Course Name: Engine System and Performance
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Week - 10
Lecture – 36

Lec 36: Introduction to Superchargers

I welcome you all to the session on engine systems and performance. Today, we shall discuss superchargers. We have learned from our internal combustion engine course that superchargers are typically used to compress the intake air, and this particular device or component has manifold applications. But today, we shall discuss this particular element in the context of internal combustion engines. We shall see what different methods are available to increase or compress the intake air following this method, which is supercharging. Then, if we use a supercharger, or if we make an effort to supercharge the intake air, what are the possible outcomes? What are the different detrimental effects as well? And finally, we shall discuss a typical layout of this particular component—supercharging. So, to start with, superchargers are not a recent invention. Rather, the history of supercharging dates back to eminent scientists and engineers in this field—Gottlieb Daimler and Rudolf Diesel.

So, Gottlieb Daimler first supercharged his engine, as is apparent from the patent filed by him way back in the year 1885. What he did to achieve supercharging, we shall discuss this part. But supercharging of high-speed gasoline engines is as old as its type.

It is Gottlieb. Damlier and. Rudolf. Diesel. They used supercharging essentially with a specific objective in mind and that is essentially to increase the density of the intake air, so that the mass of air can be increased. Because if we can recall the amount of air that will be drawn into the engine cylinder is essentially the volume or displacement volume. So, if you would like to increase the mass of air, we have to increase the density of intake air. So, for a fixed volume, if we increase the density of intake air, mass will increase. And if we can increase mass of air, larger amount of fuel can be ignited, which in turn will ensure greater power output. So, Gottlieb Daimler, he first, he had supercharged his engine, his first engine, as reported in his patent. This patent number is DRP 34926 in the year 1885. So, you can see that the concept of supercharging high-speed gasoline engines started long ago. So, this is not really a new invention. And we have also seen that supercharging high-speed gasoline engines is as old as the engine itself.

So what Daimler actually did? If you recall, utilized the movement of the piston between two centers: top dead center and bottom dead center. Now, when the piston is returning from TDC to BDC, perhaps during the intake stroke or power stroke, So, if we can use the downward movement of the piston—or rather, the end of the piston—the piston top will always be in contact with fresh charge, combustion products, etc. But at the bottom end of the piston, if we can use the downward movement of the piston, to compress the charge—that means if we can have a cavity in the crankcase and allow charging of mass through it—that compression can be achieved by the downward movement of the piston. That is what Daimler did long ago, and he found that utilizing the downward movement of the piston is essentially what we know as a two-stroke engine. So, the downward movement of the piston, if used to compress the charge, can eventually result in a greater mass of air being drawn into the engine cylinder. So, that was his concept. Now, we know very well that supercharging certainly has a few advantages. On the contrary, it also has a few disadvantages.

So, supercharging, first experienced an absolute peak in regard to the power output. So, we can tell that having, supercharging of the intake here, it was really possible to increase the power output. So, it experienced a peak in the output. In regard to the power, also, if it is the engine which is used, say, it was seen if we go back to the next slide, that means we all know that if the engine attains a higher altitude, certainly the density of the air will reduce, so then the mass of air that will be entrapped during the intake stroke inside the cylinder volume will be less because of low density.

So, at high altitude, the supercharging experienced even better performance, and that was observed during the Second World War. So, supercharging the intake air is not new; rather, the concept has started long back, and supercharging, experienced a peak in power development. It helped to have better performance even when engines attained higher altitudes and it is because of the supercharging of high-speed gasoline engines, rather I should say engines because engines experienced the first absolute peak in regard to output power. It helps to obtain better performance when engines or attained high altitude. It has been observed that the brake mean effective pressure (BMEP) that is, brake mean effective pressure values were obtained or reached up to 23 bar. Now the question is, so supercharging has all these favorable aspects. Supercharging can be done by using a mechanical compressor, which means a mechanical turbo compressor. So, we need to have one mechanical element or device that is essentially a compressor to compress the air. We can also use root blowers. So, we need to compress the air and increase its pressure. That can be done by using a mechanical turbo compressor, or there is a provision of using exhaust gases to run a compressor via a small turbine. So, that is known as turbocharging. Essentially, if we use exhaust gases that come out from the engine cylinder, and if it is possible to run a small turbine using these exhaust gases, the turbine can be connected to the compressor. This particular combination or arrangement is known as an exhaust gas-operated compressor.

So, whether the compressor, which will be used to increase the pressure or mass of intake air, enhancing the density of intake air, this compressor can be run either by using a mechanical turbo system or by using exhaust gases. So, using exhaust gases to run a compressor was first introduced in the market in the US, and that was first introduced by Chevrolet. Chevrolet is an automobile company, and that company first introduced an exhaust gas-operated supercharging unit in the year 1960. So, what we can understand is that our objective should be to increase the consumption of air. Since the displacement volume—that is, the movement of the piston from top dead center to bottom dead center—remains the same, for a given displacement, if we need to increase the mass of air, we certainly need to increase its density. This can be done by using supercharging, and the greater the mass that can be entrapped inside the cylinder, the larger the amount of fuel that can be ignited, resulting in higher power output. We know that if a larger amount of fuel can be burned, the indicated power output will be greater. Thus, the indicated power output is directly proportional to the mass of air consumption.

Brake horsepower is not directly related to the mass of air or air conjunction, but still that for a given other losses like frictional losses, this brake horsepower is also indirectly dependent on the mass of air to be entrapped inside the cylinder because higher is the indicated power higher will be the brake power. So, though brake horsepower is not directly dependent on the consumption of mass or air that should be entrapped inside the engine cylinder, but it is indirectly dependent on-air consumption. So, we shall now see what are the different methods by which mass of air or air consumption or air to be entrapped inside the engine cylinder during intake stroke can be increased.

So, possible methods available to increase the air consumption of an engine. So, if we list down the method, first one is essentially increasing the displacement volume. So, the greater amount of mass can be drawn into or inducted in the cylinder. So, but if we try to increase the piston displacement, understand that the engine will be bulky cylinder will be having not little rather larger length and weight. Also, will be more so this is again not a viable option probably to increase the air consumption of an engine. So, this is not possible, we will increase weight of the engine and if we increase the engine cylinder length cooling of engine will be a problematic issue. Then number two option could be running the engine at higher speed, this could also be an option to increase air consumption of an engine but if we allow engine to run at a higher speed then certainly frictional losses will be more over and above large inertia stress will be imposed on several parts of the engine because piston is moving at a high speed. So, the stress due to inertia will be more and that is known as inertia stress, that will be rather larger inertia stress will be imposed on several parts of the body so this is again not a viable option.

Larger frictional loss, and this is called greater inertia stress—stress due to higher inertia. So, these two problems or problematic issues are there. And the third option, that is increasing the density of air or charge, rather. That is what I was talking about in the beginning of today's class. So, if we can increase density, even if we keep the displacement volume fixed, speed fixed, density will ensure a higher mass of air to be consumed during the intake stroke, and we can really ignite or burn a larger amount of fuel. So, this could be a viable option because, if we use this, then we can write that greater mass of air or charge can be inducted into the same volume associated with piston displacement, associated with the piston displacement and speed.

As such, this option is a viable option. So, out of these three different methods that we have discussed—that is, increasing the density of air or charge—if we can increase density, then a greater amount of charge can be inducted into the same volume associated with the piston displacement and engine speed. And this option can be a viable option among the three options that we have discussed so far. So, essentially, this is nothing but the use of supercharging. So, that means we need to use a mechanical device to increase the density of the incoming air or intake air.

And that can be done by increasing the pressure. So, if we increase the pressure, essentially, density will increase. And if the density increases, a greater mass of air consumption will be higher, and a higher amount of fuel can be burned. So, this is what it is. Now, the question is, if you'd like to increase the density of air, essentially, to obtain a higher amount or higher mass of air to be consumed or to be inducted, is there any specific requirement for the engine designs needing to be changed? No, because the designs of the engine will remain more or less the same, but what we need to ensure is that If we increase or if we need to increase density, via the development of pressure using a compressor, it must be ensured that engine components or parts will be able to withstand that high pressure. So, this is what is important to ensure. The device used to increase the density of inlet air is known as a supercharger. So, we will be using this particular method to increase the air consumption rate or mass of air to be drawn into the engine cylinder. Now, this particular device or component can be used for both two-stroke and four-stroke cycle SI and CI engines. But there are some issues, one important point in this context: we all know that the compression ratio of an SI engine is limited to 10 or even 12.

So, why it is so? Because for the SI engine if we increase the compression ratio, then the rise in pressure at the end of the compression stroke will be more, and that pressure may lead to an important undesirable event, which is known as detonation. So essentially to prevent detonation, what is done is the compression ratio is kept low. So, now question is if we use supercharger for the SI engines, whether it is a two-stroke cycle or four-stroke cycle engine, then supercharging will increase the pressure of the intake air.

Now, that air pressure again will increase during compression stroke. So, as I said that compression ratio is limited or it is low for the SI engines. And hence, if we again increase the pressure of the incoming air, then probability of having detonation will be more for the SI engines if we use supercharging. So, what is typically superchargers are used for SI engines when SI engines are taken at higher altitude. Because at higher altitude, air density will reduce automatically with height and at that height even if we increase supercharger or supercharging, the rise in pressure will not create any harmful effect to the engine in terms of detonation. So, that means supercharging can be used for SI engines, if the SI engines are taken for higher altitude applications. That means SI engine-based aircraft or airplane can use superchargers without inviting problem associated with detonation. So, this is most important point that I had mentioned. Now, let us discuss about if we really need to go for supercharging, though it will be having some points or some effects in the context of SI engine operation that superchargers cannot always be used for SI engines. If the SI engine or SI engines are used for high-altitude applications like SI engine-based airplanes or aircraft, then certainly superchargers can be used even without inviting, problems associated with detonation. But then, if we need to go for supercharging, what are the different favorable aspects or even detrimental effects associated with this? So, effects due to supercharging that means, if we use a supercharger, we will have several favorable and even several detrimental effects. So, let us now list down. Number one is, it increases the mixing of the air-fuel mixture or mixing of the air-fuel mixture, so that means it helps to obtain a homogeneous mixture of air-fuel for the SI engines or even for the modern CI engines.

Since the fuel is injected through a multi-port fuel injection system. Sometimes, fuels are injected even in the intake manifold. And the fuel which is injected is allowed to mix with the incoming air stream. So, we need to ensure that the fuel which is sprayed into the incoming air stream should be able to mix with the air stream properly to ensure a homogeneous mixture.

If we need to ensure that, certainly some chaotic environment should be there in the intake manifold. To some extent, the flow environment should be of a turbulent type. Now, if we use supercharging, the entire purpose is to increase the pressure. If pressure increases, certainly the velocity of the intake air will increase, though the intake diameter remains the same. That higher velocity will create a turbulent flow environment, which in turn will ensure chaotic mixing of the air-fuel mixture. Rather, it will promote the mixing quality of these two different streams, that is, the air stream with the injected fuel. So, that is the favorable aspect. So, it will ensure uniform distribution of charge in the cylinders, and number two is a homogeneous mixture of air-fuel. So, this is the favorable aspect.

Now, number two is the temperature of the charge will increase. The temperature of the charge will increase because compression will lead to an increase in pressure accompanied by an increase in temperature. So, this temperature will have some favorable and some detrimental effects. Favorable effects are, that if we increase temperature, since fuel is injected in the intake manifold, the higher temperature will allow fuel droplets to completely vaporize. So, it will help better vaporization of the fuel droplets spread into the intake manifold. Now, what would be the detrimental effect?

With increasing temperature, the charge or incoming charge temperature will be higher. And if the temperature becomes higher, that may become vulnerable for the initiation of detonation when that charge is compressed further. Because the charge will be drawn into the cylinder, and that will be drawn into the engine cylinder during the intake stroke. In the intake stroke itself, if the charge has a higher temperature, the temperature will increase even further during the compression stroke. So, essentially, the chance of having detonation will be greater with this particular effect, that is, increasing the temperature of the charge.

So, despite the favorable aspect of better or more efficient vaporization of fuel droplets in the intake manifold, higher temperature may lead to the possibility of detonation in SI engines. On the other hand, this higher temperature can reduce the possibility of detonation in CI engines. So, these two things are there.

Then, the third is the most important, that is power consumption, the power required to drive the superchargers. So, for this, a rise in temperature has several effects. It will promote vaporization of fuel droplets in the intake manifold, but a rise in temperature will increase the temperature of the charge even further during the compression stroke, and that eventual rise in temperature may become vulnerable for the onset of detonation in SI engines, but this rise in temperature will reduce the possibility of detonation in CI engines.

Now, let us come to this particular point: the power required to drive the superchargers. Now, we know that it is a mechanical element, a mechanical device. So, the compressor will run, taking some power. So, we need to supply a certain amount of energy in the form of work to drive the compressor. So, that work will be consumed by the compressor from the output work of the engine itself.

So, we are trying to increase the power output by increasing the air consumption rate. Now, out of this, output power will be increased because of the supercharging. If the compressor itself consumes a significant part of this power, eventually, it will have no gain. But on the other hand, what will happen? That the entire assembly of the supercharging unit will increase the overall weight of the engine. The operation will be noisy, so what will happen? We will be having, a little larger weight of the engine. The operation will be noisy, but on the other hand, the total gain will be zero. So, this part should be taken into account when someone is designing the supercharging unit, so that the power consumption by this unit should not be equal to or more than the power being developed because of this unit.

Supercharging. So, to increase the density of the intake air, we need to have some mechanical components. Typically, these are compressors, or you can use root blowers. So, now, let us look into the mechanical devices which are typically used to compress the intake air, essentially to have better performance of the engine.

But, this part should be taken into account so that the effective power output can be realized with the use of superchargers. So, if we go to the next slide, then we can see, typically, this is the schematic depiction of root blowers. So, positive displacement type compressors or even root blowers. These are used. You can understand this is two lobes.

Probably you have studied all these things in the context of turbomachines because all these are turbomachines and this is three-lobe. So, what happens that, this is the, intake air. So, air is taken to this turbine. Blower through this path and when air is passing through this device the movement of the lobe, this is two lobes, so in the course of its motion inside this device, it pressure increases essentially, rotating devices and we are trying to rotate this taking some power from the engine shaft.

So, that power is imported to the incoming stream that is air stream and receiving that power, air density or air pressure increases. So, this is two lobe roots blower, this is three lobes. All these are positive displacement type as if there is a positive displacement of the air volume, which was entrapped and then there will be positive displacement of the air volume which is entrapped here and then it would be the air volume will be compressed. So, this is root blower. So, superchargers may include a positive displacement type of compressor or even centrifugal compressor, coupled, directly to the engine.

So, it is not always a necessity that superchargers should include only positive displacement type. It may also include centrifugal type compressor coupled directly to the engine. So, in the schematic depiction, we are showing roots blower and these are positive displacement type.

In a positive displacement type, we have studied perhaps in our undergraduate fluid machinery course, there is a positive displacement of the system boundary. So, if we take at a given particular instant of time, a certain amount of working fluid will be taken inside the machine, and the entire volume will be displaced by the moving boundary of the machine in another stroke. So, in one stroke, a certain volume or certain amount of working fluid will be drawn into the machine, and in the next stroke, the entire volume will be displaced or discharged by the moving boundary of the machine, hence there is a positive displacement of the working fluid. Hence, the name positive displacement comes.

Here, the next slide, in the schematic, we have shown a roots blower, and it has two lobes. Roots blowers can have two lobes or three lobes also. So, if I go to the positive displacement type, though you have studied this particular topic in your fluid machinery course, positive displacement of volume of the working fluid, typically air for this case or gas, positive displacement of volume of gas is entrapped in one stroke and is compressed during another stroke by the movement of the device's boundary elements.

So, these devices can be a blower or a compressor. So, in one stroke, the working fluid will be drawn into the machine, and during another stroke, the entire will be entrapped. In another stroke, the entrapped mass will be compressed and discharged by the movement of the device's boundary elements. So, this is all about the positive displacement type. So, for the superchargers or for supercharging, superchargers may include a positive displacement type compressor or even a roots blower, or superchargers may include a centrifugal type compressor as well, and that is shown in this slide.

So, this is a centrifugal-type compressor. Probably, you have studied centrifugal pumps. So, the mechanism of raising pressure is the same. The only difference is we have studied centrifugal pumps, wherein the working fluid is an incompressible liquid. But here, the working fluid is gas or air.

Now, in a centrifugal pump, we have studied that blades are mounted on a hub, and the hub plus blades assembly is known as an impeller. While for a compressor, a few blades are mounted on a hub, or this hub plus blades entire assembly is known as a rotor for the compressor. So, in a centrifugal compressor, what is done—you can see from the schematic depiction—is that let me show the arrow here, which will help you understand. So, this is basically the direction of air entry into the machine.

So, this is air enters in a direction known as the axial direction. So, this is the axis of the machine. So, parallel to the axis of the machine, air enters into this device. And then the blades are rotating or vanes are rotating and eventually, air is discharged in the radial direction.

So, you can see this is the radial direction. So, this is basically a directional change in the fluid stream, and here the device is using centrifugal action to increase the pressure, which in turn will increase the density of the intake air, and hence this is known as a centrifugal compressor. So, centrifugal compressor, this type of compressor is characterized by axial inlet of air and its subsequent radial discharge, it has this hub plus vanes.

All blades—this is known as the rotor of the compressor, the rotor of the compressor, and it has a group of diffuser vanes where the total pressure rise is equally divided, or the total pressure rise is equally shared by these elements. So, there are a group of vanes which are mounted on this hub and the entire assembly is known as the rotor, and the total pressure developed or total pressure rise is shared equally by all these vanes or blades. So, this is all about the centrifugal compressor, and superchargers may use or may include both positive displacement type compressors, or they may include centrifugal compressors to increase the density or pressure of the intake air. So now, let us look into a typical arrangement of the layout of the supercharger. So, that you can see from the schematic depiction that this is the compressor which is there in the circuit. This compressor can be a positive displacement type; this compressor can be a centrifugal type. But what we can see from this arrangement or block diagram is that the compressor shaft is connected to the shaft of the engine. That means the engine and compressor are mechanically coupled. So, these two components are mechanically coupled, and the output power of the engine will be used to feed some power to run the compressor.

So that means to run this compressor, engine or engines need to supply certain amount of power and that is what i was talking about designers should be careful so that to run the compressor, the consumption of power should not be equal to the additional power being developed because of this. So, this is what it is now and then this compressed air is fed into the engine cylinder essentially to increase the mass of air or air consumption rate. So, let us now, consider this particular unit and we shall briefly recapitulate our basic understanding of thermodynamics to have some mathematical expression of the total pressure rise or total change in or total the amount of power that we need to supply to compressor for its operation. So, if we take out this particular element and if we can say that advantages of this particular arrangement. Advantage is that we can really increase the mass of air to be drawn into the engine cylinder. Disadvantages it can consume significant part of power output so designers will be careful but best thing is it can adjust to change or change in load condition. So, if any engine needs to experience or if the engines experience changing load condition, then this particular part is very very helpful to adjust to the changed load condition. So, if we take out this particular component and draw here. So, we all know the compressor, if we draw quickly we shall solve this, we shall work out this because we have studied all this. So, this is inlet to the compressor and this is outlet of the compressor. If we consider pressure is p i temperature is T i these two different or enthalpy is h i and outlet quantities are P_0, h_0 and T_0, so, that means this compressor is having the shaft and the shaft is connected to the shaft of the engine. So, the incoming air which is taken from the ambience will be compressed to increase its density.

So now we know that if we apply or if we draw the control volume that you have studied in your undergraduate basic thermodynamics course and if we go to the next slide, and if we apply steady flow energy equation, SFEE applied to the control volume around the compressor. So, this is the control volume. We are assuming that the compression process is reversible adiabatic process. So, there is no heat interaction.

So, around the compressor, an adiabatic process compression. So, there is no heat interaction. What we need to do is supply a certain amount of work into the system or into the control volume to get this process done, which means to increase the pressure of the working substance. So, if we apply the steady flow energy equation quickly,

$$\dot{Q} + \sum \dot{m}_i \left(h_i + \frac{v_i^2}{2} + g z_i \right) = \dot{W} + \sum \dot{m}_e \left(h_e + \frac{v_e^2}{2} + g z_e \right)$$

So, this is the steady flow energy equation. We have only one inlet and one exit. So, we can assume that

$$\sum \dot{m}_i = \sum \dot{m}_e = \dot{m}$$

So, we can write these two equations. These two equations are applicable to describe the process that takes place across this compressor. And if we ignore, but we can consider that the change in kinetic energy and potential energy is negligible as compared to the change in enthalpy. So, the height of the compressor is very low, so the change in elevation—that is, the change in potential energy—can be ignored as compared to the change in enthalpy. We are also going to consider the change in kinetic energy small as compared to the change in enthalpy. If it is so, then from there, is no heat interaction between the compressor and surroundings, so that means we can write $\dot{Q} = 0$. If this is the case, then we can use this equation

$$\dot{m}_i h_i = \dot{W} + \dot{m}_e h_e$$

$$\dot{W} = \dot{m}(h_i - h_e)$$

So, that is the work, which is needed to run the compressor. Typically, in thermodynamics, we consider work added to the system as negative and heat added to the system as positive. So, the convention we consider is heat added to the system is positive and work done by the system is positive. So, this work, which we need to supply to the control volume, should be

$$\dot{W} = \dot{m}(h_e - h_i)$$

So, because this is negative work. So, this is the $\dot{W}$ for the compression process. So, this is the work to be added. So, this is work to be added to the compressor. Addition of energy in the form of work and now if we assume that air has a specific heat remaining constant, then W dot, and that is not the actual work. So, this is the ideal work that equals

$$\dot{W}_{ideal} = \dot{m}C_p(T_0 - T_i)$$

Because while we have estimated this work, which is needed to be added to the compressor for its operation, we did not take into account the frictional losses. So, this is the ideal work done. If we go to the next slide, then we can figure out the mechanical efficiency of the compressor.

So, the mechanical efficiency of the compressor equals to,

$$\eta_{mech,compressor} = \frac{\dot{m}C_p(T_0 - T_i)}{\dot{W}_{actual}}$$

Because the actual work which we need to supply to the compressor should be much higher than this amount because of frictional loss; we need to supply a certain amount of work to overcome the frictional losses. So, from there we can calculate

$$\dot{W}_{actual} = \frac{\dot{m}C_p(T_0 - T_i)}{\eta_{mech,compressor}}$$

Now the question is, to close this equation, we can have, so essentially, we need to know the rise in temperature, that is, what is $(T_0 - T_i)$. So, for that, let us consider the expansion or compression of the air. So, if we consider these two pressures, so this is P_i, this is P_0.

So, this is T, this is s. So, this is basically 1 to O_s, whereas the actual compression process will be like this. So, this is O. So now, this is the actual compression process. While this is the isentropic compression process. So, from there we can define the isentropic efficiency of the compressor, which is nothing but the rise in temperature following the isentropic process divided by the rise in temperature following the actual process. So that means if we would like to close this equation, we need to know about this quantity $(T_0 - T_i)$. Then we can write from here that

$$(T_0 - T_i) = \frac{(T_{0s} - T_i)}{\eta_s}$$

and we can write one step further that is

$$(T_0 - T_i) = \frac{T_i}{\eta_s}\left(\frac{T_{0s}}{T_i} - 1\right)$$

So, for this reversible adiabatic process, so for the isentropic process, we can write

$$\frac{T_{0s}}{T_i} = \left(\frac{P_0}{P_i}\right)^{\frac{\gamma-1}{\gamma}}$$

So, if we plug in this expression, then we can write

$$(T_0 - T_i) = \frac{T_i}{\eta_s}\left(\left(\frac{P_0}{P_i}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)$$

So that is the change in temperature. So, this

$$\frac{P_0}{P_i} = r_p$$

So, this is known as the pressure ratio. So, the rise in pressure is because of this compression process.

So, try to understand, had we followed this compression process, we could somehow, if we go back to the previous slide wherein we have written w dot actual that we need to supply to the compressor, it should be equal to this quantity. If we know the mechanical efficiency and $(T_0 - T_i)$ that we have, established in terms of the pressure ratio. So, these mathematical expressions will be needed for solving a few numerical problems that we will do in our next class.

So, if you would like to summarize today's discussion, we have discussed superchargers, the need for supercharging in the context of internal combustion engines, what are the different avenues by which we can go for increasing the density of intake air, and we have discussed the merits and demerits associated with each and every method.

Thereafter, we have discussed the outcomes of a particular method, that is, supercharging, which means if we need to go for compressing or compression of the intake air. Then we have discussed the different types of compressors used for supercharging. And finally, we have mathematically established some expressions. Those are very useful for solving numerical problems. With this, I will stop here today and we shall continue our discussion in the next class.

Thank you.