POWER PLANT SYSTEM ENGINEERING

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Module 4

Lec 3: Wind Energy - Part I

Dear learners, greetings from IIT Guwahati. We are in the MOOCs course that is Power Plant System Engineering module 4. The title of this module is Hydro and Renewable Energy Power Generation System. So, previous lectures we have emphasized about the hydro energy part. From now onwards we will focus our attention to renewable energy systems.

The first category of renewable energy system is the wind energy. So, in this lecture we will emphasize on some specific topics. First let us see, what this renewable energy resources is and why wind energy falls under this category. Then we will see the impact of wind energy in the recent days. So, we will try to see the energy potential that can be achieved through wind. Then we will try to understand the fluid mechanics principles for harnessing wind power. And to have this wind power, we require some machines and we call them as wind machines or wind turbines. Now, when these wind turbines are installed at a particular geographical location, we must understand about its structural stability that means, the way of analysis of calculating forces on wind turbine blades. So, these are the topics of discussions in the today's lectures. Now, let us go one by one.

So, the first point that I need to emphasize is renewable energy resources. In recent days energy has been basic necessity for the existence of mankind and in every sectors and in fact, in all developing countries, the role of energy is vital in every sectors requires such as agriculture, industry, residential and in the commercial firms. Since the energy demands are very huge, the conventional energy resources are not sufficient to meet these demands. Hence, we need to explore other category of energy resources. So, nature has given us plenty of opportunities and based on that, we can classify them as conventional sources or non-renewable energy resources, and the other category is non-conventional resources or renewable energy resources.

So, by considering this, when you use the word renewable, essentially the energy is formed by nature itself. So, in other words, the way we consume our energy or the rate at which we consume the energy must be generated by the nature itself. So, in such cases we classify those energy resources as renewable form of energy. So, one point of time conventional sources of energies was thought to be renewable, maybe 200 years back, because demands were less, but energy formation was more that means, resources were available in plenty. But in recent days, the fossil fuel based energy resources has shown sharp decline, hence they fall under the category of non-renewable sources of energy.

Then, as of now whatever the renewable sources of energy available, that we are going to quantify them and try to see if they have some potential to meet the today's energy needs. So, essentially the classification of energy resources are of two type, conventional sources, and non-conventional sources. Under the conventional sources, we have commercial or non-commercial category. Under Commercial category, we have a coal, petroleum, natural gas, and then electricity. In non-commercial category, we have firewood, straw, dried cow dungs. And those are the non-commercial form of energy.

But that part we have already discussed, at the time of discussing about the steam power plant where coal was the major source of energy. But now our attention will be more focused towards the renewable source of energy. Under these heading, we have bio energy, solar energy, wind energy, tidal energy, energy from urban waste and geothermal energy. We will try to touch upon all in subsequent lectures and in this module some of these energy resources will be covered. Now, let us understand very specific things that, what is this renewable form of energy which means that nature creates these energy on its own. And the rate at which it is consumed is much less than the rate at which it is formed.

So, that is the way, we say that these renewable energy are clean, they are abundant resources and continuously reproduced by the nature. They have the capability of self-renewing the resources. And under these headings, we have sunlight, wind, flowing water, biomass and earth's internal heat. As you see in this figure, in this solar energy category, we have solar cells or solar photovoltaics, we have wind machines or wind turbines, we have hydro power plants. Geothermal energy is another category of harnessing energy from earth resources or earth crust and bio energy which is mainly derived from bio derived plants. Then we will move on to this wind energy and before you go for the theoretical background let us see whether this wind energy has sufficient potential or not.

So, as you see in this figure over the last 25 years, there is a steady growth in the requirement of wind energy as a supplement for other resources of energy. Now with last 25 years the steady growth shows that wind has a tremendous potentials in supplementing the main or primary energy resources. Moreover since the energy is abundant, clean, and cheap, it can be also supplement to the major conventional energy resources, as a mainstream electricity generation, the wind has an immense potential which can be considered for the next generation's demands. So, with a projection by international Energy Agency, the road map says that about 25% of global energy demand will be made by wind energy by the end of 2050. So, that means, we must try to understand the capability of wind as a potential renewable energy resources.

So, having said this let us try to understand why this wind flows. Although we normally know that wind is the flow of air, but why this air flows. So, essential background for the wind formation is the solar heating. So, sun is the major resources of energy. When the sun's radiations falls on the earth surface, then it falls on lands as well as on all water bodies. So, the very basic difference between these is that, when it falls on the lands, the land gets heated. So, what happens is that the air associated with the ground or land mass also gets heated. So, once this gets heated, its density drops. So, it moves up. On the other hand when it falls on the water bodies, some of the radiation gets absorbed by the water itself and some gets evaporated from the water surface. So, that way the evaporated water goes up. Basically speaking, as you know that evaporation causes cooling, so, normally when water gets evaporated, it tries to supplement the heated air which is on the ground. So, because of this temperature difference between the water bodies and land mass, the air tries to move from higher density medium, which is the water to the lower density medium that is land. So, in this process the wind blows or air flows. So, we call it as wind. Now, in general, there are two types of wind, one is planetary wind and the other is the local wind.

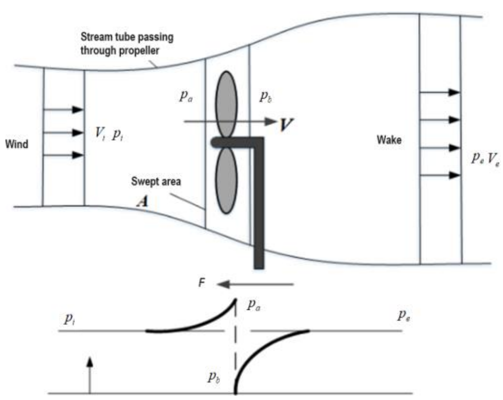
So, if you look at this particular figure as I mentioned earlier, here we have cool surface & warm surface. So, cool dense air comes down and warm light air goes off because warm means its density is less. So, air moves up. So, this circulation keeps on happening and the convection and advection currents tries to supplement each other.

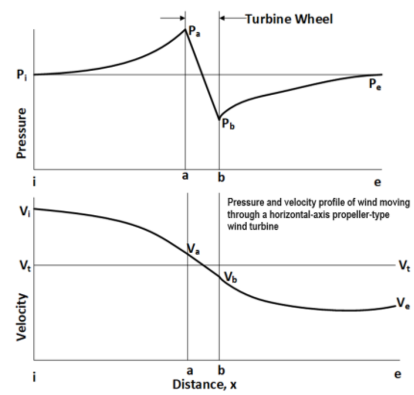
Now, the first category of wind is planetary wind. So, if you look at earth, this planetary wind comes from the effect of rotation of earth and the solar heating that falls on the entire earth mass. So, entire earth mass constitutes of water bodies as well as land mass. Now, looking at the earth surface, if you can say, we have equator somewhere in the middle of the earth and we have North Pole and South Pole. So, eventually the hot air region is mainly the equator area and the cold air regions are North Pole as well as South Pole. When the radiance of the Sun falls on earth surface and at the same time earth is rotating on its own axis as well as around the sun. So, we all know that eventually the equator regions gets the enough solar radiation. That means, air becomes hotter there. So, this hot air moves up and goes to the upper atmosphere and it tries to flow towards the North Pole or South Pole and we call this as air movement from equator to poles. Side by side when it goes in that direction, it pushes the cooler air in the vicinity of earth surface and they try to move towards equator and hence such solar heating causes the planetary wind. In fact, one becomes cool surface, other becomes hot surface. Dense air cools on the surface itself whereas, the hot air moves in the upper atmosphere. So, this is what we call as planetary wind.

Other kind of wind is called as local wind. So, essentially if you want to see local wind at one particular location, then there are water bodies as well as land mass. So, in this figure, we have water bodies here and we have land mass. And we have daytime as well as the night time. So, let us try to see, what happens. The radiation from the Sun falls on both these land mass and water bodies. Eventually, when the radiation of the Sun falls on the water surface, some of the water vapours get evaporated. So, that means, it goes to the atmosphere, but that is replaced by the hot air that comes from the land mass. So, this circulation keeps on happening because hot air has a less density, it flows towards the water medium at little bit higher altitude and finally, it comes and replaces the cool air. So, thereby the circulation happens. Air heated over the land keeps rising and this warm air gets cooled in the upper atmosphere and keeps descending, then this cool air over water bodies again comes back to the land. So, the heated air is again replaced by this cool air that comes from the water bodies. So, this particular concept happens in the day time. So, exactly reverse phenomena operates in the night time. Normally we have warm air that comes from water & they are replaced by air mass that comes from the land. So, this phenomena is normally called as land breeze, when the cool air from land flows towards the warmer water bodies. And other term we use is the sea breeze, when wind blows from water bodies to warmer lands.

In the second mechanism of local wind formation, we have hills and because of this land mass in the form of hills. Now because of the slopes of this hills, air above the slope heats up during the day and cools in down in the night more rapidly than the lowlands. So, because of this the heated air during the day keeps rising along the slopes and relatively cool air flows down during night. So, this is what we call as an elevation difference that causes the wind blowing or air motion and we call this as wind. So, having said this wind formation, we have now a standard a survey which shows that about 2% of solar radiation falling on the earth surface is converted to kinetic energy in the atmosphere and 30% of this kinetic energy occurs at the low elevation that is within 1km above earth surface. Now if it is possible to harness this 30% kinetic energy of this air, which normally happens at lower elevations then it can satisfy the energy requirements of any developed countries. So, because of these reasons, wind power has a compelling positive arguments, as a potential resource for future energy generation and moreover it is pollution free and also available in free.

Now, let us understand the fluid mechanics principle of wind power. To harness wind energy, we require some structures and we call that as wind machines. And this particular figure below is the schematic view of a wind machine or typically a wind turbine. And it is installed at a particular geographical location. Depending on the size, we can fix its area that is called as swept area. That means, we are capturing the wind which is available at any geographical location. So, based on the area which is perpendicular to the wind direction, we call this as swept area and considering this area we can imagine to have a stream tube passing through this turbine or propeller. Why you call this as a propeller because turbine is connected to some shaft and rotor arrangement that means, wind drives this propeller.





Now, there are two things, one is a particular segment that is from a to b in which the turbines or wind machines are installed and another is the free stream area that is between i and e that is inlet and exit. So, this is the entire domain in which we are bothered about the location of the wind machines. So, wind flows at some velocity V_i and pressure p_i. This condition is inlet condition. And wind leaves from the turbines that is far away from this location that is from point B. So, we call this as exit condition and its pressure is p_e and velocity is V_e. Obviously, to understand about the pressure and velocity distribution, one can simply apply the Bernoulli's equation.

So, initially when the wind approaches towards the turbine, the initial pressure keeps rising and suddenly entire energy gets converted to power. So, pressure suddenly drops from p_a to p_b. So, ideally or hypothetically, it will be sharp drop, but eventually what happens is that the drop in pressure from p_a to p_b is not exactly sharp, it is an inclined straight line. Whereas, we do not see the same sharp drop in case of velocity drop. The velocity drop takes place in a gradual manner and it starts dropping from ‘a’ to ‘b’. So, essentially between the locations ‘a’ and ‘b’, we have the turbine installation or wind machine installation. So, power is being harnessed at these locations. So, because of this reason pressure and velocity drops. Now, eventually after point b, the wind starts gaining its further momentum and tries to expand and because of this reason, pressure further increases, and final velocity V_e is nothing, but your exit velocity. So, there are three velocities, one is inlet velocity of the wind, second is exit velocity of the wind and there is a velocity at which the rotor rotates and that is called as turbine velocity V_t. And in fact, for a given wind speed we have a fixed value of V_t. So, we need to find out what is the relation between p_a, V_a, p_b & V_b and at a particular stream velocity, V_i what should be the value of V_t.

So, this is the essential theme of the principle of power generation. So, essentially speaking the outgoing exit stream must have less pressure and velocity because that will give you the maximum wind potential in this captured area. And in this zone wind kinetic energy will be maximized for rotation of the rotor or shaft. So, we need to understand the mathematical expression for this complete domain. So, we take this complete domain as i and e that is inlet and exit and location ‘a’ and ‘b’ as the location at which the turbine is being installed. And since there is a sharp drop in the pressure, we will keep aside ‘a’ and ‘b’ separately and try to apply Bernoulli's equation between ‘i and a’ and subsequently between ‘b’ and ‘e’. So, based on this we write these general energy equations from ‘i to a’ and ‘b to e’. And we also neglect the elevation difference.

General energy equation for inlet $(i\ to\ a)$ and exit region $(b\ \ to\ e)$:

$$p_i + \frac{1}{2}\rho V_i^2 = p_a + \frac{1}{2}\rho V_a^2; p_e + \frac{1}{2}\rho V_e^2 = p_b + \frac{1}{2}\rho V_b^2$$

Here we assume the density of air to remain constant because its flow is incompressible. And as you see from the figure

Since, $V_i > V_a$ & $V_b > V_e \Rightarrow p_a > p_i$ & $p_b < p_e$

Now, from these two equations, we can get an expression for pressure difference between 'a' & 'b' in terms of initial pressure, exit pressure, initial velocity, exit velocity and velocities at 'a' & 'b'.

After subtraction, $p_a - p_b = \left(p_i + \rho \frac{V_i^2 - V_a^2}{2}\right) - \left(p_e + \rho \frac{V_e^2 - V_b^2}{2}\right)$

And here we will try to impose some assumptions which are more realistic way of estimating. So, for example, if you assume that $p_e = p_i$ that means, we give a sufficient domain so that the wake effects of wind that goes out from the turbine will not affect the main free stream wind. So, for that reason, we allowed this velocity to drop gradually and it finally, matches with the free stream. And similarly pressure drop also happens to be in a gradual manner and finally, the pressure becomes p_e. So, since there is no wake effects then we can make an assumptions that $p_e \approx p_i$ & $V_a \approx V_b \approx V_{t,}$ they are also approximately equal and that part is essentially is nothing but your turbine velocity that eventually happens at this location at which the wind turbine or wind machine is installed. So, with this assumptions, we get an expression for pressure difference between 'a' and 'b'

Far away from turbine, $p_e \approx p_i$ & $V_a \approx V_b \approx V_t \Rightarrow p_a - p_b = \frac{1}{2}\rho(V_i^2 - V_e^2)$

Now, next thing is that from these expressions we need to find out the axial force. The axial force is nothing, but the force that is being experienced by this wind machines due to this pressure difference captured over the swept area 'a' and we can write this as

Axial force, $F_x = (p_a - p_b)A = \frac{1}{2}\rho A(V_i^2 - V_e^2)$

So, this is one way of finding this axial force through pressure difference. Other way of calculating this axial force is through mass flow rate of the turbine. So, normally the force is nothing, but rate of change of momentum. So, rate of change of momentum is nothing, but mass times velocity difference. So, this velocity difference can be found from initial velocity and the exit velocity.

Mass flow rate through the turbine, $\dot{m} = \rho A V_t$

Axial force, $F_x = \dot{m}(\Delta V) = \rho A V_t (V_i - V_e)$

So, we have two expressions for axial force in two different forms, one is through Newton's law & other is from the energy equations. So, by equating both the equations, we get an expression for the turbine velocity, V_t

Equating F_x and solving for V_t, $V_t = \frac{1}{2}(V_i + V_e)$

So, once you know V_t then next job is to find out the steady flow work, which is nothing, but the difference in the kinetic energy and when you multiply this into mass that is $\dot{m}$ it will give you the total work.

Steady flow work for thermodynamic system bounded by i and e, $W = \frac{1}{2}(V_i^2 - V_e^2) \times \dot{m}$

Now, from this work transferred we can get the steady flow power.

Steady flow power, $P = \frac{\dot{m}}{2}(V_i^2 - V_e^2) \Rightarrow P = \frac{\rho A V_t}{2}(V_i^2 - V_e^2)$

$$= \frac{1}{4}\rho A (V_i + V_e)(V_i^2 - V_e^2)$$

V_i & V_e:Inlet and exit wind velocity; V_t:Velocity within the turbine; ρ:Density of air

$(p_a - p_b)$:Pressure difference across turbine wheel;

A:Projected area perpendicular to wind stream

So, you have to pause your attention right now here, because we have to find out which are typically constants. So, from this expressions we can say that ρ is a constant quantity does not change in this entire domain between ‘i and e’. ‘a’ is the swept area that is fixed by the installation location, V_i is your wind speed at that particular location, but we do not have any control on it. But our job is to maximize the wind kinetic energy. So, only control that we have is on V_e. So, eventually if you can reduce this V_e as small as possible then the negative term in Power expression will come down. So, that means, we need to bring down this V_e term, because it kills the wind power.

To do that mathematically we have to differentiate Power with respect to V_e and we get a quadratic equation. And from this we obtain the optimum exit velocity. And putting this optimum value we get the expression for maximum power as stated below..

Wind power, $P = \frac{1}{4}\rho A\left(V_i + V_e\right)\left(V_i^2 - V_e^2\right)$

For maximum power at given incoming wind velocity, $\frac{dP}{dV_e} = 0$

$\Rightarrow 3V_e^2 + 2V_i V_e - V_i^2 = 0$ so that, $V_{e,opt} = \frac{V_i}{3}$ & $P_{\max} = \frac{8}{27}\rho A V_i^3$

V_i & V_e : Inlet and exit wind velocity; $\left(p_a - p_b\right)$: Pressure difference across turbine wheel
V_t : Velocity within the turbine; A: Projected area perpenticular to wind stream; ρ: Density of air

So, this is how we get the power from this turbine. Let us find out what is the total power available in the wind.

Total power avaialble in the wind stream,

$P_t = \frac{1}{2}\dot{m}\, V_i^2 = \frac{1}{2}\left(\rho A V_i\right) V_i^2 = \frac{1}{2}\rho A V_i^3$

Now, if you take this ratio between these two terms, we call it as a maximum efficiency of the turbine or maximum power coefficient.

Maximum efficiency/Maximum power coefficient, $\eta_{\max} = \frac{P_{\max}}{P_t} = \frac{16}{27} = 0.593$

So, this number is a very significant. That means maximum efficiency, that can be captured from any turbine does not exceed beyond 59.3%. So, efficiency of any turbine cannot go beyond this number. So, this is a particular limiting case and this comes out from theory and it is a fixed number. And since it comes from a theoretical analysis, so none of the turbines can harness a power coefficient more than this number. So, this particular number is most familiarly known as Betz number or we put this limit as Betz limit which does not exceed 59.3% or 0.593. So, for all wind turbine analysis Betz number is the most or optimum number that any turbine cannot approach. So, this is called as an ideal propeller type turbine.

Now, there are two efficiencies, one we have already explained as the maximum efficiency, but for any realistic turbine, we can write its efficiency as the below expression.

Maximum efficiency/Maximum power coefficient,

$$\eta_{\max} = \frac{P_{\max}}{P_t} = \frac{16}{27} = 0.593$$

Efficiency of wind turbine,

$$\eta = \frac{P}{P_t} \Rightarrow P = \eta\left(\frac{1}{2}\rho V_i^3\right); \eta \approx 30\% \text{ to } 40\%$$

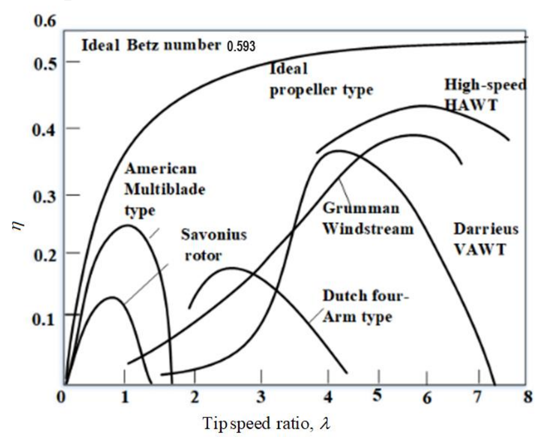
Here this efficiency normally range from 30% - 40%. So, conventional wind turbines have efficiency in this range. We also define another term which is called as a tip speed ratio expressed as λ.

Tip speed ratio, $\lambda = \frac{\omega r}{V_i}$; V_i : Inlet wind velocity

ω : Rotational velocity of the turbine

r : Radius of wind turbine

So, if you look at this turbine rotor from the tip, the velocity varies drastically because the structure is huge. So, we call this as a tip speed ratio. Now, depending on the location or installed structures, the tip speed ratio can vary. It can vary with r which depends on the size of the rotor. It can also vary with respect to V_i that is initial wind speed or it can also vary with the rotation of the rotor. The tip speed ratio is plotted against the efficiency term in the below graph.



So, as a theoretical number this λ is a fixed number that is efficiency approach to the Betz limit 0.593.30 00. So, we call this as an ideal propeller type turbine. Now, any other realistic type turbine that we operate at a particular location are classified as horizontal axis turbine, HAWT, vertical axis turbine and any kind of rotors we call as savonius rotor and we can have multi blade type rotor. So, and all these are nothing, but the wind machines, but if you can see here they have different numbers in terms of the tip speed ratio. For example, this multi blade type of rotor can have tip ratio close to maximum 1.8, but if you look at vertical axis type of wind turbines, they can go up to a tip speed ratio close to 7.5. So, this is how it can vary. And in fact, when you go for higher efficiency range then we can explore high speed horizontal axis wind turbines. So, depending on the particular geographical location at which this wind is available, the choice of rotor or turbines can be done. So, this analysis gives the more realistic view for applying wind machines at a particular location.

Another important point that I need to emphasize here that the power coefficient is strongly dependent on blade-to-wind speed ratio that reaches its maximum value at about 0.6 only at the maximum speed. And the blade speed drops rapidly at blade tip to wind speed ratio which is below 2. That means, if you take this particular multi blade type rotor for which the tip speed ratio does not exceeds 1.2, then you can see efficiency at a particular point drop drastically when the tip speed ratio is less than 2.

The next important analysis that we are going to discuss in the same wind power philosophy is the analysis of forces. So, in a particular propeller type wind turbine, there are two forces that act, one is the circumferential force which is in the direction of wheel rotation and in fact, it generates the torque for the rotor and for which electric power generation is possible. Another force, which is nothing, but your axial force that is generated due to wind stream and your turbine structure must withstand this axial force for harnessing the power. So, this particular axial force does not lead to any power production term rather our structure should withstand this axial force to harness the power from the wind. So, this gives the structural stability of the wind machines. And for this reason, axial force needs to be evaluated.

Now, all this analysis we must do at the maximum point of efficiency or optimum exit velocity. So, for that reasons let us try to understand what is the torque developed by the turbines.

Torque developed the wind turbine, $T = \frac{P}{\omega} = \frac{P}{\pi D N}$

It can be also represented in terms of rpm of the rotor.

$$\Rightarrow T = \eta\left(\frac{\rho D V_i^3}{8N}\right); P = \frac{\eta}{2} A V_i^3; A = \frac{\pi}{4} D^2$$

And at maximum efficiency, the maximum torque can be simplified to this number.

At maximum efficiency, $\eta_{\max} = \frac{16}{27}$

$$\Rightarrow T_{\max} = \frac{2}{27}\left(\frac{\rho D V_i^3}{N}\right)$$

And again we can find out the axial thrust on the turbine which is derived from your earlier expressions and it is expressed in terms of diameter of the rotor inlet wind velocity and exit wind velocity.

Axial thrust on wind turbine, $F_x = \frac{1}{2}\rho A(V_i^2 - V_e^2) = \frac{\pi}{8}\rho D^2(V_i^2 - V_e^2)$

And at maximum efficiency point we can get maximum axial thrust as below stated expression.

At maximum efficiency, $V_e = \frac{V_i}{3} \Rightarrow F_{x,\max} = \frac{4}{9}\rho A V_i^2 = \frac{\pi}{9}\rho D^2 V_i^2$

ω:Angular velocity; N: Revolutions per unit time; ρ:Density of air

D:Diameter of turbine wheel; V_i:Inlet wind velocity

So, these two terms are very vital one for torque generation and other for maintaining the structural stability. So, this is all about the wind theory. Another important point I need to emphasize here is that referring to this particular expressions like $F_{x,\max} = \frac{\pi}{9}\rho D^2 V_i^2$, one can minutely observe that the maximum thrust is proportional to the square root of diameter. Normally this diameter or the swept area is kept very large to capture more wind power, but at the same time while enlarging this diameter your axial thrust also goes up. That means, we need a heavy structure to harness this wind power. So, one has to make a balance design approach so that the maximum structure we can install based on the available wind speed at that particular location. So, for that reason the upper limit diameter is determined through appropriate design by ensuring the stability of this structure and other economical considerations. Of course, geographical site location is also more important. So, this is all about the contents that I need to deliver in this lecture. So, let us try to solve a numerical problem based on our analysis that is principle of wind power.

Q1. The average wind flow at a certain geographical location is 8 m/s at standard atmosphere. Calculate, (a) total power density of wind stream; (b) maximum power density obtainable from a wind turbine; (c) realistic power density obtainable from a horizontal axis wind turbine; (d) total power produced from 100 m turbine wheel dimeter; (e) torque and axial thrust when the turbine is operating at 40 rpm at maximum efficiency.

It is a simple problem. So, the problem statement goes like this, the average wind speed at a particular geographical location is 8 m/s and it is at a standard atmosphere that means, we know pressure and temperature at that location. So, based on this we need to calculate the total power density which is available in the wind with respect to this 8 m/s. Then we need to find out the maximum power density which can be obtainable from wind turbine through Betz limit. Third is to find out the realistic density obtainable from a horizontal axis wind turbine. So, in that location if you want to put a wind machines which is nothing, but a horizontal axis wind turbine, then you need to assume certain efficiency. To assume certain efficiency, we can refer this particular curve. That means if you take a particular horizontal axis wind turbine, typically its average efficiency falls at 0.4. So, that is what we say it is a realistic power density corresponding to this efficiency. And at same location, if you use a vertical axis, efficiency may probably come down below 0.4. So, in this case we can say realistic power density for which efficiency can be treated as 0.4 or 40%. Then we need to find out the total power produced by 100 m wheel diameter. That means, if that horizontal axis wind turbine has wheel diameter of 100 m, then you have to find the total power. Now, for this total power we need to find the maximum thrust and torque, when the turbine rotates at 40 rpm at maximum efficiency point. So, for all these things we have derived this established formula. This problem also explains how you can just introduce that formula and keep calculating. So, let us understand the given data.

$V_i = 8\frac{\text{m}}{\text{s}};$

At Atmosperic condition, $p = 1 \text{ atm} = 101325\frac{\text{N}}{\text{m}^2}; T = 15° = 288 \text{ K}$

For Air, we take $R = 287\frac{\text{J}}{\text{kg.K}}$

So, keeping ideal gas equation we can find out the density of air ρ.

$$\rho = \frac{p}{RT} = \frac{101325}{287 \times 288} = 1.23\frac{\text{kg}}{\text{m}^3}$$

So, the first quantity to find out is the.

a) Total power density $\frac{P_{tot}}{A} = \frac{1}{2}\rho V_i^3 = 315\frac{\text{W}}{\text{m}^2}$

b) Maximum power density, $\frac{P_{max}}{A} = \frac{8}{27}\rho V_i^3 = 186.6\frac{\text{W}}{\text{m}^2}$ (By using Betz limit)

c) Realistic power density, $\frac{P}{A} = \eta\left(\frac{P_{max}}{A}\right); \eta$

$= 40\%$ for a horizontal axis wind turbine.

$$\Rightarrow \frac{P}{A} = 0.4 \times 186.6 = 74.6\ \frac{\text{W}}{\text{m}^2}$$

d) Total power produced from a 100 m turbine wheel, P

$D = 100\ m; A = \frac{\pi}{4}D^2$; And $P = A(74.6) = 602 \text{ kW}$

e) Torque at operating speed of 40 rpm

$$N = 40 \text{ rpm} = \frac{40}{60}\text{rev}/s; \text{Maximum Torque, } T_{max} = \frac{2}{27}\frac{\rho D V_i^3}{N} = \frac{2}{27}\frac{1.2 \times 1000 \times 8^3}{40/60}$$

$$= 7\ kN$$

f) Maximum axial thrust, $F_{x,\text{max}} = \frac{\pi}{9}\rho D^2 V_i^2 = 275 \text{ kN}$

So, let us try to understand the physics behind this. So, to produce a power of 600 kW by a wheel diameter of 100 m, we are able to get a maximum torque of 7 N. That means, for that torque we can say that the structural stability of that wind machine or horizontal axis wind turbine will have ability to withstand a force of at least 275 kN. Hence, the size of wind turbine is big. So, the requirement for wind turbine study is that, we need to understand the structural stability of the machines at a particular location. With this I conclude. Thank you for your attention.