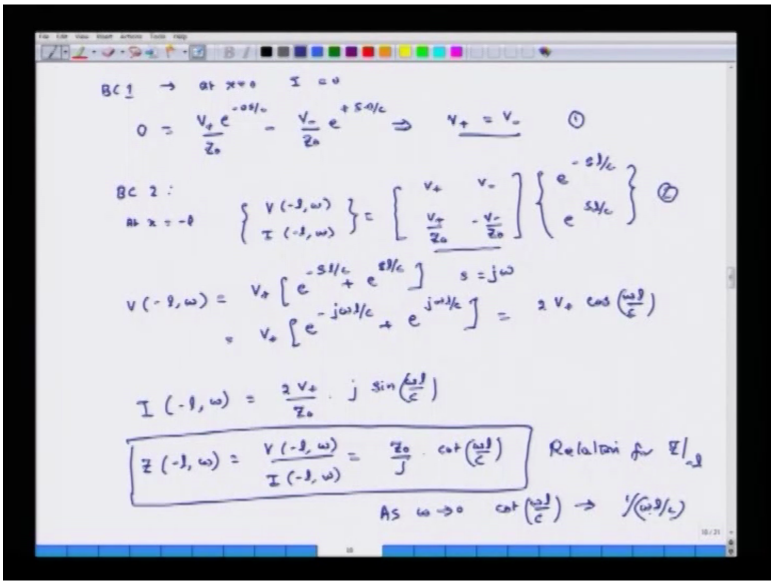

So if that is the case then Z minus L omega as omega approaches 0, I make that approximation in the exact relation which is this and then what I get is instead of this equal sign I have almost equal to or approximate to 1 over j times 1 over omega times L over Z 0 c. So this is my approximate relation for complex impedance. And this is my exact relation.

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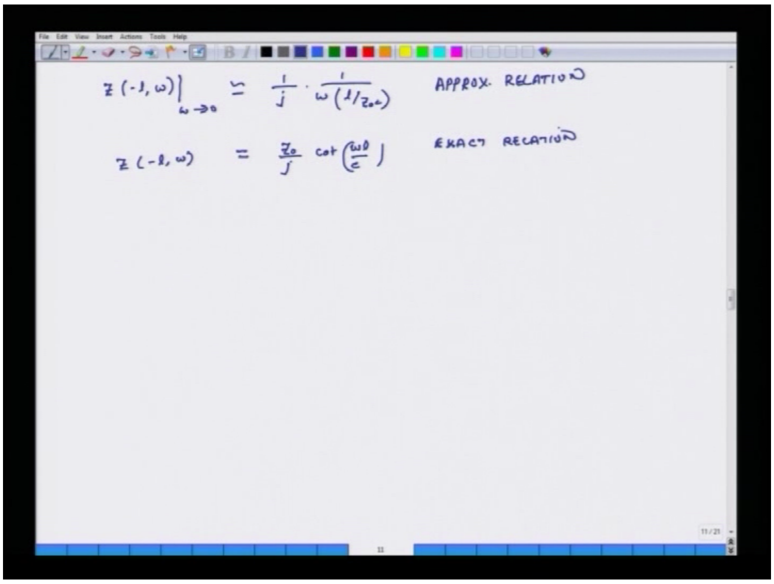

So, what I am now interested in finding out the second question that how good is our approximation? So what value of omega is at good within 5 percent? So I want to know that what is the difference between exact and approximate answers and up to what values of omega will this difference not exceed 5 percent of the exact number? So all we have to do is plug in different values of omega, calculate exact value, calculate approximate value and then see, is the difference exceeding 5 percent or not?

And so that is what we are going to do. So I am just going to write these tables. I have already calculated these results. So in first column I have omega then my approximate value. So what I am going to do is I am going to eliminate Z 0 and I am going to compare c over omega L with respect to cotangent of omega L over c. So the second column is going to be c over omega L. And then the third column is and then this is my error. So this is exact and this is approximate.

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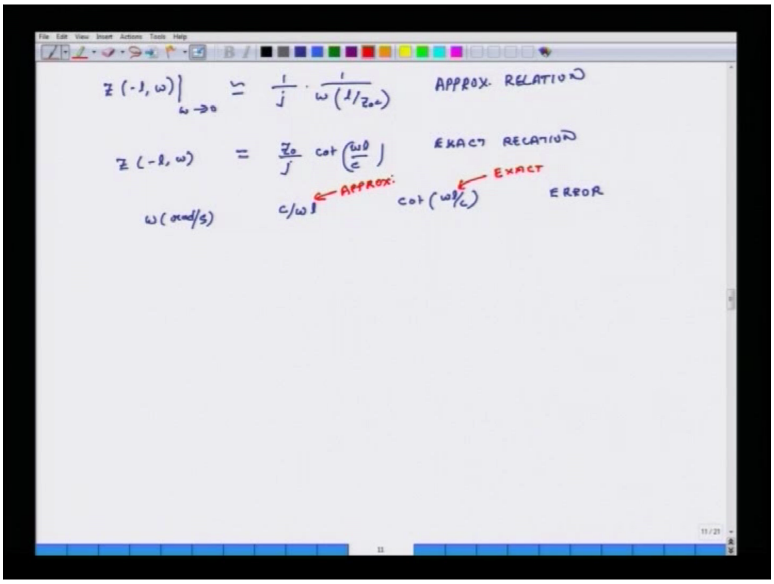

So we pick up a frequency. Let us say omega equals 10 to the power 4 radians per second. So I find that c over omega L is 10 to the power 5. Same thing cotangent and error is approximately equal to 0. I pick up a higher number, omega is 1 megahertz 10 to the power 6 radians per second, c over omega L goes down to 1000, same cotangent and again my error is still 0. So I increase my omega to 10 to the power 8, c over omega L further drops down and it becomes 10 and we now start seeing some difference.

Cotangent omega L over c is 9 point 967 and my error is 0 point 33 percent. And then I do a hit and trial and try to find for what value of omega this error is approximately equal to 5 percent? So this is the number. 10 to the power 9 times 0 point 38, my c over omega L is 2 point 63 and this value cotangent of omega L over c is 2 point 5 and this is about 5 percent.

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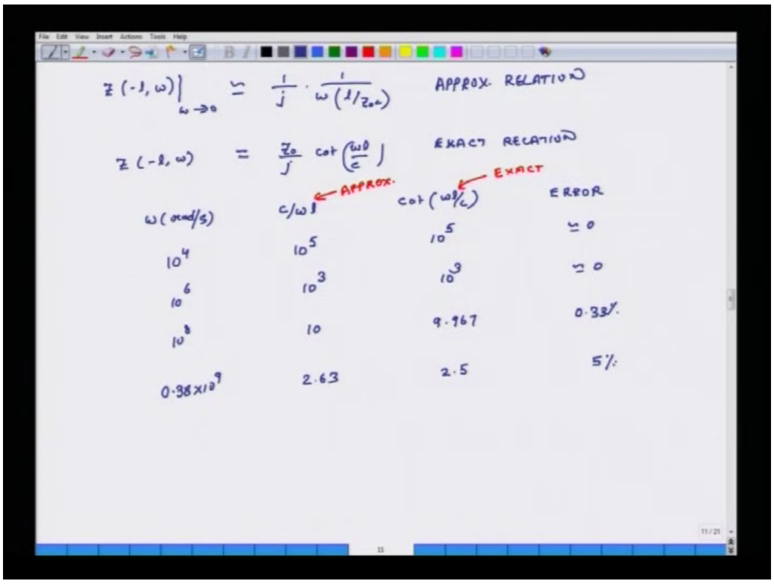

So the reason I wanted to make this illustration, do this example is because later in acoustics low course we will be making a lot of approximation and we will be making statements like omega is getting very small or this number is becoming very large. Then we have to be cognizant of the fact that what is the context in which we are talking about things which are small or things which were large? Now in a Layman's view all these values of omega, 10 to the power 4, 10 to the power 6, 10 to the power 8 and so on and so forth they are extremely large.

But it turns out that are approximate and exact results, are fairly close as long as my omega is less than point 38 times 10 to the power of 9, okay. So this number it corresponds to 60 point 5 megahertz, okay.

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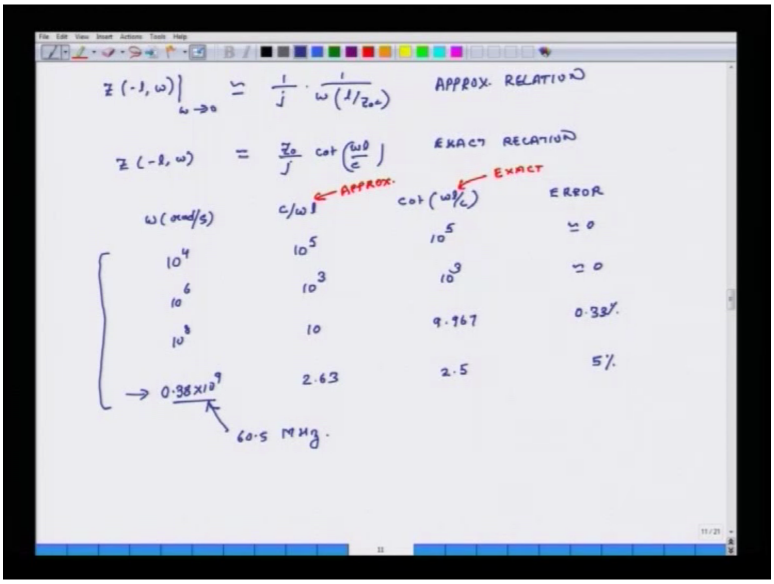

The last question we were interested in finding is that I know from electrical engineering theory and physics that capacitance between two plates which are parallel to each other is A area of the plates times epsilon permittivity constant divided by d. If this is my capacitance can I find the value of Z 0? So that is what I am going to do in next few minutes. So we know that c equals A epsilon over d.

The other thing we know is that when I see this approximate relation my Z is impedance V over I at x equal to minus L. It is basically 1 over j times omega times L over Z 0 c. Now we know that for a capacitor the impedance that is V over I is 1 over j times 1 over omega c. So you know capacitance is equal to 1 over omega c impedance. So this term is also capacitance.

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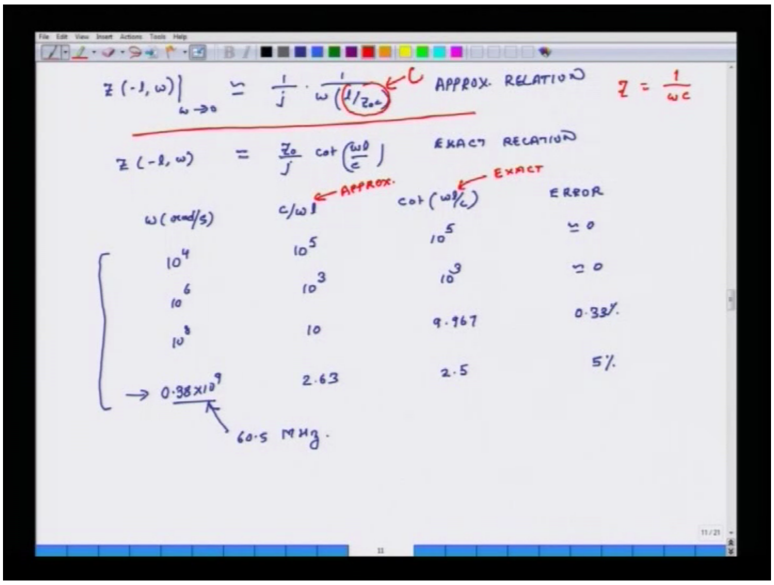

So this is my one relation for capacitance and then the other relation for capacitance is L over c Z 0. From here I am getting and now from this block I can find the value of Z 0.

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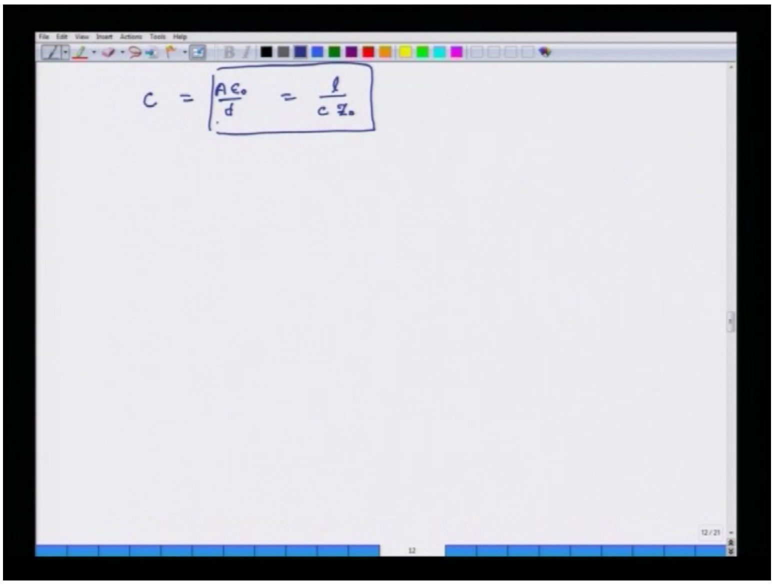

And Z 0 is equal to L over A epsilon times d over c where L is the length of the plate, d is the distance between these two plates, c is velocity of electromagnetic waves which is 3 times 10 to the power 8, A is the area of these plates and epsilon is permittivity of free air. So with this approach we are able to calculate the value of Z 0.

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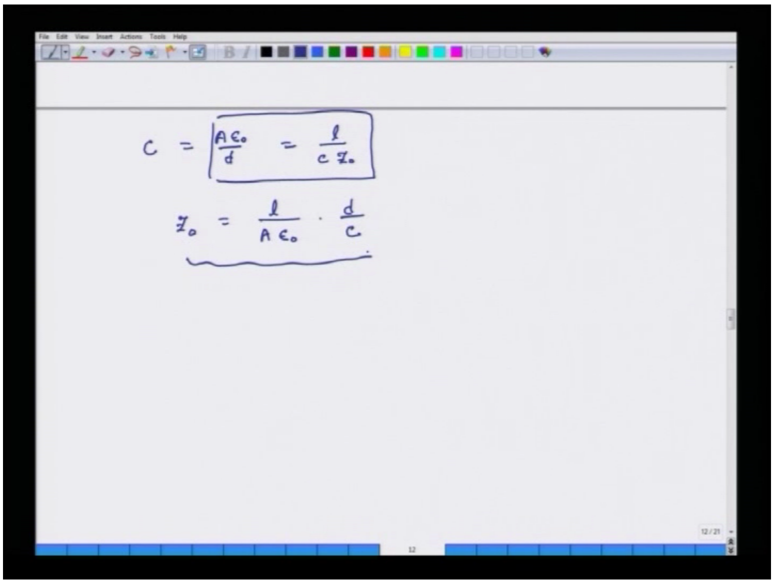

So that concludes my today's lecture and what we have talked about is basically a review of all these concepts which we have done in past several lectures through four examples. And with this I conclude my today's lecture and look forward to seeing you in the next class. Thank you.