

POWER PLANT SYSTEM ENGINEERING

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Module 4

Lec 4: Wind Energy - Part II

Dear learners greetings from IIT, Guwahati. We are in the MOOCs course Power Plant System Engineering, module 4 that is Hydro and Renewable Energy Generation Systems. So, in our last lecture we are focusing on wind energy part 1. Now in continuation of this wind energy, we are going to discuss some new concepts of harnessing wind energy. So, in our previous lectures I emphasized about harnessing wind power in a typical propeller driven wind turbine and it is mainly governed through steady flow energy equations or in turn Bernoulli's theorem can be applied everywhere in the flow field except the location at which the energy is being harnessed by the turbine. Now similar concepts can be extended for other types of turbines. To have a better design of those kind of innovative wind turbine blades for localized power or where the velocity of the wind is relatively less, we need to think about the governing aerodynamics principles of it.

So, for that reason, we are going to introduce a topic, called as Magnus effect, which is a kind of aerodynamic principle that is being used for innovative designs of wind turbine blades. In fact, Magnus effect is a kind of a situation, where if you are able to rotate a cylinder in a free stream medium, it says that the pressure difference across this cylinder surface will create a lift. So, essentially in other words, we design the blades in such a way that irrespective of the direction of the wind if we can ensure a lift force, then we will be able to rotate the cylinder. So, this is the basic philosophy or fluid mechanics principles or in fact, aerodynamics principles behind the vertical axis wind turbines, which we are going to discuss now. Based on these two concepts like Bernoulli's theorem and Magnus effects, the entire wind turbine family is classified and we call them as wind machines. And lastly I will just try to emphasize when you install a wind turbine at a particular location, it is

expected that based on the uniform speed of that location, it should be able to perform in a manner that the total rated capacity of the power can be generated. So, I will give a brief introduction on these topics.

Now, let us start with brief introductions. In our last class I mentioned that wind has immense potentials, it falls under the roof of renewable energy. Apart from solar energy, hydro, geothermal, and bio energy, wind also has a definite potential for harnessing power. And it is treated as a renewables. And since it has been a renewable kind of an energy resource, so it is being projected as a major source of power by end of 2050.

So, a lot of researches have been put on this topic. However, in our study we will just focus on some basic aspects of this wind. So, in our last lecture, we have discussed about a propeller driven or horizontal axis wind turbine, where we say that the turbine is installed in a manner that direction of the wind and axis of this rotation are parallel to each other. So, in other words wind direction and the horizontal axis are in parallel. So, based on our analysis, we have imagined as if there is a stream tube which passes through the propeller and wind has its initial velocity, at which it enters into this stream tube and leaves at some exit pressure and velocity.

So, through this process we apply Bernoulli's equations at two locations that means, at a free stream second at the location at which the turbine is mounted that is between 'a' and 'b'. So, Bernoulli's equation is applied between 'i and a' and further between a, b and e. So, that way we can see an increase in the pressure when the stream approaches the turbine again when the flow leaves the turbine there is also increase in the pressure. So, what happens in between? So, this is the zone, where entire kinetic energy of the flow is being taken by this rotor or the propeller. So, that way the pressure drops from p_a to p_b and also at that point of time, we have bring back this velocity V_i to V_e velocity at this location.

So, what we have seen here is that through this analysis we have defined the term total power that is available in the wind. And through this particular arrangement, what is the maximum power that we can extract. The difference between them gives rise to the

maximum power coefficient. And this maximum power coefficient is independent of any kind of wind turbines, blades or independent of materials or independent of the wind velocity and all. So, that is the reason, we write this maximum power coefficient as Betz number that number happens to be 0.593. And the Betz limit indicates that, the maximum power that can be extracted from wind is independent of the design in an open flow environment because it is a fixed number.

So, that way one can plot the power coefficient as a function of tip speed ratio. But what happens here is that when you install a particular turbine at a given location, it depends on three major factors. One is the maximum swept area or capture area that we can achieve. So, that is achieved through the rotor area or wind turbine area and it is defined by its radius r . And second one is ω value which represents that with given wind speed velocity of V_i , what is the rotational velocity of the turbine or that is angular velocity. So, considering all three numbers we define a term called as tip speed ratio and this tip speed ratio for any given turbines has certain limit. So, for that reason, there are different class of turbines as you can see in the figure like multi blade type, maybe we can say four-arm type, Darrieus type, horizontal axis type. And we can see that they have some limits in which they operate in a particular tip speed ratio. Whereas, if you look at ideal propeller type turbine, it is independent of all these internal designs. So, the efficiency initial increases and it keeps on till it becomes a fixed value and that as this number 0.593. So, all other turbines always operate below that number. So, that is the reason we say it as Betz limit.

So, in other words, what we can say is that wind has a potential to work up to the Betz number, but for a given wind turbine, at a particular geographical location, it is not possible to reach that number. So, that is why we have different ways to design the blades of the wind turbine to achieve this. So, we have understood the fact that a typical horizontal axis wind turbine which normally falls under the category of propeller or a wind turbine, has its own limitation. Because it operates in the tip speed ratio between 4 to let us say 7. But is there any other possibility to design any other kind of turbines which can also work at low wind speed? So, to answer this question, we have to look into some other options or other designs.

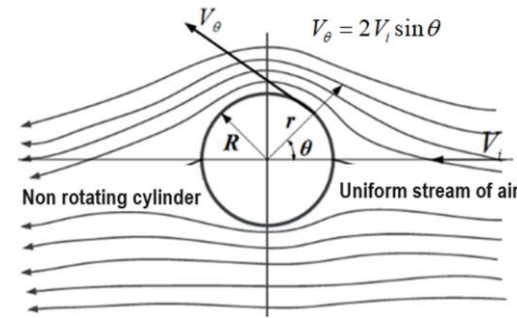
So, obviously, when you move on to other kind of designs, the concept that we apply for propeller type may not fit. So, for that reason, we have to find out some other principles. And second reason is that when you look at the high speed horizontal axis wind turbines they are generally operated at wind speed of more than 100 kilowatt or above. Now, if the wind speed is there at a particular location, but we do not have the possibility of installing such a big capacity turbine, then can we think of installing a small capacity turbines which will incur less structural stability? Because horizontal axis wind turbines are very prone for axial thrust as they operate at higher wind speed. So, for that reasons, we have to go for some other alternative solutions. So, that is where this Magnus effect comes into pictures. So, this Magnus effect is a typical aerodynamic principle, which says that we can think of a lift based and wind turbine rather than conventional propeller driven turbine. So, this lift based wind turbines are governed by this Magnus effects. Now the advantage of those turbines are that they can be useful or they can be considered as small wind turbine novel designs and the power generation may be less than 100 kW.

So, they operate at low wind speed and they fall under category of rotor type turbine like Savonius rotor, Darrieus rotors type. So, for those turbines, we can design the blades and it should be lift based designs. That means, if wind speed at a given locations is available then you can design some turbine blades which can generate lift on those designs, then obviously, the system will rotate or this rotor will rotate. So, this is what exactly Magnus effects talks about. So, for the time being let us not talk about the turbine itself rather we will just talk about what does this Magnus effect tells us or what inference we get from the Magnus effects.

So, the Magnus effect demonstrates that a lift force is generated if a horizontal cylinder is rotated about its axis and moves through the still wind. Conversely if a stationary horizontal cylinder is rotated about its axis in a cross wind, it will generate a lift force. So this is what about the horizontal cylinder. Now, we will try to see if this Magnus effect happens in a vertical cylinders. So, effect is equally applicable to a vertical cylinders when it is rotated about an axis in a cross winds. So, it will experience a force perpendicular to its axis which will cause it to move in the direction perpendicular to the wind. So, in other words we can think of a vertical cylinder placed in a cross wind and if we are able to generate a lift then

obviously, the cylinder will rotate. So, this is what exactly we want in a vertical axis wind turbine. Another thing is that to make this analysis more simplified we can imagine the length of the cylinder to be much greater than its diameter, so that the flow around it can be typically considered to be two dimensional because length effect will not come into picture.

So, we will discuss this Magnus effect in details. So, to demonstrate this Magnus effects we will try to first consider simple aerodynamic principles, in which we take a non-rotating vertical cylinder, placed in a non-viscous flow. So, ideally it is an inviscid flow. This is a vertical cylinder placed in an inviscid flow. What we expect is that when it is placed in a uniform stream, we will find the streamlines approaching in this manner and typically the streamlines from upper and lower part of the cylinder will be symmetrical in nature. What we see here is that when the flow approaches the cylinder all the streamlines will be symmetrical about its axis.



So, we can say that the cylinder geometry is defined by this angle θ radius is defined by this R and from the center we define a radius at any location downstream because this $r > R$. So, now if you want to find out the velocity at any point on the surface you can draw a tangent. So, we will have a velocity V_θ and that will define its velocity. Now, once you know this velocity, we can also apply Bernoulli's equation to find out the pressure. So, there are two locations, one is location i and other is at location θ . Here θ means on the surface.

So, we apply this Bernoulli's equations by neglecting potential energy effects at two locations to find its pressure. But velocity on the surface can be defined as

Velocity of air stream at any point on cylinder surface, $V_\theta = 2V_i \sin \theta$

Apply Bernoulli's equation at same position for calculating pressure,

$$\frac{p_i}{\rho} + \frac{V_i^2}{2} = \frac{p_\theta}{\rho} + \frac{V_\theta^2}{2} \Rightarrow p_\theta = p_i + \frac{1}{2} \rho V_i^2 (1 - 4 \sin^2 \theta)$$

Now once we know this theta we can also define a non-dimensional pressure. Because $\left(\frac{1}{2}\rho V_i^2\right)$ remains constant, and this term is called as dynamic pressure and for a given wind velocity this dynamic pressure is a fixed quantity. So, non-dimensional pressure can be defined as the ratio of the pressure difference between at any point on the surface and the free stream pressure to this dynamic pressure.

$$\text{Non-dimensional pressure, } p_\theta^* = \frac{p_\theta - p_i}{\left(\frac{1}{2}\rho V_i^2\right)} = 1 - 4 \sin^2 \theta$$

So, a close look on this analysis, we can see the quite obvious fact that, when velocity is low, pressure is higher & when velocity is more, pressure is lower. So, obviously, from this analysis we will have stagnation points at two locations.

Stagnation point, $V = 0$; at $\theta = 0^\circ$ & 180°

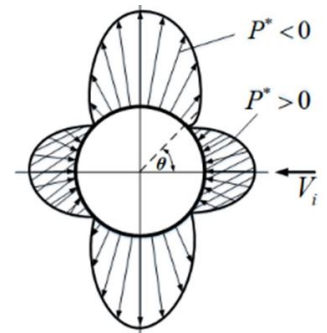
So, stagnation point means this flow velocity on the surface becomes 0. So, when we have velocity = 0, at that point pressure happens to be maximum. So, that way we can say that at stagnation point your $p_\theta^* = 1$ and $V = 0$.

$$p_\theta = p_i + \frac{1}{2}\rho V_i^2; p_\theta^* = 1; \rho: \text{Density of air}$$

θ : Polar angle measured from stagnation point

p_i & V_i : Incoming uniform air pressure & velocity far away from cylinder

Now, moving on, if you can find streamlines and pressure distribution pattern for this horizontal cylinder, for a given velocity V_i you will see that there will be a uniform pattern of pressures on the entire surface of this cylinder & along its axis. Since the flow pattern is symmetric in axis so obviously, we can draw a conclusion is that, there is no resultant force either parallel or perpendicular to the undisturbed streams. Hence, there is no drag on the cylinder or there is no lift force. So, the drag and lift is nothing, but aerodynamic term. Any force which acts perpendicular to the free stream direction of the flow, like here it is the wind speed, is the lift and any force that acts in the



Streamlines and pressure distribution for two dimensional non-viscous flow past non-rotating cylinder

opposite to the direction of the wind speed is the drag. So, obviously, drag and lift are perpendicular to each other and we will use these same term for analysis of this Magnus effect. So, we see here the streamline patterns and pressure distribution pattern for a non-viscous flow past a non-rotating cylinders.

Now, let us move to the next effect. Here instead of keeping the cylinder in non-rotating mode you start rotating the cylinder at certain velocity, but keep this airstream as stationary. That means, you are just rotating a cylinder in an ambient air or stationary air. There is no air velocity or free stream velocity. So, that means, you are inducing a circulatory flow around a rotating cylinder. So, the air also feels the effect of some circulating actions due to this rotation of cylinder. Of course, the effect is more predominant in the vicinity of the cylinder, we have free stream velocity V_i pressure p_i and far away from this side also free stream pressure and free stream velocity remain same. So, that means, far stream is not affected, but the vicinity of the cylinder gets affected through this rotation of the cylinder. So, what we observe here is that the rotation of the cylinder produces circulatory flow around it, but the air, far away from the stream is still at rest. Now, question is at what velocity it rotates. To do that it can be shown that the velocity is inversely proportional to the distance from the cylinder axis and for which we can put a dimensional constant and that dimensional constant is termed as the circulation.

So, if you say the radius of the cylinder is R and any location which is at distance r , beyond this cylinder radius, we can find out the velocity in two parts. One is the cylinder peripheral velocity that is V_p , other is velocity at any distance r from this axis V_r . And typically peripheral velocity is a constant quantity and you term this as circulation.

Cylinder rotation produces circulatory flow, $2\pi r V_r = 2\pi R V_p = \Gamma$

Velocity at distance r from cylinder axis, $V_r = \frac{\Gamma}{2\pi r}$

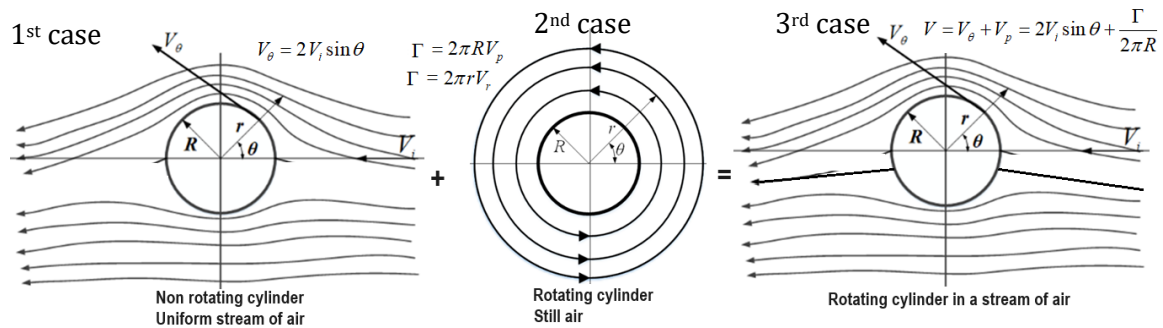
Cylinder peripheral velocity, $V_p = \frac{\Gamma}{2\pi R}$; $V_p = 2\pi R N$

Now, for a given wind speed if it makes number of revolutions, so we can multiply N to it. So, accordingly we can have the circulations constant as

Circulation constant, $\Gamma = (2\pi R)^2 N$

R :Cylinder radius N :Number of revolution per unit time

So, here what we observe is that, when a cylinder rotates in still air, it produces a circulating effect or flow around the cylinder. Now we merge these or superimpose these two effects. So, first thing is non-rotating flow in a uniform stream of air and second thing is rotating flow in a still air. So, we have analyzed both the things separately we have calculated V_θ and we have circulation. When you superimpose these two things, we will have a net effect which is nothing, but called as a rotating cylinder in a stream of air. So, when we have a rotating cylinder that is a stream of flow then we have these effects. And there we can have both the effects together and this velocity on the surface can be defined at any location θ in two parts one is V_θ & other is V_p . Here V_p stands as this peripheral velocity and this



peripheral velocity is a fixed quantity that is $\frac{\Gamma}{2\pi R}$. So, that way the most significant effect that we can observe here is the non-uniform pressure and velocity on the upper part and lower part of the cylinders.

So, if you look at this particular figure here and in the 1st case the streamline have uniform pattern and there is a flow symmetry. In the 2nd case also we have flow symmetry about this axis, but in the 3rd case, it is non-symmetric. By non-symmetric means we will have different values of pressure distribution and velocity pattern. So, what does this mean? In the 1st case we have two stagnation points as A and B, which we have already defined and in the 2nd case here, we will not have a stagnation point because the cylinder is continuously rotating. Now, in the 3rd case, where these previous two effects will be merged, then there will be two situations, one for the upper part of the cylinder and other for lower part. Now, the upper surface of the cylinder will experience a higher velocity because the velocity gets

added whereas, the lower part of the cylinder will be at lower velocity because the velocities here oppose the direction. So, the combined velocity pattern will be like they will reinforce in the same directions in the top and they will oppose each other in the bottom. So, net effect is to increase the velocity at the top and decrease the velocity at the bottom. So, net effect would be that initially there are two stagnation points in case 1st case and this point for a symmetric case (3rd) will try to move towards each other. That means, point A and B will move towards each other and we will have a stagnation point maybe at point C and that point we will see at $\theta = 90^\circ$.

Velocity on the surface of the cylinder, $V = V_\theta + V_p = 2V_i \sin \theta + \frac{\Gamma}{2\pi R}$

On top surface, (between $\theta = 0^\circ$ & 180°) $\Rightarrow \sin \theta$ is positive

On bottom surface, (between $\theta = 0^\circ$ & -180°) $\Rightarrow \sin \theta$ is negative

At stagnation point, $V = 0; \Rightarrow \theta = -90^\circ; V_p = 2V_i$ & $\Gamma = 4\pi R V_i$

And from this analysis we can say why the velocity will be higher in the top because this expression will say that $\sin \theta$ will be positive when θ is between 0° & 180° and for the bottom surface θ is between 0° & -180° so $\sin \theta$ is negative. So, this gives the fact that net velocity increase in the top surface & there will be decrease in the velocity in the bottom surface.

So, the other effect of this velocity is that the pressure of the cylinder will be higher in the bottom and lower on the top. Now, through this process, what we arrive at here is that, there is a pressure difference between top and the bottom surface and this pressure difference will give you an upward force and this upward force is known as lift force. So, what we can conclude from this Magnus effect analysis is that when a rotating cylinder is placed in a stream of air, then higher pressure is felt on the bottom surface and it induces an upward force and this upward force acts in the direction perpendicular to the stream direction and this upward force is known as lift force. And this lift force is nothing, but the combined effect of what we say integrating the pressure over the projected area. So, in net

effect of this pressure difference multiplied by area will give you the force and we are going to calculate this pressure through this Bernoulli's equations.

So, we are going to see here we are going to use this Bernoulli's equation at two locations. One at any point on the surface which is defined by this angle θ . So, we say p_θ & V_θ and any location we say p_i & V_i . So, this equations are applied here.

Bernoulli's equation,

$$\frac{p_i}{\rho} + \frac{V_i^2}{2} = \frac{p}{\rho} + \frac{V^2}{2} \Rightarrow p = p_i + \frac{1}{2}\rho(V_i^2 - V^2)$$

So, we find V at any point on the surface and as it is a rotating cylinder in a stream of air, so

$$\text{Velocity on the surface of the cylinder, } V = V_\theta + V_p = 2V_i \sin \theta + \frac{\Gamma}{2\pi R}$$

So, by putting this velocity term in this pressure equations we have the pressure on the surface of the cylinders.

$$\text{Pressure on cylinder surface, } p = p_i + \frac{1}{2}\rho \left[V_i^2 - \left(2V_i \sin \theta + \frac{\Gamma}{2\pi R} \right)^2 \right]$$

And we integrate this pressure over this frontal area. And this frontal area or projected area can be calculated considering the following things. If you look at take projections of the cylinder to find out the projected area, we will have a rectangle with $2R$ length and this height, H with respect to cylinder this is height.

A : Projected area ($= 2RH$)

So, when you put this, we form an equation that is lift force

$$\text{Lift force, } F_L = \int_{-\pi}^{\pi} -(p \sin \theta) R d\theta H = 2\pi \rho R H V_p V_i = 4\pi^2 \rho R^2 H N V_i$$

R : Radius of cylinder; H : Length (height) of cylinder; ρ : Density of air

V_i :Incoming uniform air velocity; V_p :Peripheral velocity ($= 2\pi RN$)

N :Number of revolutions per unit time

This is one way we find the lift force by integrating pressure over the surface of the cylinder. Now, there are other ways that lift force can be expressed in the form of lift coefficient that is C_L .

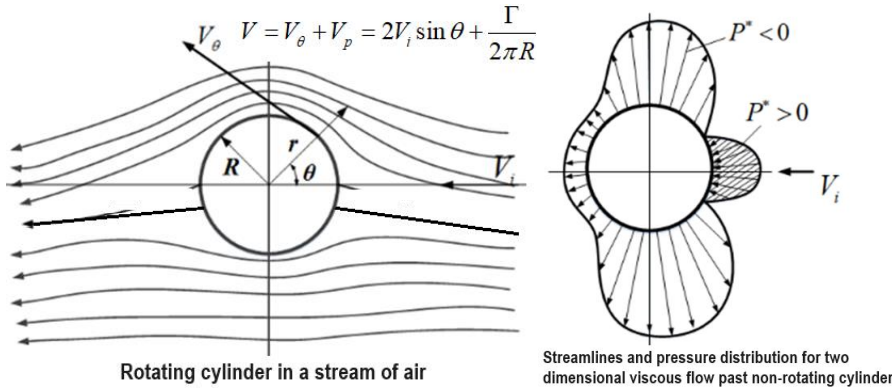
$$\text{Lift force, } F_L = C_L A \left(\frac{1}{2} \rho V_i^2 \right); \text{ Lift coefficient, } C_L = 2\pi \left(\frac{V_p}{V_i} \right)$$

So, by combining these two we get a term what is called as lift coefficients C_L . So, here important point to be noted is that if we have V_i velocity here in the free stream, but when it approaches we have V_p velocity which is on the surface. So, this is the circumferential velocity on this surface and C_L is basically function of $\frac{V_p}{V_i}$ and this $\frac{V_p}{V_i}$ is known as velocity ratio. At the same time we also have

$$\text{Free stream dynamic pressure, } q_\infty = \frac{1}{2} \rho V_i^2$$

Now, this entire analysis shows that the pressure is non-uniform on the surface. So, as a result we will have a non-uniform pattern in which your p^* which is non-dimensional pressure, will be different at different locations.

So, this non-uniformity of this velocity and pressure distribution and streamlines pattern is shown here and it is for the case of non-rotating cylinder placed in a viscous flow. So, this is a case where we can say that there will be pressure and velocity non-uniformity on the surface.

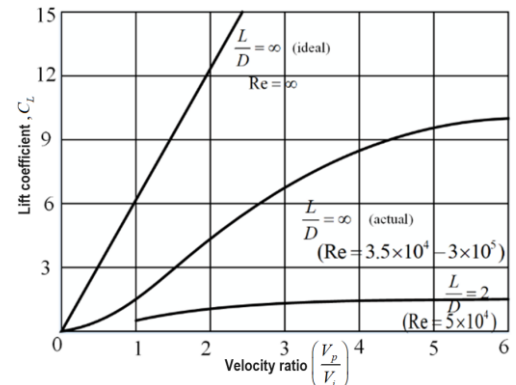


What we have done so far is an ideal theory or idealistic theory and that is called as ideal non-viscous flow for which we find the lift force. And this lift coefficient is defined by $C_L = 2\pi \left(\frac{V_p}{V_i} \right)$. And if $V_p = V_i$, so C_L becomes a fixed number. So, this is what the ideal theory is all about. But in reality this is not the case because there are issues like effect of viscosity, boundary layer separations that makes the flow to have non-uniform pressure distribution. So, obviously, we will not try to use this ideal theory for a realistic applications. And if it happens to be for wind turbine design then we have to be more realistic or careful.

So, for that reasons we define the term Reynolds number

$$\text{Reynolds number, } R_e = \frac{\rho V_i D}{\mu}$$

So, Reynolds number is defined for the flow which is available in the free stream which is a combined effect of the wind velocity and if you harness this flow through a diameter D and μ stands for viscosity then we can calculate this Reynolds number R_e for the flow. And for a given Reynolds number, we can plot various values of velocity ratio with respect to lift coefficients and in fact, this is what we call as experimental observation or experimental trend. So, this trend depends on two factors one is L/D ratio that is lift to drag ratio. And if it becomes higher and higher that means, lift is much higher than the drag or lift to drag is a kind of a fixed number, lift is almost two times of drag for a given Reynolds number case. So, that way one can



say what will be the upper limit of this lift coefficients. So, for an ideal theory, when lift to drag ratio goes to infinity, C_L will keep on increasing, but that is not the trend. But for a particular velocity number we can get a big difference between real and ideal number. So, for example, at velocity ratio of 2 the ideal theory will give predictive lift coefficient as 12, but your experimental evidence will show lift coefficient as something close to maybe 4.5. So, that much difference comes into picture for a non-uniform pressure distribution case when you do the analysis for a realistic flow. So, that is the reason, we normally take the data from the experimental evidence for wind turbine calculations and that is dependent on this velocity ratio.

Now, having said this lift force, we will try to introduce another term which is called as drag. Drag is nothing, but the force that acts in the direction of the stream. So, in other words, when we say that a rotating cylinder is placed in a stream of air, the drag force is the force in the direction of the stream. Here in this particular case your drag force will be 0 because the Magnus effect does not give the indication of the drag force because when you say velocity pattern and pressure distribution in the direction of the stream. But in general way we say this drag force is as per below expression. Essentially this drag force is also a function of Reynolds number.

$$\text{Drag force, } F_D = C_D A \left(\frac{1}{2} \rho V_i^2 \right)$$

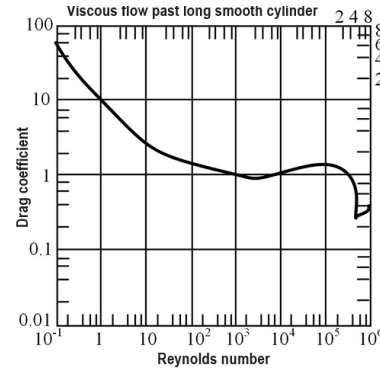
$$\text{Drag coefficient; } C_D = f(R_e)$$

$$\text{Reynolds number, } R_e = \frac{(2R)V_i\rho}{\mu}$$

μ :Viscosity of air; ρ : Density of air V_i :Incoming uniform air velocity

A :Projected area(= $2RH$) R : Radius of cylinder H : Length (height) of cylinder

So, in an ideal theory, we have non-uniform pressure distribution. So, of course, in an ideal theory there is case where $C_L = 2\pi$ when $V_p = V_i$ and for that case your drag force will be 0. That is the prediction from the ideal theory. But in realistic case when there is a non-uniform pressure difference, still there will be a drag force. That is the reason, for real fluid, the pressure imbalance results a drag force in the direction of free stream. And, that is a function of again Reynolds number. So, essentially both lift and drag coefficients are function of Reynolds number and that has to be found out from the experimental observations. So, a typical trend is shown here where we present the drag coefficient as a function of Reynolds number.



Drag coefficient; $C_D = f(R_e)$

$$\text{Reynolds number, } R_e = \frac{(2R)V_i\rho}{\mu}$$

μ :Viscosity of air; ρ : Density of air R :Radius of cylinder V_i :Uniform air velocity

And in more and more analysis, when the Reynolds number is much higher in the order of 10^5 to 10^6 , then this is a close number of C_D is approximately 0.4. So, this is all about the drag force.

Knowing all these lift and drag force concepts, we are now in a position to define various designs of wind machines. So, wind machines are typically conventional machines which captures wind kinetic energy and convert it to power or mechanical work. So, we have shown two fluid mechanics principles on how we can harness power from wind by using these machines, one is through Bernoulli's theorem & other is through Magnus effects. Typically, wind machines are known as wind turbines because these machines produce power. So, whatever machine produces power, we call it as a turbine. So, specifically we call them as wind turbines and they are classified in four major category based on axis of the rotation, installation sites, operation scheme and aerodynamic force. So, under axis of rotation, we have put two categories, horizontal axis wind turbine, and vertical axis wind turbine. Based on aerodynamic force which is the other extreme we have 3 types, one is called as a drag based turbine, 2nd one is lift based turbine, & other is the combined type. Now, moving on to other category, for commercial point of view we have classification

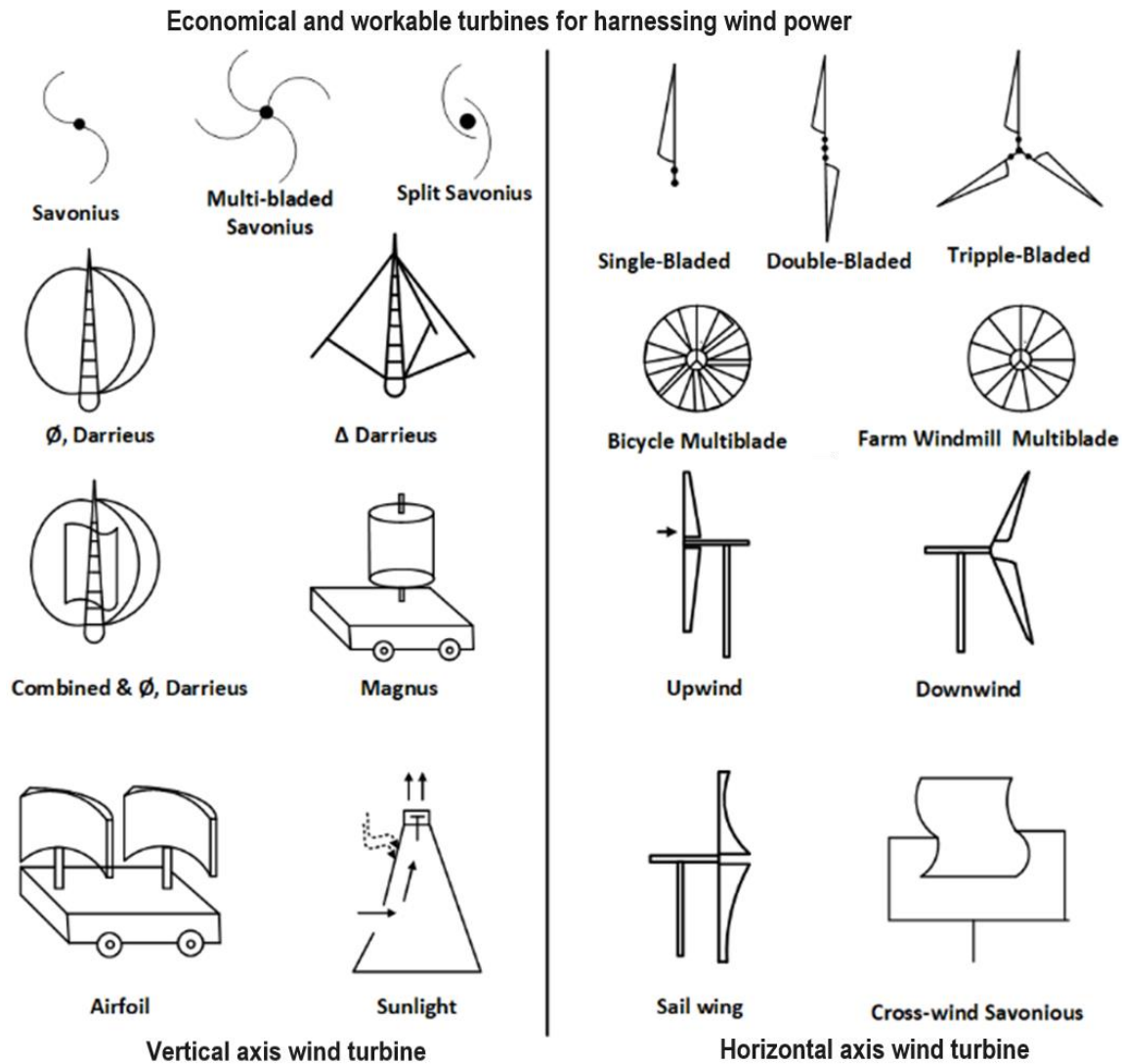
under the heading, installation sites. Here, we can have offshore turbine or onshore turbine that means, some turbines can be put at close to sea & other is far away from the sea. Now, under operation scheme, it can be fixed control type or variable control type.

Other main important aspect is rotor diameter and power rating. So, under rotor diameter and power rating, turbines fall under different roofs from micro to large scales. And typically when you go for large scale type, it is taken care by horizontal axis wind turbines, but when you go for lower side, this is taken care by vertical axis wind turbines. But under that roof there are multiple ways we can go for innovative designs. But the each of them has their own advantage and disadvantage. For example, when you say horizontal axis wind turbines, they are very good for more power generation that is more than 100 kW, but they require a very safe structural site because they operate at higher wind speed. So, obviously, they are very prone for stability of the structure. And again at a given location, there is possibility that wind may not flow in a particular direction. The horizontal axis wind turbines are very prone to the direction of the wind. So, you have to find the wind locations otherwise it will give a detrimental effect. But in case of the vertical axis wind turbines, their direction can be changed based on the direction of the wind because of their geometric features.

So, in a horizontal axis wind turbine the axis is perpendicular to this plane and in a vertical axis wind turbine if you can see in the design, the wind direction keeps on changing from the entire surface. So, under horizontal wind turbines, we have upwind rotor & downwind rotor. Under vertical axis we have Savonius, Darrieus and Giromill. Apart from this there are many other kinds like we have multi rotor, four-arm rotor, & many other designs. And most important fact is that each of the design has a limitation of different tip speed ratio. So, for a given wind speed, we require a fixed tip speed ratio. So, this is all about the basic difference because each horizontal axis and vertical axis wind turbine has its own merits and demerits.

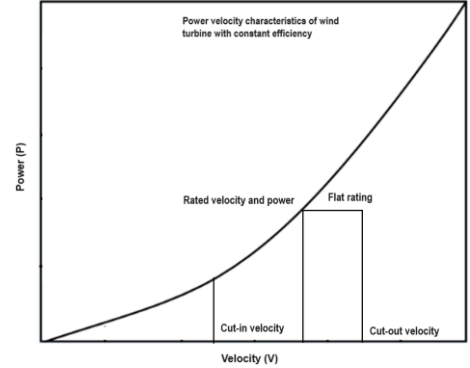
And ultimately, if you classify the wind machines based on these horizontal and vertical axis type, we have this particular pictorial table that shows that most economical and workable turbines for harnessing wind power. And moreover all lift based turbines are

based on Magnus effects and all drag based wind turbines are based on the Bernoulli's theorem from a basic fluid mechanics or aerodynamics point of view.



And the last part we want to focus is that when you think about horizontal or vertical wind turbines, we have shown this particular classical figures which says the power coefficient as a function of tip speed ratio and each design of these wind turbines have their limitations in their tip speed ratio. And this is because from power generation viewpoint, we expect that the turbine should operate in a smooth manner and we have seen here that the power is function of density, area and V_i^3 , & the maximum axial force is a function of diameter and V_i^2 which means any fluctuation in the velocity will have a detrimental effect on the power as well as on the axial thrust. Axial thrust denotes about the stability of the structure.

So, it means that for wind turbine operations, we have a narrow range of wind speed in which we should operate. So, that is the reason, we normally plot this power and velocity characteristics curve. We can define a cut-in-velocity and cut-out-velocity. So, cut-in-velocity and cut-out-velocity is nothing, but your range of V_i at a given location. This V_i range fluctuates at a given location and it keeps on fluctuating over the day or month as well as a year. So, based on this we find mean wind velocity at that particular location.



So, from this we drop another vertical which we call as \bar{V} which is nothing, but average wind velocity. So, it means that, irrespective of the availability of wind speed much above V_i , we put our rating of the wind turbines in a range, which we call as a flat rating. That means, a turbine should operate within that range for continuous supply of power, irrespective of fluctuation in the wind speed.

$$\text{Mean wind velocity, } \bar{V} = \frac{\sum_i^n V_i}{n}; n: \text{Number of observations}$$

So, this is the summary of this wind turbine operations. The cost effective method to design a windmill is to produce rated power less than maximum prevailing wind velocity using smaller turbine and generator unit. So, a flat rating is defined for any turbine machine assembly that produces constant output at all wind speed. Since there is a loss in efficiency and power at low velocity, wind turbine is designed to operate at a minimum wind speed. And then we define a term which is known as cut-in-velocity. And in order to protect the wind turbine wheel against the damage at high end velocity, we also put up the upper limit which is called as a cut-out-velocity. So, through this cut-in and cut-out velocity, we should be able to produce wind power continuously. So, the wind turbine must have a capability to operate at variable load over a narrow range to deliver a constant power. So, you have to operate at various wind speed loads, but to give a constant power. So, this is the entire summary of turbines irrespective of what we are going to use. So, now we will move on to the last point that is solving a numerical problems based on our discussion so far.

Q1. The average uniform wind flow at a certain geographical location is 13 m/s at standard atmosphere. Calculate the lift, drag and resultant forces on 1.8 m diameter 90 m long smooth cylinder rotating at 140 rpm in the wind stream.

So, the problem statement goes like this. The average wind flow in a certain geographical location is 13 m/s at standard atmosphere, we need to find out lift force, drag force and resultant force, using a 1.8 m diameter and 90 m long smooth cylinder that rotates at 140 rpm in the wind speed. So, the problem statement is simple, but here to get this lift, drag forces, we need to rely on this classical graphs, lift coefficient and drag coefficients as a function of Reynolds number and velocity ratio.

So, for the solution first thing that you see is Standard atmospheric conditions

$$p = 1 \text{ atm}; T = 15^\circ\text{C} = 288 \text{ K}$$

$$\Rightarrow \rho = \frac{p}{RT}; R = 287 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$\Rightarrow \text{Density of air, } \rho = 1.23 \frac{\text{kg}}{\text{m}^3}$$

And at this atmospheric pressure and temperatures, we can refer the data table to find the viscosity of air.

$$\mu = 19.7 \times 10^{-6} \text{ N} \cdot \text{s}/\text{m}^2$$

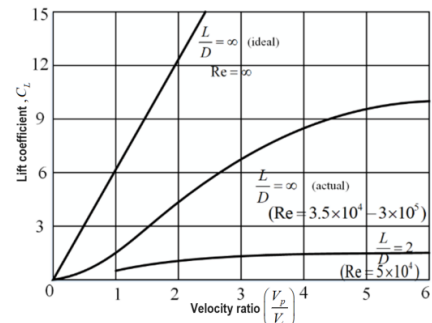
$$\text{Data given, } D = 1.8 \text{ m}, N = 140 \text{ rpm}$$

Now, with this data we will be able to solve this problem.

$$\text{Peripheral velocity, } V_p = 2\pi RN = 13.2 \text{ m/s}$$

$$\text{From given data, } V_i = 13 \frac{\text{m}}{\text{s}}; \text{ So we can see that, } \frac{V_p}{V_i} \approx 1$$

$$\begin{aligned} \text{For this case, Reynolds number, } R_e &= \frac{\rho V_i D}{\mu} = \frac{1.23 \times 13 \times 1.8}{19.7 \times 10^{-6}} \\ &= 1.46 \times 10^6 \end{aligned}$$



$$\text{Dynamic pressure, } q_{\infty} = \frac{1}{2} \rho V_i^2 = \frac{1}{2} \times 1.23 \times 13^2 = 104 \text{ N/m}^2$$

So, let us go to the basic expressions

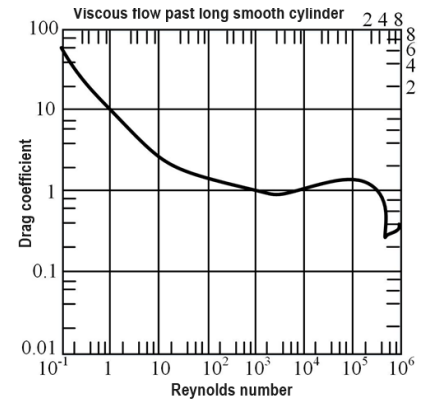
$$\text{Lift coefficient, } C_L = 2\pi \left(\frac{V_p}{V_i} \right)$$

$$\text{Lift force, } F_L = C_L A \left(\frac{1}{2} \rho V_i^2 \right); A = \text{projected area} = 2RH = 162 \text{ m}^2$$

So, to find C_L we require this graph. If you take an experimental observation graph, we have to take this middle plot in this because that plot fits closely to our Reynolds number value. So, for velocity ratio close to 1, at this location if you drop it, so approximately we can say C_L can be 1.

$$\text{Now Lift force, } F_L = C_L A \left(\frac{1}{2} \rho V_i^2 \right) = 1 \times 162 \times 104 = 16848 \text{ N}$$

$$\text{And Drag force, } F_D = C_D A \left(\frac{1}{2} \rho V_i^2 \right)$$



Now, for Reynolds number in the range of 10^6 , if you check this particular graph,

$$C_D \approx 0.4; \text{ So Drag force, } F_D = 6739 \text{ N}$$

$$\text{Resultant Force, } F = \sqrt{F_D^2 + F_L^2} = 18146 \text{ N}$$

And resultant force make an angle ϕ with respect to wind direction.

$$\phi = \tan^{-1} \left(\frac{F_D}{F_L} \right) = 21^\circ$$

So, we have lift force, drag force and resultant force. However, the theoretical value of lift & drag values are

$$(F_D)_{th} = 0;$$

$$(F_L)_{th} = 2\pi \times 162 \times 104 = 105860 \text{ N} ; \text{ because, } \frac{V_p}{V_L} = 1, \text{ so } C_L = 2\pi$$

So, what we understood here is that, your realistic object that is the cylinder experience 16848 Newton, but actually $(F_L)_{th}$ value would be 105860 Newton. Correspondingly $(F_D)_{exp}$ is 6439 Newton, but $(F_D)_{th} = 0$. So, the net effect is that the ideal theory does not give the real replication of experimental observations. So, we must use the experimental data as a benchmark to calculate the C_L & C_D when you design a wind turbine. So, this is all about these problem. Thank you for your attention. With this I conclude this lecture.