Acoustics
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Lecture 4
Module 2
Thermodynamic Processes During
Sound Transmission

Good afternoon, in last several lectures we have till so far covered a slew of topics and we began this journey in the area of acoustics we started with some reviews, we reviewed a bunch of concepts which we will see that they will come in handy as we walk down this course. One particular area which we reviewed was the term linearity, what is linearity?

Then we moved further and we had a review of complex numbers that was followed by review of poles and zeros then pole and zero plots phase and magnitude plots for transfer functions and then bode plots. So that was pretty much the overall theme for our reviews and then we started developing equations for sound propagation specifically in one dimension.

So we developed a pressure wave equation and then after that we started developing transmission line equations as sound propagates through medium and finally we started applying these transmission line equations and pressure wave equation in context of 1-D propagation and we played with a set of boundary conditions and then we also developed a mathematical basis for (()) (1:56).

So that is what we have covered till so far and today we will be doing a bunch of problems or examples which will further our understanding of whatever we have learnt till so far. So what we will start with is a problem and here if you remember that when we were developing 1-D wave equation we had assumed based on our understanding of reality that the compression in air as sound propagates this compression and rarefaction process is essentially an adiabatic process.

This assumption is based on our understanding of reality and it is based and it has been validated subsequently by a very large number of experiments and experimental data. What we will do today is, we will again revisit that equation and assume in this case it will not be an accurate assumption but we will still assume that suppose sound propagation was instead of an adiabatic process it was an isothermal process. So had that been the case then what would be the implication in terms of pressure wave equation and also how would sound propagates and how fast it would propagate? What would be the velocity of sound in media?

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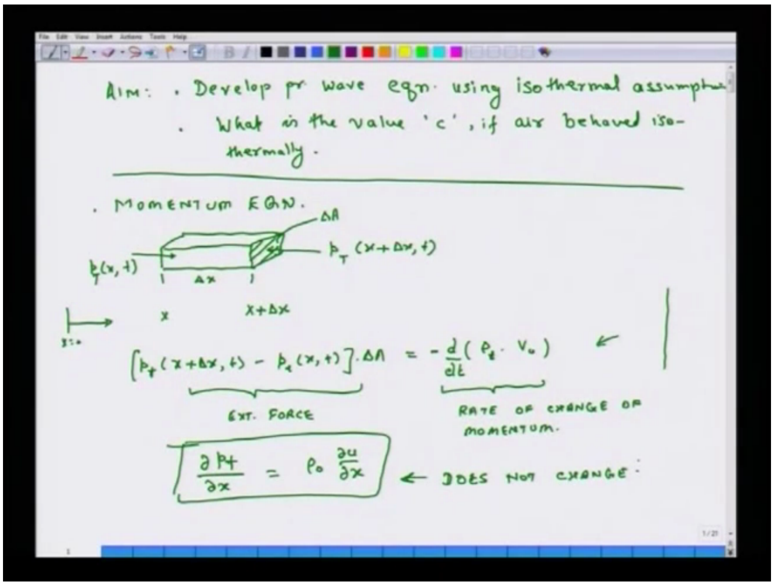

So that is what we are going to do today, so our aim is develop pressure wave equation using isothermal assumption. So this is 1 aim and then the second aim is what is the value of ‘c’? That is velocity of sound if air behaved isothermally, okay. So if we recall the way we have developed these equations of sound propagation, pressure wave equation and velocity wave equation was by first developing 3 individual equations.

The first equation was a momentum equation or a force balance equation where we took a small piece of unit of small volume of air and we applied differential pressure on it as a consequence of this pressure this small volume of air would accelerate governed by Newton’s first law of motion and we expressed this phenomena through the moment equation.

The second equation which we developed was material constitutive it related material constitutive behaviour and there we assumed that the air in question was behaving in an adiabatic way, so that was the second one and then the third one was about conservation of mass and using principles of conservation of its magnet mass we developed this continue equation.

So we will again revisit all these equations and see which particular equations get impacted by it and how it ultimately affects our pressure wave equation. So we will start by momentum equation and we had assumed that if there was a small volume of air and it was about Delta x long and I have an external pressure. So x is equal to 0 here, x is equal to Delta x here.

So your total pressure is going to be x plus Delta x and it is also going to be a function of time and here total pressure is going to be a function of x excuse me, so I will just set up my coordination system here. Here the value of, so this is my reference x is equal to 0, so here the value of x is x and here the value of x is x plus delta x. So the net force on this piece of air is essentially the difference of PT at x plus delta x and PT multiplied by this area, okay.

And let's call this area as delta A, so net force on this is, so this is the net force and this should equal the rate of change of momentum of this fluid volume. So the moment of this fluid volume is density times volume and I will call this volume as Vu and if I have to do it rate of change of momentum then I do a take a time derivative of it, so it is d over dt and the reason I have this negative sign as we had explained earlier was that pressure x inverts.

So if pressure on the right side is more compared to pressure on the left side of this fluid volume then this whole thing is going to accelerate in the negative x direction. So that is the reason I have a negative sign here. So once again this is my external force and this is rate of change of momentum. So this is essentially Newton's second law and this equation we had seen earlier that we were able to simplify this to this form and as we see that in development of this momentum equation there is no influence of the behaviour of gas, it does not matter whether gas is behaving adiabatically or isothermally. So this momentum equation it does not change.

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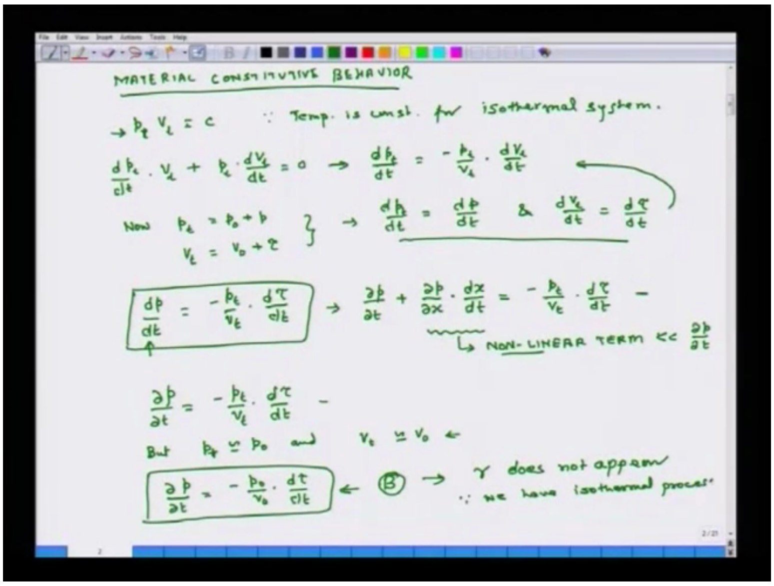

Next we will look at the material constitutive behaviour. So we have earlier we assumed that it was adiabatic but now we are going to use a isothermal gas law. So the isothermal gas law says that pressure times volume constant because temperature is constant for isothermal system. So now if I differentiate this particular equation and time I get dPt over dt times volume plus pressure times time derivative of volume and that equals 0.

And if I rearrange this what I get is time derivative of pressure with respect time derivative of pressure is nothing but equal to negative of Pt over Vt times time derivative of volume. Now we know that Pt is the final pressure and that is equal to initial pressure which is atmospheric pressure P not plus some incremental pressure and please note that this incremental pressure is extremely small.

And similarly Vt equals V not plus Tau, so from these I can say that time derivative of pressure is same as dP over dt and time derivative of volume is same as time derivative of incremental volume. So using these relations if I plug it into here I get dP over dt equals minus Pt over Vt times dTau over dt.

I process this relation further and what I do is, I break this up into its specific components. So what I get is, this is the total derivative of pressure with respect to time and this I break it up into partial derivatives. So what I get is, partial derivative of pressure with respect to time plus partial derivative of pressure with respect to x times dx over dt is equal to minus Pt over Vt times d Tau over dt.

Now this term is a non-linear term and what we know is that this term is extremely small compared to del P over del t because the increments in pressure and velocities they are extremely small. So with this assumption what we do is, we simplify this relation and what we get is partial derivative of pressure with respect to time equals minus Pt over Vt times dTau over dt.

And we know further we know but Pt is approximately equal to P not, once again because we have seen that the process of sound propagation is such that incremental pressures are extremely small compared to the ambient static pressure which is atmospheric pressure and similarly incremental changes or changes in volume and we know that Vt is for similar reason is approximately equal to V not. So using these approximations what we get is partial derivative of pressure with respect to time is minus P not over V not times d Tau over dt.

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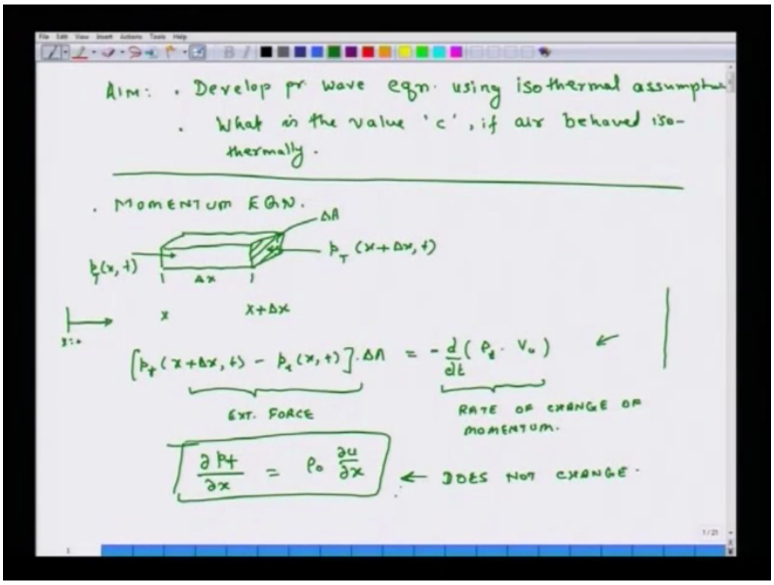

So we have developed the force balance equation or the momentum equation and we have developed the equation which dictates how the gas is behave in this case and this equation is called equation A and this equation as B, what we find is that the material constitutive behaviour has changed because here gamma does not appear because we have isothermal process.

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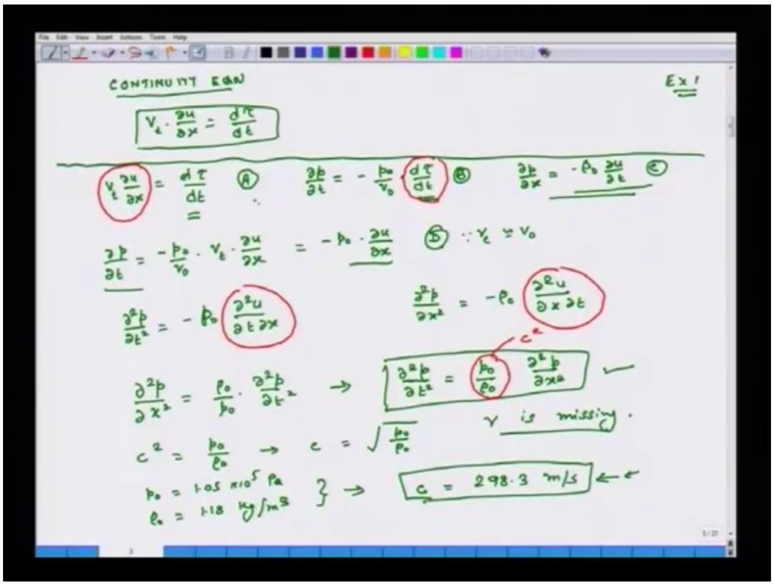

And finally we have a very quick look at the continuity equation and if we do the maths correctly we will find that the continuity equation also does not change. So the only question which is changing is the material constitutive equation. So the continuity equation remains as is and it can be written as Vt times del u over del x equals total time derivative of volume with respect to total time derivative of volume.

So we have these 3 equations, this is my material constitutive equation let's call this equation A then I have partial derivative of P with respect to t equals minus P not over V not times d Tau or dt this is B and the third equation is del P over del x equals minus delta del u over del t this is equation C, so these are the 3 equations and what we are going to do is that from these 3 equations we will solve for pressure in terms of x and we are going to eliminate Tau.

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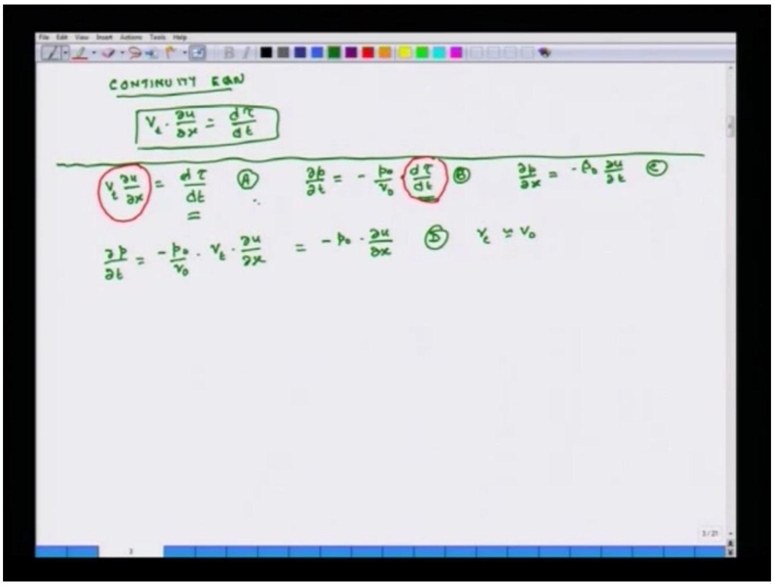

So if we looked equations A and B we have this d Tau over dt here and d Tau over dt here, so from these 2 I'm going to eliminate Tau and what I get is from this equation I get del P del t times and then this d Tau is same as this number, so I'm going to replace d Tau over dt and replace it by velocity times du over dx. We also know that Vt is approximately equal to V not. So what I get is minus P not times Px, so let's call this equation as D.

Now what we do is we differentiate D with respect t, so we differentiate this relation with respect to time and then this relation with respect to x and then we again try to eliminate u from these 2 relations. So if I differentiate equation D with respect to x with respect to time what I get is second derivative of pressure with respect to time is equal to minus and then from this equation what we get is, so now we are going to differentiate C with respect to x.

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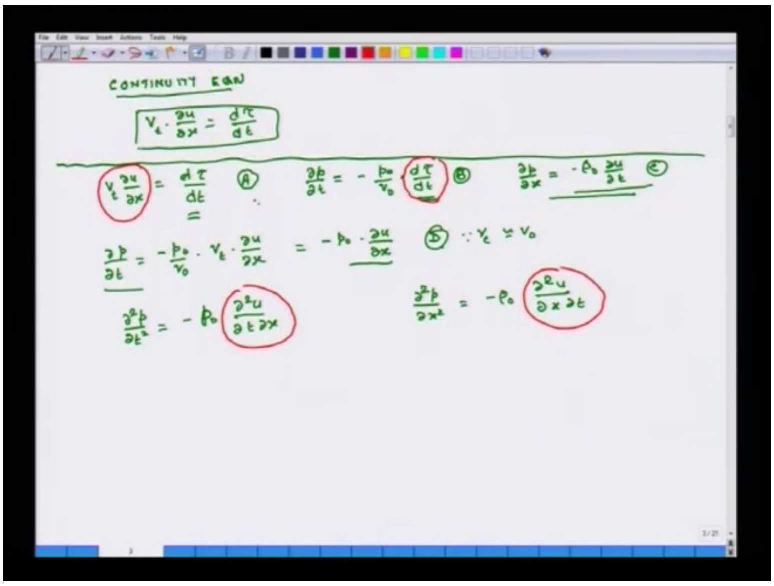

So what I get is, now once again these 2 terms, this term and this term they are going to be same as long as my u the function u and its derivatives are continuous in both in space and in time. So for that particular condition partial derivative of u with respect to t and x is same as partial derivative of u with respect to x and t and if that is the case then I can eliminate this term partial derivative of u with respect to x and t.

And what I get is, so this is my wave equation for pressure and once again we notice that gamma is missing from this case and what that mean is that the system is isothermal and that is why gamma is missing and this is how the sound is going to get propagated.

If sound propagation was a purely isothermal phenomena and the other thing we have to we had to find was what is the value of C? So just as what we had found out earlier in case of adiabatic system in this case c square is nothing but this term, so c square equals P not over rho not or c equals P not over rho not the whole thing square root and because P not equals 1.05 times 10 to the power of 5 pascals and rho not equals 1.18 kg per cubic meters what I get is c equals 298.3 meters per second. So this is the value of c which Newton estimated that this is how fast sound is going to be travelling.

But of course his prediction did not match with reality and as a consequence people had to relook at this whole procedure and what they found was that in this whole thing this term gamma was missing because the assumption that gas behaves isothermally as sound propagates was an inaccurate assumption and as a consequence what does prediction for sound's velocity comes out be incorrect. So this is the first example of problem wanted to cover in today's class.

So the second example which we are going to do is about a cylinder. So what we have is a cylinder and this is at room temperature there is air inside the cylinder and what we are going to do is, we are going to, so this is cylinder which is closed on one end and open on the other end. So at the open and we are going to put a piston which is airtight, so no air comes out of it.

And we are going to have a piston with a finite mass and because of this mass the air is going to get compressed a little bit because the piston moves downwards and as it moves it settles down and we are going to analyze this system and we will try to find out what is a natural frequency of the system? So let me draw a picture for illustration purposes.

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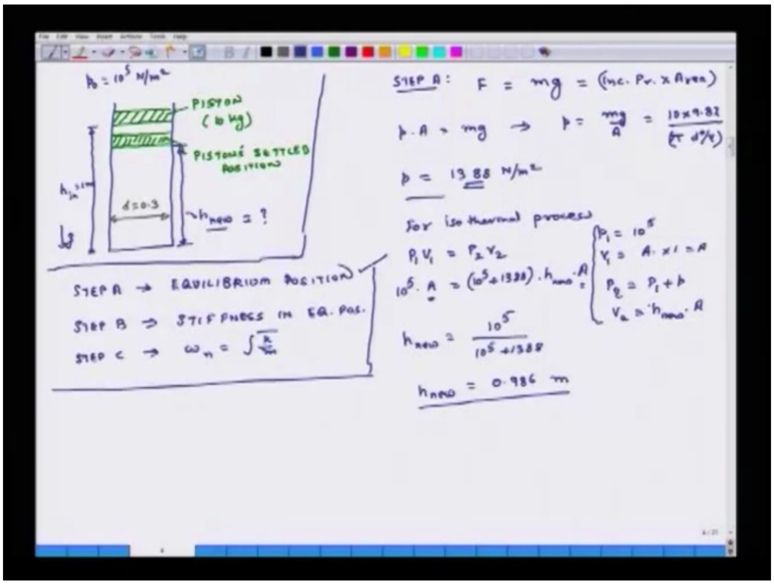

So this is my cylinder and in this cylinder I have a piston, the initial height of the piston, so I call it h initial is 1 meter and of course gravity is acting in the vertical direction downwards. Right now when the height is 1 meter the piston is not released and at this point of time the pressure outside is 10 to the power of 5 Newton's per square meter. So pressure outside then also in the inside is 10 to the power of 5 newtons per meter square.

Now I am going to release this piston, so the mass of this piston is 10 KGs and I am going to release this piston and as a consequence this piston is going to move downwards and this is going to be the settled position of the piston and the new height is going to be h new. Now once it settles is there in mind that as it moves down it moves very slowly and then it settles down at a final position.

So once it has settled down to the final position then I want to find that if I excite this system the piston because there is air inside it, it will act as a spring so it will try to force the piston out, so it will act as a spring mass system where mass is going to be predominantly the mass of the piston and the stiffness is going to come from the stiffness of air. So it is going to act as a single degree of freedom spring mass system and it will have some national frequency.

So our goal is to figure out what is the value of this natural frequency, so how are we going to solve this problem? So what we will do is, we will solve this problem in 3 stages, step A we are going to find equilibrium position, step B for this equilibrium position we are going to find stiffness exerted by air and then step C we are going to find omega, natural frequency equals k over m and this is how we are going to find out.

So we will start with step A, so that extra force which is being impressed upon air is this external force is F and that equals mg and this force is being balanced by the incremental pressure because once this mass moves downwards, the pressure inside the cylinder is going to increase so there will be some incremental pressure, so this force is going to be equal to incremental pressure times area.

And the area, so this diameter again I have to mention this diameter is 0.3 meters, so we plug-in the values for area mass g and then through that process we can find incremental pressure. So if incremental pressure is P then P times area equals mg and what I get is P equals mg over a and that is equal to 10 times 9.81 divided by pi d square over 4 where d is equal to 0.3 meters and thus what I find is my incremental pressure is equal to 1388 newtons per meter square.

So now I know the pressure in this (()) (31:07) settled position and now I want to know what is the value of h new? So that is my equilibrium position, now we know that this piston is settling slowly and what that means in thermodynamic terms is that all the extra heat as this air is getting compressed it is able to get out and the temperature of air outside the cylinder and inside the cylinder is because it has to remain, so the overall temperature of cylinder does not change, so this is an isothermal process.

So once it is an isothermal process then I can use this relation P1 V1 equals P2 V2 and from that I can find what is going to be my new volume? So for isothermal process PI V1, P1 is initial pressure, total initial pressure not incremental pressure V1 is initial volume is same as equal to final pressure times final volume. Now P1 is equal to 10 to the power of 5, V1 is equal to the volume of the cylinder, so that is A times height and the height is 1, so that is A.

P2 equals P1 plus incremental pressure which is 1388 and then V2 is equal to h new times area. So I put in all these values in this equation and what I get is 10 to the power of 5 times A equals 10 to the power of 5 plus 1388 times h new times A, so A goes away and what I get is h new equals 10 to the power of 5 over 10 to the power of 5 plus 1388 and this what I get is h new equals 0.986 meters. So now I have found I have completed my first step that is step A.

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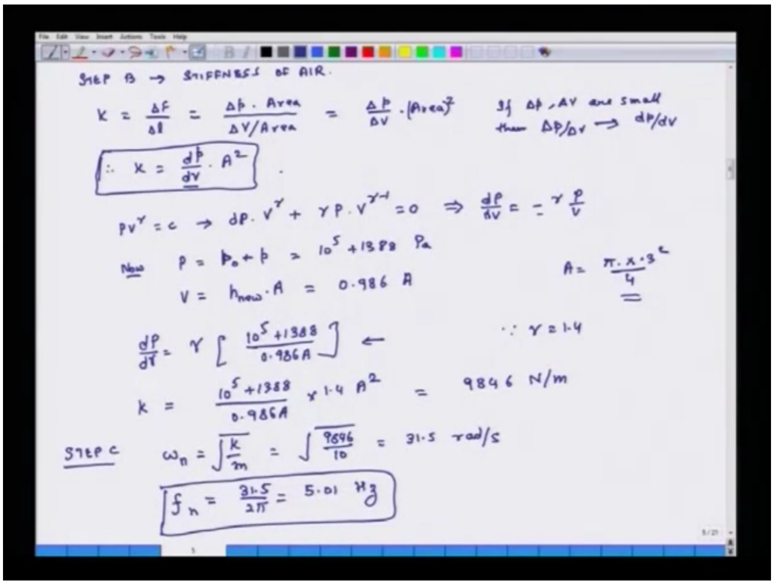

So my next step is to find step B stiffness of the system, stiffness of an air column which is 0.986 meters tall and whose internal pressure is 10 to the power of 5 plus 1388 meters per meter square. So what we know is, that stiffness is equal to small change in force divided by small change in length. So I have an air column I press it a little bit there is an incremental force and also the length shrinks by small amount, so the ratio of this change in force and the change in length that will give me stiffness.

Now this Delta F is nothing but change in pressure times area and this Delta l is basically Delta V change in volume divided by area, so what I get is, now if these changes are extremely small, if Delta P Delta V are small then Delta P over Delta V in the limit of this thing going to 0 approaches dP over dV, so therefore K is equal to dP over dV times A square.

So what we have developed is a relation for stiffness that is fitness equals dP over dV times square of area. So now we have to find what is the value of dP over dV because we already know what is A. Now we know that the compression of gas or air in the cylinder, when this 10 kilogram of mass was settling down it was an isothermal process, now it has settled down and now we want to know whether when the vibration happens the gas is going to get again compressed and you know it will compress and expand and compress and expand.

So again we have to make an assumption we have to make an assessment whether this expansion and compression process is adiabatic process or is it an isothermal process because we may get different results based on our assumption as we saw in the case of speed of sound also. So right now what we will do is, we will assume that this process is an adiabatic process and later we will check the validity of our assumption based on some based on some other approach.

So we will assume that this process of expansion and contraction in gas as these vibrations are happening it is an adiabatic process and with this assumption we will develop a relation for dP over dV. So for an adiabatic process Pv to the power of gamma equals c and if I differentiate it I get dP times V to the power of gamma plus gamma P times V to the power of gamma minus 1 equals 0 and this gives me dP over dV equals minus gamma P over V and the negative sign once again as we saw earlier it indicates that as pressure it is increasing in the system volume contracts and as pressure goes down volume expands. So that is the reason why I have a negative sign here.

So now in our case P is equal to P not plus incremental pressure and that is 10 to the power of 5 plus 1388 pascals, volume equals the height of air column h new times area and that is 0.986 times area. So dP is equal to gamma and I am dropping the negative sign for the purposes of convenience equals, so gamma times 10 to the power of 5 plus 1388 divided by 0.986A and once again A is equal to pi times 0.3 square over 4.

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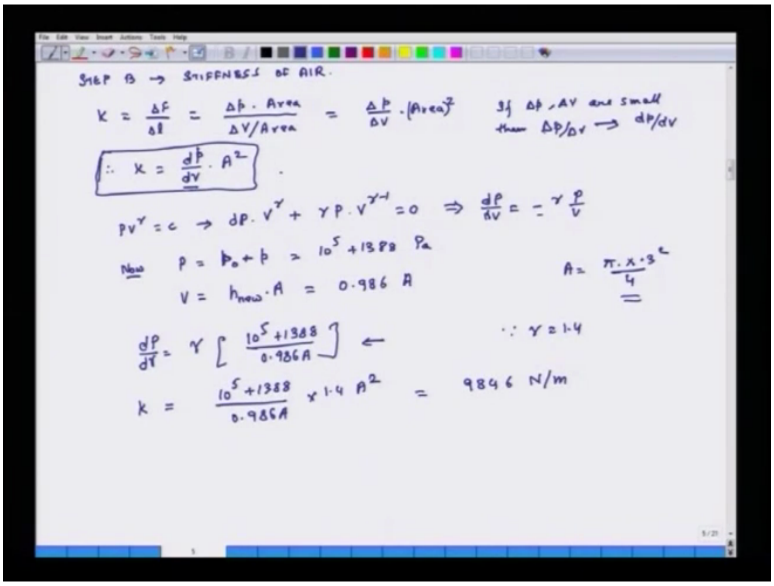

So if I use this relation into this then I get my k equals 10 to the power of 5 plus 1388 over 0.986A times 1.4A square because they are my equals gamma equals 1.4 is a gas constant and if I plot the value of A as this thing then this comes out of the 946 newtons per meter. So now I have completed my second step which is step B, so now I do my step C and that is omega n equals k over m equals 9846 divided by 10 because 10 is the mass of the piston I am ignoring the mass of the air column because if I calculate the mass of air column it comes to be less than 0.8 percent of the mass of state, so mass of the air column is going to have negligible impact on the vibrational characteristic of the system.

So stiffness is 9846 coming from air, mass is 10 coming from piston and this comes to be 31.5 radiance per second and then frequency natural frequency is equal 31.5 over 2pi equals 5.01 hertz, so that is my natural frequency and with this I conclude my today's lecture and I look forward to seeing you in the next class, thank you.