POWER PLANT SYSTEM ENGINEERING

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Module 4

Lec 2: Hydro-Power System: Part-II

Dear learners, greetings from IIT, Guwahati. We are in the MOOCs course Power Plant System Engineering module 4 that is Hydro and Renewable Energy Power Generation System. In our last lecture we discussed about hydropower system part 1 in which we have given emphasis on water power and specifically power generation through Pelton wheel. So, moving further, in the second part of the lecture, we will introduce some more topics mainly the degree of reaction, then we will also introduce another class of turbine which is known as Francis Turbine. Then we have some basic introduction for Propeller and Kaplan Turbine. Then for comparison of turbines across various applications, we are going to define the specific speeds for all types of hydraulic turbines. And finally, when these turbines are being used as a prototype in a realistic hydro power system, there is also a need that similar type of turbine has to be tested in the laboratory and to do that we need to maintain the model, size, designs. And for that reason, we are going to define certain parameters like scale ratio, unit speed, unit power and unit discharge. So, this is the overall philosophy for our lecture.

Just to give the brief insight, in our last class we have emphasized water power is a renewable form of energy, but the only criticality of this power generation is that we need to have a potential height. Another positive side of water is that water has a density of 1000 kg/m^3 and if a large slug of mass is available at certain height, then electricity production will be in a huge amount, given that the turbine is supposed to run throughout the year. So, this is the very basic theme of hydroelectric power plant. Although the initial infrastructure is very high, yet the running cost is low and we do not require high skilled manpower. Anyway in our last lecture, we have emphasized a class of turbine which was categorically told as impulse turbine. It is a Pelton wheel. And Pelton wheel is a very basic & old concept, also known as a water wheel. People used to harness water power through this wheel. But one important thing in this Pelton wheel is that, starting from the height, where the water is available to the turbine point that is where the wheel is positioned, the entire flow passage is almost atmospheric. So, we have to first convert the potential head to kinetic energy and that kinetic energy through this nozzles that was the essentially arrangement in a Pelton wheel and nozzle gives the water jets that in turn rotates the wheel. So, this is the basic aspect for why we call this an impulse turbine.

Now, let us try to understand the theoretical background of this. Now, if you look at this Bernoulli's equations at any inlet and outlet of the turbines, then we can write that equation

Apply Bernoulli's equation at inlet & outlet of turbine,

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}=E+\frac{p_2}{\rho g}+\frac{V_2^2}{2g};\ E\text{:Energy transferred from fluid to rotor}$$

So here, E is nothing, but the energy transfer from fluid to water in short this is your potential head.

When, $p_1 = p_2$(atmospheric) $\Rightarrow E = \frac{V_1^2 - V_2^2}{2g}$ (Impulse turbine)

So, this is nothing, but your velocity head and this is exactly what we do in an impulse turbine or in a Pelton wheel. So, the entire water always experiences atmospheric pressure throughout its path or the entire velocity head is being converted for power generation or that is the energy transferred from fluid to rotor.

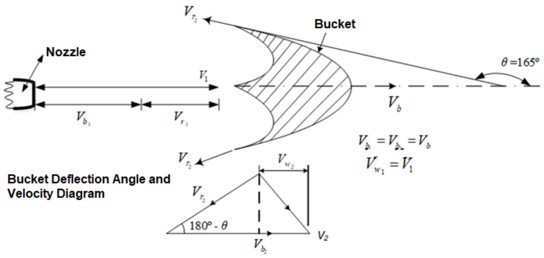
When,$V_1 = V_2 \Rightarrow E = \frac{p_1 - p_2}{\rho g}$ (Pure reaction turbine)

Now, another category of turbine that can be thought of is that, let us say instead of making $p_1 = p_2$, let us say $V_1 = V_2$. So, in that case your velocity head will vanish. Now, what we will arrive at is E that is energy transferred from fluid to rotor is nothing, but your pressure head. So, ideally this is called as a pure reaction turbine.

However, in our basic structure of turbine manufacturing, we cannot have a pure reaction. Although we can have complete impulse turbine as you see in case of Pelton wheel, but reaction turbine is not possible. So, there has to be some compromise between velocity head as well as the pressure head and such turbine is called as mixed flow turbine. So, in mixed flow turbine both velocity head and pressure head take part in the energy transferring from fluid to rotor.

Now, considering this aspect we define a term which is called as a degree of reaction for a hydraulic turbine, which is defined as the ratio of energy transferred due to pressure drop to the total energy transfer. And considering this we call the turbine either a pure impulse or pure reaction. So, if you take pure impulse it is a Pelton wheel and for this Pelton wheel we can say, its degree of reaction is 0. Now, when you say pure reaction we get the energy transferred from the fluid to rotor by virtue of the pressure drop, thereby exerting a reaction by the Newton's third law of motion. And such a turbine is called as a reaction turbine. So, here if you see this particular diagram for a Pelton wheel, we can see that there is a nozzle which essentially gives the necessary water jet for running the bucket of water. So, that means, velocity is getting converted to energy that is transferred to the rotor.

Now, considering this, since we cannot have complete pure reaction turbines, so we define this degree of reaction. From Euler equation which we derived in the last class that is



Energy transferred from fluid to rotor, $E = \frac{1}{g}(V_{b1}V_{w1} - V_{b2}V_{w2})$

So, in this particular diagrams V_1 stands for either bucket velocity or blade velocity. For a Pelton wheel V_{b1}& V_{b2} are equal. But since we are looking for a reaction turbine, we say that there will be both the term V_{b1} and V_2/ V_{b1}& V_{b2}. Now, if you take this particular expression, you will see that there is a term $V_{b2}V_{w2}$ which is typically negative quantity.

So, if somehow we make this quantity to 0 that means, by designing a suitable blade, if you can make V_{w2} component to be 0 then we can maximize this energy. So, Euler's equations now becomes

$$E = \frac{1}{g}(V_{b1}V_{w1} - V_{b2}V_{w2}) = \frac{V_{b1}V_{w1}}{g}$$

V_{b1}&V_{w1}:Blade and whirl velocity at the inlet

Now, this is the maximum energy which can be transferred from fluid to rotor, but degree of reactions that means, the component of energy due to pressure drop is nothing, but

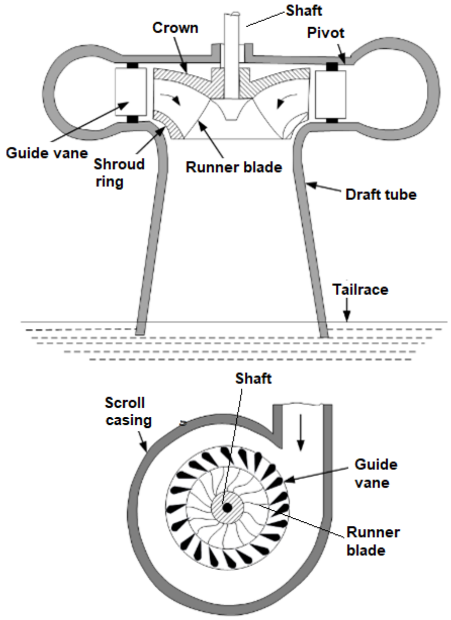
$$\text{Degree of reaction}, R = \frac{(p_1 - p_2)/_{\rho g}}{E}$$

$$\Rightarrow R = \frac{E - \left(\frac{V_1^2 - V_2^2}{2g}\right)}{E} = 1 - \frac{V_1^2 - V_2^2}{2gE}$$

$$\Rightarrow R = 1 - \frac{V_1^2 - V_2^2}{2V_{b1}V_{w1}}$$

So, this equations can be simplified by to find the expression for degree of reaction in terms of absolute velocity V_1, V_2, V_{b1}& V_{w1}. So, this expression is normally used if it is a mixed flow type of turbine or it is a reaction turbine. So, we have told that there is no pure reaction turbine. So, there has to be some component in which the energy is transferred due to both pressure head as well as the velocity head.

Now, the most versatile turbine under a reaction turbine is called as Francis turbine. So, here we can say that the energy transfer from water to runner takes place due to the static pressure drop as well as the velocity head and they are mostly preferred for medium head and medium discharge. And since it is not a pure reaction turbine so, it is categorically told as a mixed flow variety with radial entry and axial outlet. Now, how it is radial entry and axial outlet, you can understand from this figure. So, water from the penstock enters to this scroll. This,



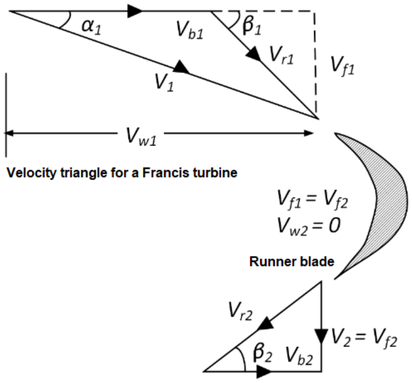
Schematic view of Francis turbine

penstock is nothing, but the water pipe or conduit in which the high head water is allowed to flow from certain location to the location of the turbine.

So, when it enters from the penstock, it passes through a casing which is called as scroll casing and this scroll casing surrounds the runner and this runner is integrated with two types of vanes, one is called guide vane & other is called runner blade. So, entire water gets interacted with all the guide vanes. So, slug of water that enters to the penstock & it interacts with all guide vanes. The role of the guide vane is to allow the suitable passage for the water to corresponding blade which is called as runner blade. So, this is to give the suitable velocity positions and velocity triangles for maximizing energy of water. So, this is what we call as runner blades. Now, it is discharged vertically to this plane. So, you can see water that comes from the guide vane to this, it comes through in this top view you can say how water is allowed to get discharge after this runner blade and it passes in a controlled way in a tube which is called as a draft tube. So, normally we say that the Francis turbine is controlled through the pressure difference because it is a reaction turbine. So, that is the reasons in order to have a smooth passage of water to maximize the energy available from the water, it has to pass in a controlled way through this draft tube to the tailrace. Tailrace is nothing, but the discharge point of water.

So, this is the overall picture. Now, if you look at the runner, they have number of curved blades normally 12 to 22 and they are welded to the shroud which means that these runner blades are fixed blades and for a given turbine, they cannot be deflected or they cannot be disturbed. So, there are 12 to 22 number of curved blades.

Then another important point that I have already mentioned is that, in a Pelton wheel we say that complete passage of water is purely atmospheric, but when the water jet comes that water jet hits at a particular time to fix number of buckets. And all buckets of Pelton wheel are not exposed to the water jet. So, sequentially one by one bucket gets exposed to this water jet. But in a Francis turbine you can see that all guide vanes of this runner are exposed to the complete slug of water and we say that it is a pressure regulated device. And that is the reason, entire thing happens in a closed tube and before the water gets discharged to the tailrace. So, this is the basic difference between the Pelton wheel and the Francis turbine. Now, the free end of the draft tube which is the discharge passage of water is submerged deep into the tailrace so that the water does not see any kind of atmospheric pressure in its path.



Velocity triangle for a Francis turbine

Now, let us try to understand how energy is harnessed from the water by looking at the velocity triangles that is drawn for a Francis turbine. So, for a fixed particular runner blade, we have drawn this velocity diagram. We have V_1that stands for absolute velocity of water getting discharged from the guide vanes. Now, runner is also rotating so when the water gets discharged we see its relative velocity is V_{r2}. The first point is that there are angles in the velocity diagram, guide vane angle is α_1 and runner blade angle is β_1. And then if you look at this particular velocity triangle, we have two components V_{f1}& V_{w1}. This V_{w1} consists of entire length. So, it is basically $V_{w1}$is directed opposite to this so that we get this velocity V_1 in these direction. So this is the inlet velocity triangle consisting of $V_1, V_{w1},$ & V_{f1}.

Now the angles are so designed that we need to ensure that V_{w2} should be 0 to harness maximum energy.

So when $\mathrm{V_{w2}} = 0;$ then essentially we have $V_{f1} = V_{f2}$.

What are V_{f1}& V_{f2}? So V_{f1}& V_{f2} for this Francis turbine, are called as a flow velocities or Meridional velocities. And $V_{f1} = V_{f2} = V_2$. Another important point is that, there are two velocity V_{b1}that is at inlet and V_{b2} at the outlet.

At inlet, whirl velocity $V_{w1} = V_{f1} \cot \alpha_1$

Blade of blade at inlet, $V_{b1} = V_{f1} \cot \alpha_1 (\cot \alpha_1 - \cot \beta_1)$

$V_{f1} = V_{f2}$:Meridional or flow velocity(constant); α_1:Exit angle of guide vane

$V_{w2} = 0; \alpha_2 = 0$:Axial discharge(no whirl component); β_1:Inlet blade angle

So, through this process we can find out the expressions for

Work done per second per unit mass, $E = V_{b1} V_{w1} = V_{f1}^2 \cot \alpha_1 (\cot \alpha_1 - \cot \beta_1)$

And looking at this velocity triangles this E can be expressed in terms of V_{f1}, & angles α_1& β_1 using this velocity diagram. Ultimately at the end, our main analysis is to find out degree of reaction, blading or diagram efficiency, hydraulic efficiency, and overall efficiency. These are the three efficiencies we are interested in. First is the blade or diagram efficiency which can be expressed as below.

Blading or diagram efficiency, $\eta_b = \dfrac{E}{E - E_1} = 1 - \dfrac{E_1}{E + E_1}$

$$= 1 - \frac{1}{1 + 2 \cot \alpha_1 (\cot \alpha_1 - \cot \beta_1)}$$

Loss of kinetic energy, $E_1 = \dfrac{1}{2}(V_{r2}^2 - V_{r1}^2)$;

Since blade is rotating and there is a difference in the relative velocity of water that is V_{r2}& V_{r1}. So E_1 is nothing, but the loss accounted in terms of loss in kinetic energy. Now, from this also we can find the expressions for degree of reaction and the hydraulic efficiency as per below equation.

Degree of reaction, $R = 1 - \dfrac{V_{f1}^2 \cot^2 \alpha_1}{E}$

Hydraulic efficiency, $\eta_h = \dfrac{E}{H} = \dfrac{V_{w1} V_{b1}}{gH}$; Overall efficiency, $\eta_o = \dfrac{P}{\rho Q g H}$

P:Total power output; Q:Discharge; H:Net head;α_1:Exit angle of guide vane

V_{r1}&V_{r2}:Relative velocity at inlet & exit of blade; β_1:Inlet blade angle

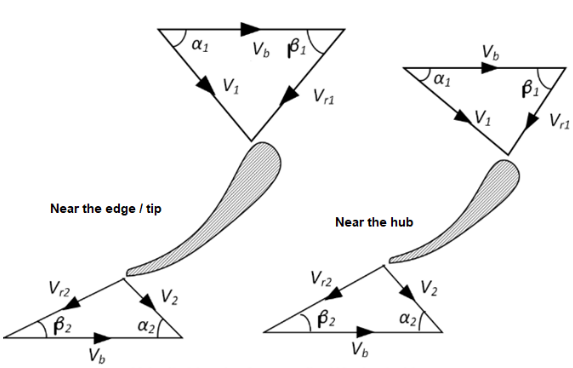
V_{b1}:Blade of blade at inlet; V_{w1}:Whirl velocity at inlet

$V_{f1} = V_{f2}$:Meridional or flow velocity (constant)

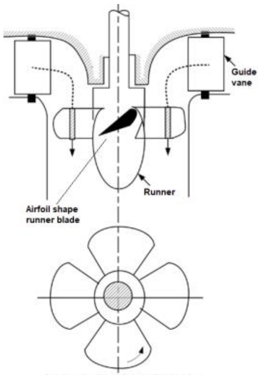
So, these are the some mathematical expressions that can be laid down for a Francis turbine. Then we will move to next segment of turbines which is propeller or Kaplan turbine. So, the propeller turbines are generally axial flow device. So, they are preferred for axial flow. And main intention of this turbines is that they provide large flow area by utilizing larger volume of water with small velocities. Here, possible way of harnessing power is through two ways, one is high head low discharge or low head high discharge. So, here we have larger volume of water, but small discharge. So, there are some locations where large quantity of water is available, but net head is less. For such cases we use propeller turbines. And actually this propeller turbines have axial flow runner with 4 to 6 blades of airfoil shapes and moreover there is a difference in this blades of propeller turbines with axial flow and the Francis turbines the blades are fixed, but here we have adjustable beds. propeller turbines are reaction turbines which are preferred for low heads that is in the range of 4m- 80m, but higher specific speed.

A special class of turbines, in the propeller turbine category is the Kaplan turbine. So, the Kaplan turbines have an advantage that the design features or manufacturing features of Kaplan turbines is such that, the turbine runner blades can be adjusted depending on the load changes. For example, at the down side of this, electric power is generated through the rotation from this turbine & when there is a change in the electric load in the grids then it automatically affects the location at which power is generated which is nothing, but at the location of the turbine. Now, for that if there is change in load, we need to adjust the power. For adjusting the power we have to look into the adjustment of flow rate. And the flow rate adjustment is mainly relied on the adjustment of guide vanes. So, for this Kaplan turbines there are possible advantage of adjustable runner blades, which can cater the load changes at the time of need. And why we say this? Because efficiency of the reaction turbine depends on the inlet blade angles. Since the Kaplan turbines allow the variation of inlet blade angles with change in the load, the turbines can run at maximum efficiency at all loads. And their hydraulic efficiency is about 90% and their specific speed varies from 260 to 280.

So, these are the some velocity triangle features for a propeller and Kaplan turbines.



Velocity triangles for propeller / Kaplan turbine



Schematic view of propeller turbine

Here we can see that the exit guide vane angle α_1 varies from 45°- 70° while inlet blade angle β_1 varies from 20° - 60°. So, these are some numbers, but if you see the outlet triangle, here V_2 is no longer V_{f2}. So, here we have V_{f1}& V_{f2} and these two numbers are not equal. And of course, we have V_2 as well. There is absolute velocity of the jet, then relative velocity. Another significant feature is that in the design of the blade there are two locations that is the hub and at the tip. So, at these two locations the change is very gradual from the hub to tip and at every locations we can have velocity triangles. So, this allows the change in the power by adjusting the blades. But we are not going deep into those aspects because we will have a series of velocity triangles at different locations of this blade that is from tip to hub.

The next segment of the topic of our discussion is the specific speeds. Now, we know all types of turbines mainly Pelton wheel, Francis turbines, propeller and Kaplan turbines and we know their working conditions, velocity triangles. But when these turbines are supposed to be used, their models has to be tested effectively. Now, for the testing the model, we may not have facility for generating a very high head and also we do not have a facility for generating large power. So, for that reasons, normally what has been done is that an analogous parameter is defined and that analogous parameter remains same for the model as well as prototypes. So, here we use the concept of dynamic similarity or similarity rules based on which the models and prototypes can be compared.

Now, one such parameter in this case is the specific speed. To define the specific speed, we know that there are three important parameters desired for power generations. One is head of water, second is rotational speed, and then third is how much power we are going to generate. So, these three things has to be considered together to define this specific speed and that has to remain same for the model as well as the prototype. So, by considering this the specific speed of a turbine is defined as the speed of operations of a geometrically similar model turbine which produces 1kW power while operating a head of 1m.

There are two things here, we need to focus. First thing is that it can be a non-dimensional number or dimensional number and through this process or through variety of similarity, consideration of similarity rule the specific speed of a turbine is defined in following ways.

Specific speed, $N_s = \dfrac{N\sqrt{P}}{H^{\frac{5}{4}}}$ (dimensional form); $N_s' = \dfrac{N\sqrt{P}}{\rho^{\frac{1}{2}}(gH)^{\frac{5}{4}}}$ (non-dimensional form)

N:Normal working speed(rpm); g:Acceleration due to gravity $\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)$

P:Poweroutput(kW); ρ:Density of water $\left(\frac{\mathrm{kg}}{\mathrm{m}^3}\right)$; H:Net head(m)

Now, analogous term to the dimensional is a non-dimensional form. To make it unit less we multiply $\rho^{\frac{1}{2}}g^{\frac{5}{4}}$. So, this is another way of expression. However, at the end of our analysis, it is just a matter of scaling, but we will mostly preferred for this expression for dimensional form and intentionally we will not write the units. So, we will just say that the specific speeds for all types of hydraulic turbines vary from 4 to 1100. So, main significance of the specific speed is that lower specific speed denotes slow runner, high specific speed implies fast runner.

So, in our last lectures I have given the particular table which talks about class of turbines depending on the flow directions, and specific speeds. Now, we are more focused considering the types of turbines based on specific speed and they fall in three categories, low, medium and high. And all turbines also can be useful in these three category. And based on their specific natures, they can be operated for maximum and minimum power and maximum diameters and suitability of a particular locations.

Then another significant features considering model and prototype is the scale ratio. Scale ratio corresponds to unit speed, unit power and unit discharge and it is mainly related with respect to model and prototype. But previously we have defined a term called speed ratio, which is the ratio of blade velocity to water velocity and for different types of turbines like for Pelton wheel, it is 0.4 to 0.47, for Francis, it is 0.5 to 1 and for propeller, it is 1.5 to 3. And also with respect to Pelton wheel specifically, a jet ratio is defined which is typically maintained in the range 10 to 24. But these parameters are specific to respective turbine.

Now, if this prototype turbine has to be tested with respect to model then we define this term which is called as a scale ratio. So, the scale ratio represents the ratio of diameter of the model turbine to its prototype. But when you make this comparison we must say they should have same specific speed. Now, looking at the comparison between these two what is the most common? One is V_b, blade velocity or bucket velocity and other is velocity of water.

Blade Velocity, $V_b = \frac{\pi DN}{60}$; Velocity of water, $V = C_v\sqrt{2gH}$

For model(subscript m) and prototypes(subscript p), $V_b \alpha V$

These two expressions says $V_b \alpha DN; V \alpha \sqrt{H}$

So, that way looking at the similarity rule we can write $\Rightarrow DN \alpha \sqrt{H};$

So, considering this we can find the expressions for scale ratio, which is the ratio of diameter of the model to the diameter of prototype. And that is proportional to this speed range for the prototype and the model as well as $\sqrt{H}$ of model and the prototype.

$\frac{D_m}{D_p} = \frac{N_p}{N_m}\sqrt{\frac{H_m}{H_p}}$ (scale ratio)

D:Diameter; H: Head; N: RPM;C_v: Coefficient of velocity

Now, moving further, we apply same logic to define the unit terms. They are nothing, but the operational characteristics of hydraulic turbines. The unit speed is defined as the speed of a geometrically similar working turbine which works under a head of 1m.

Unit power is the power generated by geometrically similar turbines working under a head of 1m. Unit discharge is the flow rate of the turbine working under the head of 1m.

Blade Velocity,$V_b = \frac{\pi DN}{60}$; Velocity of water, $V = C_v\sqrt{2gH}$

So, first thing is from same logic, when you say

$V_b \ \alpha \ V \Rightarrow N \ \alpha \sqrt{H} \Rightarrow N = K_1\sqrt{H}$; At $H = 1$m; $N = K_1 = N_u$; Unit speed,$N_u = \frac{N}{\sqrt{H}}$

With same logic when you define the power,

Power developed by turbine, $P = \rho QgH = \rho(AV)gH = \rho A\left(C_v\sqrt{2gH}\right)gH$

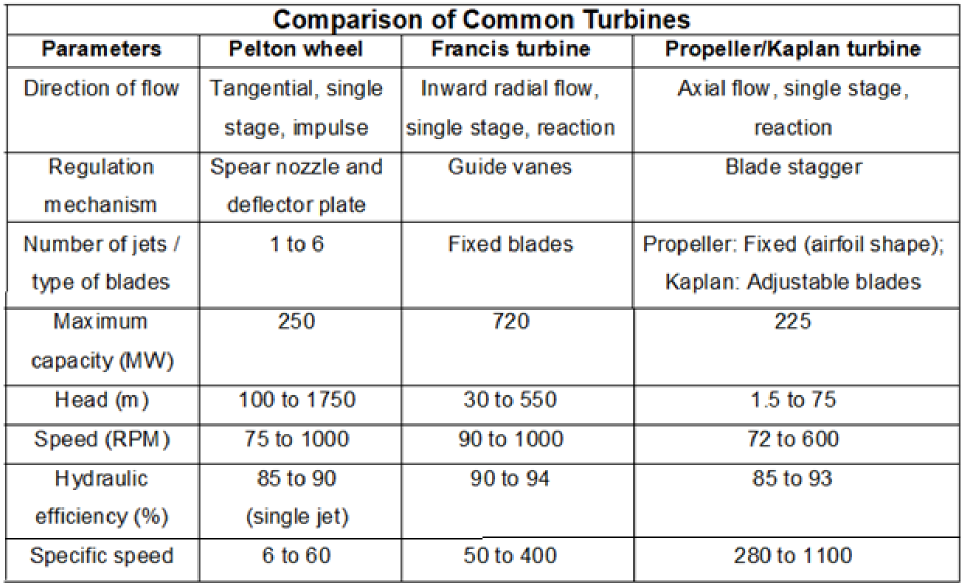
$\Rightarrow P\alpha(H)^{\frac{3}{2}}; P = K_2(H)^{\frac{3}{2}}$ At $H = 1$m, $P = K_2 = P_u$; Unit power,$P_u = \frac{P}{(H)^{\frac{3}{2}}}$

Similarly Flow rate for turbine, $Q = AV = A\left(C_v\sqrt{2gH}\right); Q \ \alpha \ (H)^{\frac{1}{2}}; Q = K_3(H)^{\frac{1}{2}}$

At $H = 1$m, $Q = K_3 = Q_u$; Unit discharge, $Q_u = \frac{Q}{(H)^{\frac{1}{2}}}$

A:Area; D:Diameter; H:Head; N:RPM; ρ:Density; C_v:Coefficient of velocity

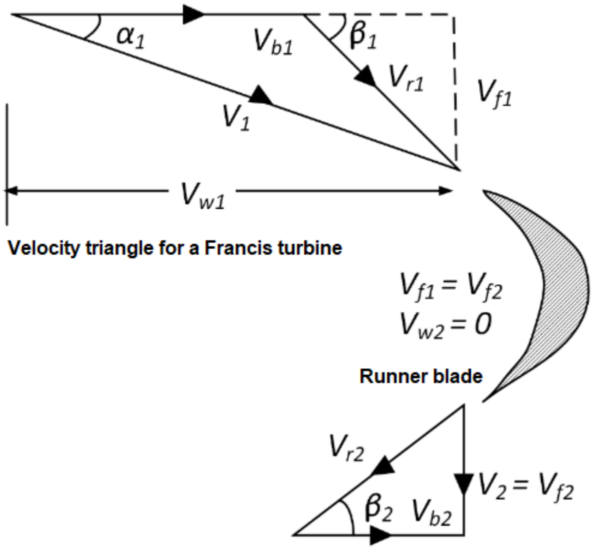
So, these parameters are mainly required when you discuss about the model as well as the prototypes. Whatever we have discussed so far, the complete comparison of specific parameters among Pelton wheel, Francis, Propeller/Kaplan turbines is presented here.



Comparison of Common Turbines			
Parameters	**Pelton wheel**	**Francis turbine**	**Propeller/Kaplan turbine**
Direction of flow	Tangential, single stage, impulse	Inward radial flow, single stage, reaction	Axial flow, single stage, reaction
Regulation mechanism	Spear nozzle and deflector plate	Guide vanes	Blade stagger
Number of jets / type of blades	1 to 6	Fixed blades	Propeller: Fixed (airfoil shape); Kaplan: Adjustable blades
Maximum capacity (MW)	250	720	225
Head (m)	100 to 1750	30 to 550	1.5 to 75
Speed (RPM)	75 to 1000	90 to 1000	72 to 600
Hydraulic efficiency (%)	85 to 90 (single jet)	90 to 94	85 to 93
Specific speed	6 to 60	50 to 400	280 to 1100

So, this you can say is the summary sheet of the all types of hydraulic turbines, their respective efficiency, electric speed, power generation capability, head and rpm. So, with this we complete this hydraulic component of hydroelectric power systems. So, before you conclude this lectures let us try to solve some numerical problems based on our discussion in this lecture.

Q1. A runner of a Francis turbine has inner diameter 0.75 m and outer diameter 1.5 m. It operates under a head of 150 m with a specific speed 120 and generates power of 15 MW. The water enters at an angle 10° and leaves radially with no whirl component. Calculate the inlet and outlet blade angle. Assume, hydraulic efficiency of 90%.



Velocity triangle for a Francis turbine

So, the first problem is about the Francis turbine. We discussed this Francis turbine and here we present the velocity triangles that are inlet and outlet velocity triangle. These velocity triangles are useful to find the energy which is getting transferred from fluid to rotor.

So, two important things for this Francis turbines we have this V_{w1} component and V_{f1} component that constitutes the velocity vector of V_1. Correspondingly this V_{w1} is associated with V_{b1} which is there in the inlet triangle. Then we have blade angles α_1 and guide vane angle β_1. For the outlet triangle, we have similar terms beta β_2 and α_2 and here essential feature is that V_{f1}& V_{f2} that is flow velocity remains same from inlet to outlet. Now, with this summary we are going to solve the problem.

So, from the given data, it has the inner diameter 0.75m and outer diameter 1.5m. So, these two diameters will decide V_{b1} & V_{b2}. There is a head 150m, specific speed 120, but what we do not know is the speed or rpm. So, to get this rpm let us take this specific speed expression.

Specific speed, $N_s = \frac{N\sqrt{P}}{H^{\frac{5}{4}}} = 120$

Now, while using this expressions one must ensure the power should be in kW and H should be in meter.

$P = 15 \text{ MW} = 15000\text{kW}; H = 150 \text{ m}$

So, inserting this number we can get

$$\Rightarrow N = \frac{(150)^{1.25} \times 120}{\sqrt{15000}} = 514 \text{ rpm}$$

Now, $V_{b1} = \frac{\pi D_1 N}{60}$; Given data, $D_1 = 1.5\text{m}$

$$\Rightarrow V_{b1} = 40.4 \text{ m/s}$$

Now, Hydraulic efficiency, $\eta_h = \frac{V_{w1}V_{b1}}{gH} = 0.9$

$$\Rightarrow V_{w1} = \frac{0.9 \times 9.81 \times 150}{40.4} = 32.8 \text{ m/s}$$

Now, looking at the inlet triangle we can write,

$\tan\alpha_1 = \frac{V_{f1}}{V_{w1}}$; Data given, $\alpha_1 = 10°$

$\Rightarrow V_{f1} = 5.8\ \mathrm{m/s}$

Since $V_{f1} = V_{f2} \Rightarrow V_{f2} = 5.8\ \mathrm{m/s}$

Another expression we can write which is

$\tan\beta_1 = \frac{V_{f1}}{(-V_{w1}) + V_{b1}}$; Here, V_{w1} is negative as it is in the opposite direction of V_{b1}.

$$\Rightarrow \tan\beta_1 = \frac{5.8}{(-40.4) + 32.8} = 0.763 \ \Rightarrow \beta_1 = 37°$$

So, we have inlet and outlet blade angles. So, we have $\beta_1 = 37°$, which is inlet blade angle. Now, for outlet blade angles we know that $V_{b2} = V_{b1}/2$ because $D_2 = D_1/2$.

So $V_{b2} = 20.2\ \mathrm{m/s}$

Now, from the outlet blade triangle we can say

$$\tan\beta_2 = \frac{V_{f2}}{V_{b2} - V_{w2}} = 0.287 \ \Rightarrow \beta_2 = 16°$$

So, we have inlet blade angle 37° and outlet blade angle is 16°. So, this is the first problem.

Q2. A Francis turbine with specific speed of 200 has to develop 30MW power operating at 200 rpm. An experimental model is made to operate at flow rate 0.6 m^3/s and head 4.5 m. Assuming an overall efficiency of 90%, estimate the speed, power and scale ratio for the model. With same efficiency, calculate the flow rate through the turbine.

The second problem is based on the specific speed. So, we have a model and we have prototype. So, the parameter associated for model & prototype are as below.

Parameter	Model	Prototype

Power	P_m	P_p
Head	H_m	H_p
Diameter	D_m	D_p
Speed	N_m	N_p
Specific speed	N_s	

So, for the solution of the problem, we can start with the first thing that is model. So, we are asked about the speed, power and scale ratio for the model. So, we will do it first for the model.

$$P_m = \rho Q_m g H_m \eta \times 10^{-3}\ ; \quad \eta = 90\%, H_m = 4.5\ \mathrm{m}, Q_m = 0.6\,\frac{\mathrm{m}^3}{\mathrm{s}}, \rho = 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}, g = 9.8\ \mathrm{m}/s$$

This power has to be in kW. So, you have to multiply 10^{-3} kW. So, by inserting these numbers we can find out $P_m = 23.83$ kW. First answer we get is the power developed by the model.

And specific speed remains same then we need to find out speed of the model.

$$\text{Specific speed}, N_s = \frac{N_m\sqrt{P_m}}{H_m^{\frac{5}{4}}} = 200$$

$$N_m = \frac{200(4.5)^{1.25}}{\sqrt{23.83}} = 268.4\ \mathrm{rpm}$$

So, this is the second answer.

Third answer is scale ratio.

$$\text{Scale Ratio}, \frac{D_p}{D_m} = \frac{N_m}{N_p}\sqrt{\frac{H_p}{H_m}}$$

But here we do not know the terms associated with prototype. Now for the prototype we have $P_p = 30\text{MW} = 30000$ kW, $N_p = 200$ rpm.

We can say, $N_s = \dfrac{N_p\sqrt{P_p}}{H_p^{\frac{5}{4}}} = 200$

$\Rightarrow H_p = 61.8$ m

Now Scale Ratio, $\dfrac{D_p}{D_m} = \dfrac{N_m}{N_p}\sqrt{\dfrac{H_p}{H_m}} \quad \Rightarrow \dfrac{D_p}{D_m} = \left(\dfrac{268.4}{200}\right)\left(\sqrt{\dfrac{61.8}{4.5}}\right) = 5 \quad \Rightarrow D_m = \dfrac{1}{5}D_p$

So, this is the third answer. Then with same efficiency we need to calculate the flow rate for the turbine. So, flow rate for prototype

$$Q_p = \frac{P_p}{\rho g H_p \eta}\ ; P_p = 30000\ \text{kW}, H_p = 61.8\ \text{m}, \eta = 90\%$$

$$\Rightarrow Q_p = 55\frac{m^3}{s}$$

So, prototype flow rate is 55 meter cube per second.

Q3. In a hydro power station, water is available at a rate of 200 m^3/s under a head of 20 m. The available turbines should run at 150 rpm with overall efficiency of 85%. Find the number of turbines required: Case I – Francis turbine with maximum specific speed of 460; Case II – Kaplan turbine with maximum specific speed of 350.

And the last problem is about the comparison of turbines. So, what we have is that in a hydropower system, water is available at a flow rate of 200 m^3/s and head 20m.

So, we can say the power available with water or water power is

$P = \rho g H Q \eta \times 10^{-3}$ kW

$\Rightarrow P = 1000 \times 9.81 \times 20 \times 0.85 \times 200 \times 10^{-3} = 33354$ kW

So, this is the power available for water or water power. Now, we have choice for the turbines and this turbine should run at rpm of 50.

Case-1: Francis Turbine,

$$N = 150 \text{ rpm}, N_s = 460 \text{ rpm} = \frac{N\sqrt{P}}{H^{\frac{5}{4}}}; H = 20 \text{ m}$$

$$P = 16823 \text{ kW}$$

So, a single Francis turbine will give you 16823 kW power. So, number of turbines will be

$$n = \frac{33354}{16823} \approx 2$$

So, we need to have two number of Francis turbines.

Case-2: For same situation if you look for Kaplan with same rpm,

$$N = 150 \text{ rpm}, N_s = 350 \text{ rpm} = \frac{N\sqrt{P}}{H^{\frac{5}{4}}}; H = 20 \text{ m} \Rightarrow P = 9739.3 \text{ kW}$$

So, from this specific speed expression, we can find out power developed by one Kaplan turbine will be 9739.3 kW.

$$\text{Number of Kaplan Turbines} = \frac{33354}{9739.3} \approx 4$$

So, to produce water power of 33.354 MW, if we use Francis turbines, we require two Francis turbines or if we use Kaplan turbines, we need to have four number of Kaplan turbines. So, this makes an initial judgment in a hydropower stations, what is the requirement of turbines, based on the power available from the water. So, with this I complete this hydropower module or hydropower segment. Thank you for your attention.