Course Name: Engine System and Performance
Professor Name: Pranab Kumar Mondal
Department Name: Mechanical engineering
Institute Name: Indian Institute of Technology, Guwahati
Week - 09
Lecture – 34

Lec 34: Performance map, Analytical method of performance estimation

I welcome you all to the session on engine system and performance. And in today's class, we shall discuss an important map, known as the performance map, which is used to compare the performance of two dissimilar SI engines. Afterward, we will solve or rather derive some analytical expressions which are used to predict the performance of SI engines under varied ambient conditions. So, if we recall what we discussed in the last class, it was the air consumption of SI engines, followed by performance curves of SI engines.

Then, engine characteristics of SI engines—that means we tried to map or observe the variation of engine power output with engine speed while keeping the load constant, and then with different engine loads while keeping speed constant. What we did not discuss is the performance comparison of two dissimilar engines—that means if the engines have different cylinder sizes and shapes. It is very important to have a particular curve or map that can be used to compare the performance of two dissimilar engines. Now, the question is, as I have repeatedly mentioned, the performance curves we have seen and discussed in the last class are not suitable for comparing the performance of one engine with another engine of a different size. So, if we need a particular map, which can be used to compare the performance of two dissimilar engines—that means engines of different cylinder sizes and shapes—then we need to have some performance data, a presentation of performance data based on generalized parameters like BSFC (brake-specific fuel consumption), BMEP (brake mean effective pressure), and mean piston speed. Because these parameters are essentially used to represent the performance of engines of different sizes. Rather, these parameters—that is, BSFC, BMEP, and mean piston speed—are all independent of engine cylinder shapes and sizes. So, if we examine the map used to compare the performance of two dissimilar engines, the map looks like this.

What we can see from this particular map is, that along the x-axis, engine speed is described. It can be mean piston speed as well. So, that is speed and along y-axis, output that is brake mean effective pressure, this particular parameter is described here. So, this is BMEP. And in this curve, we also have mean piston speed. I mean, as I said, along x-axis, in the bottom, we have engine speed. Along x-axis, at the top, we have mean piston speed. So, what we can see essentially this map shows the contour or contours of constant BSFC. So, all these are BSFC. Now, as I said you that we wanted to have presentation or representation of performance data on the basis of some generalized parameters. Those are brake mean effective pressure.

BSFC which are shown in this map by these contours and in piston speed. So, in this figure or in this map which is known as performance map we can see these contours rather this map shows the contours of constant BSFC. So, for the sake of completeness then this map plots the contours of constant BSFC in the load speed plane, because brake mean effective pressure is essentially the representation of load. Now this particular curve or this particular map is obtained by performing test on a SI engine having cylinder volume 2 dm^3.

So, this map plots data set, obtained from experiments conducted on SI engine having cylinder volume 2 dm^3. So that is important to know. So basically, now question is if we try to have another performance map of another SI engine, the qualitative rather the map should be similar both qualitatively as well as quantitatively if we maintain almost that cylinder volume same.

So, essentially what we can tell from this particular map is, this map is very useful to compare the performance of two different engines, that is engines of different cylinder sizes. So, as I said you that this is an experimental data obtained on SI engine having 2 dm^3 and 4 cylinders. Now, this is very important. What we can see from this map, and all these contours are basically, contours of constant BSFC.

And if we look at this map carefully, we can see there exists a region of a single minimum, single valued minimum BSFC region. So, if we look at this map, we can see clearly this map, if I just try to mark this map, is having or there exists a region of single valued minimum BSFC region or single valued minimum BSFC island. So, this particular region or region is known as single valued minimum BSFC island. Now, this is having rather this particular the contour corresponds to BSFC 275. This BSFC 275 is not very clear from the contour plot, but this particular BSFC contour, or the contour corresponding to BSFC 275 gm/kW-hr. Now, I would like to discuss a few salient points of this particular map, and these salient points are as follows. So, starting at this minimum BSFC island, we can see BSFC or fuel consumption rate increases in all other directions because all other contours have BSFC values higher than 275. That means, starting at this minimum BSFC contour or minimum BSFC region, in all other directions, mass flow rate or fuel increases. Another two important salient points are if one would like to traverse towards the right from this minimum BSFC island at a constant load. This particular region corresponds to minimum engine speed and moderate to high engine load. So, if I take this, these two lines and if we take the projection of this line on the x-axis, then we can see this particular island corresponds to engine speed which is not very high, while this particular regime corresponds to engine load which is moderately high.

Now, if someone would like to traverse from this particular regime towards the right for a constant load, then again, fuel consumption rate increases. This is attributed to the frictional effect because if engine speed increases, frictional loss will increase, necessitating higher fuel consumption. On the other hand, at a constant load, if we reduce engine speed—that means if someone would like to traverse from this point towards the left—then again, we can see fuel consumption rate increases. And that we have discussed because, at low engine speed, the engine cylinder will get sufficient time to transfer heat to the coolant or to the ambience. And sufficient heat transfer rate will again necessitate or demand higher fuel consumption.

The last or final salient point that I would like to discuss is, instead of that constant load, someone would like to traverse at a constant speed. So, if someone would like to keep the speed constant and traverse toward the upper side of this map, that means if load increases, the fuel consumption rate will increase. That is quite obvious because we need to enrich the charge. That is, we need to supply a greater amount of fuel. But if someone would like to traverse toward the bottom part of this map, that is, keeping speed constant if load decreases.

Then again, we can see the fuel consumption is becoming greater. And that is, when the load is less, a significant part of this is used to overcome the frictional losses, and that is why fuel consumption increases. One particular point I should mention before I go on to discuss the analytical method of engine performance. Now, our analytical expression, which we shall derive in today's class. The upper part of this map essentially corresponds to full-load operating characteristics, while the lower part of this curve corresponds to part-load operating characteristics.

So, with this, we can say that this particular map is really very useful and suitable for comparing the performance of two engines having different cylinder sizes and shapes. Next, we shall discuss the analytical method, or we shall try to derive some expressions analytically, and those expressions can be used to compare or predict the performance of engines. Under different operating conditions. So, let us start our discussion on this particular part, that is, the analytical method of performance estimation. So, analytical method of performance estimation.

So that is Important, it is not possible to perform tests if the operating condition changes. So, if we have a particular engine and if we know the performance, characteristics of that engine at a given operating condition, then using these analytical expressions, we can easily predict the performance of that particular engine when the operating condition is changed or under different operating conditions. So, let us start with, what we shall consider.

So, we will consider a four-stroke cycle, naturally aspirated engine. So, what is a naturally aspirated engine? Because for this particular engine, there is no need for a supercharger or turbocharger. That means intake air will be drawn into the engine cylinder only due to the atmospheric pressure difference, that is, the atmospheric pressure minus cylinder pressure. So, there is no supercharging unit to push or, to drive air into the engine cylinder. So, that is called a naturally aspirated engine.

Naturally aspirated engines have a few advantages, like they are cheap, reliable engines, Easy to maintain, but the disadvantage is low, power-to-weight ratio. So, there are many advantages, as I said, very cheap, easy to maintain, and reliable, while the disadvantage is low power-to-weight ratio. So, now the question is, if we consider a four-stroke cycle naturally aspirated engine, and if we try to have or try to drive the Expression which can be used to predict the performance of an engine or engines under varied operating conditions, and essentially, we are trying to derive all these expressions analytically. So, to start with. So, we know that when we try to extract power output from an engine, we get such power output at the cost of some input energy. And that input energy is essentially the energy which is being supplied to the engine with the fuel. So, if we consider that supplied energy to the engine,

$$= Q_{cv} \times \dot{m}_f$$

So, this Q_{cv}, is the calorific value of fuel.

So, basically this is the energy that we are supplying to the engine. Now, we can write this expression again. So, this is

$$= Q_{cv} \times F \times \dot{m}_a$$

where *F* is the fuel-air ratio. So, that is

$$F = \frac{\dot{m}_f}{\dot{m}_a}$$

So, if you go to the next slide. So, knowing this is basically, this $\dot{m}_a$ is the mass of air per cylinder per cycle. So, this is the mass flow rate of air per cylinder per per cycle. So, if you go to the next slide, then you can write this

$$\dot{m}_a = \eta_v \times V_s \times \rho_i$$

We know that is mass per cylinder per cycle can be determined if we know the volumetric efficiency into cylinder volume into inlet air density. So, that is the actual. This is the theoretical mass flow rate of air that we discussed in our previous class. So, that is the cylinder volume or stroke volume also.

So, we can write the indicated work done. So, we know the energy supplied to the engine, and if we try to have indicated work done. So, the indicated work done by this energy supplied to the engine is equal to

$$= \eta_{ind} \times \text{energy supplied to the engine}$$

If we go back to the previous slide, then we can write that equal to

$$= \eta_{ind} \times Q_{cv} \times F \times \eta_v \times V_s \times \rho_i$$

So, this term is the indicated thermal efficiency. So, if we go to the next slide, so it is very clear.

Once we have the indicated work, that is work available inside the cylinder from there we can easily write some mathematical expression of indicated mean effective pressure from the work done. So, Pdv that is indicated work done is essentially. So, indicated mean effective pressure is continuously changing inside the engine cylinder that is why the concept of mean effective pressure was introduced.

So, if we know the indicated work done we can have some estimate of indicated mean effective pressure. So, if we go to the next slide then we can write indicated mean effective pressure. So, this quantity is indicated mean effective pressure into stroke volume so or cylinder volume. It is also stroke volume, so if we now go to this, so this is essentially the indicated work done that we have already derived here.

So, this indicates work done, which is eta indicated into this. So, this quantity is nothing but indicated mean effective pressure into cylinder volume or stroke volume. So, if we go to the next slide, then we can write this equal to

$$imep \times V_s = Q_{cv} \times F \times \rho_i \times V_s \times \eta_v \times \eta_{ind}$$

$$imep = Q_{cv} \times F \times \rho_i \times \eta_v \times \eta_{ind}$$

$$imep \propto Q_{cv} \times F \times \rho_i \times \eta_v \times \eta_{ind} \tag{1}$$

So, this is a very important relation. Using this relation, we can derive. The information about indicated mean effective pressure under two different conditions.

So, using this expression, we can have for two different operating conditions or two operating conditions, we can write using Eq (1),

$$\frac{(imep)_2}{(imep)_1} = \frac{F_2 \times \rho_2 \times \eta_{v_2} \times \eta_{ind2}}{F_1 \times \rho_1 \times \eta_{v_1} \times \eta_{ind1}} = R_i(\text{say}) \tag{2}$$

Here, 1 and 2 denote two operating conditions. So, having an analytical expression of indicated mean effective pressure, we can now try to have brake mean effective pressure or then we can have brake horsepower. So, we can write that

$$BHP = bmep \times V_s \times \frac{N}{2}$$

brake horsepower (bhp) equals, brake mean effective pressure into volume into *N* by 2. Four cylinders, so four torque cycle engines. So, there are two revolutions per cycle. So, *N* by 2, from there we can write this is the brake horsepower. So, that is brake work multiplied with this we can have power, that is brake horsepower. So, now that means we can write

$$BHP \propto bmep \propto N$$

Now, what can we do? Again, we can have this expression for two operating conditions, that is 1 and 2, so between two states, 1 and 2, we can write

$$\frac{BHP_2}{BHP_1} = \frac{bmep_2}{bmep_1} \times \frac{N_2}{N_1} \tag{3}$$

So, the question is, say, if the speed is assumed to remain the same. So, N_1 equal to N_2. So, say speed is constant, that means N_1 equal to N_2equal to N. Then what we can write is

$$BP = IP - FP$$

$$bmep = imep - fmep$$

So, indicated power is not available; this power is available inside the engine cylinder, while the power available at the shaft is brake power, which is indicated power minus the friction, minus friction power. So, then we can write brake mean effective pressure equal to indicated mean effective pressure minus frictional mean effective pressure. So, that means what you can write now is

$$\frac{bmep_2}{bmep_1} = \frac{imep_2 - fmep_2}{imep_1 - fmep_1}$$

Can we write, this in terms of indicated mean effective pressure and frictional mean effective pressure. While frictional mean effective pressure be the same for the same speed, but indicated mean effective pressure between these two states. Already we have some expression. we can write one step further.

$$\frac{bmep_2}{bmep_1} = \frac{\frac{imep_2}{imep_1} - \frac{fmep_2}{imep_1}}{1 - \frac{fmep_1}{imep_1}}$$

So, this is the expression. Now, if the speed is constant, so for a constant speed that is N_1 equal to N_2 equal to N, this $fmep_2$ equal to $fmep_1$ equal to frictional mean effective pressure. So, with this we can further write this expression here that

$$\frac{bmep_2}{bmep_1} = \frac{R_i - \frac{fmep_2}{imep_{1_1}}}{1 - \frac{fmep_1}{imep_1}} \tag{4}$$

We are trying to derive some expressions analytically, and these expressions can be used to measure the performance of an engine under two operating conditions. It is not really possible to perform tests for each condition. So, performing a test on an engine at a given condition, we can use these expressions to predict the performance under different conditions. So, this is what we have written. Now, the question is, we have relations between two operating conditions in terms of indicated mean effective pressure and brake mean effective pressure. If we try to recall today, in the beginning, we discussed that if we need to compare the performance of two dissimilar engines, it is convenient to represent performance data in terms of generalized parameters like BSFC, BMEP, etc.

So now let us consider what BSFC is. So, if we try to establish a relationship between the two states 1 and 2 in terms of their respective BSFCs, then we can write that

$$\frac{bsfc}{isfc} = \frac{IHP}{BHP} = \frac{imep}{imep - fmep}$$

Try to understand:

$$bsfc = \frac{\dot{m}_f}{BHP}$$

$$isfc = \frac{\dot{m}_f}{IHP}$$

This is also brake mean effective pressure into stroke volume into *N* by 2. So, stroke volume into *N* by 2 is common in both numerator and denominator.

So, if we consider this relation for two conditions, state 1 and state 2, we can now write that

$$\frac{bsfc_2}{isfc_2} = \frac{imep_2}{imep_2 - fmep_2} \text{and}$$

$$\frac{bsfc_1}{isfc_1} = \frac{imep_1}{imep_1 - fmep_1}$$

So, this is what we can write based on the expression or based on the above expression. And so, if we divide one by the other, then we can obtain

$$\frac{bsfc_2}{bsfc_1} = \frac{isfc_2}{isfc_1} \times \left[\frac{1 - \frac{fmep_1}{imep_1}}{1 - \frac{1}{R_i}\frac{fmep_2}{imep_2}} \right] \tag{5}$$

So essentially what we did, we had tried to establish the expression of indicated mean effective pressure, brake mean effective pressure, and finally indicated horsepower, brake horsepower, and finally brake specific fuel consumption. We have tried to relate BSFC with ISFC. Now this is what we have done.

All these derivations of all these expressions analytically. Now the question is next we will consider the effect of ambient conditions, if the ambient conditions change on the performance of a four-stroke cycle naturally aspirated engine. So, it will not take much time. So now let us consider, the effect of ambient conditions on the performance of a four-stroke cycle naturally aspirated. SI engine or naturally aspirated engine. We did not specify whether it is an SI engine or CI engine because these expressions are valid for both SI and CI engines, but it is valid for naturally aspirated engines. So, the engine is not using any supercharging unit. So, what we can do for this, we have to first understand to this P_i, T_i so subscript i denotes inlet condition. So, these are the inlet pressure and temperature. So, the question is, let us assume that inlet mixture which is not. This is a mixture of dry air, wet air, also the fuel vapor.

So, if it is a SI engine, certainly, this would be mixed with the fuel vapor. So, even for the modern CI engines, fuel is spread into the intake manifold. So, this P_i, that is the pressure of the mixture, assuming that the component gases, that is dry air, wet air and fuel vapor are all these component gases are behave like ideal gases and then using Dalton's law of partial pressure, this equal to

$$P_i = P_a + P_m + P_f$$

a is dry air, *m* is moist air, that is wet air, and *f* stands for fuel vapor. So, these subscripts are used to denote pressure of the dry air, pressure of the moist air, pressure of the fuel vapor that is present in the mixture. And we could write these expressions using Dalton's law of partial pressure. Now, if we consider *N* and *M*, *N* is the number of moles and *M* is the molar masses of each of these constituents.

Then we can write this equation P_i that is inlet pressure equal

$$P_i = \frac{N_a + N_m + N_f}{V} \bar{R}_a T$$

So, one step further that equal to

$$P_i = \frac{N_a}{V}\left[1 + \frac{N_m}{N_a} + \frac{N_f}{N_a}\right]\bar{R}_a T_i$$

So, this term can understand easily that, this equal to

$$\frac{N_f}{N_a} = \frac{m_f}{m_a} \times \frac{M_a}{M_f} = F\,\frac{M_a}{M_f}$$

That is the second term, this can be written equal to

$$\frac{N_f}{N_a} = \frac{m_f}{m_a} \times \frac{M_a}{M_f} = 1.6G$$

$$P_i = \left[1 + 1.6G + F\,\frac{M_a}{M_f}\right]$$

where G is the specific humidity of air, F, that is fuel air ratio. So, this is the expression that we can have. So, having established the expression of inlet air that is mixture of dry air, moist air and fuel vapor, density of inlet air mixture is

$$\rho_i = \frac{P_i}{\bar{R}_a T_i}$$

So now the question is, when investigating the effect of atmosphere, we are trying to examine the effect of ambient conditions. We assume that speed and fuel-air ratio, these two quantities do not change. So, let me tell you one thing. Essentially, through this exercise, we are trying to investigate the effect of ambient conditions on performance. Assuming that the fuel-air ratio F and engine speed, these two quantities do not change.

So, if we try to recall expression or Eq (2), where we could write the relation of indicated mean effective pressure at two different states in terms of air density and two other things, that is, volumetric efficiency and indicated thermal efficiency. Try to understand F_2 and F_1, these two are basically fuel-air ratios. We are not going to change the fuel-air ratio, and also, we are not going to change the speed.

So, if we use this expression to derive the expression at this case, the expression for this case rather, so that means we shall use Eq (2) that I had shown, and we assume that engine speed and fuel air ratio do not change. So, if that is the case, then we can rewrite

Eq (2) in a reduced form, and we can write indicated mean effective pressure at 2 divided by indicated mean effective pressure at 1. So, *F* will not change. Then, if the speed is not going to change. Then, indicated thermal will be the same. So, the failure ratio is not changing. Speed is not changing. Then, indicated thermal efficiency will not change. Rewrite this expression. For this particular case. And it will be

$$\frac{(imep)_2}{(imep)_1} = \frac{\rho_2 \times \eta_{v_2}}{\rho_1 \times \eta_{v_1}} = R_i \qquad (6)$$

So, we know volumetric efficiency is essentially the actual mass flow rate of air. This is defined as the ratio of the actual mass flow rate of air to the theoretically calculated mass flow rate of air or volume flow rate. So, it is not mass flow rate; it is also volume flow rate.

If you multiply with ρ, then you will get mass flow rate; otherwise, volume flow rate. So, then let us recall. So, if we go to the slide, then here η. The final expression that we have. So, you can understand that this quantity, theoretically calculated volume or flow rate of air is the same, that is stroke volume, which is fixed. So, it is proportional to the actual volume flow rate of air, which in turn depends on that is

$$= \frac{1}{P_i \rho_i}$$

So, essentially,

$$\eta_v \propto \sqrt{T_i}$$

So, actual mass flow rate means, the velocity, that is,

$$= 2\sqrt{\frac{\Delta P}{\rho_i}}$$

So, the

$$\rho_i = \frac{P_i}{\bar{R}_a T_i}$$

So, this is proportional to

$$\rho_i \propto \frac{P_i}{T_i}$$

So, if it is the case, it would be proportional to

$$\eta_v \propto \sqrt{T_i}$$

$$\rho \times \eta_v \propto \frac{P_i}{\sqrt{T_i}}$$

So, then this Eq (6) can be written further here

$$\frac{(imep)_2}{(imep)_1} = \frac{\rho_2 \times \eta_{v_2}}{\rho_1 \times \eta_{v_1}} = \frac{P_2}{P_1}\sqrt{\frac{T_1}{T_2}} = R_i \tag{7}$$

So, what we did essentially, we have tried to estimate this quantity, that is, the ratio of indicated mean effective pressure at 2 different states in terms of pressure and temperature, which will vary if the ambient condition changes, and this volumetric efficiency is essentially the actual volume flow rate of air, which in turn depends on the velocity of air at the orifice meter that you have discussed, that is,

$$= 2\sqrt{\frac{\Delta P}{\rho_i}}$$

From there we could write the final expression of this quantity, which can be used to again predict the performance, under different or for different operating conditions.

So, if you would like to summarize today's discussion, we can say that we have discussed a technique or method to obtain some expressions analytically, and those expressions are useful to predict the performance of a particular engine under different operating conditions. We shall try to solve two different problems essentially to illustrate the concept that we have discussed in today's class, and that part we will do in the next class.

Thank you very much.