POWER PLANT SYSTEM ENGINEERING

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Module 3

Lec 1: Fundamentals of Gas turbine systems

Dear learners greetings from IIT, Guwahati we are in the MOOCs course Power Plant System Engineering module number 3. This is the new module that we are going to start after taking the discussions on steam power system. So, in this module number 3 we will emphasize on another kind of technique or technology to generate power. So, that is gas turbines and combined power system. So, basically in this module we will try to emphasize that same combustion process. If you can do another technology or another method what is the possibility that we can have.

One possibility is that you can have based on gas turbines system is that it can serve for power generations as well as for thrust generation. In fact, gas turbine is the backbone of aircraft industries, because they generate thrust through this gas turbine technology. So, in our module, we will emphasize on two different themes; one is gas turbine for production of power, other is gas turbine for production of thrust. So, let us start this first lecture.

So, in this lecture we will touch upon fundamentals of gas turbine systems and here we will try to emphasize on various gas turbine cycles with their possibilities. Then one important thing is that for gas turbines, we take Brayton cycles as reference cycle. For example, for steam power systems, the reference cycle was ideal Rankine cycle. And that was treated as steam power cycle and for gas turbines, the corresponding reference cycle is the Brayton cycles. Then subsequently we can analyze its thermodynamic aspects to know the work output, power developed and subsequently efficiency of the cycles.

Also we will touch upon the possible working fluids that can be used for gas turbine system. Let me give the brief introduction on gas turbine cycles. Gas turbines provide wide range of services mainly for industrial plants for driving mechanical devices like pumps, compressors, electric generators and these have also capability of producing electric power for peak load requirements. Normally whenever the steam power fails, additional power can be supplied through gas turbines and that enhances the peak load requirement. Also other application of gas turbines that normally the steam power plants do not have, is the thrust generations.

So, all aircraft engines are powered by this gas turbine technology and they are mainly used for thrust generation. Now, apart from that additional advantage of this gas turbine unit is that they are used in combined cycle plants. That means, in combination with steam turbines, regenerators, heat recovery boilers; there are various augmentation techniques that steam turbine can be combined with gas turbine technology for a combined cycle power generation. Many a times we also call this as co-generation. With this background let us see that some of the advantage of gas turbine plants.

1. They are small in size, requires less mass, low initial cost per unit power output.
2. They are available with short delivery times that means, once you start this engine then within no time it can start and also it has a smooth running as compared to steam system.
3. They have compatibility with wide range of liquids and gaseous fuels, synthetic fuels. They have flexibility for the supply of process needs that means, in a gas turbine unit the compressor is used for compressing the air, but for process needs, wherever there is a high pressure air requirement, we can tap this compressed air for use. And also this gas turbine plants have very less environmental restrictions.

However, apart from these advantages, there are some disadvantages as well.

1. They have difficulties in using as primary base-load prime movers. That means, although they can be used safely for augmentation plant or for supplying the extra

energy, but for primary requirement you cannot have this gas turbine unit because the starting torque is normally less.

2. They are incompatible with solid fuels, and they are only suitable for peak load requirements.
3. They have lower efficiency compared to steam power plants, but if you want to improve the efficiency it requires some boosting that means, initial gas temperature needs to be higher. And to be achieved that, it needs an extra cost.

So, these are some of the disadvantages, however, gas turbine technology is a very useful technology and it has been commonly used in many applications. And main applications is that most of the marine engines are also driven by the gas turbine plants for production of power. Aircraft engines require the gas turbine technology for thrust generation.

Side by side steam power systems requires gas turbine units for its operation in combined mode, which can produce the peak load requirements. So, these are some of the themes or advantages that gas turbine systems have. Now, let us try to understand the thermal circuits for a gas turbine cycles. The first category or first type of configurations for this gas turbine is single shaft configurations. The other type of configuration, it can have is two shaft configuration. These are the two possibilities normally gas turbines operates.

So, let us try to understand what these two things mean. Normally if you look at a gas turbine units, there are four basic components; one is compressor, second is turbine, third is combustion chamber and then fourth one is the shaft. So, what happens is that air enters to the compressors at state 1 and normally it is atmospheric air. Now after compression, it enters into the combustion chamber that is at state 2 and in this combustion chamber, normally we add fuel. So, this is similar to IC engines where fuel and compressed air get combusted and from this, we get the combustion products at state 3. So, this combustion products expands in the turbine and exhaust comes at state 4. And the power is getting developed by these turbines. So, here the turbine has two roles to play; one is, it can give the load requirement for the plant, second is, this turbine also drives this compressor through the shaft. So, ideally speaking the turbine speed and compressor speed should match, so that single shaft can be connected and at the same rate the turbine also produces the power.

So, power development(W_T) comes from the turbine and compressor takes the work input(W_C). So, part of this turbine work goes to compressors to drive it and rest of the work or the balance part that is $W_T - W_C$ is your resultant power that is being produced. So, this is what the single shaft arrangement needs to have. And we call it as a direct open cycle because both the sides like inlet and exhaust they are atmospheric in nature, there is no additional or secondary fluid comes into picture.

In two shaft arrangement the process remains same as it is. Air enters at state 1, after compression it leaves at state 2 and in the combustion chamber fuel plus air gets added. So, fuel gets added to the high compressed air. So, it leaves at state 3. Now, here we have the changes; first is we have a high pressure turbine and this high pressure turbine produces power just to drive the compressor. So, we have W_{HP} (High pressure turbine) & W_{LP} (Low pressure turbine) that are the two turbines. So, high pressure turbine drives the compressors.

So I can say Total turbine power, $W_T = W_{HP} + W_{LP}; and\ W_{HP} = W_C$

That means, compressor power requirement is achieved through this high pressure turbine. And expansion is still not complete, we take out this combustion products and further expand in another turbines. So, essentially the outside load requirement is achieved through low pressure turbine. So, advantage of this is that, in the single shaft arrangement we cannot run the system at varied load, but here in a two shaft arrangement, since the two shafts are decoupled to each other, that means, only compressor and high pressure turbine can run at same rpm whereas, the turbine connected for load requirement can run at different rpm. So, this allows the loads to be driven at variable speeds. So, this is the single shaft and two shaft arrangements and we call both of them as direct open gas turbine cycle.

Now, another point I need to emphasize here that in this figure, we have shown here that, these gas turbines are used for power generation, but there are possibilities that instead of getting power through turbines, the exhaust gas can also expand further in a nozzle for thrust generation, which essentially happens in an aircraft engine. So, the useful power can be supplied by the turbine or allowing the gas to expand further in a nozzle that can provide propulsion or thrust for this aircraft engine. This is another advantage of the gas turbine systems.

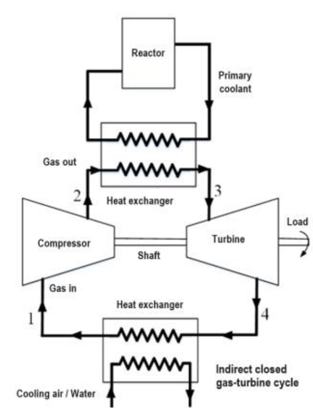
In fact, all aircraft engines rely on gas turbine units. So, gas turbine technology is the backbone for aircraft systems. And again we call this as open cycle because we you need to rely on atmospheric air which is continuously taken. Since we are dependent on atmosphere, air is the only feasible choice. This is about direct open gas turbine cycles.

Another thermal circuit possibility that we can have is indirect open gas turbine cycle. The word indirect is being used here because we do not have a combustion chamber rather we have a heat exchanger. And normally this is used when the gas turbine technology used in the combined power plant port. So, in this arrangement we have another kind of working fluid and we call this as coolant and this is produced from any other reacting system. Initially air is taken from the atmosphere and it enters in the compressor at state 1 and after compression it goes to state 2. Instead of air getting combusted it takes heat from another medium or another working fluid, normally called as a secondary fluid or coolant for the cycle and there it only takes the heat. Thereby, it becomes high pressure and high temperature air and that enters into the turbine then expands. So, the load requirement is again same W_T and here W_C is the compressor work. So, difference between them is the work that is getting produced $W_T - W_C$, but here only the difference is that the air, the primary fluid and the secondary fluid do not mix with each other; only heat is getting tapped.

So, this is all about the indirect open gas turbine cycle. So, the word open means atmosphere, we rely on atmospheric air. The word indirect means the air is not used directly rather heat is taken to the air from another fluid. The next thermal circuit arrangement is direct close gas turbine cycle. In Direct close gas turbine cycle, we have a reactor of course, this reactor can be interpreted as combustion chamber as well. So, in this case what happens is that, we have air plus fuel which are mixed in this reactor and this compressor and turbine have single shaft and the turbine drives the compressor as well as it produces the power, but extra change that we have here is that we do not rely on atmospheric air. So, basically here instead of air we have a possible choice of any other gases that can continuously be circulated in this cycles. So, let us say there is some gas that enters in the compressor, after compression it goes to state 2, it enters to a reactor where basically the reactor or the combustion chamber is where the heat addition takes place. Then after heat addition this high pressure and high temperature gas enters to the turbine at state 3 and after expansion the gas comes out from the turbine, but here instead of going out to the atmosphere, the same gas gets recirculated. So, the piping arrangements are different, one for the working fluid circuit other for the rest of the components in the circuit. So, this is what we call as a direct closed gas turbine cycles.

And next and most important arrangement that we are going to discussed in our cycle is the Brayton cycle. It is an indirect closed gas turbine cycle. So, what you take? We take the techniques or thermal circuits from a direct closed gas turbine cycle and indirect open gas turbine cycles, combine them then we get this indirect closed gas turbine cycles. So, the indirect gas turbine cycle combines the indirect open cycle with direct closed cycle, so, that the reactor is separated from the working fluid by a heat exchanger and the working gas rejects heat to the atmosphere via heat exchangers. So, we have one working fluid which revolves in the circuit another working fluid in this reactor circuits. So, basically two different working fluids and they do not mix with each other. And this is the most efficient gas turbine cycle we can have. The main advantage for this is that, it is mainly used for power generation. Other advantage is that, we are not supposed to rely on one particular gas, we can have multiple possibilities of gas to optimize the power and efficiency requirement.

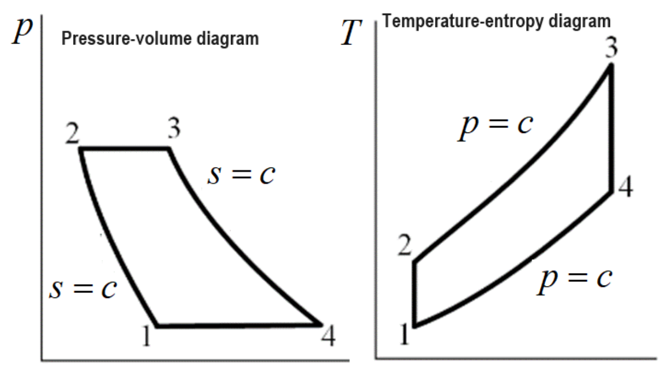
This is what we call as a Brayton cycles. So, the indirect closed gas turbine cycle is the benchmark or analysis part for a Brayton cycles. So, the ideal cycle for a gas turbine power output is the Brayton cycle. You can see here, we have state 1 where gas enters, after compression the gas comes out. So, this process is



called as compression process that is from 1 to 2. So, here pressure shoots up and it is an isentropic process. Then from 2 to 3, it enters to a heat exchanger where heat is getting added to the gas that we say is Q_{in}. So, Q_{in} is for the process 2-3. So, it is a heat addition process at constant pressure and here we have compressor that takes W_C. Then after heat addition process the gas enters to the turbine for expansions.

So, heat addition process is constant pressure process and the turbine process is denoted by 3 -4 and this process is isentropic process that is constant entropy process that is in the turbine. And again finally, heat rejection process that takes place at constant pressure. So, effective work that we get is from expansion in the turbine and heat addition process that takes place is Q_{in} that is in the heat exchanger. So, this is the very basic bottom line for an ideal Brayton cycle and here the components are treated to run as a steady flow manner with negligible change in the kinetic energy. So the work done from the turbine is equal to change in the enthalpy.

Now, here we need to do some assumptions. The specific heat for the gas that continuously enters in the compressor and turbines, should be assumed to be constant for the range of operations that we are looking at. So, with constant specific heat assumptions the analysis becomes a simple air-standard cycle with air as working fluid.



Brayton Cycle

Further we need to do this thermodynamic analysis. For this thermodynamic analysis let us refer to this pressure-volume and temperature-entropy diagrams, process 1-2 is compression, where W_C work enters. The process 2- 3 is constant pressure heat addition. 3- 4 is expansion in the turbine. 4- 1 is heat rejection that is Q_{out}. So, this is what we see in the pressure volume diagram. Similarly, 1- 2 is isentropic process in the compressor, 2- 3 is the heat addition process in the combustion chamber, 3 - 4 is the expansion process in the turbine and 4 - 1 is the heat rejection process at constant pressure.

So, with this analysis we can recall some of the fundamental parameters. First thing we have to define the pressure ratio for the turbine that means, what is the range of pressure it should work. For that, we define a term which is called as pressure ratio for turbine. There are possibility that the pressure ratio in the compressor and turbine may or may not be equal. However, for our analysis, we will say that since the compressor is driven by the turbine, so we say that the pressure ratios need to remain same. And for isentropic expressions, we can have the temperature ratio that can be expressed in terms of pressure ratio.

Pressure ratio in turbine, $r_p = \frac{p_3}{p_4}$; Isentropic expansion in turbine, $\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$

Then by using this pressure-volume and temperature-entropy diagrams we can calculate the work done in the turbine. The calculation is as such because in gas mode we can write enthalpy as C_p times T. Enthalpy is a function of temperatures. So, we can arrive at work done from the turbine and subsequently this can be interpreted in terms of pressure ratio. Now, one important expression I need to emphasize here that is the expression for work done in the turbines.

Work done in turbine, $\dot{W}_T = \dot{m}c_p(T_3 - T_4) = \dot{m}c_pT_3\left(1 - \frac{T_4}{T_3}\right) = \dot{m}c_pT_3\left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)$

With no loss in pressure in the cycle,pressure ratio in turbine, $r_p = \frac{p_2}{p_1}$

Isentropic compression in compressor, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$; $k = \frac{c_p}{c_v}$; $R = c_p - c_v$

c_p:Specific heat at constant pressure; c_v:Specific heat at constant pressure; R:Characteristic gas constant

But one thing I need to emphasize here that power developed from the turbine depends on the maximum cycle temperature. So, in this diagram if you see T_3 is the T_{max}. So, higher you go to the maximum temperature, power requirement will be high. And second thing is that it also depends on the pressure ratio. Now if you can go for higher pressure ratio, power that can be developed is more. In similar analogy, we can also find the work that is to be given for the compressor. And T_2 is the temperature of the gas that comes out after the compressions.

Work input for compressor, $\dot{W}_C = \dot{m}c_p(T_2 - T_1) = \dot{m}c_p T_2\left(1 - \frac{T_1}{T_2}\right)$

$$= \dot{m}c_p T_2\left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)$$

So, we have work done in the turbine, work done for the compressors then we can find out the net work done for the cycles.

Net work rate for cycle, $\dot{W}_n = \dot{W}_T - \dot{W}_C = \dot{m}c_p(T_3 - T_2)\left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)$

Isentropic compression in compressor, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}; T_2 = T_1 r_p^{\frac{k-1}{k}}$

And we also can have heat added to the cycles in 2-3.

Heat added to the cycle, $\dot{Q}_A = \dot{m}c_p(T_3 - T_2);$

And from noting down the net work and heat added we can define the thermal efficiency for the cycles. So, thermal efficiency is the given by

Thermal efficiency of the cycle, $\eta_{th} = \frac{\dot{W}_n}{\dot{Q}_A} = \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)$

Net specific work for cycle, $\frac{\dot{W}_n}{\dot{m}} = c_p\left(T_3 - T_1 r_p^{\frac{k-1}{k}}\right)\left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)$

$$\Rightarrow \frac{\dot{W}_n}{\dot{m}} = c_p \left[T_1 \left(1 - r_p^{\frac{k-1}{k}} \right) + T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}} \right) \right]$$

Pressure ratio in turbine, $r_p = \frac{p_3}{p_4}$

Now the net work is also retained in. Here we have expressions in terms of T_3& T_2 and T_2 is an intermediate number which normally we do not know. However we know the minimum temperature and maximum temperature.

$T_3 = T_{max}$ & $T_1 = T_{min}$

So, an idealistic way or most effective way of representing net work is in terms of T_3 & T_1 which is the highest and the lowest temperature of the cycle. And also pressure ratio is another parameter that needs to be accounted. So, basically net work is a function of maximum temperature T_3 minimum temperature T_1 and pressure ratio. Additionally it also depends on the working fluid that is the function of C_P. So, the characteristics of working fluid here is with respect to specific heat ratio k and C_P. These two numbers is very vital for the choice of the gas, which is being used as a working fluid.

So, with this we will try to emphasize what are the different inferences that we get. So, basically whatever expressions we have done so far, we can just plot them graphically. We have parameters like pressure ratio, maximum temperature, minimum temperature then we have working fluid. So, based on this we can find the curve for efficiency and specific work. So, if you keep on doing it the curves normally looks like this.

So, the first inference that we get from this is that the thermal efficiency of the cycle for a given working fluid with constant C_P and specific heat ratio, increases asymptotically. So, basically in this curve, with common X axis that is pressure ratio, the efficiency is plotted here. So, this is plotted for air as well as for helium. And the dotted line refers to the efficiency curve and solid line refers to the specific work. So, if you look at dotted line that is for efficiency curve, we can see that helium always happens to be more efficient than air as working fluid. And this curve continuously increases and in fact, it does not depend on the cycle temperature. You can see here in this expression. For a given value of k, just keep on increasing r_p and we will have this efficiency curve.

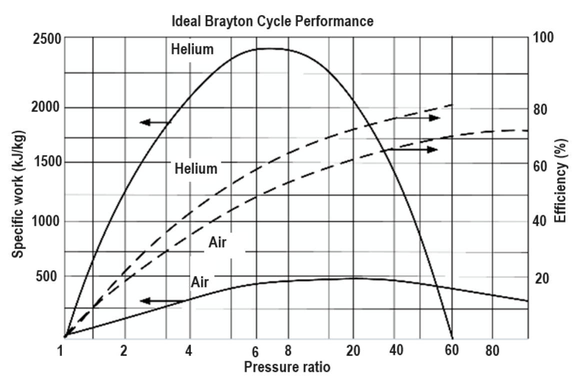
Then for second inference, we will look into the specific work. Now, when you look at the specific work using this expressions we find is that specific work initially it keeps on increasing and one particular point it starts dropping.

Net specific work for cycle, $\frac{\dot{W}_n}{\dot{m}} = c_p \left[T_1 \left(1 - r_p^{\frac{k-1}{k}} \right) + T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}} \right) \right]$

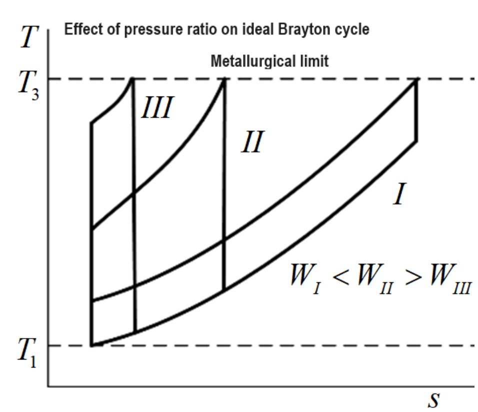
For maximum work, $\frac{d\left(\frac{\dot{W}_n}{\dot{m}}\right)}{dr_p} = 0 \Rightarrow T_2 = (T_1 T_3)^{\frac{1}{2}}$

At same pressure ratio and maximum work, $\frac{T_2}{T_1} = \frac{T_3}{T_4} = r_p^{\frac{k-1}{k}} \& (T_2 = T_4)_{opt}$

Optimum pressure ratio, $r_{p,opt} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$



That means, if you keep on increasing the pressure ratio after some point of the pressure ratio, the work output comes down. And this is true for helium as well as air. Now, comparing helium and air, you see that helium has high value of C_P & k. In fact, its C_P value is almost 5 times more. If we can see peak power for helium it is almost more than 5 times. So, this is the advantage. However, the way it rises, it also falls suddenly. So, that means, if you have a choice between air and helium, it is better to choose the helium as working fluid, because at a given pressure ratio helium will have a higher capability of producing work.



This particular plot of temperature entropy diagram shows the effect of pressure ratio in the Brayton cycles. So, basically T_3 is the maximum temperature. The turbine supporting system are metallurgical requirement, and accordingly blades need to be operated. So, that is fixed by maximum temperature. Now, when we have minimum temperature that is fixed by the atmospheric conditions. So, within this maximum temperature and minimum temperature there can be n number of Brayton cycles. That means in the curve, abscissa & ordinates value may be different for each cycle. But the curve for the work output shows that for whatever working fluid, either air or helium, there is an initial rise, then followed by a peak work and then it has to come down. So, this particular T-s diagram shows that there are three cycles; cycle 1, cycle 2 and cycle 3. Cycle 1 has a wider range and cycle 2 has smaller range. But we will get the optimum work in the cycle 2 only. So, basically this is true because we want to operate the cycle only for a maximum power, for which the pressure ratio is optimum. So, that is the reason, we normally operate the gas turbine cycle for maximum power and that maximum power can be obtained by differentiating with respect to pressure ratio and it turns out to be a condition as mentioned below.

Net specific work for cycle, $\frac{\dot{W}_n}{\dot{m}} = c_p \left[T_1 \left(1 - r_p^{\frac{k-1}{k}} \right) + T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}} \right) \right]$

For maximum work, $\frac{d\left(\frac{\dot{W}_n}{\dot{m}}\right)}{dr_p} = 0 \Rightarrow T_2 = (T_1 T_3)^{\frac{1}{2}}$

At same pressure ratio and maximum work, $\frac{T_2}{T_1} = \frac{T_3}{T_4} = r_p^{\frac{k-1}{k}} \& (T_2 = T_4)_{opt}$

So, based on this we arrive at the expressions for optimum power ratio $r_{p,opt}$ which is a function of maximum temperature, minimum temperature and *k*.

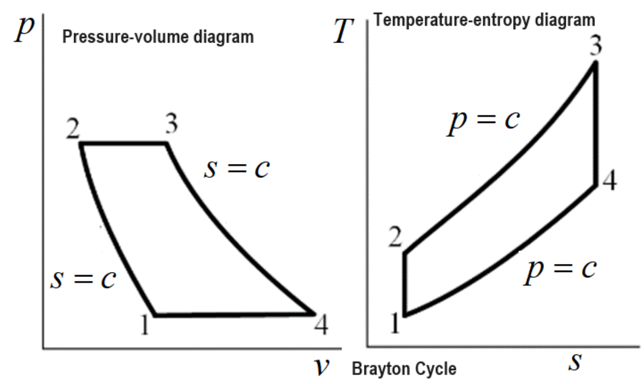
Optimum pressure ratio, $r_{p,opt} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$

So, very basic summary is that use of working fluid in a gas turbine unit or the choice of working fluid in the gas turbine unit is to design this Brayton cycle in such a way that, it should produce maximum power and for that maximum power we have to use this optimum conditions of pressure ratio. So, that is the advantage. In fact, for other pressure ratios, the cycle can be operated, but the better approach is to choose a working fluid which has more capability in terms of specific heat and specific heat ratio. So, I have already explained that at same value of temperature limits, pressure ratio and specific heat ratio, the net specific work is a direct function of specific heat which means helium can produce 5 times higher power than that of air.

Similarly for same values of temperature limits, pressure ratio and specific heat, working fluid with higher value of specific heat ratio have advantage of producing more specific work. So $r_p^{\frac{k-1}{k}}$, this ratio is a very vital term because specific heat ratio for helium is higher. Another point that I need to emphasize in terms of working fluid, we have only told about air and helium then what are the other possibilities? We say helium is a mono atomic gas, air is a diatomic gas, air or nitrogen can be diatomic gas. So, we can also have triatomic gas. So, there are possibilities that we can have different types of gas that can work in the Brayton cycle. But for a given pressure ratio, let us take this pressure ratio as 4, at this pressure ratio with air as working fluid, it can produce closely about 350 kJ/kg of work, but at same pressure helium has the capability to produce closely 2800 kJ/kg work. But if you want to produce same work for air, we have to go for higher pressure ratio. So, it means that for reduction of the plant size, we need to rely on mono atomic gas for a given pressure ratio because mono atomic gas has advantage of higher specific heat and also will have higher values of k. So, this is the background summary for the thermodynamic analysis. So, now we will solve a numerical problem which is based on the working principle of an ideal Brayton cycle.

Q1. Find the pressure ratio required for an ideal Brayton cycle to produce a net work of 1400 kJ/kg. The cycle has minimum and maximum temperature of 288 K & 1380 K. Also, calculate optimum pressure ratio and maximum work. The choice of working fluid are, helium and air.

Here we need to find out the pressure ratio required for an ideal Brayton cycle to produce net work of 1400 kJ/kg and we are given with data that cycle has maximum and minimum temperature of 288 K and 1380 K. We also need to find out optimum pressure ratio and maximum work. So, we have given the choice of working fluid with helium and air. To solve the problem, first thing we need to do is to draw this pressure volume and temperature entropy diagram.



So, that is 1 - 2 is the compression. 2 - 3 is the combustion. 3 - 4 is the expansion in the turbine. 4 - 1 is the heat rejection. And in this temperature entropy diagrams the cycle can be drawn in this manner like 1-2-3-4.

So, here we say *s* = *C* that means entropy is constant & here *p*=*C*, pressure is equal to constant. So, basically we have Q_{in} and we have $\dot{W}_C$ that enters into the system. $\dot{W}_T$, is power that is being produced. So, with this notations the data that is given is as follows.

$$T_1 = 288K; T_3 = 1380K$$

And we are supposed to we produce power that is,

$$\frac{\dot{W}_n}{m} = 1400 \text{ kJ/kg}$$

For working fluid, we have air and Helium. So, let us start with the working fluid, helium. If you take working fluid helium and try to find out the optimum pressure and maximum work then we need to note down the value specific heat ratio for helium that is as follows.

$$\text{For Working Fluid, He}; k = 1.66; C_P = 5.2\frac{kJ}{kg.K}$$

We also know the optimum pressure ratio, $r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$

By inserting the values of T_3, T_1 & k, for He, $r_{p,opt} = 6.6$

$$\text{Now, } \left(\frac{\dot{W}_n}{m}\right)_{max} = c_p \left[T_1 \left(1 - r_p^{\frac{k-1}{k}}\right) + T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right) \right]$$

So, we have all the numbers here.

$$\text{So, } \left(\frac{\dot{W}_n}{m}\right)_{max} = 3242.5 \text{ kJ/kg}$$

So, this answer gives you the fact that we can use helium as working fluid and with this optimum pressure ratio, 6.6 it can give maximum power of 3424.5 kJ/kg.

So, these two answer we got, but our requirement is that we need to produce 1400 kJ/kg then for that we need to find the pressure ratio. So, again for that answer if I put

$$\left(\frac{\dot{W}_n}{m}\right) = 1400 \text{ kJ/kg}$$

Now we need to calculate (r_p).

$$\text{We know, } \left(\frac{\dot{W}_n}{m}\right) = c_p \left[T_1 \left(1 - r_p^{\frac{k-1}{k}}\right) + T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right) \right]$$

$$\Rightarrow 1400 = 5.2 \left[288 \left(1 - r_p^{\frac{1.66-1}{1.66}}\right) + 1380 \left(1 - \frac{1}{r_p^{\frac{1.66-1}{1.66}}}\right) \right]$$

Then find r_p by iterative process. So, to start this iterative process, we use different values of r_p. But we all know that for r_p of 6.6, W_{net} is 3242.5. But our requirement for W_{net} is 1400. So, if you keep some values which is lesser than 6.6 and keep drive finding whether left hand side expression matches with the right hand side that is 1400 and then that will be your answer. So, by iterative process if you keep on iterating and $r_p \approx 2.2$. So, the pressure ratio required for helium is 2.2.

Now, same thing let's try for air.

For Working Fluid, air; $k = 1.4$; $C_P = 1.005 \frac{kJ}{kg.K}$

We can arrive at optimum pressure ratio, $r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} = \left(\frac{1380}{288}\right)^{\frac{1.4}{2(1.4-1)}} = 4.77$

Now, at this optimum pressure ratio, $\left(\frac{\dot{W}_n}{m}\right) = c_p \left[T_1 \left(1 - r_p^{\frac{k-1}{k}}\right) + T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)\right]$

$$\Rightarrow \left(\frac{\dot{W}_n}{m}\right) = 1.005 \left[288 \left(1 - r_p^{\frac{k-1}{k}}\right) + 1380 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)\right]$$

Take $r_p = r_{p,opt}$

$$\Rightarrow \left(\frac{\dot{W}_n}{m}\right) = 335.8 \text{ kJ/kg}$$

So, we need to have important inference here that with air as working fluid, when you operate this cycle at an optimum pressure ratio of 4.77, we can have maximum power of 335.8 kJ/kg that means, air cannot produce 1400 kJ/kg. So, for this problem the option of air is ruled out. So, we must use helium as working fluid at a pressure ratio of 2.2. So, this particular thing, we can also verify graphically if you look in the graph for pressure ratio of 4.77 somewhere close to 5, the possible value of specific work for air, we can drop down which is close to 335. And there is no possibility that with air we can have specific work beyond this value that is 500, but with helium with this ratio of 2.2, we can easily go up to 1400. So, this is cross verified from this graph. So, this is all about for the lecture today. Thank you for your attention.