POWER PLANT SYSTEM ENGINEERING

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Module 2

Lec 09: Steam Nozzles (Part I)

Dear learners greetings from IIT, Guwahati. We are in the MOOCs course Power Plant System Engineering module 2 that is Vapor Power System part 2. So, in this lecture we will start a new topic and that is nothing, but your steam nozzles and the steam nozzles are the essential components for steam turbines. But in this lecture we are going to study the theoretical aspects of steam nozzle. Before you start that thing, the theoretical component consists of the thermodynamic aspects of the effect of area change and flow properties with respect to thermodynamic parameters. Now based on this we will try to see that how the flow properties are going to be affected, when there is a change in area.

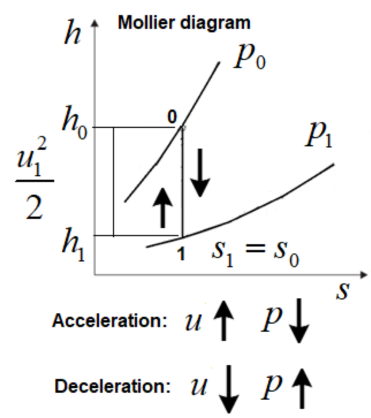
And this change in area; in our term we call this as shape of the nozzles; how the shape of the nozzles we have to design. While talking about the shape of the nozzle then we will do some theoretical treatments of nozzle shapes finding its optimum configurations and when we design such nozzles what should be the critical pressure ratio. These are the important topics of today's lecture. So, let us try to understand the effect of area change and the flow properties.

Ideally speaking we are talking about the area change with respect to nozzles. So, in our previous studies, when you dealt with impulse and reaction turbines, we have seen that nozzles are the integral components. For example, in a single stage impulse turbines, nozzles feeds the steam for rotation of the blades. That happens in a single stage turbine and when you go for compounding also a single nozzle supplies the steam to all the moving blades and they are arranged in a particular fashion. So, that is called as a Curtis turbines or velocity compounded impulse turbines.

Now, moving for the pressure compounding impulse turbine that is Rateau turbine where each of this moving blades needs a separate nozzles for feeding of steam. And most important fact is that in all these impulse turbines or all these arrangements, what we see is that most of the pressure drops takes only in the nozzles. So, at the cost of pressure drop we can see the increase in the flow velocity and of course, when there is a pressure drop, the enthalpy also drops. So, what happens is that all these drop in the total enthalpy of the steam is compensated through the pressure drop.

However, when you do this expansion in the reaction turbines, although there is no separate component of nozzle, they have fixed blades or guide vanes that takes care about the pressure drop. But most important fact is that the passage of those fixed blades are such that they perform a nozzle action. Nozzle actions, I will explain in the subsequent slides. It is nothing, but flow has to expand at the same time the velocity must increase and pressure must drop. So, this is what we call as a nozzle actions and this can be achieved with suitable change in the area of the passage. So, either you can have a separate arrangement for that what we call nozzles or you can have same nozzle actions by designing the flow passage. So, in any case, area change is the driving factor.

So, this is what I have explained here for impulse turbines, nozzles are the integral components for reaction turbines, nozzle action is more importance and they are achieved through fixed blade or guide vanes arrangements. Now, let us try to understand more theoretical aspects, what is a nozzle and with respect to our situation like when we talk about steam superheated steam and the flow expands and in any case the flow is typically either liquid or steam or vapor and it mostly falls in the incompressible range. So for that, if such a fluid has to expand what should be the shape? So, this is the main theme of the discussion for today's lecture.



So, by definition we can say that nozzle is a duct of smoothly varying cross sectional area in which steadily flowing fluid can be accelerated at the expense of pressure drop. So, this is first thing, and the pressure drop has to happen because steam is in the superheated condition of very high pressure and temperature. So, the need of the nozzle has many applications in practice where we require high velocity of steam of fluid and for that nozzles are the best choice. So, we have steam & gas turbine engines, jet engines; in fact, in the jet engines nozzle gives the adequate thrust at the expense of pressure drop because it accelerates the flow and at the same time, the reaction force from the exhaust gives the necessary force for the body or the jet engines to fly. And in the category of opposite action that is when the fluid is decelerated in the duct causing the rise in the pressure in the steam, it is called as a diffuser. And typical application includes that we have the passage that involves centrifugal compressors, ramjet engines where the intake systems are designed such that flow has to decelerate and causing the rise in the pressure.

Now, our analysis we will try to stick to a one dimensional flow and while talking about one dimensional flow we can say that we can have a converging passage where the area decreases along the direction of the flow or we can have a diverging passage where area increases along the direction of the flow. And of course, we do not look into the effect of friction, which is ignored. So, it is a frictionless flow and we also have some assumption like the fluid velocity remains constant at the mean value, although the area changes. When you take the fluid velocity at any particular section, we take the mean value along that particular sections. That means, at each sections we have one value of velocity. So, that is the assumption we make.

Now, let us try to understand this same concept with respect to the Mollier diagram, because most of our analysis is referred with respect to Mollier diagram when the flow expands. When a flow expansion takes place from higher pressure to lower pressure, you can use this one dimensional equation where enthalpy remains same at two locations and at one particular location the fluid starts with zero velocity. So, the total enthalpy is nothing, but the static enthalpy.

$$h_2 + \frac{u_2^2}{2} = h_1 + \frac{u_1^2}{2}; \text{At } u_2 = 0 \Rightarrow h_2 = h_0 \Rightarrow h_0 = h_1 + \frac{u_1^2}{2}$$

So, basically speaking, at initial state we have the total enthalpy which is available in the fluid and it expands to certain pressure. Now, during this expansion process and if you assume this expansion process to be isentropic then what we can see here is like this in the diagram.

If you see that the flow expansion takes place from 0 to 1, that means, the arrow that shows towards the bottom then what is going to happen is that your velocity will increase and pressure will drop. That means, we are going for decreasing the pressure, but at the increase in the velocity or the fluid gets accelerated. Now, if you go from 1 to 0 that means, fluid is decelerated and the pressure increases. So, this is the very basic theory for nozzle actions and diffuser actions and this thermodynamic behavior can be replicated by designing this flow passage. Now, in either case the flow passage can be converging or diverging; whether the expansion has to take place or compression has to take place or in other words whether this diverging passage will behave as a nozzle or this diverging passage can also behave as a diffuser. So, all it depends on the nature of the flow. That means, if the expansion has to happen that means, if it is a nozzle action then the fluid must accelerate, velocity must increase, pressure must decrease and if the flow is incompressible then we will end up having a converging passage. If the flow is compressible then we may have a diverging passage.

So, this depends on whether the initial flow is subsonic or supersonic. So, that means, if your initial flow is supersonic, a converging passage will act as a diffuser action or when the fluid is initially subsonic, the converging passage will act as a nozzle action. However, in our philosophy, since we will be dealing with incompressible flow and initial situation is always subsonic, our main focus will be on the converging passage. So, in the other side of the story that we can get the compression process in the Mollier diagram and the flow compression along the path involves the increase in the pressure with change in the area of the flow patches. So, in the process of compression the static enthalpy increases and the flow velocity drops.

The diffuser action is just opposite to the nozzle actions. This is not part of our discussions. So, just for the sake of curiosity that flow passage or how the area change affects the flow properties; in that context, this analysis is more important. Now, whatever we discussed if you can summarize, when a fluid is allowed to enter a passage in which area can change then we can achieve compression or expansion of the fluid in the direction of the flow. If it is an expansion of the flowing fluid then we call this as a nozzle action. If it is a compression of the flowing fluid then we call it as a diffuser action.

Then let us try to understand what should be the shape of the nozzles. All this depends on the area of the passage. So, to understand this concept let us try to think about a stream of fluid at certain pressure p_1, enthalpy h_1 and flow velocity C_1 which enters in a duct. For the time being let us stick to this converging passage. So, we can apply the steady flow energy equation at any section x-x. So, we can mark it as any section x-x.

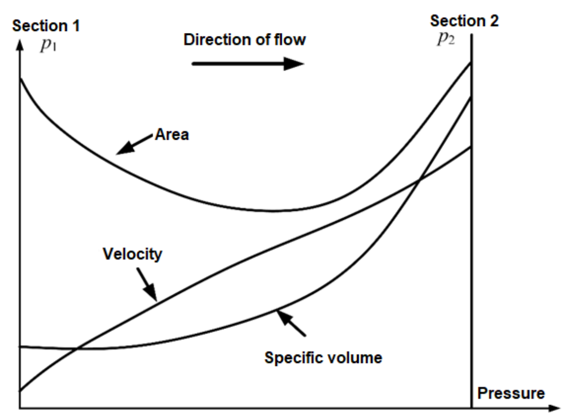
We need to see that how the pressure or the flow properties values are being detected at the section x-x or since it is a nozzle action, the pressure has to drop that means, we need to see how that pressure falls along the length of the duct. So, for that reason we consider the steady flow energy equations where the total energy at the inlet stage is equal to total energy at the outlet. Hence, or Total energy is conserved, hence

$$\dot{m}\left(h + \frac{C^2}{2} + gZ\right) + \dot{Q} + \dot{W} = \dot{m}\left(h_1 + \frac{C_1^2}{2} + gZ_1\right)$$

Where heat transfer is $\dot{Q}$ and work transfer is $\dot{W}$. For the time being we are going to neglect this heat transfer and work transfer terms. Now, if you have the same mass flow rates so, it gets cancelled and then we arrive at an equation where,

$$\Rightarrow h + \frac{C^2}{2} = h_1 + \frac{C_1^2}{2}$$

Which means $h_1 + \frac{C_1^2}{2}$ is your inlet and $h + \frac{C^2}{2}$, any value referring to section x-x.



So, from this expression for at any section x-x we can calculate what the velocity of the flow is and that is,

$$C = \sqrt{2(h_1 - h) + C_1^2}$$

Now, let us try to understand the same concept in a different philosophy or represent in a graphical figure. So, try to understand this particular equations. One is,

$$C = \sqrt{2(h_1 - h) + C_1^2}$$

Other is mass flow rate. If you can see, mass flow rate at any cross section is constant.

$$\dot{m} = \rho AC = \frac{1}{v} AC$$

$\Rightarrow$ Area per unit mass flow, $\dfrac{A}{\dot{m}} = \dfrac{v}{C}$

$$\Rightarrow \frac{A}{\dot{m}} = \frac{v}{\sqrt{2(h_1 - h) + C_1^2}}$$

That means, area per unit mass flow rate is depends on the specific volume of the fluid, enthalpy difference and the initial velocity. So, for the time being if you just neglect that initial velocity component to be 0, so basically it depends on the specific volume and the enthalpy drop.

Now, let us try to understand this particular diagram with respect to this expression. If you take the two sections that is section 1 and section 2, and in the sections if you try to plot this particular curve. That means, you can say how area varies with pressure, then how velocity varies with pressure and how specific volume changes with pressure and remember we have fluids, it is either steam or liquid. So, we see that the generic graph detects the decreases in the pressure. That means, your pressure decreases as we have performed the nozzle actions. With decrease in the pressure your area drops down. That means, area drops to a particular value then further it increases. And the velocity continuously increasing that is anyway our expectation, velocity must increase with the pressure must drop.

Then your specific volume; in the beginning stage, normally there is not much change, but after that specific volume suddenly increases. So, this gives an indication that, the change in the area is often detected by rate at which velocity is increasing or rate at which specific volume is increasing depends on this. So, the dominance of velocity and specific volume also detects the area change. So, it can be seen here in a close look that from section 1 to 2, the area decreases initially and reaches to a minimum and then increases again. So, this decrease part is often detected by the rate at which velocity is increasing and rate at which the specific volume is increasing. In the initial part you can see here that when v increases less rapidly than C, then area decreases, when v increases more rapidly than C then area increases. So, this decrease in the area depends on the relative dominance of area velocity and specific volume and because of this reason, we have a land of minimum section and that minimum area is known as the throat of the nozzle.

Now, referring to diagram; from this initial things, we can say that the throat is the minimum area. Another point if I recall

$$\dot{m} = \rho AC = \frac{1}{v} AC \ \Rightarrow \frac{A}{\dot{m}} = \frac{v}{C}$$

Now, from this expression you can see that, when C is very high or C close to 0 means A goes to maximum that means A goes to infinity. So, this word infinity means that in most of the cases, we say that initial velocity at the nozzle inlet is 0 by assuming the fact that we start this with a very larger area and suddenly drops to a minimum value. So, this sudden drop in the area gives you the fact that your C_1 goes to 0 and with this concept the shape of the nozzle is designed. So, in some sense, we get that what should be our inlet shape and what should be our outlet shape and this diagram gives you the throat of the nozzle that is the minimum area.

So, with this analysis or this mathematical expressions are now sufficient to analyze further to quantify what happens to velocity and other parameters. So, having said this we say that in most of the practical applications, velocity at the inlet of the nozzle is negligibly small in comparison to the exit velocity that we can see from the equation that when C is higher A is larger. That means, in most of the cases the nozzles are shaped in such a way that nozzle converges rapidly over the first fraction of the length and then we arrive at the minimum area and from there the diverging action starts or passage starts. Now, for example, for the time being, if you analyze that you are expanding a perfect gas, where $h = c_p T$ & $v = \frac{RT}{p}$, then the area for unit mass flow rate

$$\frac{A}{\dot{m}} = \frac{v}{\sqrt{2c_p(T_1 - T)}} = \frac{\left(\frac{RT}{p}\right)}{\sqrt{2c_p T_1 \left(1 - \frac{T}{T_1}\right)}}$$

Then we can recall our isentropic process for a perfect gas between pressure and temperature relationship. Now, for any arbitrary section x-x, where the properties values are detected by pressure p, temperature T, specific volume v and flow velocity $C$and inlet conditions are specified as pressure p_1, temperature T_1, specific volume v_1 and flow velocity C_1. So, putting these we can find

Isentropic process for a perfect gas: $\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = x^{\frac{(\gamma-1)}{\gamma}}$

Area per unit mass flow, $\frac{A}{\dot{m}} = \frac{\left(\frac{RT}{p}\right)}{\sqrt{2c_p T_1 \left(1 - \frac{T}{T_1}\right)}}$; Take $\frac{p}{p_1} = x$

$\Rightarrow \frac{A}{\dot{m}} = \frac{C_n}{\sqrt{x^{\frac{2}{\gamma}} - x^{\frac{(\gamma+1)}{\gamma}}}}$; C_n:Constant for fixed inlet conditions (p_1, T_1)

At minimum area, $\frac{d}{dx}\left(\frac{A}{\dot{m}}\right) = 0 \Rightarrow x = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{(\gamma-1)}}$

Now here this expression will give you a conditions for minimum area. So, in other word it says that any arbitrary condition, pressure is detected with respect to its initial condition p_1. Now, when you have a minimum area, there we will have maximum flow rate, which we will come back later, but at this minimum area conditions which is nothing but the throat area conditions, the values are defined with its critical parameters.

Critical pressure ratio, $\frac{p_c}{p_1} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{(\gamma-1)}}$;

Critical temperature ratio, $\frac{T_c}{T_1} = \left(\frac{2}{\gamma + 1}\right)$;

Once you have the critical pressure ratio you can as well find out the critical temperature ratio. Here critical condition means, we have a minimum area where the conditions are p_c, T_c, specific volume v_c and flow velocity C_c. So, this is the conditions for minimum area situations.

Now, putting $\gamma = 1.4$, that is for air,

Pressure ratio, $\frac{p_c}{p_1} = 0.5283$ & Temperature ratio, $\frac{T_c}{T_1} = 0.8333$

So, based on this minimum area conditions, we are now in a position to define the terms like critical velocity. So, the critical velocity is the velocity at the throat of a correctly designed convergent-divergent nozzles. Now, why I say convergent-divergent nozzles, because in our previous analysis we say that area decreases initially and then increases further. So, by putting this concept, we say, initially it is a convergent then a divergent nozzle. When the pressure ratio of the nozzle is the critical pressure ratio.

Now, here one of the main assumption is that the specific volume of liquid is constant over a wide range. Therefore, the nozzle for liquid is always convergent even at high velocities. So, the critical velocity is the velocity at the exit of a convergent nozzle when the pressure ratio across the nozzle is at critical pressure ratio. So, this is the basic definition of a convergent-divergent nozzles.

So, we have already derived this expression that is velocity of the flow at any section x-x is C .

Critical pressure ratio, $\frac{p_c}{p_1} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{(\gamma - 1)}}$;

Critical temperature ratio, $\frac{T_c}{T_1} = \left(\frac{2}{\gamma + 1}\right)$

Velocity of flow at any section $x - x$: $C = \sqrt{2(h_1 - h)} = \sqrt{2c_p T\left(\frac{T_1}{T} - 1\right)}$

Then from this we can find out the critical velocity at the throat by putting this expression here by putting enthalpy term and neglecting $C_1 \to 0$.

Critical velocity at throat: $C_c = \sqrt{2(h_1 - h)} = \sqrt{2c_p T_c\left(\frac{T_1}{T_c} - 1\right)}$

Then from this we can find out critical velocity at the throat considering the fact that $c_p = \frac{\gamma R}{\gamma - 1}$ & $\frac{T_1}{T_c} = \left(\frac{2}{\gamma + 1}\right)$; Critical velocity at throat: $C_c = \sqrt{\gamma R T_c}$

So, when we think the fluid as a perfect gas, then one can also define further relation with a parameter what is called as speed of sound. The speed of sound by definition at constant s, is nothing but

Speed of sound: $a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$; density, $\rho = \frac{1}{v}$ & $pv^\gamma = \text{constant}$(Isentropic process)

Now, if you just simplify this expression and find the value of a, we can arrive at a conclusion that speed of sound also has same relation what we have for the critical velocity at the throat.

$\Rightarrow a = \sqrt{\gamma p v} = \sqrt{\gamma R T}; pv = RT$(Perferctgasequation)

So, we can say that critical velocity of a perfect gas in a convergent-divergent nozzle is equal to speed of the sound in the gas at critical temperature. And this speed of sound is very vital when we design the nozzles for supersonic flow, but that is not part of our analysis. We will stop our discussions by considering the fact that we are only bothered about the two things- One is what happens at the inlet conditions where typically $C_1 \rightarrow 0$ and initial conditions will be given p_1, v_1, T_1.

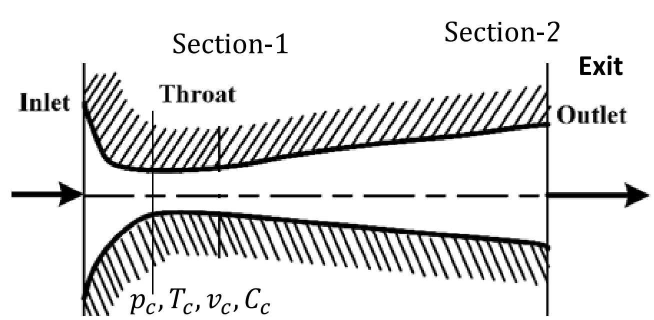
And what is the throat conditions where we have minimum area. Here at inlet, the area goes to infinity, at the throat, we have minimum area and properties are defined by its critical pressure, critical temperature, critical specific volume and followed by critical velocity. And any other conditions are defined at any section x-x, can be outlet or exit and that value we detected by any arbitrary condition p, T, v & C.

So, with this I have explained the basic theoretical aspects for a nozzle and whatever discussions what we have so far let us try to understand with the help of one particular problem based on the analysis of nozzle.

Q1. Air at 8.5 bar and 200°C expands in a convergent-divergent nozzle into a space maintained 1 bar. The mass flow rate of air in the nozzle is 4.5 kg/s. Assuming that the inlet velocity is negligible, calculate the throat and exit cross-sectional area of the nozzle.

This is a simple clear cut problem of our discussion so far today. So, before you solve the problem, let us first try to draw the schematic diagram how the nozzle should look like.

That means, we have section 1, section 2 and we say it as inlet and exit. And since it is a convergent-divergent nozzle we have minimum



area at the throat. At this of course, we have critical conditions p_c, T_c, v_c & C_c.

Now, the flow is your air and it enters at

$p_1 = 8.5$ bar, $T_1 = 200°\text{C} = 473$ K & $\dot{m} = 4.5$ kg/s.

And exit condition we know only $p_2 = 1$ bar, we do not know the value of T_2, v_2 & C_2. And the question asked is to calculate the throat area, A_{min} and exit area A_2.

Since we have air, we can say $\gamma = 1.4$,

We also know, R = 0.287 kJ/kg.K; $C_p = 1.005$ kJ/kg.K

So, this information are for air. Now, we need to know what critical pressure ratio is. Because critical pressure ratio is required when you need to find out the minimum area.

Critical pressure ratio, $\dfrac{p_c}{p_1} = \left(\dfrac{2}{\gamma + 1}\right)^{\frac{\gamma}{(\gamma-1)}}$

For $\gamma = 1.4$; Critical pressure ratio, $\frac{p_c}{p_1}$ = 0.5283

And similarly

Critical temperature ratio, $\dfrac{T_c}{T_1} = \left(\dfrac{2}{\gamma + 1}\right) = \dfrac{2}{2.4} = \dfrac{1}{1.4}$

$$T_c = \frac{200 + 273}{1.2} = 394 \text{ K}$$

$$P_c = 8.5 \times 0.5283 = 4.49 \text{ bar}$$

$$v_c = \frac{RT_c}{p_c} = \frac{287 \times 394}{4.49 \times 101325} = 0.25 \text{ m}^3/\text{kg}$$

$$C_c = \sqrt{2(h_1 - h_c)} = \sqrt{2C_p(T_1 - T_c)} = \sqrt{2 \times 1005(473 - 394)} = 398.5 \text{ m/s}$$

a) Now Throat area, $A_c = \frac{\dot{m} v_c}{C_c} = \frac{4.5 \times 0.25}{398.5} = 0.0028 \text{ m}^2 \approx 2800 \text{ mm}^2$

   So, this minimum area that is throat area is 2800 mm^2.

b) Then Exit area,

We know the inlet conditions. Since the process can be treated as isentropic, we can say

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = (8.5)^{0.4/1.4} = 1.84$$

$$\Rightarrow T_2 = \frac{473}{1.84} = 257 \text{ K}$$

$$v_2 = \frac{RT_2}{p_2} = \frac{287 \times 257}{101325} = 0.728 \text{ m}^3/\text{kg}$$

$$C_2 = \sqrt{2C_p(T_1 - T_2)} = \sqrt{2 \times 1005(473 - 257)} = 658.9 \text{ m}^3/\text{kg}$$

$$A_2 = \frac{\dot{m}v_2}{C_2} = \frac{4.5 \times 0.728}{658.9} = 0.00497 \text{ m}^2 \approx 4970 \text{ mm}^2$$

So, we got the answer as the throat area of the nozzle is 2800 mm^2 and the exit area of the nozzle is 4970mm^2. So, this is all about the conceptual picture of nozzle and its analysis. With this I conclude. Thank you for your attention.