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Lec 37: Performance Map of Superchargers and Numerical Problems

I welcome you all to the session on engine system and performance. Today, we shall discuss the performance map of superchargers, and then we shall solve one numerical problem to illustrate the performance of superchargers, following the technical discussion we had in our previous class. So, we have discussed the performance map of SI engines, and if you can recall, this particular map is very useful for comparing the performance of two dissimilar SI engines. We have discussed this part in one of our previous classes. Today, in the context of superchargers, we shall see why this map is important and what information is depicted from experimental observations or measurements. We shall see whether this performance map can provide us with a qualitative or quantitative estimate about the design parameters or the compressor.

So, if we try to recall, we discussed superchargers and saw that supercharging internal combustion engines, especially high-speed ones, is very important, primarily for better efficiency. We also saw that supercharging is one of the feasible methods by which it is possible to increase the air consumption rate inducted into the cylinder during the intake stroke. What we discussed is that the density of intake air should be increased to raise the mass of air for a given displacement volume of the cylinder, as well as a given speed. If we increase the speed, we can certainly induct a larger mass of air, but increasing the engine speed also raises frictional losses. So, let us discuss what this particular map represents in the context of superchargers. If you try to recall, supercharging is nothing but increasing the pressure of intake air using a mechanical device, which is compression. So, when we talk about the performance map of superchargers, it is essentially the performance map of a compressor or other compressors.

So, if we try to draw the block diagram, let us first discuss this. So, supercharging—This is done by increasing the pressure of the intake air via a compressor. If you recall from the previous class, we discussed that to run the compressor, we again need some energy to be supplied to it. Now, this compressor can be run or operated using a mechanical device, which is called a mechanically driven compressor. So, we need one compressor to increase the pressure of the intake air, and to run the compressor, we again need to supply some amount of energy in the form of work.

That energy should be borrowed from the engine itself. There may be a case where we can use the exhaust gases of the engine to run the compressor through a small turbine, and that is called turbocharging. So, it can be a mechanically charged, mechanically assisted, or mechanically driven compressor or we can have a turbocharged system. So, if we try to draw the block diagram, let us draw it. If this is the engine block, and exhaust gas is taken to run a turbine. From there, exhaust gases come out from the turbine. Now, if we can have a small compressor connected to the shaft of this turbine, so that we can have this compressor (C), which takes air from the ambience, and that compressed air is now fed to the engine cylinder. So, this is the block diagram of a supercharging unit. But essentially, from the schematic depiction, you can understand this is turbocharged. So, if this is the engine, this is turbine (T), and then this is the compressor. So, the idea is to use the exhaust gases.

So, this exhaust gases are used to run this turbine, small turbine and compressor is connected to the shaft of this turbine. So, as the turbine is rotating, this turbine is acting to drive the compressor rather the power output from the turbine is used to run the compressor. And compressor is used to compress the air and that compressed air is supplied to the engine. So, this entire unit is known as turbocharged. Sometimes that I should mention here that here one particular device is used and that is known as After cooler. So, this is the turbocharged. Essentially, we are talking about performance map of the superchargers means performance mark of the compressor itself. Because we need to know about this particular device, that means we need to supply certain amount of energy to run this unit, that is this compressor. At the cost of that input energy, what is the rise in pressure of the intake air and that is important to know.

So, if we look at this map and this map is essentially the performance map of the superchargers. As I told you before, performance map of superchargers is nothing but the performance map of compressors. So, what you can see from this performance map? Essentially, idea is to obtain pressure ratio that you can see along y axis. We need to know the pressure ratio.

So, we need to know the pressure ratio that is

$$r_p = \frac{P_e}{P_i}$$

So, exit pressure of air to the inlet pressure of air. So, as the air, passes through the compressor, there is a rise in pressure. So, the air pressure at the exit of the compressor to the pressure of air at the inlet to the compressor, this is known as pressure ratio or r_p.

So, what we are trying to have in this map to obtain the pressure ratio for different values of mass flow rates. So, that means if we keep on increasing mass flow rate of intake air then what would be the value of pressure ratio for different speeds of the compressor and if we can collect all the data and if we can represent all this data and that map is which will be obtained is known as performance map. So, if we look at this performance map carefully we can see that in this map, there is a maximum efficiency line so if we mark this, so this is the maximum efficiency line that we can see there are isoefficiency lines are presented using green colored line and in addition to these two different lines that is maximum efficiency line and isoefficiency lines, in this map, ISO speed line, these two lines are also predicted or depicted in this map.

Most importantly, if we look at, we can see there is a particular boundary, which is a line denoted by dotted red colored line which is known as surge line. We shall discuss about this particular line in this context. Also, you can see a choke line which is shown here using blue colored dotted line. So, these two lines are very important to know before we go to solve a particular numerical problem.

Now, as I told you, let me explain once again what is done. The entire objective is to obtain a pressure ratio with a change in mass flow rate. And if we do this at a given speed, we will get one particular line. If we perform this task for different speeds, then we will get this map, which is the performance map.

So that means the first objective is to get the pressure ratio, experimentally of course, or the rise in pressure of air as it passes through the compressor for different values of mass flow rate at a constant speed. If we do these experiments or if we perform this task, for different speeds, then this map can be obtained. In this map, in addition to these isoefficiency lines, there are two different lines as well that we can see from this map. One is the maximum efficiency line, and the other lines are isospeed lines.

What we can see towards the extreme left of this map, there is an important line, denoted by a red-colored dotted line. And this line is known as the surge line. So, this is essentially a check mark or check line because a designer or operator should not allow the compressor to run beyond this line. So that means this is the check line.

While this line, which is known as the choke line, shown here using a blue-colored dotted line, is also very important to know. As such, these two different lines are the border lines of compressor operation. So, the operator should run the compressor between these two lines, at any point between these two lines. Now, let us talk about the significance of these two lines, that is, the surge line and the choke line.

The surge line is very important because surging is really an undesirable phenomenon in the context of compressor operation. It is nothing but oscillation; as the air is allowed to pass through the compressor, it oscillates between the inlet and exit of the compressor, and this phenomenon is associated with flow reversal as well. So, this is not desirable at all. So, the surge line is obtained. So, we need to know a particular mass flow rate for a given speed for which we will obtain this surge line.

So, if we take any point on this line, the compressor will not be able to run stably. So, it would be unstable operation. So, if we go further leftwards from this line, then it is highly unstable zone. Stable zone is rightward of this particular line that is surge line. Now let us come to this particular line that is choke line. So, choke line is essentially a line which is also demarcating compression operation from non-operation zone to smooth operation zone.

What is this line? We have studied about choking in the context of nozzle. So, we have studied that in a convergent divergent nozzle, if we decrease the pressure at the exit, that information should reach at the inlet of the nozzle to increase the mass flow rate. Because flow rate is dependent on the pressure drop.

So, if we create even further drop in pressure at the exit of the nozzle, that information will not be sensed by the nozzle inlet to increase the mass flow rate because that is called, sonic state will be achieved. So a further decrease in, so there is a threshold value of pressure ratio exit pressure and if we reduce exit pressure beyond that threshold value there will be no change in mass flow rate through the nozzle and the nozzle is said to be choked similarly here also it is if we increase the mass flow rate the rise in pressure ratio will not be there rather the pressure ratio drops so what we can see, this particular line is representative of those points where a further rise in mass flow rate will not allow the rise in pressure of that air mass. And if we can take the locus of all these points, then we will get this line that is choke line.

So, that means if we allow the compressor to handle a larger amount of air mass, but the the rise in pressure will not be there; rather, the pressure rise drops. So, there is a sudden drop in pressure rise with increasing mass flow rate of air. And with increasing mass flow rate of air, the compressor speed will increase, frictional losses will increase, but the eventual gain will not be there because you can see from this, if we traverse along this line—you can see though the mass flow rate increases, but the pressure rise is not that significant, so pressure rise drops. So, this is again a very important line to be taken into account for the compressor operation. So, what we can see from this performance map is that this map gives us qualitative information about the compressor operation zone.

So, the operator should know that the compressor can be operated anywhere between these two lines, that is, the surge line and the choke line. So, this map is very important to give us some qualitative information about the safe operation or safety operation of the compressor.

So, with this, let us move to solve a numerical problem to illustrate the theoretical part that we have discussed in the previous class in the context of supercharging. So, if we go to the problem statement, let me read the problem statement first, then we shall solve this.

Problem: An 8-cylinder, 4-stroke Diesel engine operates at 2400 rpm. It is provided with a turbocharger and an aftercooler. The bore and stroke length of the engine are 125 mm and 155 mm, respectively. The volumetric efficiency is based on inlet manifold conditions of 1.8 atm and 325 K temperature after the aftercooler is 0.85. The compressor and turbine isentropic efficiencies are 0.75 and 0.6. The ambient conditions are 1 atm, 300 K. Calculate the following: the power required to drive the compressor of the turbocharger. If the exhaust gas temperature from the engine is 950 K, calculate the pressure at the turbine inlet. Assume that the turbine exhausts to the atmosphere.

Solution: Many data are given in this problem statement. As such, we can gather much information from this problem statement. So, what we can do first is let me schematically draw the turbocharging unit, or supercharger, which is equipped with a supercharger or turbocharging. Basically, the compressor will be operated using a small turbine, and the turbine will be driven by the exhaust gases of the engine. That is what is important. So, let me solve this problem.

If we draw the schematic we will have one turbine, and this turbine is operated or driven by the exhaust gases. These exhaust gases are coming from the engine. Then, it is given that the turbine exhaust goes to the atmosphere. So, this is the turbine exhaust. The turbine is connected. The small turbine will now operate the compressor.

So, typically, turbines and compressors are schematically shown like this. In the turbine gases expand, while in the compressor gases are compressed. So, this is the compressor inlet intake air from the atmosphere. Then, the compressor compresses the air, and that air is taken to the aftercooler. Finally, it is taken to this engine.

So, this is the block diagram. This is the compressor, and this is the common shaft, and this is the aftercooler. This aftercooler is provided, if you can recall the discussion we had in our last class. When the compressor compresses the intake air, the temperature also increases. If the temperature increases, the rise in temperature will subsequently increase in the engine itself. Because this air will be taken during the intake stroke to the engine, so this is the engine.

And if the temperature of the intake air is higher before it enters the engine, subsequently its temperature will rise further and it will create some operational issues for the SI engines because it will create knock or detonation. So, that is not a desirable phenomenon at all. So, what is done, is this particular unit that is after the cooler is incorporated here to reduce the temperature of the intake air before it enters the engine.

So, this is the schematic depiction and this entire unit is known as a turbocharger or turbocharger. So, this is a turbocharged compressor. The compressor can be mechanically charged or mechanically driven. But in this case, the compressor is driven by this turbine, a small turbine.

And this is known as a turbocharged compressor. So, let me write turbocharged compressor. So, if we go to solve the problem, what we need to first calculate is the stroke volume. Stroke volume

$$V_s = \frac{\pi}{4}(d)^2 L$$

So, the bore diameter is 125 mm and the length is 150 mm. So, if we use this value, that is

$$V_s = \frac{\pi}{4}(0.125)^2 \times 0.15 = 1.83 \times 10^{-3}\ \mathrm{m}^3$$

So, that is the stroke volume. Now, if we can calculate the inlet air density based on the manifold condition. Go back to this statement once again. It is given the volumetric efficiency based on the inlet manifold conditions are 1.8 atm pressure, 325 K temperature, and the volumetric efficiency is 0.85. So, what we can do is, we can calculate the density of inlet air based on the manifold conditions, this

$$\rho_{in} = \frac{P_{in}}{RT_{in}} = \frac{1.8 \times 10^5}{287 \times 330} = 1.9 \text{ kg/m}^3$$

So, this is the density of the inlet air based on the manifold conditions. Now, having calculated the density of inlet air based on manifold conditions and knowing the stroke volume or displacement volume—we can calculate the amount of mass to be inducted theoretically into this cylinder or the engine. Theoretically, the mass flow rate of air per cylinder per cycle would be how much? The stroke volume or displacement volume, that is

$$= 1.83 \times 10^{-3} \times 1.9 = 0.003477 \text{ kg}$$

So, this is the theoretically calculated mass of air to be drawn into the engine cylinder per cycle. So, now let us calculate the mass of air. So, this is the theoretical mass of air. Now, if you would like to calculate the mass flow rate, then we need to know the RPM. So, if we go to the next slide, having calculated the theoretical mass of air, now we can calculate the actual mass of air per cylinder per cycle. That equals, because we know the volumetric efficiency, so we know this volumetric efficiency equals the actual mass of air. This is defined as the ratio of the actual mass of air or mass flow rate of air to be drawn into the cylinder to the theoretically calculated mass of air. So, if we remove now this mass flow rate, so this is the volumetric efficiency,

$$\eta_{vol} = \frac{m_{actual}}{m_{theoretical}}$$

$$m_{actual} = \eta_{vol} \times m_{theoretical} = 0.85 \times 0.003477 \text{ kg} = 2.95 \times 10^{-3} \text{ kg}$$

This is the actual mass of air per cylinder per cycle, that is very important. If we know in the problem statement, it is given there are eight cylinders. So, if we go to the next slide, so eight-cylinder, four-stroke cycle engine and then we can calculate the actual mass of air per cycle, that is,

$$= 8 \times 2.95 \times 10^{-3} = 0.0236 \text{ kg}$$

And it is coming as 0.0236 kg. So, this is again the mass of or actual mass of air per cycle.

Now, the number of cycles per second. We know the RPM, which is 2400. A four-stroke cycle engine means there are two revolutions in one cycle. So, this is

$$\frac{2400}{60 \times 2} = 20\left(\frac{1}{\text{sec}}\right)$$

So then, we can calculate from this the actual mass flow rate. So, we have written here the mass of air, but now we are writing the mass flow rate of air through the engine per cycle. It is not per cycle, rather per second. We can write here in terms of unit. So, this is

$$= 20 \times 0.0236 = 0.472 \text{ kg/s}$$

So, we get it, the actual mass flow rate of air. Now let us draw an *h-s* diagram because if we look at the unit, if we can recall the unit, we can see there are two devices. First of all, the turbine is there, and then the compressor is there. So, this is the compression of air, and then we have the turbine. So, these two are connected to this common shaft. So, this is the exhaust gas and this is compressor air in, air out.

So, this is the compressor, and this is the turbine. So, this is exhaust gas in, and exhaust gas is out. So, what we can see here is that air is compressed in the compressor while the exhaust gases expand in the turbine. So, if we draw the *h-s* diagram here, what we can see is the actual compression process along with the isentropic one.

So, if we try to draw the *h-s* diagram just for the sake of completeness, if it is *s*, this is *h*, and this is the isentropic 1 to 2s. So, if this is P_2, and if this is P_1, certainly P_2 is greater than P_1, then the actual process would be 2. So, this is the compression process. Similarly, if we try to draw the *h-s* diagram for the turbine, this is for the compressor. This is for the turbine.

And we can define the isentropic efficiency of the compressor as we know that the actual work needed to compress the air should be much higher than the isentropic one. So, it is

$$\eta_C = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} = \frac{(T_{2s} - T_1)}{(T_2 - T_1)}$$

While C_p for air remains constant.

So, for the turbine, if we draw again these processes, then what we can see is this is P_1' prime, this is P_2'. Certainly, you can understand that $P_1' > P_2'$. So, this is the isentropic expansion from 1 prime to $2s'$. While the actual expansion will be like this. So, this is $2'$.

So similarly, you can define here η_T. You can understand that if we use isentropic expansion of the gas, then the amount of work that we will get will be higher as compared to the frictionless or real compression, real expansion process. So that is, it is essentially

$$\eta_T = \frac{(h_1' - h_2')}{(h_1' - h_{2s}')} = \frac{(T_1' - T_2')}{(T_1' - T_{2s}')}$$

So again, if we consider that C_p of, because the turbine handles exhaust gases. So, if we assume C_p of exhaust gas is constant.

So, these two are very important because we will be using these two expressions to solve the problem further. So, as I told you, for a constant C_p of air, we can write compression efficiency as equal to

$$\eta_s = \frac{(T_{2s} - T_1)}{(T_2 - T_1)}$$

So, from there, we can calculate T_2, which is the exit temperature of air at the exit of the compressor, and that would be equal to

$$T_2 = \frac{(T_{2s} - T_1)}{\eta_s} + T_1$$

In the problem statement, it is given that P_1 equals 1 atm pressure. The power required to drive the compressor. Now, ambient conditions. So, these are ambient conditions. T_1 equals 300 K. P_2 equals 1.8 atm. That is the manifold pressure. And T_2 equals 330 K. So, these are given. So, these are the manifold conditions, and these are the ambient conditions. So, if we plug in the value here, we get this is and so then we can have

$$T_{2s} = 300 \times \left(\frac{1.8}{1}\right)^{\frac{0.4}{1.4}} = 354 \text{ K}$$

So, if we plug in the value over here, then we will get

$$T_2 = \frac{(354 - 300)}{0.75} + 300 = 372 \text{ K}$$

So, this is coming as. 372 Kelvin. This is very important. So, if we go to the next slide. Then, we can calculate. The power input needed for the compressor, that equals,

$$Power_{in} = \dot{m}_{air} C_p (T_2 - T_1)$$

$$= 0.472 \times 1.005 \times (372 - 300) = 34.15 \text{ kW}$$

If we go to the previous slide, we can see that actual compression is from 1 to 2. And these two conditions are the manifold conditions that we have mentioned over here. So that means, the power input to the compressor for its operation should be this, and if we plug in the value, because we already have m dot air calculated actual. We have already calculated 0.472 kg per second is the actual mass flow rate of air through the engine, it comes out as 34.15 kilowatts. So, this is the answer to the first part of the question. Then, we have to calculate the second part of the question, which is to determine the exhaust gas temperature from the engine, given as 950 K.

The exhaust gas temperature is 950 K. Then, you have to calculate the pressure at the turbine inlet. So, what we can do is assume this is given. We can assume that there is no loss of power, as the power developed by the turbine is transmitted and given to the compressor. So, there is no loss of power. If we assume that, then, considering the *T-h* diagram for the turbine. This is the power output or the power input needed for the compressor. We are assuming that the input power of the compressor, or the power given to the compressor, is exactly the output power of the turbine.

If that is the case, then we can write

$$34.15 \text{ kW} = \dot{m}_{exhaust} C_{p,exha,gas} (T_{1'} - T_{2'})$$

Actual. So not isentropic. So, this is the isentropic. This is the actual expansion in the turbine equal to 34.15. So, let me draw the *h-s* diagram once again. So, this is what we could consider. So, this is 1′, this is 2′. So, we are just to avoid confusion, we are giving 1. So, for example, the expansion is taking place from 1. So, this is $P_{1'}$, this is $P_{2'}$, this is 1′, and this is isentropic expansion while this is actual expansion. So, we are going to consider the actual one because frictional losses will be there. So, this is the expression. Now, from here we can calculate easily that the

$$\frac{34.15 \text{ kW}}{\dot{m}_{exhaust} C_{p,exha,gas}} = (T_{1'} - T_{2'})$$

We know that mass flow rate of air. So, this is again

$$\frac{34.15\ \text{kW}}{0.472 \times 1.147} = (T_{1'} - T_{2'})$$

$$(T_{1'} - T_{2'}) = 63.08\ \text{K}$$

So, we are assuming that mass flow rate of exhaust gas, so assuming that mass flow rate of the exhaust gas is equal to the mass flow rate of air. Certainly, during intake strokes, the mass of air will be inducted, then certain amount of fuel will be spread, but the amount of fuel is not very high. So, we are assuming that mass flow rate of air that is drawn in to the engine cylinder, inducted in the engine cylinder is equal to mass flow rate of exhaust gas. So, if that is the case, then we can calculate this is coming as 63.08 and that equal to $(T_{1'} - T_{2'})$. So, if we talk about isentropic efficiency then from this we can calculate this equal to

$$(T_{1'} - T_{2s'}) = \frac{(T_{1'} - T_{2'})}{\eta_{s,T}}$$

That is actual expansion divided by isentropic expansion. So, from there we can calculate easily, here it is given $T_{1'}$ equal to 950 K that is exhaust gas temperature that is the inlet temperature of exhaust gas or exhaust gas temperature at the inlet of the turbine is 950 K and so this is given and then already we have calculated $(T_{1'} - T_{2'})$ from this expression. We know the isentropic efficiency of the turbine from there, we can calculate this fellow. So, if you go to the next slide, we can easily calculate

$$(T_{1'} - T_{2s'}) = \frac{63.08}{0.6} = 105.14\ \text{K}$$

It is coming as

$$T_{2s'} = (T_{1'} - 105.14) = (950 - 105.14) = 844.85\ \text{K}$$

This is the case if we assume gamma exhaust gas equal to 1.33 then we can use isentropic relation that is

$$\frac{P_{exit,T}}{P_{in,T}} = \left(\frac{T_{2s'}}{T_{1'}}\right)^{\frac{1.33}{0.33}}$$

$$P_{in,T} = P_{exit,T} \times \left(\frac{T_{1'}}{T_{2s'}}\right)^{\frac{1.33}{0.33}} = 1.58\ \text{atm}$$

It is coming as 1.58 atmospheric pressure. So, this is the final answer to this question, that is the inlet pressure that we had to calculate, what is the pressure at the inlet of that turbine inlet. So, this is 1.58 atmospheric pressure. So certainly, that exhaust gases are fed into the turbine, and the pressure of the exhaust gases is not equal to the atmospheric pressure, which you can see from this calculation as well, that the exhaust gas pressure is a little higher than the atmospheric pressure.

So, to summarize today's discussion, we have talked about the performance map of superchargers, which is nothing but the performance map of compressors. Thereafter, we have, and while we have discussed this performance map, we have seen a few salient points. Thereafter, we have solved our numerical problem to illustrate the theoretical part that we have discussed in the context of supercharging.

So, with this, I stop here today, and we shall continue our discussion in the next class.

Thank you.